For each finite sequence below, give its generating function.

a.
$$1, 4, 6, 4, 1$$

b. 1, 1, 1, 1, 1, 0, 0, 1

c.
$$0,0,0,1,2,3,4,5$$

d. 1, 1, 1, 1, 1, 1, 1

e.
$$3,0,0,1,-4,7$$

f. 0,0,0,0,1,2,-3,0,1

2. For each infinite sequence suggested below, give its generating function in closed form, i.e., not as an infinite sum. (Use the most obvious choice of form for the general term of each sequence.)

b. $1,0,0,1,0,0,1,0,0,1,0,0,1,\dots$

c.
$$1, 2, 4, 8, 16, 32, \dots$$

d. $0,0,0,0,1,1,1,1,1,1,1,1,1,1,1,1,1,\dots$

e.
$$1, -1, 1, -1, 1, -1, 1, -1, 1, -1, \dots f$$
.

 $\text{e. } 1,-1,1,-1,1,-1,1,-1,1,-1,\dots \text{f. } 2^8,2^7 \binom{8}{1},2^6 \binom{8}{2},\dots,\binom{8}{8},0,0,0,\dots$

g.
$$1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, \dots$$
 h. $0, 0, 0, 1, 2, 3, 4, 5, 6, \dots$

i.
$$3, 2, 4, 1, 1, 1, 1, 1, 1, \dots$$

j. $0, 2, 0, 0, 2, 0, 0, 2, 0, 0, 2, 0, 0, 2, \dots$

k.
$$6, 0, -6, 0, 6, 0, -6, 0, 6, \dots$$

1, 3, 6, 10, 15, ..., $\binom{n+2}{2}$, ...

3. For each generating function below, give a closed form for the $n^{\rm th}$ term of its associated sequence.

a.
$$(1+x)^{10}$$

b.
$$\frac{1}{1-x^4}$$

c.
$$\frac{x^3}{1-x^4}$$

d.
$$\frac{1-x^4}{1-x}$$

e.
$$\frac{1+x^2-x^4}{1-x}$$

f.
$$\frac{1}{1-4x}$$

g.
$$\frac{1}{1+4x}$$

h.
$$\frac{x^5}{(1-x)^4}$$

i.
$$\frac{x^2 + x + 1}{1 - x^7}$$

j.
$$3x^4 + 7x^3 - x^2 + 10 + \frac{1}{1 - x^3}$$

4. Find the coefficient on x^{10} in each of the generating functions below.

a.
$$(x^3 + x^5 + x^6)(x^4 + x^5 + x^7)(1 + x^5 + x^{10} + x^{15} + \cdots)$$

b.
$$(1+x^3)(x^3+x^4+x^5+\cdots)(x^4+x^5+x^6+x^7+x^8+\cdots)$$

c.
$$(1+x)^{12}$$

d.
$$\frac{x^5}{1 - 3x^5}$$

e.
$$\frac{1}{(1-x)^3}$$

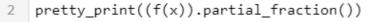
$$\text{f.} \quad \frac{1}{1 - 5x^4}$$

$$\text{g. } \frac{x}{1-2x^3}$$

h.
$$\frac{1-x^{14}}{1-x}$$

- **9.** What is the generating function for the number of ways to select a group of *n* students from a class of *p* students?
- 11. Using generating functions, find the number of ways to make change for a 100 dollar bill using only dollar coins and \$1, \$2, and \$5 bills.

1
$$f(x) = x # Generating function on right here$$





Evaluate (Sage)

▼ Hint.

Find the partial fractions expansion for your generating function. Be careful here, as you want a partial fraction expansion in which all coefficients for your denominator polynomials have integer coefficients. The partial_fraction() method in SageMath should be useful here, and pretty_print will make it easier to read. Once you have the right partial fractions expansion, you may find the following identity helpful

$$\frac{p(x)}{1+x+x^2+\cdots+x^k} = \frac{p(x)(1-x)}{1-x^{k+1}},$$

where p(x) will be a polynomial in this instance.