Laws of the Logic of Bunched Implications in Iris

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First of all, the entailment relation $\vdash$ is a preorder, giving us the reflexivity and transitivity laws

$$P \vdash P, \qquad \frac{P \vdash Q \quad Q \vdash R}{P \vdash R}.$$

Pure propositions have the straightforward introduction and elimination rules

$$\frac{\varphi}{P \vdash \ulcorner \varphi \urcorner}, \qquad \frac{\varphi \Rightarrow (True \vdash P)}{\ulcorner \varphi \urcorner \vdash P}.$$

The usual rules apply for standard conjunction, disjunction, and implication:

$$P \wedge Q \vdash P, \qquad P \wedge Q \vdash Q, \qquad \frac{P \vdash Q \quad P \vdash R}{P \vdash Q \wedge R},$$

$$P \vdash P \vee Q, \qquad Q \vdash P \vee Q, \qquad \frac{P \vdash R \quad Q \vdash R}{P \vee Q \vdash R},$$

$$\frac{P \wedge Q \vdash R}{P \vdash Q \Rightarrow R}, \qquad \frac{P \vdash Q \Rightarrow R}{P \wedge Q \vdash R}.$$