1. For each *finite* sequence below, give its generating function.

a. $1, 4, 6, 4, 1$

b. $1, 1, 1, 1, 1, 0, 0, 1$

c. $0, 0, 0, 1, 2, 3, 4, 5$

d. $1, 1, 1, 1, 1, 1, 1$

e. $3, 0, 0, 1, -4, 7$

f. $0, 0, 0, 0, 1, 2, -3, 0, 1$

2. For each *infinite* sequence suggested below, give its generating function in closed form, i.e., *not* as an infinite sum. (Use the most obvious choice of form for the general term of each sequence.)

a. $0, 1, 1, 1, 1, 1, \ldots$

b. $1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, \ldots$

c. $1, 2, 4, 8, 16, 32, \ldots$

d. $0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, \ldots$

e. $1, -1, 1, -1, 1, -1, 1, -1, 1, -1, \ldots$

f. $2^8, 2^7\binom{8}{1}, 2^6\binom{8}{2}, \ldots, \binom{8}{8}, 0, 0, 0, \ldots$

g. $1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, \ldots$

h. $0, 0, 0, 1, 2, 3, 4, 5, 6, \ldots$

i. $3, 2, 4, 1, 1, 1, 1, 1, 1, \ldots$

j. $0, 2, 0, 0, 2, 0, 0, 2, 0, 0, 2, 0, 0, 2, \ldots$

k. $6, 0, -6, 0, 6, 0, -6, 0, 6, \ldots$

l. $1, 3, 6, 10, 15, \ldots, \binom{n+2}{2}, \ldots$

3. For each generating function below, give a closed form for the n^{th} term of its associated sequence.

a. $(1+x)^{10}$

b. $\dfrac{1}{1-x^4}$

c. $\dfrac{x^3}{1-x^4}$

d. $\dfrac{1-x^4}{1-x}$

e. $\dfrac{1+x^2-x^4}{1-x}$

f. $\dfrac{1}{1-4x}$

g. $\dfrac{1}{1+4x}$

h. $\dfrac{x^5}{(1-x)^4}$

i. $\dfrac{x^2+x+1}{1-x^7}$

j. $3x^4+7x^3-x^2+10+\dfrac{1}{1-x^3}$

4. Find the coefficient on x^{10} in each of the generating functions below.

a. $(x^3+x^5+x^6)(x^4+x^5+x^7)(1+x^5+x^{10}+x^{15}+\cdots)$

b. $(1+x^3)(x^3+x^4+x^5+\cdots)(x^4+x^5+x^6+x^7+x^8+\cdots)$

c. $(1+x)^{12}$

d. $\dfrac{x^5}{1-3x^5}$

e. $\dfrac{1}{(1-x)^3}$

f. $\dfrac{1}{1-5x^4}$

g. $\dfrac{x}{1-2x^3}$

h. $\dfrac{1-x^{14}}{1-x}$

9. What is the generating function for the number of ways to select a group of n students from a class of p students?

11. Using generating functions, find the number of ways to make change for a 100 dollar bill using only dollar coins and $\$1$, $\$2$, and $\$5$ bills.

```
f(x) = x # Generating function on right here
pretty_print((f(x)).partial_fraction())
```

Evaluate (Sage)

▾ Hint.

Find the partial fractions expansion for your generating function. Be careful here, as you want a partial fraction expansion in which all coefficients for your denominator polynomials have integer coefficients. The `partial_fraction()` method in SageMath should be useful here, and `pretty_print` will make it easier to read. Once you have the right partial fractions expansion, you may find the following identity helpful

$$\frac{p(x)}{1+x+x^2+\cdots+x^k}=\frac{p(x)(1-x)}{1-x^{k+1}},$$

where $p(x)$ will be a polynomial in this instance.