1. Write each of the following recurrence equations as advancement operator equations.

a. $r_{n+2} = r_{n+1} + 2r_n$

b. $r_{n+4} = 3r_{n+3} - r_{n+2} + 2r_n$

c. $g_{n+3} = 5g_{n+1} - g_n + 3^n$

d. $h_n = h_{n-1} - 2h_{n-2} + h_{n-3}$

e. $r_n = 4r_{n-1} + r_{n-3} - 3r_{n-5} + (-1)^n$

f. $b_n = b_{n-1} + 3b_{n-2} + 2^{n+1} - n^2$

2. Solve the recurrence equation $r_{n+2} = r_{n+1} + 2r_n$ if $r_0 = 1$ and $r_2 = 3$ (Yes, we specify a value for r_2 but not for r_1).

3. Find the general solution of the recurrence equation $g_{n+2} = 3g_{n+1} - 2g_n$.

4. Solve the recurrence equation $h_{n+3} = 6h_{n+2} - 11h_{n+1} + 6h_n$ if $h_0 = 3$, $h_1 = 2$, and $h_2 = 4$.

8. Give the general solution to each advancement operator equation below.

a. $(A-4)^3(A+1)(A-7)^4(A-1)^2 f = 0$

b. $(A+2)^4(A-3)^2(A-4)(A+7)(A-5)^3 g = 0$

c. $(A-5)^2(A+3)^3(A-1)^3(A^2-1)(A-4)^3 h = 0$

9. For each nonhomogeneous advancement operator equation, find its general solution.

a. $(A-5)(A+2)f = 3^n$

b. $(A^2+3A-1)g = 2^n + (-1)^n$

c. $(A-3)^3 f = 3n+1$

d. $(A^2+3A-1)g = 2n$

e. $(A-2)(A-4)f = 3n^2 + 9^n$

f. $(A+2)(A-5)(A-1)f = 5^n$

g. $(A-3)^2(A+1)g = 2 \cdot 3^n$

h. $(A-2)(A+3)f = 5n2^n$

i. $(A-2)^2(A-1)g = 3n^2 2^n + 2^n$

j. $(A+1)^2(A-3)f = 3^n + 2n^2$

10. Find and solve a recurrence equation for the number g_n of ternary strings of length n that do not contain 102 as a substring.

11. There is a famous puzzle called the Towers of Hanoi that consists of three pegs and n circular discs, all of different sizes. The discs start on the leftmost peg, with the largest disc on the bottom, the second largest on top of it, and so on, up to the smallest disc on top. The goal is to move the discs so that they are stacked in this same order on the rightmost peg. However, you are allowed to move only one disc at a time, and you are never able to place a larger disc on top of a smaller disc. Let t_n denote the fewest moves (a move being taking a disc from one peg and placing it onto another) in which you can accomplish the goal. Determine an explicit formula for t_n.

13. Let t_n be the number of ways to tile a $2 \times n$ rectangle using 1×1 tiles and L -tiles. An L-tile is a 2×2 tile with the upper-right 1×1 square deleted. (An L tile may be rotated so that the "missing" square appears in any of the four positions.) Find a recursive formula for t_n along with enough initial conditions to get the recursion started. Use this recursive formula to find a closed formula for t_n.

17. Let $b_0 = 1$, $b_2 = 1$, and $b_3 = 4$. Use generating functions to solve the recurrence equation $b_{n+3} = 4b_{n+2} - b_{n+1} - 6b_n + 3^n$ for $n \geq 0$.

18. Use generating functions to find a closed formula for the Fibonacci numbers f_n.