Value-at-risk and expected shortfall

Oliver Sheridan-Methven

16th May 2019

1 Abstract

In this report we briefly demonstrate the behaviour of the value-at-risk and expected shortfall for the closing prices of Amazon equities from 2008 to present day. We find that the model based value-at-risk is a more conservative risk measure with a lower level of fluctuation which would likely be more suitable for use than the corresponding empirically calculated value.

2 Introduction

We define our losses as being described by some process X, where for some percentile $\alpha \in (0, 1)$ we define the *value-at-risk* $\mathrm{VaR}_\alpha(X) = \inf_l\{l\colon \mathbb{P}(X \geq l) \leq 1 - \alpha\}$. Additionally we define the *expected shortfall* $\mathrm{ES}_\alpha(X) = \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_u(X)\,\mathrm{d}u$ [Yamai et al., 2002, page 88]. With these definitions we notice that we would anticipate the value-at-risk and expected shortfall to be positive quantities for most reasonable portfolios which carry a risk of loss. (Common choices for α include 0.95 and 0.99). For a more extended treatment and definition of these risk measures we recommend the reader to Yamai et al. [2002, p. 88].

3 Empirical performance

We track the two risk measures in Figure 1 for the Amazon (AMZN) equity, where value-at-risk is both empirically calculated and also calculated from its theoretical model by having a t-distribution fitted (with 3 degrees of freedom, estimated using the MLE). Empirical estimates were computed over a 1 year interval (approximately 256 trading days).

We can make several observations:

1. The estimated shortfall appears to be appreciably larger (between 10–30% typically) than value-at-risk, so is a more conservative risk measure.

2. The empirical and fitted estimates for the value-at-risk closely track each other, although the fitted value is typically marginally larger than the empirically estimated value.

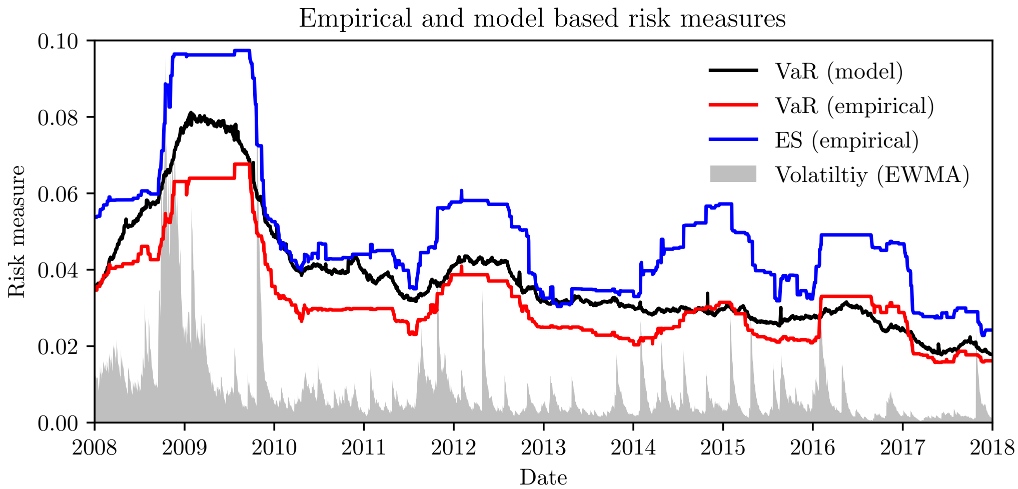

Figure 1: Emprical and model based risk measures for value-at-risk and expected shortfall at the 95$^{\text{th}}$ percentile. At the bottom (grey) we also show a rescaled exponentially weighted moving average volatility profile.

3. The fitted value-at-risk shows a smaller degree of fluctuation than the empirical values. This is particularly clear at discontinuous regions, such as early 2016.

Additionally, we can see that all the risk measures appear to capture the times of increased volatility, most appreciably the 2008 financial crisis (which appears notably between 2009 and 2010). Since then there have not been as notable or distinguished events.

4 Conclusions

Based on the results of the risk-measures as assessed on Amazon stock prices, we have seen that the empirically calculated values and model based fitted values for the value-at-risk are largely interchangeable. Practically, it may be more favourable to use the fitted values from the model based approach as these demonstrated less discontinuous behaviour compared to the empirically fitted value. This “smoother” behaviour would likely be better suited as a trading based risk measure, where it is favourable to reduce turnover costs. If a more conservative risk measure is required, then the expected shortfall may be more desirable, depending on the appropriate risk requirements for investing.

5 Code

The following code computed the various value-at-risk and empirical shortfalls estimates:

```python
def get_risk_measures(ticker='AMZN', alpha=0.95, \
    start=pd.datetime(2007, 01, 01), end=pd.datetime(2018, 01, 01)):
    """
    Author:
      Oliver Sheridan-Methven, May 2019.
    Description:
      Computes empirical and model based estimates for the value-at-risk
      (VaR) and the expected-shortfall (ES).
    Input:
      ticker: String, stock ticker.
      alpha: Float, risk quantile.
      start: Datetime, start date.
      end: Datetime, end date.
    Return:
      Dataframe, the risk measures and the appropriate data.
    """
    key = os.getenv('TIINGO_API_KEY')
    df = web_data.DataReader(ticker, 'tiingo', start, end, access_key=key)
    # I have created an account with TIINGO to gain an API access key.
    # You should also do so or use another data source.

    # For convenience we assume contiguous time samples.
    r = -np.diff(np.log(df['adjClose']))  # The log returns.
    t = df.index.get_level_values('date')[:len(r)]  # The times.
    year_length = 256  # Trading days in a year.
    dof = 3  # For t-distribution  model with 3 degrees of freedom
    var_empirical, var_model, es_empirical = np.nan* np.zeros((3, len(r)))
    for i in range(year_length, len(r)):
        x = r[i-year_length:i]
        var_empirical[i] = np.quantile(x, alpha)
        es_empirical[i] =  np.mean(x[x > np.quantile(x, alpha)])
        mean, spread = t_dist.fit(x, dof)[1:]
        var_model[i] = mean + spread * t_dist.ppf(alpha, dof)
    # Throwing the results into a nice DataFrame.
    df = pd.DataFrame({'var_empirical': var_empirical, \
    'var_model': var_model, 'es_empirical': es_empirical, \
    'returns':r, 'date': t, 'close': df['adjClose'][:-1].values})
    return df
```

References

Yasuhiro Yamai, Toshinao Yoshiba, et al. Comparative analyses of expected shortfall and value-at-risk: their estimation error, decomposition, and optimization. *Monetary and economic studies*, 20(1):87–121, 2002.