3. Consider the set $X = \{1, 2, 3, 4, 5\}$ and suppose you have two holes. Also suppose that you have 10 pigeons: the 2-element subsets of X. Can you put these 10 pigeons into the two holes in a way that there is no 3-element subset $S = \{a, b, c\} \subset X$ for which all pigeons from S go in the same hole? Then answer the same question if $X = \{1, 2, 3, 4, 5, 6\}$ with $15 = C(6, 2)$ pigeons.

1. The questions in this exercise pertain to the graph **G** shown in Figure 5.46.

a. What is the degree of vertex 8?

b. What is the degree of vertex 10?

c. How many vertices of degree 2 are there in **G**? List them.

d. Find a cycle of length 8 in **G**.

e. What is the length of a shortest path from 3 to 4?

f. What is the length of a shortest path from 8 to 7?

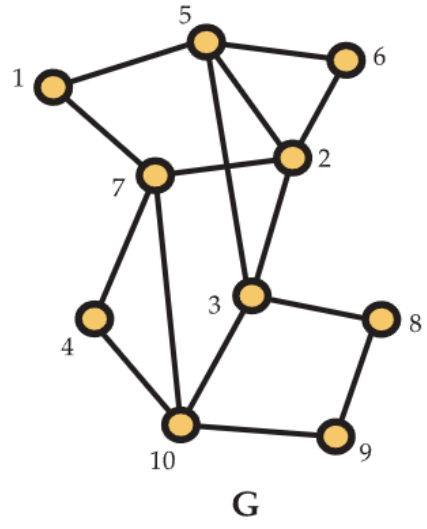g. Find a path of length 5 from vertex 4 to vertex 6.

Figure 5.46. A graph

3. Draw a graph with 6 vertices having degrees 5, 4, 4, 2, 1, and 1 or explain why such a graph does not exist.

5. For this exercise, consider the graph **G** in Figure 5.47.

   a. Let $V_1 = \{g, j, c, h, e, f\}$ and $E_1 = \{ge, jg, ch, ef\}$. Is (V_1, E_1) a subgraph of **G**?

   b. Let $V_2 = \{g, j, c, h, e, f\}$ and $E_2 = \{ge, jg, ch, ef, cj\}$. Is (V_2, E_2) a subgraph of **G**?

   c. Let $V_3 = \{a, d, c, h, b\}$ and $E_3 = \{ch, ac, ad, bc\}$. Is (V_3, E_3) an induced subgraph of **G**?

   d. Draw the subgraph of **G** induced by $\{g, j, d, a, c, i\}$.

   e. Draw the subgraph of **G** induced by $\{c, h, f, i, j\}$.

   f. Draw a subgraph of **G** having vertex set $\{e, f, b, c, h, j\}$ that is *not* an induced subgraph.

   g. Draw a spanning subgraph of **G** with exactly 10 edges.
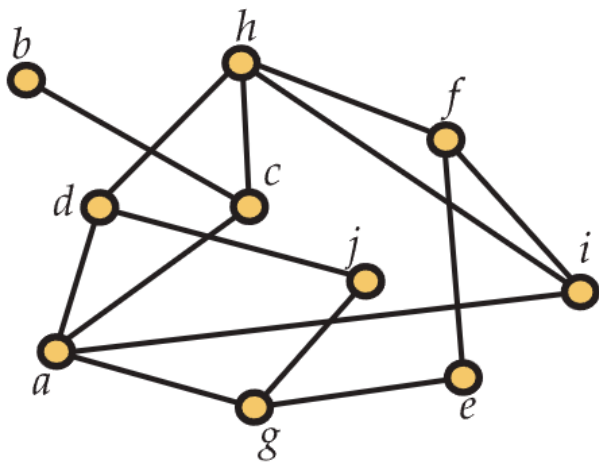
6. Prove that every tree on n vertices has exactly $n - 1$ edges.

7. Figure 5.48 contains four graphs on six vertices. Determine which (if any) pairs of graphs are isomorphic. For pairs that are isomorphic, give an isomorphism between the two graphs. For pairs that are not isomorphic, explain why.

G_1

G_2

G_3

G_4
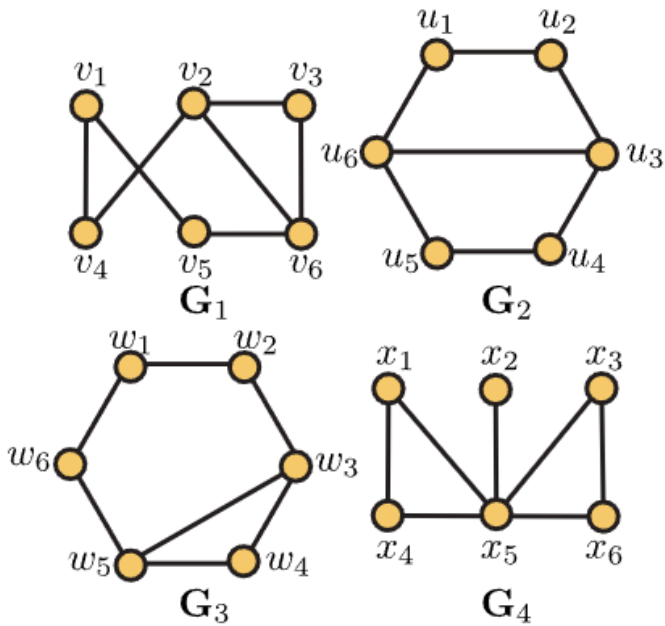
8. Find an eulerian circuit in the graph **G** in Figure 5.49 or explain why one does not exist.