1. Consider the network diagram in Figure 13.14. For each directed edge, the first number is the capacity and the second value is intended to give a flow ϕ in the network. However, the flow suggested is not valid.

a. Identify the reason(s) ϕ is not valid.

b. Without changing any of the edge capacities, modify ϕ into a valid flow $\widehat{\phi}$. Try to use as few modifications as possible.

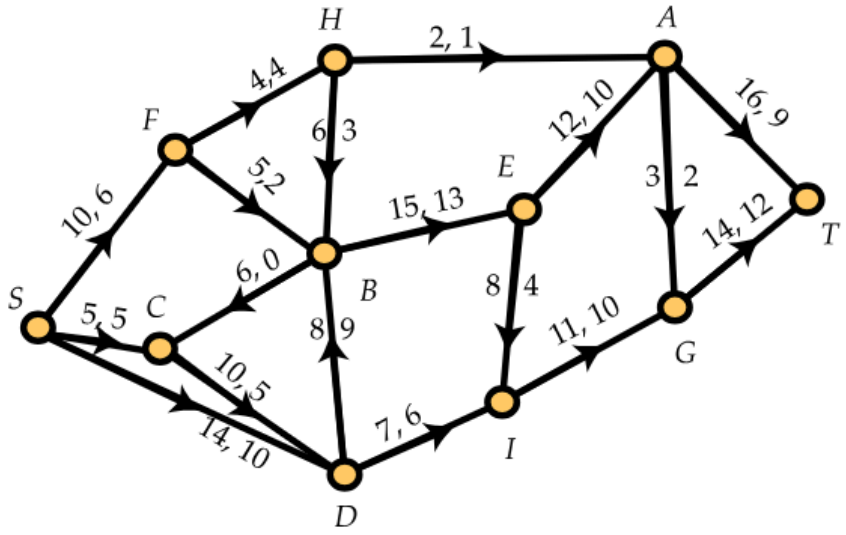

Figure 13.14. An invalid flow in a network

2. Alice claims to have found a (valid) network flow of value 20 in the network shown in Figure 13.15. Bob tells her that there's no way she's right, since no flow has value greater than 18. Who's right and why?

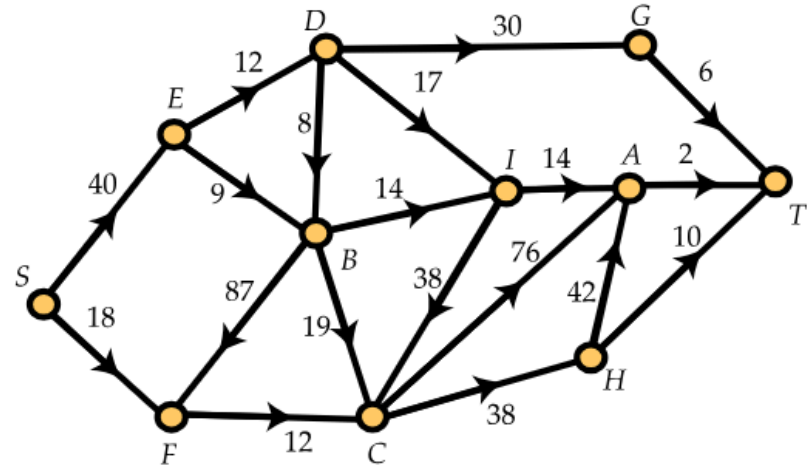

Figure 13.15. A network

3. Find an augmenting path P with *at least one backward edge* for the flow ϕ in the network shown in Figure 13.16. What is the value of δ for P? Carry out an update of ϕ using P to obtain a new flow $\hat{\phi}$. What is the value of $\hat{\phi}$?

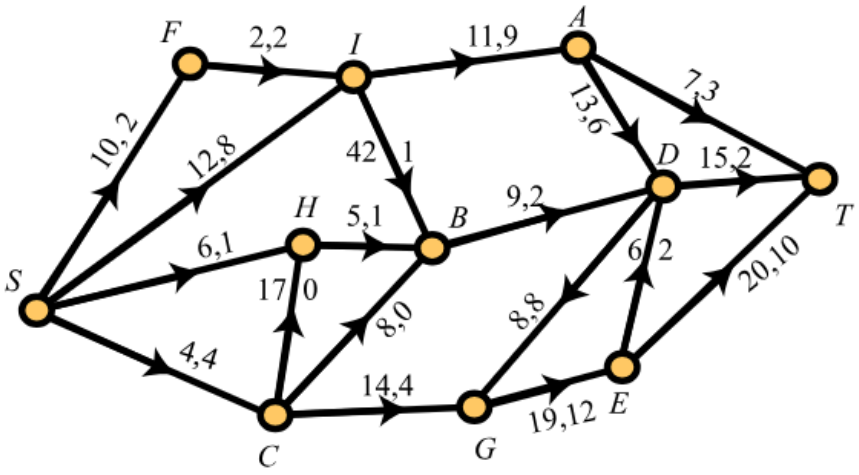

Figure 13.16. A network with flow

5. Find the capacity of the cut (L, U) with

$$L = \{S, F, H, C, B, G, I\} \quad \text{and} \quad U = \{A, D, E, T\}$$

in the network shown in Figure 13.16.

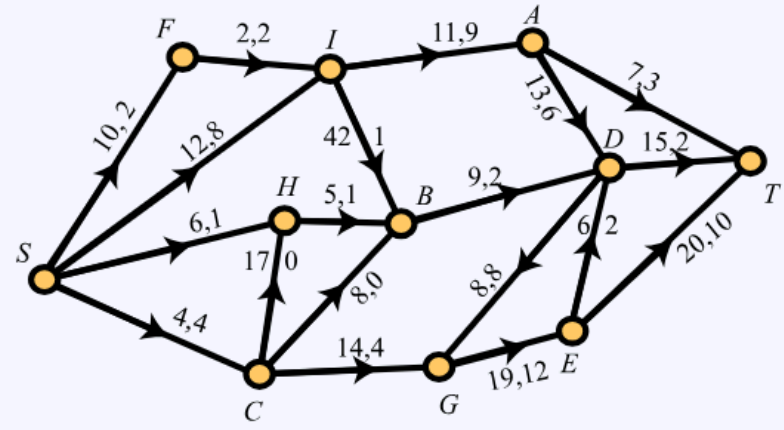

Figure 13.16. A network with flow

in-context

10. Figure 13.18 shows a network. Starting from the zero flow, i.e., the flow with $\phi(e) = 0$ for every directed edge e in the network, use the Ford-Fulkerson labeling algorithm to find a maximum flow and a minimum cut in this network.

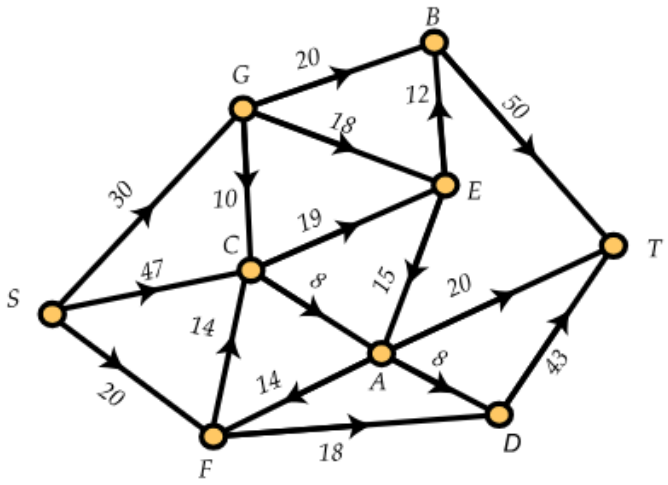

Figure 13.18. A network