16. How many integer-valued solutions are there to each of the following equations and inequalities?

a. $x_1 + x_2 + x_3 + x_4 + x_5 = 63$, all $x_i > 0$

b. $x_1 + x_2 + x_3 + x_4 + x_5 = 63$, all $x_i \geq 0$

c. $x_1 + x_2 + x_3 + x_4 + x_5 \leq 63$, all $x_i \geq 0$

d. $x_1 + x_2 + x_3 + x_4 + x_5 = 63$, all $x_i \geq 0$, $x_2 \geq 10$

e. $x_1 + x_2 + x_3 + x_4 + x_5 = 63$, all $x_i \geq 0$, $x_2 \leq 9$

17. How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 132$$

provided that $x_1 > 0$, and $x_2, x_3, x_4 \geq 0$? What if we add the restriction that $x_4 < 17$?

18. How many integer solutions are there to the inequality

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 782$$

provided that $x_1, x_2 > 0$, $x_3 \geq 0$, and $x_4, x_5 \geq 10$?

19. A teacher has 450 identical pieces of candy. He wants to distribute them to his class of 65 students, although he is willing to take some leftover candy home. (He does not insist on taking any candy home, however.) The student who won a contest in the last class is to receive at least 10 pieces of candy as a reward. Of the remaining students, 34 of them insist on receiving at least one piece of candy, while the remaining 30 students are willing to receive no candy.

a. In how many ways can he distribute the candy?

b. In how many ways can he distribute the candy if, in addition to the conditions above, one of his students is diabetic and can receive at most 7 pieces of candy? (This student is one of the 34 who insist on receiving at least one piece of candy.)

22. How many lattice paths are there from $(0, 0)$ to $(10, 12)$?

23. How many lattice paths are there from $(3, 5)$ to $(10, 12)$?

24. How many lattice paths are there from $(0, 0)$ to $(10, 12)$ that pass through $(3, 5)$?

25. How many lattice paths from $(0, 0)$ to $(17, 12)$ are there that pass through $(7, 6)$ and $(12, 9)$?

26. How many lattice paths from $(0, 0)$ to $(14, 73)$ are there that do *not* pass through $(6, 37)$?

29. Determine the coefficient on $x^{15}y^{120}z^{25}$ in $(2x + 3y^2 + z)^{100}$.

30. Determine the coefficient on $x^{12}y^{24}$ in $(x^3 + 2xy^2 + y + 3)^{18}$. (Be careful, as x and y now appear in multiple terms!)