Project A06:
Entropy, uncertainty, quantum fields and entanglement

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ISOQUANT Workshop

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Team

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Theory

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Experiments

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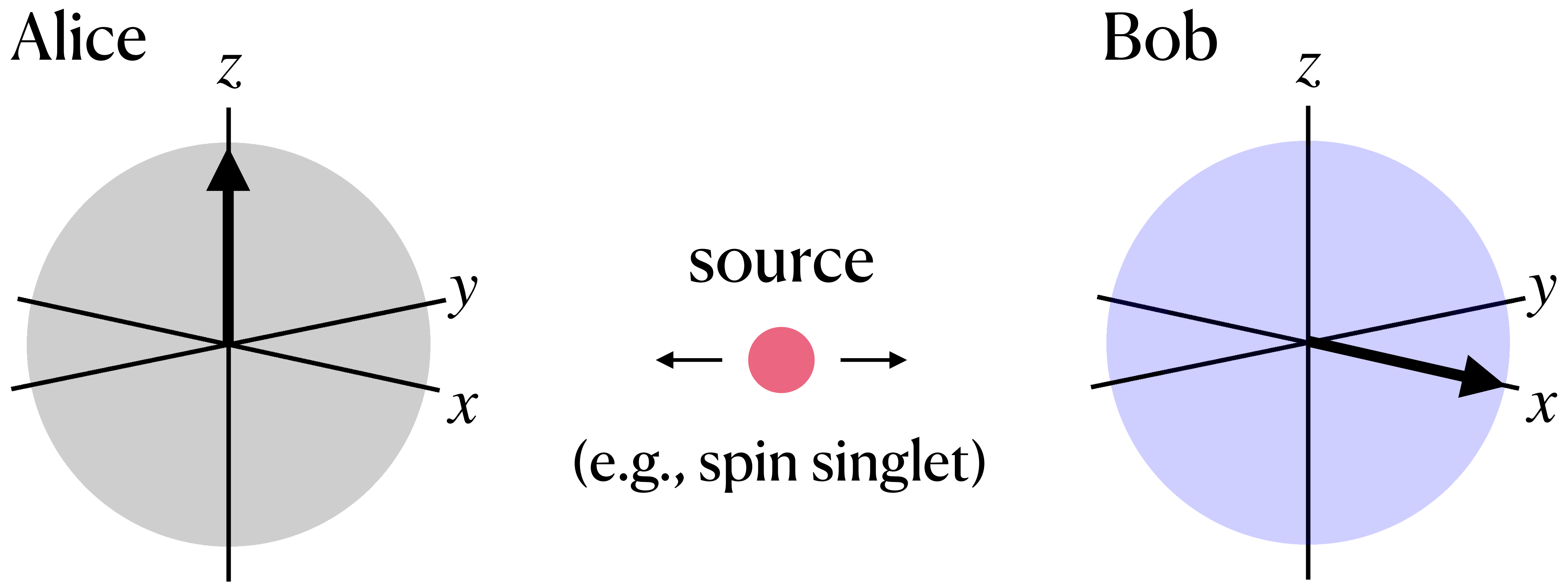
Entropy detection in continuous quantum systems

Entanglement

Mathematically: $|\psi_{AB}\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$

Conceptually: *Quantum* correlations exist

Detect through measurements in *non-commuting bases*:



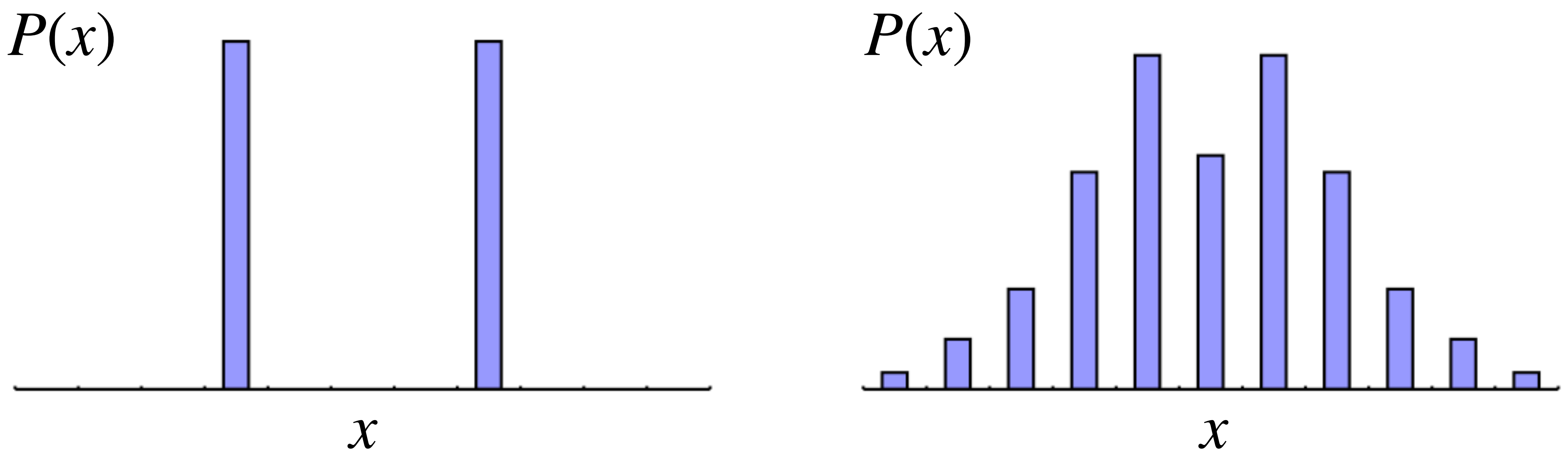
Locally, $$\sigma(X)\sigma(Z) \geq \frac{1}{2}|\langle\psi|[X,Z]|\psi\rangle|$$

Detect entanglement via uncertainty relations

$$\sigma(X_A - X_B)\sigma(Z_A + Z_B) \geq \frac{1}{2}\langle\psi|[X_A, Z_A]|\psi\rangle$$

violation of bound = entanglement

Variances $\sigma(X)$ can **miss** information



Variance only captures low-moments of distribution!

(Information) Entropy

Entropy can be thought of as:

***Measure* for the *uncertainty* about a distribution**

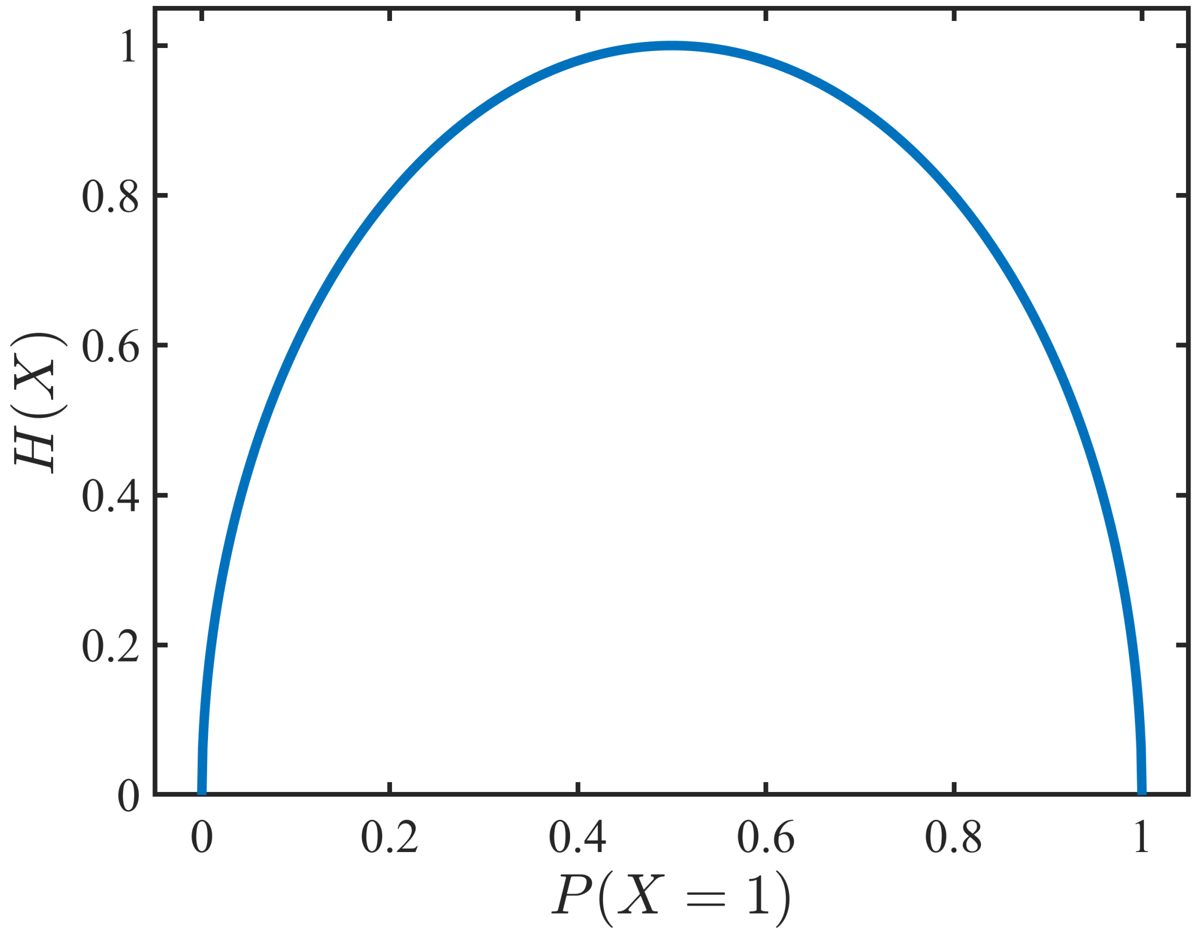
Entropy can be defined via distributions or density matrices

Shannon entropy: $H(X) = -\sum_x P(x) \, \log P(x)$

von Neumann entropy: $S(\rho) = -\mathrm{Tr}(\rho \log \rho)$

Example:
Bias coin flip

X = prob. heads



Why measure entropy:

— Underpins information theory

— A rigorous measure of uncertainty
[see, e.g., Sec. II of Coles, *et al.*, RMP **89** (2017)]

— Describes *all* moments of distribution

Entanglement detection via entropy

Measure two subsystems, extract information

Entropy most general method to do this!

$$H(X_A \,|\, X_B) + H(Z_A \,|\, Z_B) \geq -\log c + S(\rho_A \,|\, \rho_B)$$

Often called 'quantum memory'

Questions we can ask:

How does entropy build up *locally* to affect the state *globally*?

What *role* does entropy play in *universal* close-to-equilibrium dynamics

Why **measure** entanglement?

Understand out-of-equilibrium dynamics

Thermalisation of quantum systems

Many-body localisation

Resource for quantum technologies

Overlaps to other research areas

High energy physics

Floerchinger & Schwindt, PRD **102**, 093001 (2020)

Heavy-ion collisions

Outline

- Initial goals of the project
- Current state of the research
- Current problems we are facing
- Future direction/next steps

Research goals

Research goals

***Optimality** and **tightness** of entropic uncertainty bounds in experiment.*

- What *should* be measured?
- What *can* be measured?
- What are *optimal* measurements?

Bergh & Gärttner, PRL **126**, 190503 (2021).
Bergh & Gärttner, PRA **103**, 052512 (2021).

*Entropic entanglement criteria in **bosonic systems** of increasing complexity.*

- *Implement* EURs using experimentally prepared entangled states
- Explore multi-mode bosonic systems

*Entropic uncertainty relations for **quantum field** theories.*

No EURs exist yet!

- *Develop* entropic uncertainty relations for quantum fields

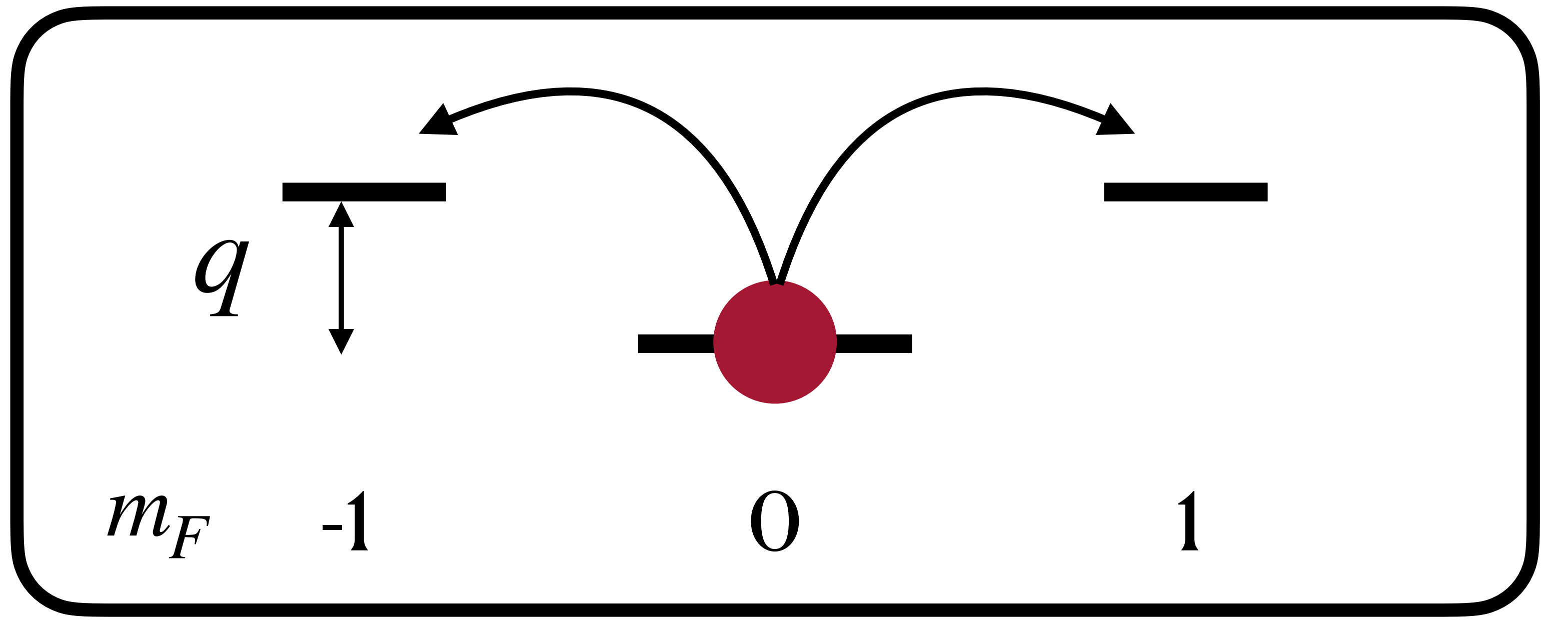
Use existing entropic uncertainty relations

$$H(X_A|X_B) + H(Z_A|Z_B) \geq -\log c + S(\rho_A|\rho_B)$$

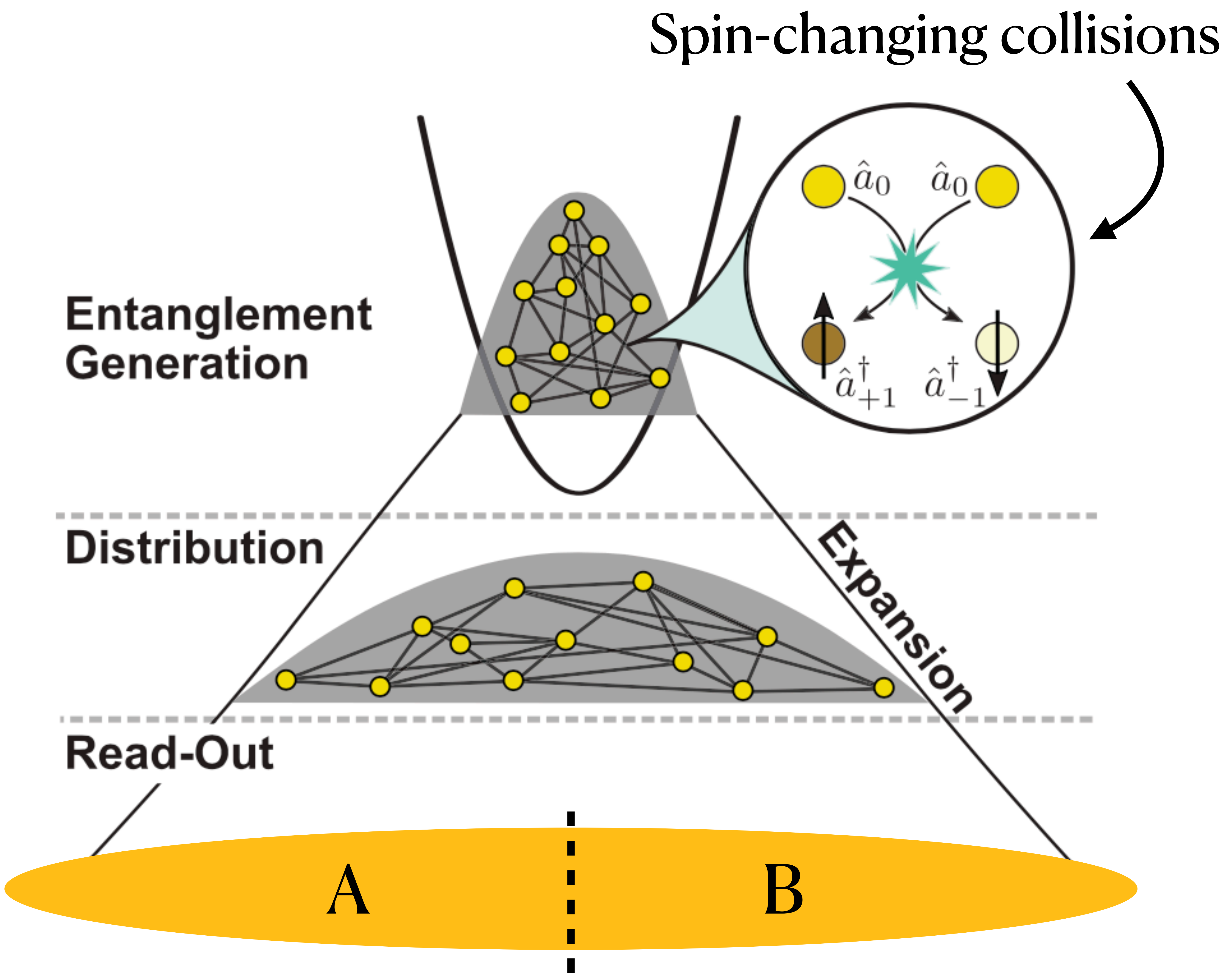
Current state of the research

The experimental system

Rubidium-87 BEC
***F* = *1* hyperfine manifold**



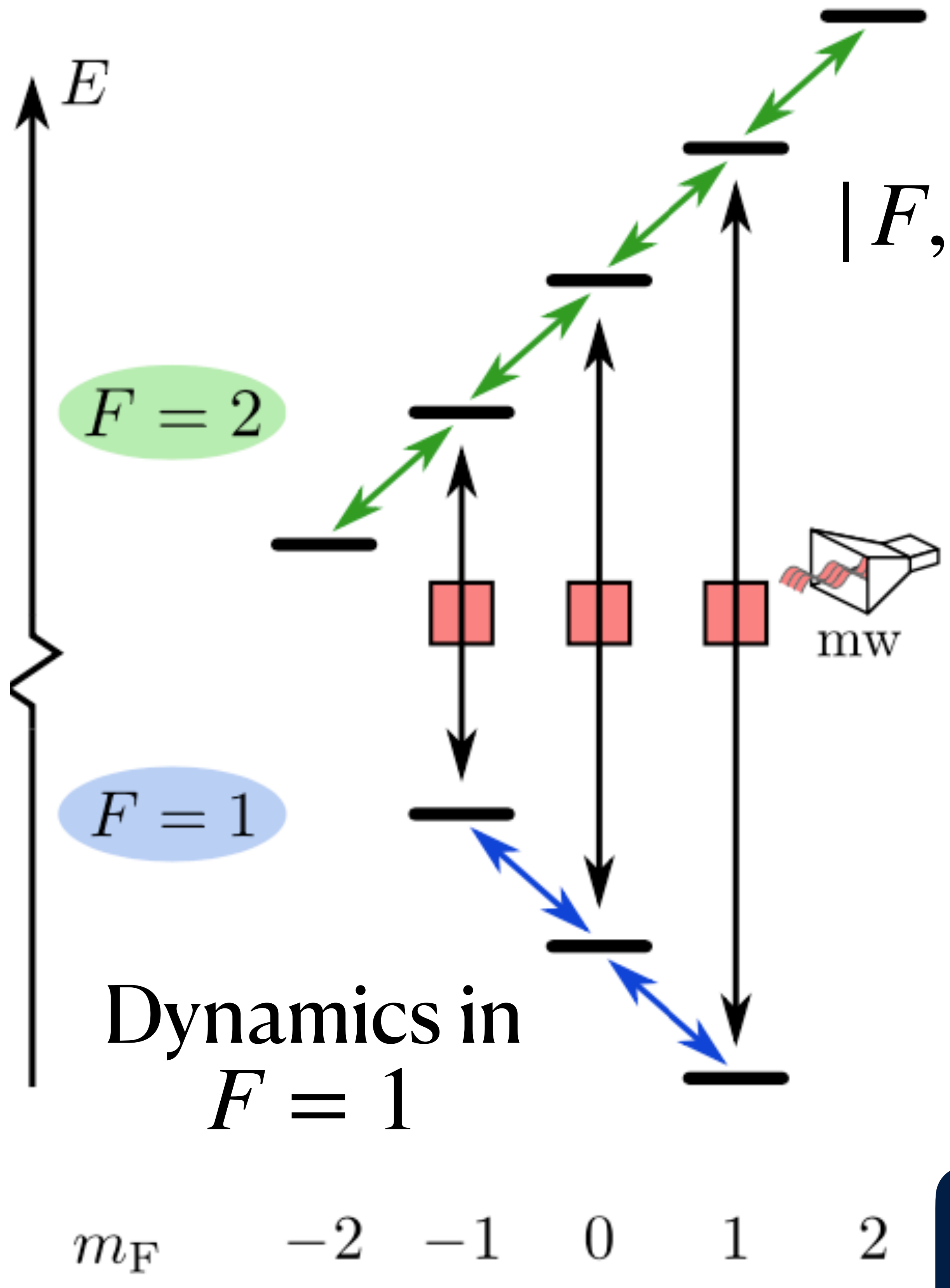
q = quadratic Zeeman shift



P. Kunkel, *et al.*, Science **360**, 413 (2018).

Measuring entropy: Experimental readout

Rubidium-87 has two hyperfine manifolds

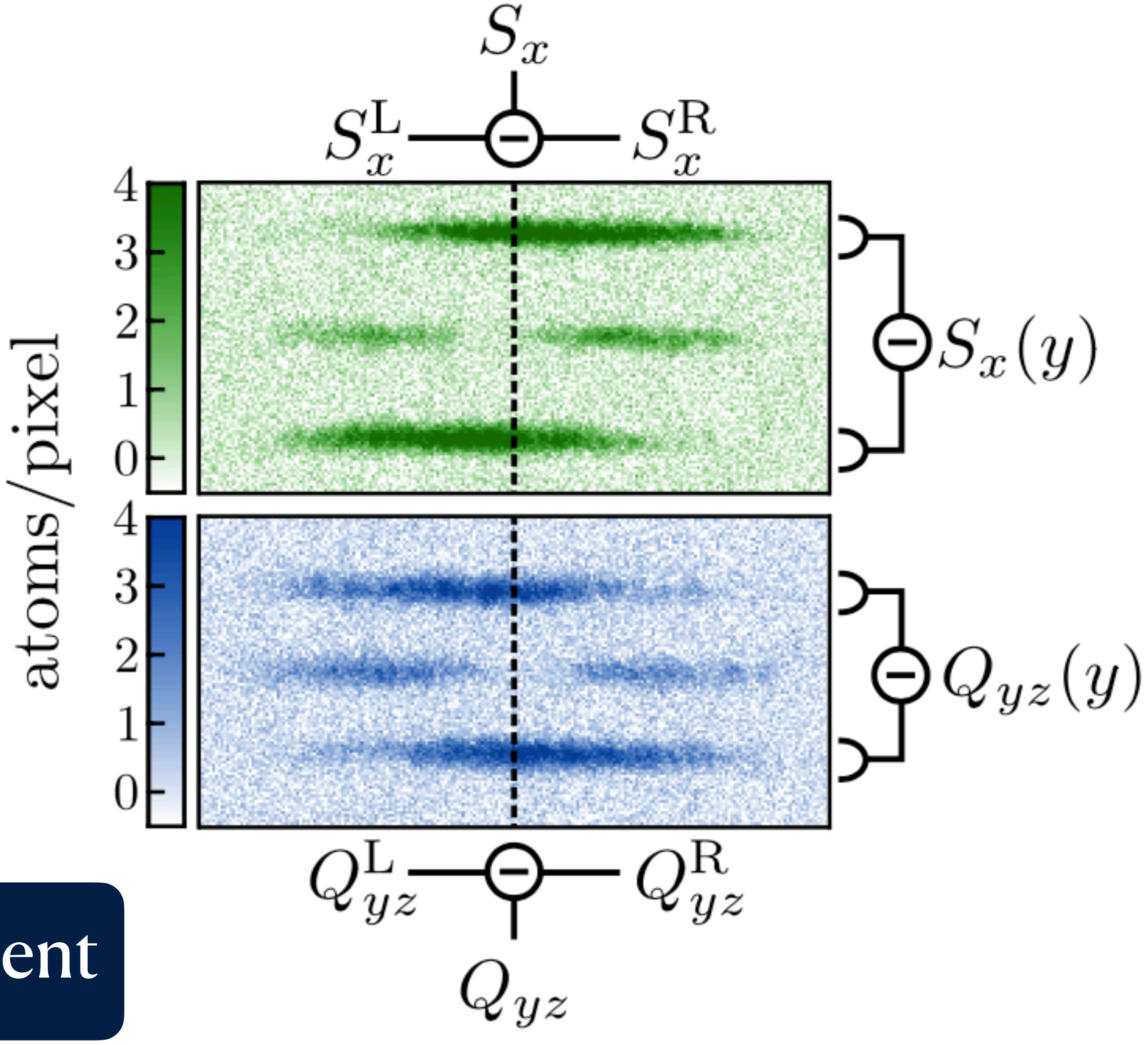


No need for state tomography!

fewer measurements

Full phase space measurement

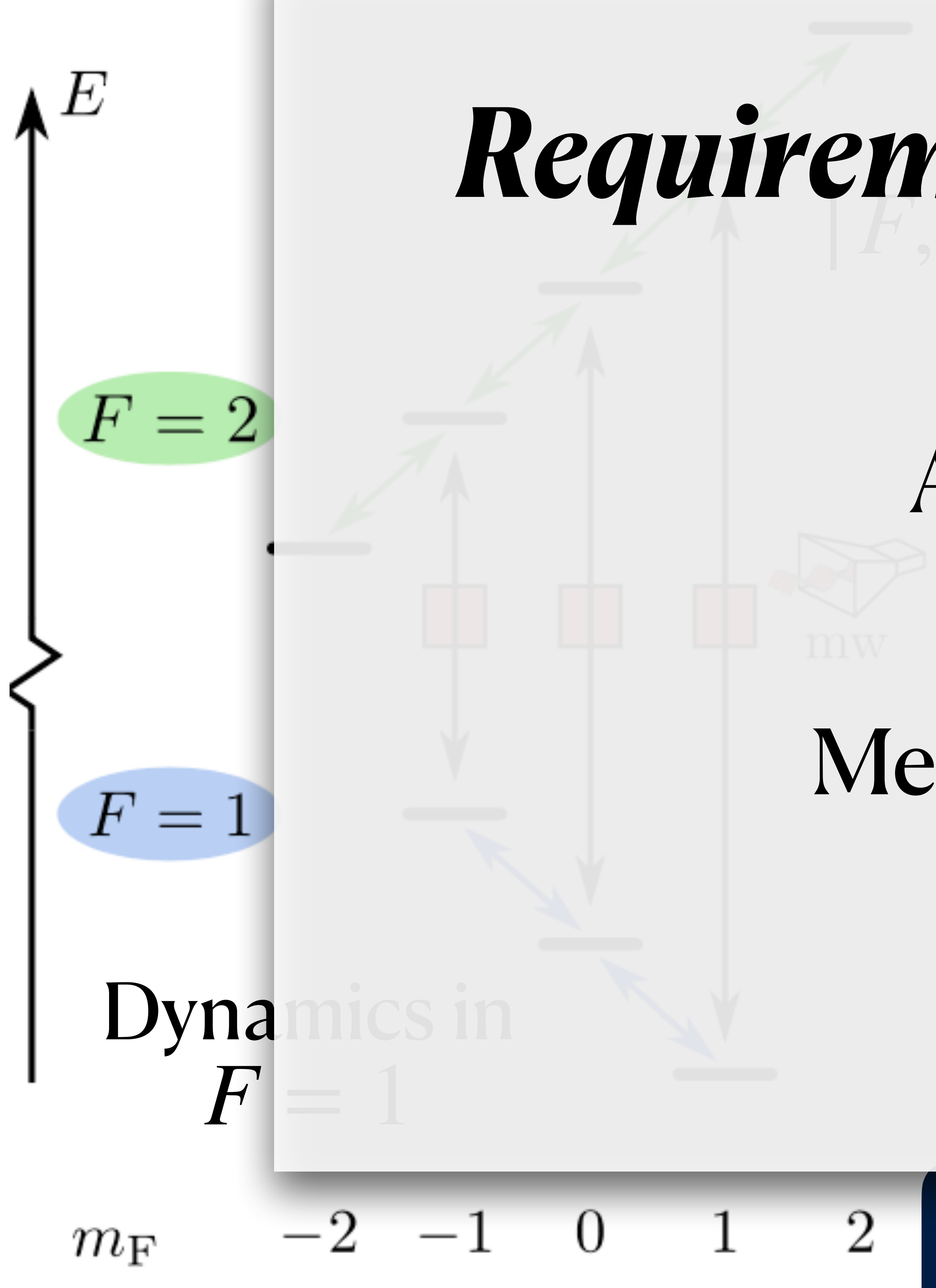
Transfer 50% atoms to *auxiliary modes*, **simultaneously** measure spin-1 observables



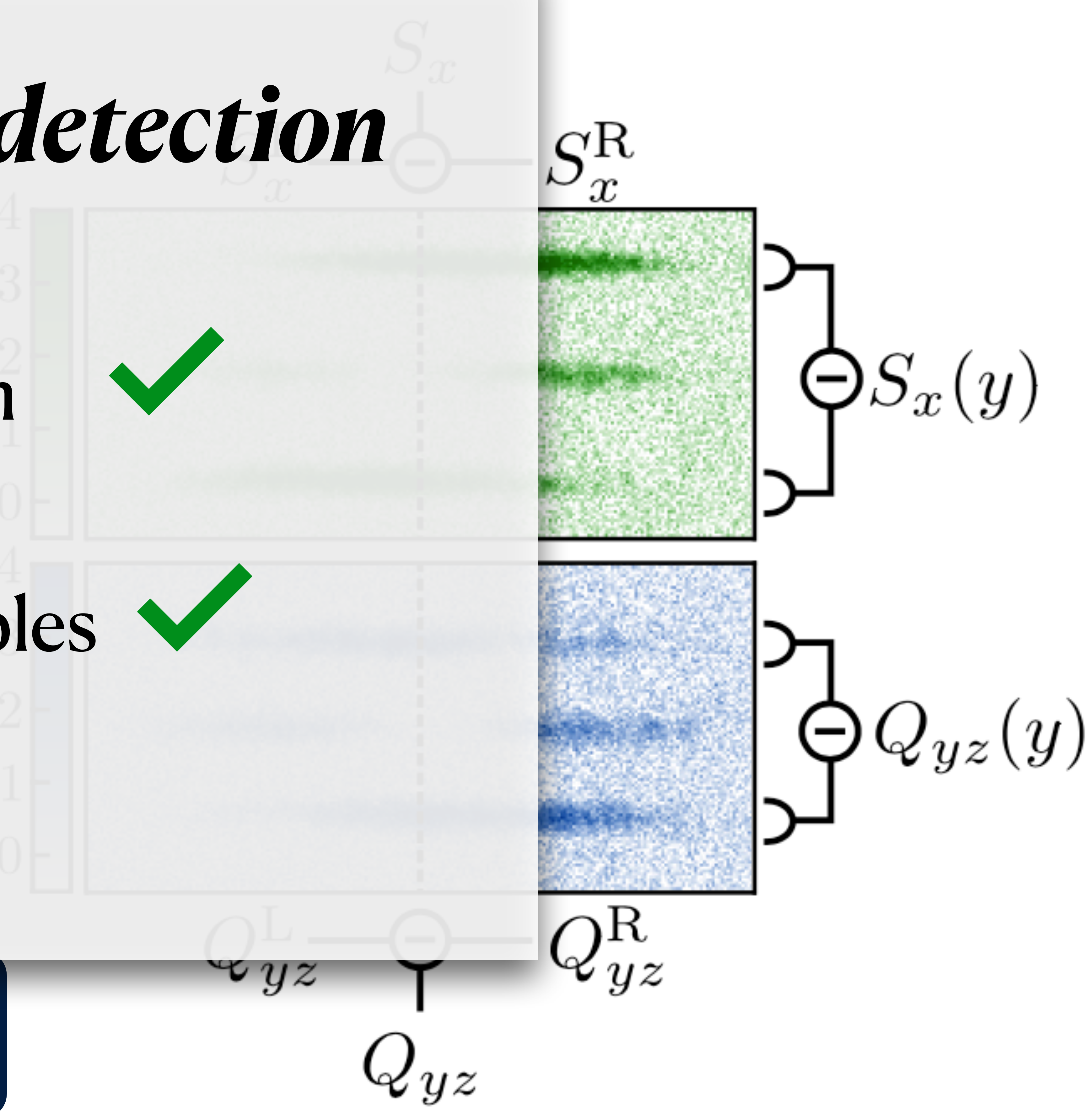
P. Kunkel, *et al.*, PRL **123**, 063603 (2019).

Measuring entropy: Experimental readout

Rubidium-87 has two hyperfine manifolds



Transfer 50% atoms to *auxiliary modes*, **simultaneously** measure spin-1 observables



Requirements for entanglement detection

Access bipartite quantum system ✓

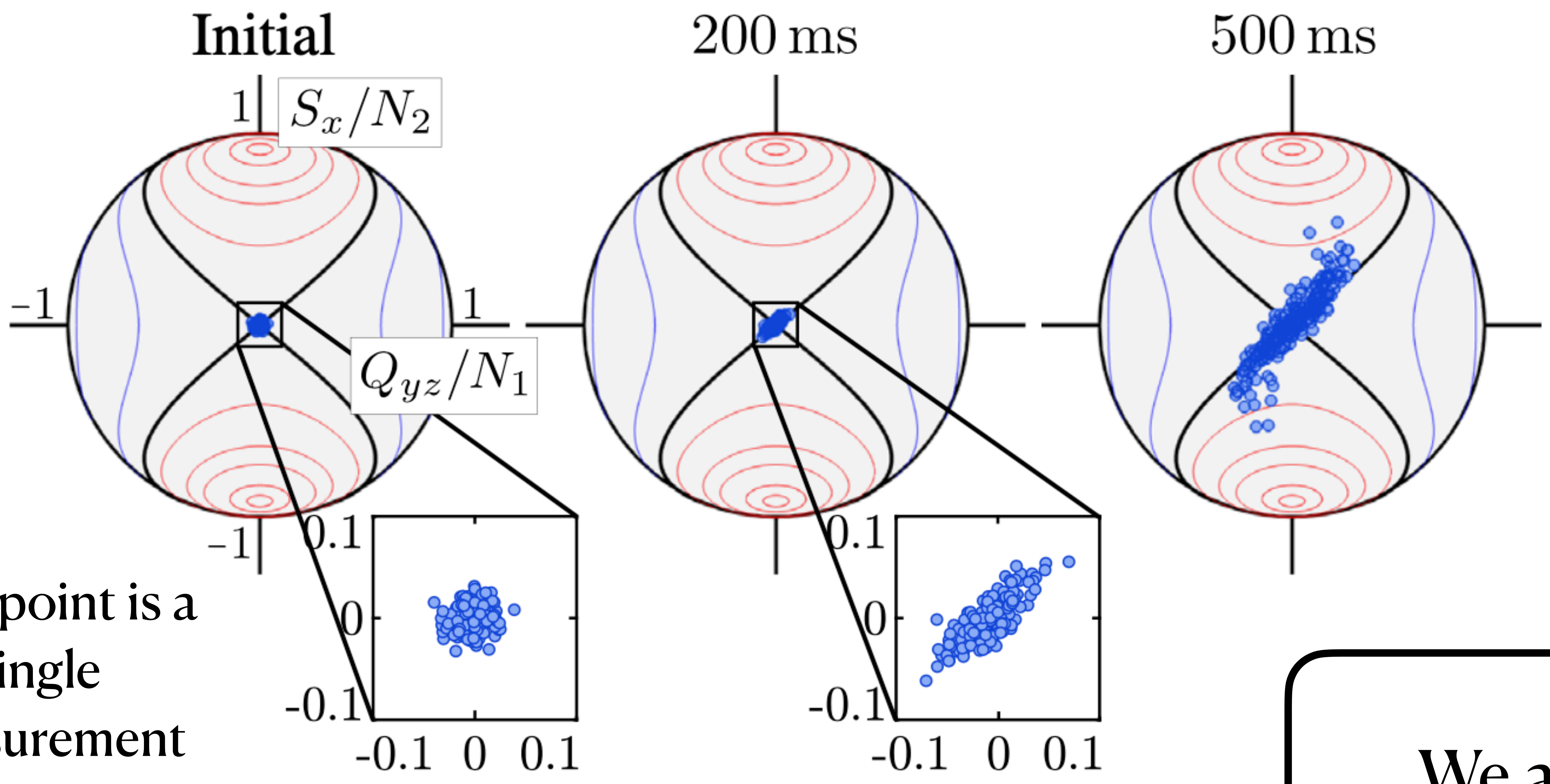
Measure non-commuting observables ✓

Full phase space measurement

P. Kunkel, *et al.*, PRL **123**, 063603 (2019).

Quasiprobability distribution

Our observables: collective spin S_x Q_{yz} Non-commuting observables!



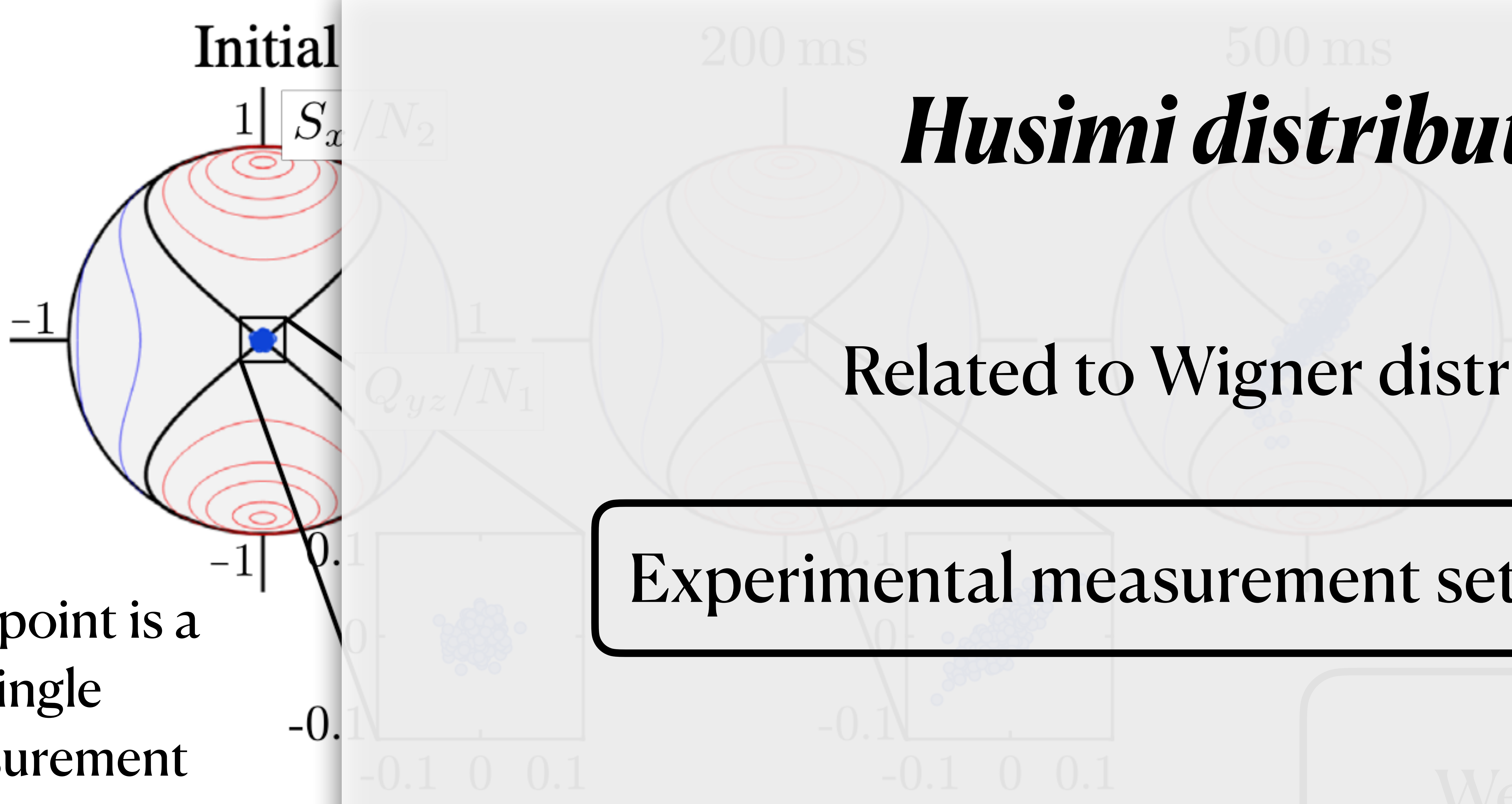
Each point is a single measurement

We associate an **entropy** S_W (Wehrl entropy) to measured distribution Q_ρ

P. Kunkel, *et al.*, PRL **123**, 063603 (2019).

Quasiprobability distribution

Our observables: collective spin S_x Q_{yz} **Non-commuting observables!**



Each point is a single measurement

Husimi distribution

Related to Wigner distribution

Experimental measurement sets the direction

We associate an **entropy** S_W (Wehrl entropy) to measured distribution Q_ρ

P. Kunkel, *et al.*, PRL **123**, 063603 (2019).

Experimental entanglement detection

- **Squeezing** in local subsystems
- **Correlations** emerge **between** local subsystems
- **Entanglement verified** using *variance-based* criteria

Detecting entanglement structure in continuous many-body quantum systems

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Martin Gärttner,[1,2,3] Helmut Strobel,[1] and Markus K. Oberthaler[1]

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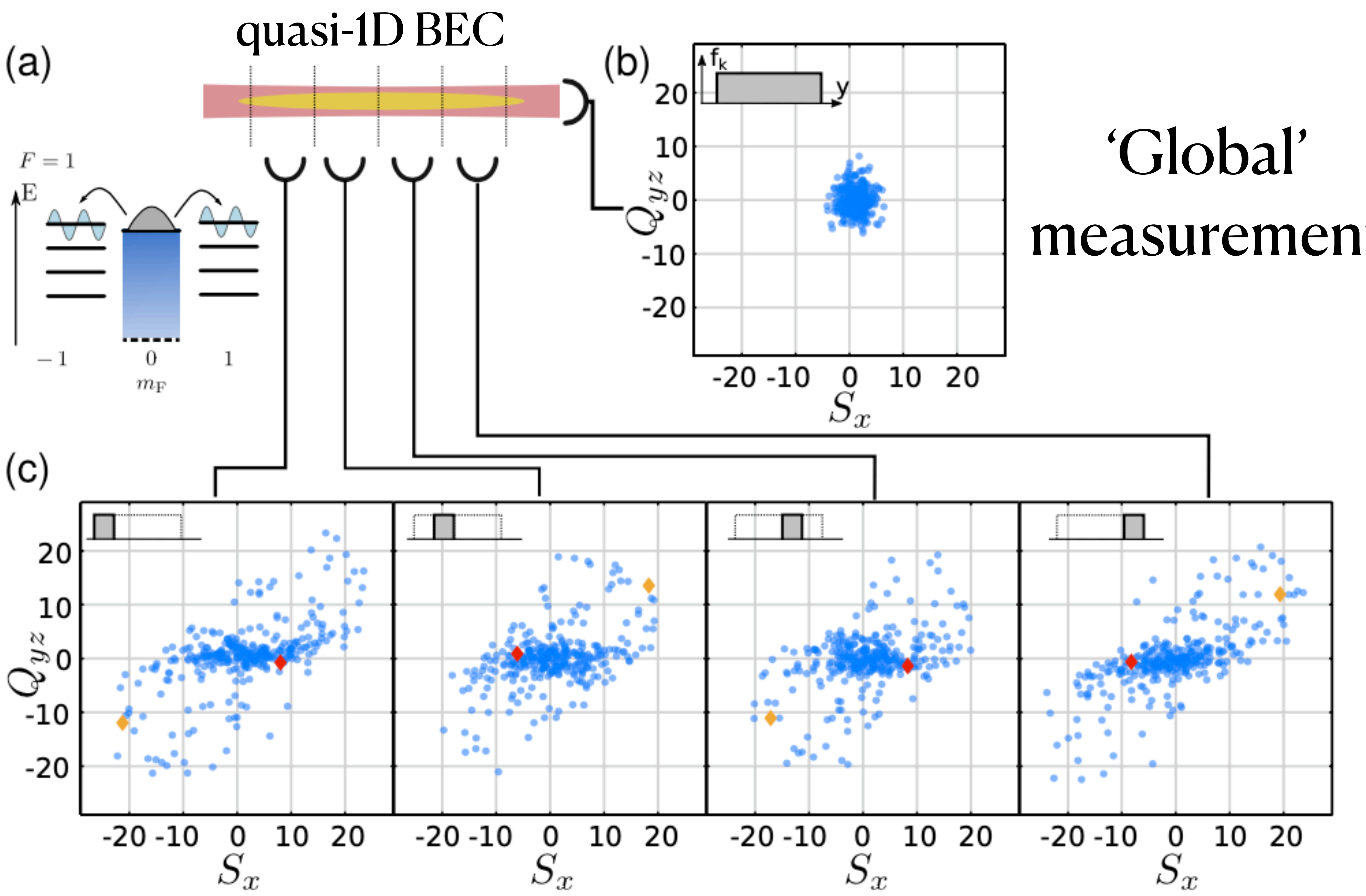
(Dated: May 27, 2021)



'Global' measurement

'local' measurements

P. Kunkel, *et al.,* arXiv:2105:12219 (2021).

Entropic entanglement detection progress

Derived *Wehrl* mutual information

$$I_W(A:B) = S_W(A) + S_W(B) - S_W(AB)$$

Measures ***total*** **correlations**

→ perfect witness for *pure states*

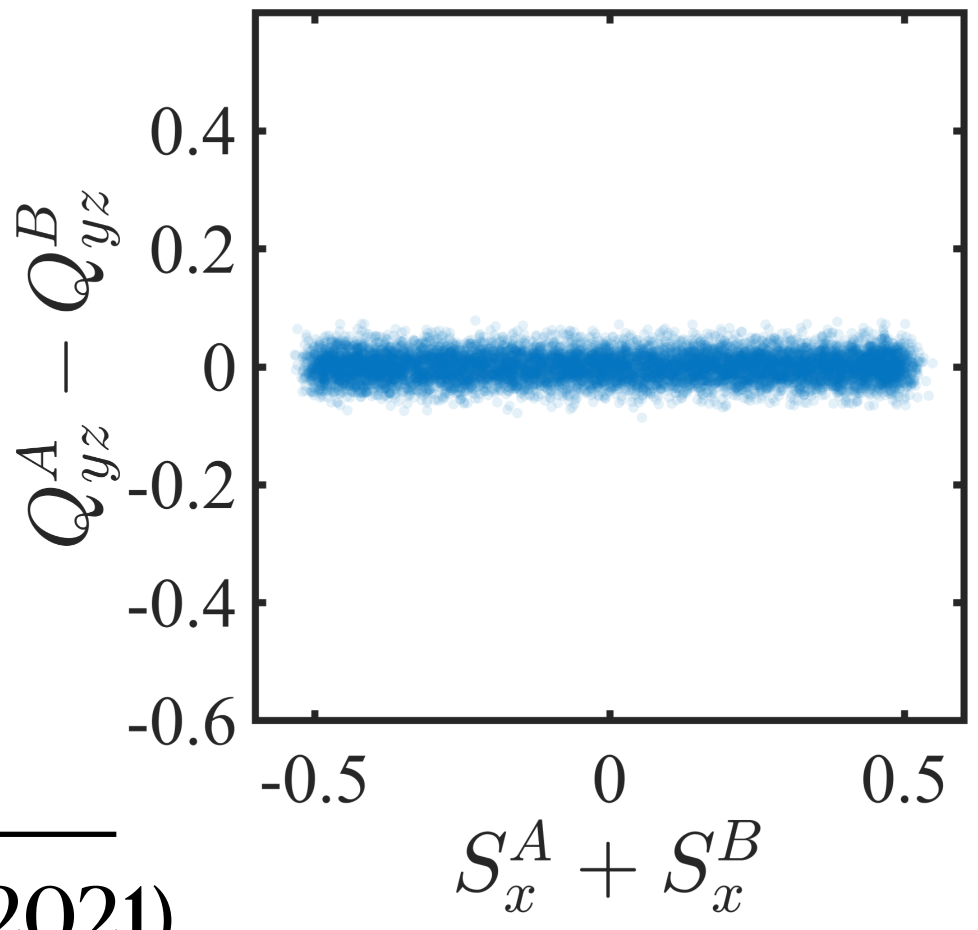
See Poster!

Floerchinger, *et al.*, PRA **103**, 062222 (2021).

Derived an entropic **entanglement witness**

Works for *general mixed states*

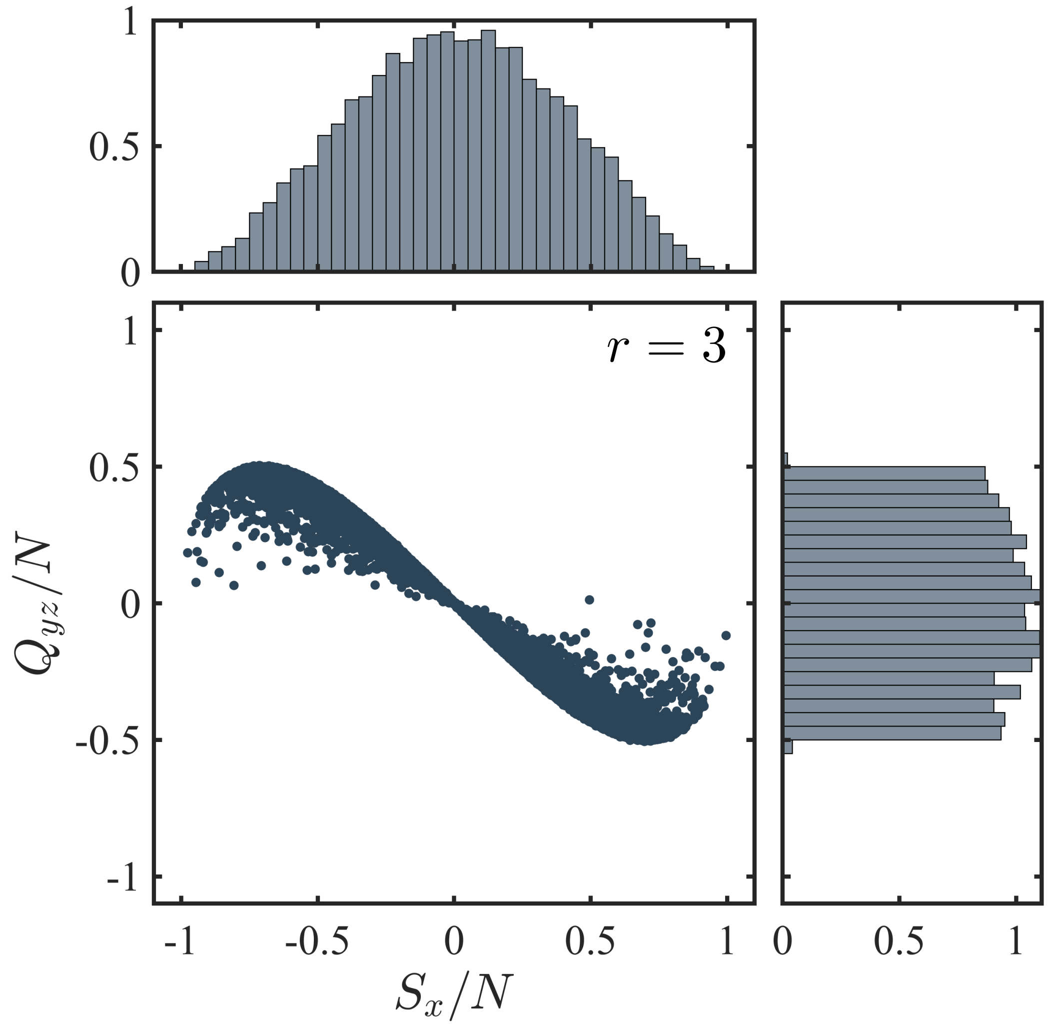
$$S_M(Q_\pm) \geq 1 + \ln 2$$



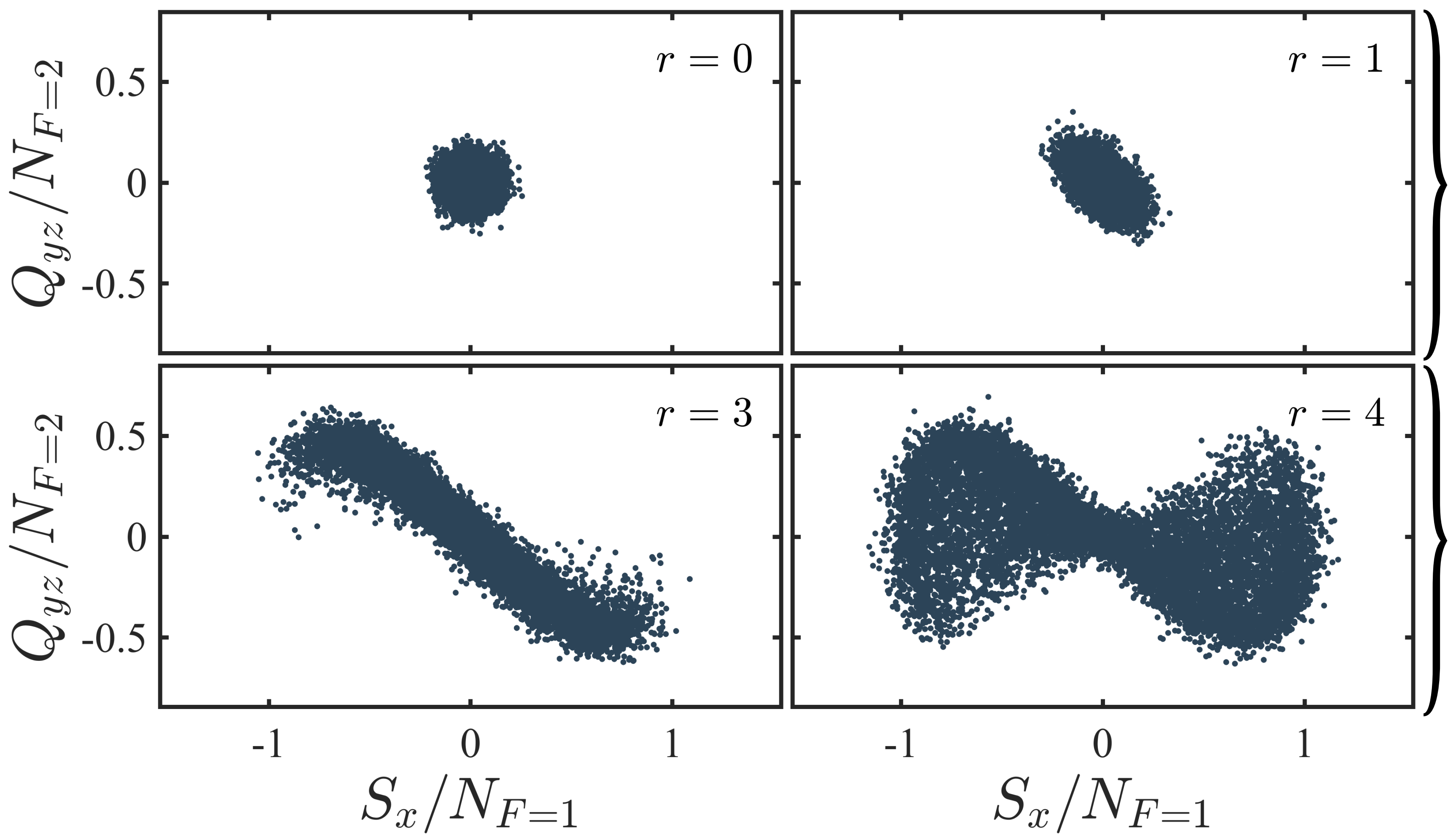
Floerchinger, *et al.*, arXiv:2106.08788 (2021).

Measuring entropy: Gaussian vs. non-Gaussian

Standard techniques: marginals of **Wigner**



Our technique: full measurement of **Husimi**



Variances

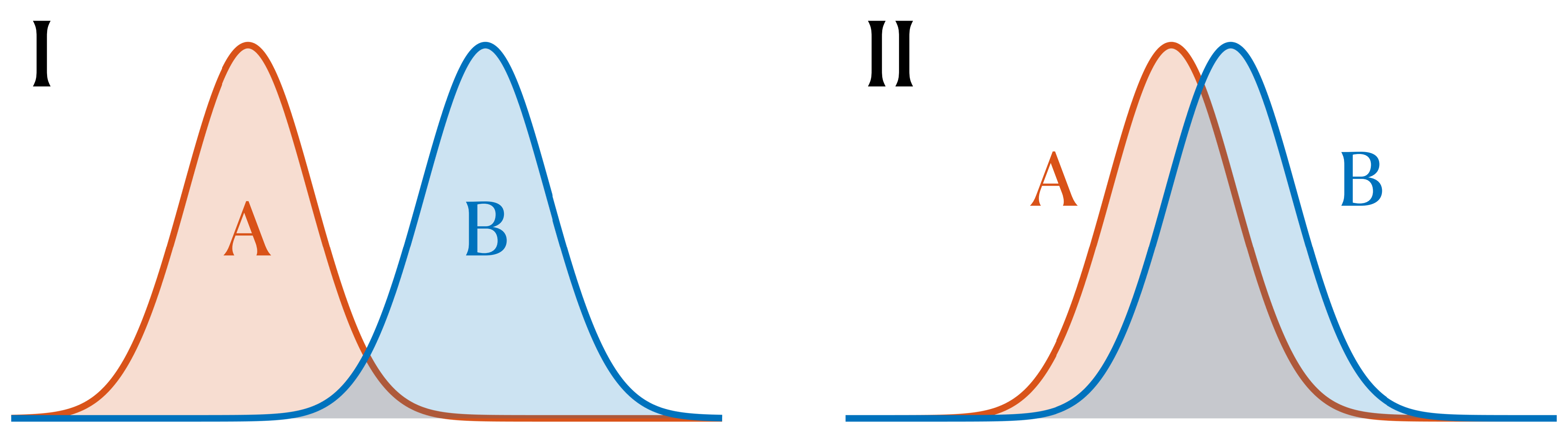
Full distribution *Challenge!*

Field theory progress

Entanglement

Consider a **free** field theory: entanglement entropy is **divergent**

Relative entropy measures distinguishability of two distributions



$$S_{\mathrm{I}}(A \parallel B) > S_{\mathrm{II}}(A \parallel B)$$

Floerchinger, *et* al., arXiv:2107.07824 (2021).

Thermodynamics of relative entropy investigated

[Floerchinger & Haas PRE **102** (5), 052117 (2020);
Dowling, Floerchinger & Haas PRD **102** (10), 105002 (2020)]

Uncertainty

Relative entropy identified as interesting quantity: w.r.t. the **vacuum** distribution

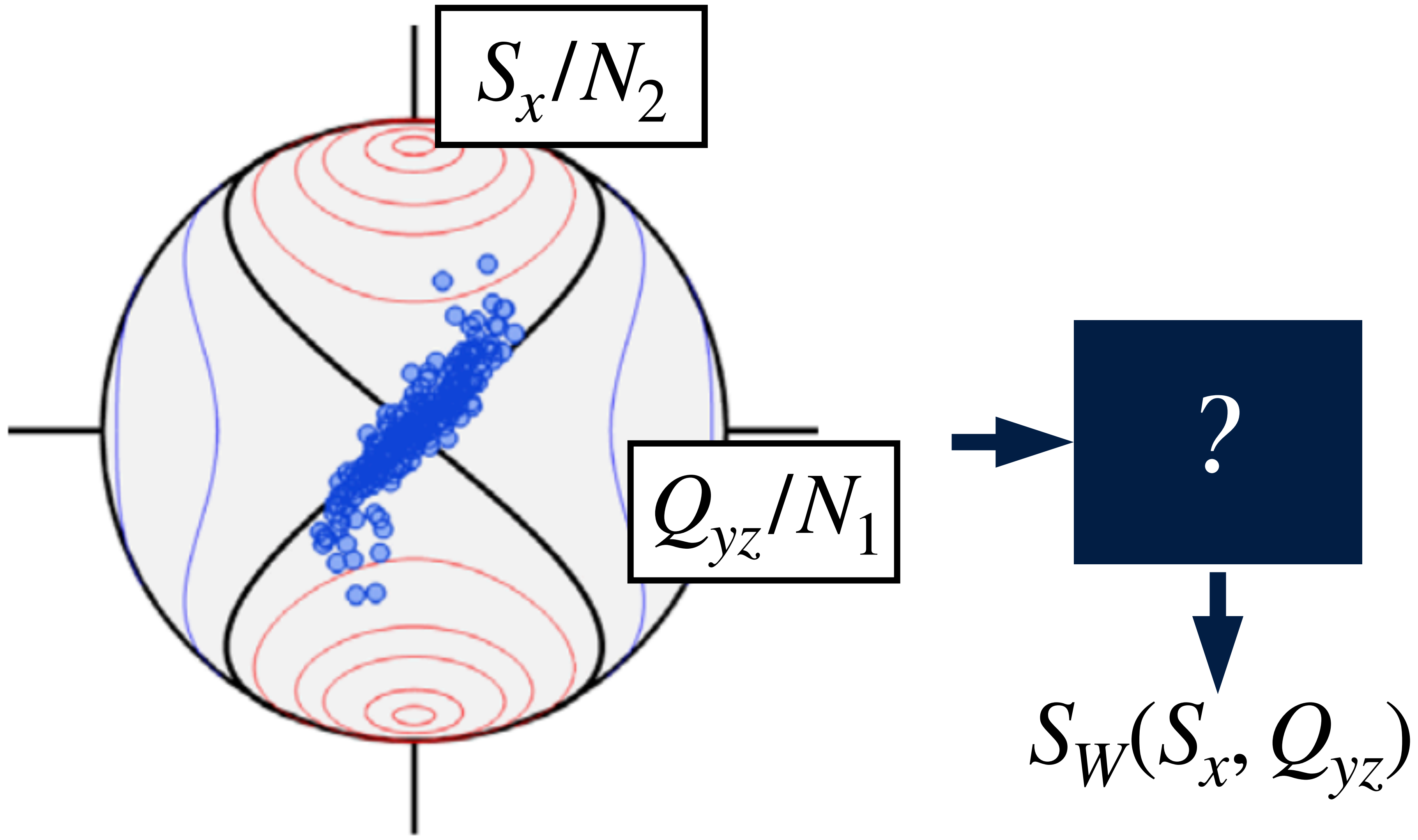
$$S[F\|\tilde{F}] = \int \mathcal{D}\phi F[\phi] \left(\ln F[\phi] - \ln \tilde{F}[\phi]\right)$$

Derived the first **entropic uncertainty relation** for a free field theory

Outstanding challenges

Entropy detection challenges

Entropy estimation



Take *sampled* distribution → calculate entropy

Possible methods:
- kernel density estimation
- machine learning methods

Experimental statistics

Increase samples from phase space distributions

Generalising witness

Our witness works for variables with

$[x_j, p_k] = i\hbar\delta_{jk}$

Make the transition:

$x_j, p_j \to S_x, Q_{yz}$

oscillator ⟶ spin systems

Field theory challenges

Generalise to bipartite systems

Formulate known EURs for field theory

Use REUR to constrain entanglement

Relative entropic uncertainty relation

Measurability of relative entropy

REUR for an interacting theory

Future direction

Research goals rethought

Optimality *and* ***tightness*** *of entropic uncertainty bounds in experiment.*

Developing ***general*** *entropic entanglement witnesses for* ***Husimi / Wehrl***

Entropic entanglement criteria in ***bosonic systems*** *of increasing complexity.*

Continuation

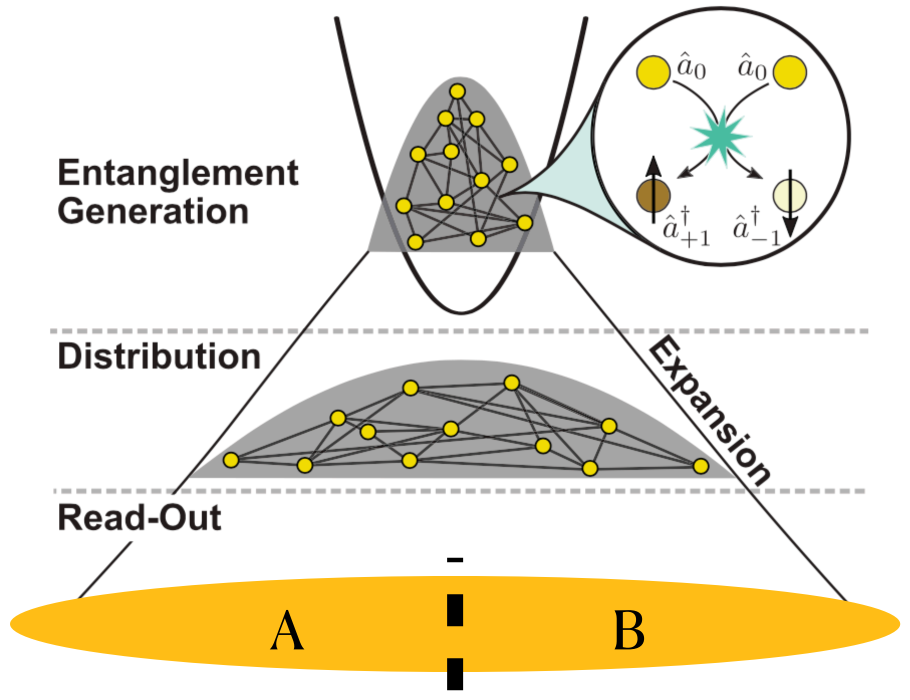
Extracting entropies from cold atom experimental data

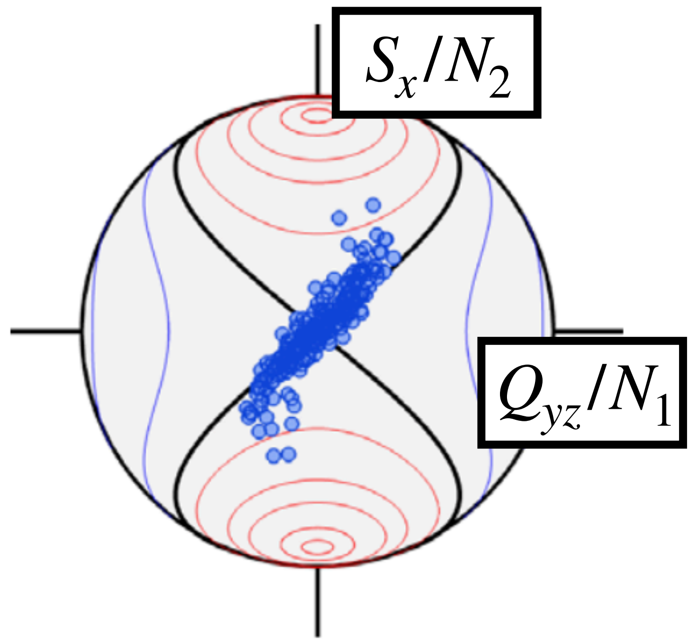
Entropic uncertainty relations for ***quantum field*** *theories.*

Continuation

Summary

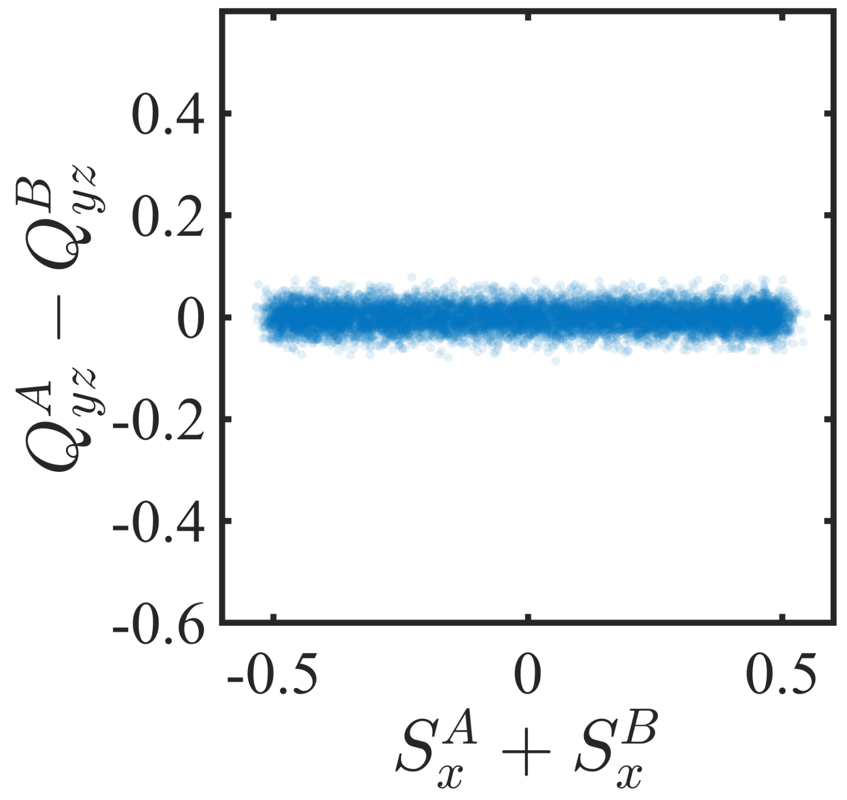
Experimental generation of entangled states





Entropic entanglement witnesses for measurement scheme

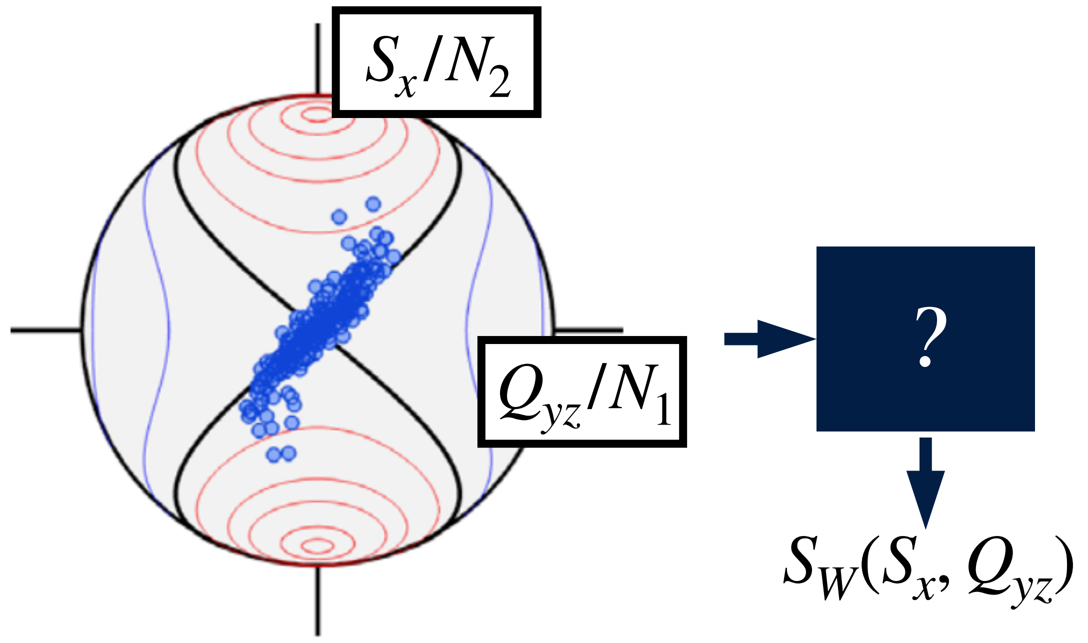
$$S_M(Q_\pm) \geq 1 + \ln 2$$



Derived entropic uncertainty relation for a **free field theory**

$$S[F \,\|\, F_\alpha] + S[G \,\|\, G_\alpha] \leq \ldots$$

Need to **calculate entropy** from **measured distributions**



Derive EUR for **interacting** fields, consider **measurability**