

**16.** List the partitions of **9**. Write a D next to each partition into distinct parts and an O next to each partition into odd parts.

**17.** Use generating functions to find the number of ways to partition **10** into odd parts.

**20.** Find the exponential generating function (in closed form, not as an infinite sum) for each infinite sequence  $\{a_n: n \geq 0\}$  whose general term is given below.

- ☐ a.  $a_n = 5^n$
- ☐ b.  $a_n = (-1)^n 2^n$
- ☐ c.  $a_n = 3^{n+2}$
- ☐ d.  $a_n = n!$
- ☐ e.  $a_n = n$
- ☐ f.  $a_n = 1/(n+1)$

**21.** For each exponential generating function below, give a formula in closed form for the sequence  $\{a_n: n \geq 0\}$  it represents.

- ☐ a.  $e^{7x}$
- ☐ b.  $x^2 e^{3x}$
- ☐ c.  $\frac{1}{1+x}$
- ☐ d.  $e^{x^4}$

**22.** Find the coefficient on  $x^{10}/10!$  in each of the exponential generating functions below.

- ☐ a.  $e^{3x}$
- ☐ b.  $\frac{e^x - e^{-x}}{2}$
- ☐ c.  $\frac{e^x + e^{-x}}{2}$
- ☐ d.  $xe^{3x} - x^2$
- ☐ e.  $\frac{1}{1-2x}$
- ☐ f.  $e^{x^2}$

**23.** Find the exponential generating function for the number of strings of length  $n$  formed from the set  $\{a, b, c, d\}$  if there must be at least one  $a$  and the number of  $c$ 's must be even. Find a closed formula for the coefficients of this exponential generating function.

**24.** Find the exponential generating function for the number of strings of length  $n$  formed from the set  $\{a, b, c, d\}$  if there must be at least one  $a$  and the number of  $c$ 's must be odd. Find a closed formula for the coefficients of this exponential generating function.

**25.** Find the exponential generating function for the number of strings of length  $n$  formed from the set  $\{a, b, c, d\}$  if there must be at least one  $a$ , the number of  $b$ 's must be odd, and the number of  $d$ 's is either 1 or 2. Find a closed formula for the coefficients of this exponential generating function.

**26.** Find the exponential generating function for the number of alphanumeric strings of length  $n$  formed from the 26 uppercase letters of the English alphabet and 10 decimal digits if

- each vowel must appear at least one time;
- the letter  $T$  must appear at least three times;
- the letter  $Z$  may appear at most three times;
- each even digit must appear an even number of times; and
- each odd digit must appear an odd number of times.

**27.** Consider the inequality

$$x_1 + x_2 + x_3 + x_4 \leq n$$

where  $x_1, x_2, x_3, x_4, n \geq 0$  are all integers. Suppose also that  $x_2 \geq 2$ ,  $x_3$  is a multiple of 4, and  $1 \leq x_4 \leq 3$ . Let  $c_n$  be the number of solutions of the inequality subject to these restrictions. Find the generating function for the sequence  $\{c_n: n \geq 0\}$  and use it to find a closed formula for  $c_n$ .

► Hint.