Entanglement in spinor Bose gases via entropic uncertainty relations

Oliver Stockdale[1], Tobi Haas[2], Martin Gärttner[1,2,3]

[1]Kirchhoff-Institut für Physik, Universität Heidelberg, Im Neuenheimer Feld 227, 69120 Heidelberg, Germany
[2]Institut für Theoretische Physik, Universität Heidelberg, Philisophenweg 16, 69120 Heidelberg, Germany
[3]Physikalisches Institut, Universität Heidelberg, Im Neuenheimer Feld 226, 69120 Heidelberg, Germany

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

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Summary

Motivation:

- Entanglement will be a **key resource** in **future quantum technologies**
- Entropy is a **tool** in **quantum information theory** for witnessing entanglement
- Bose-Einstein condensates offer a **pristine playground** for understanding quantum many-body systems

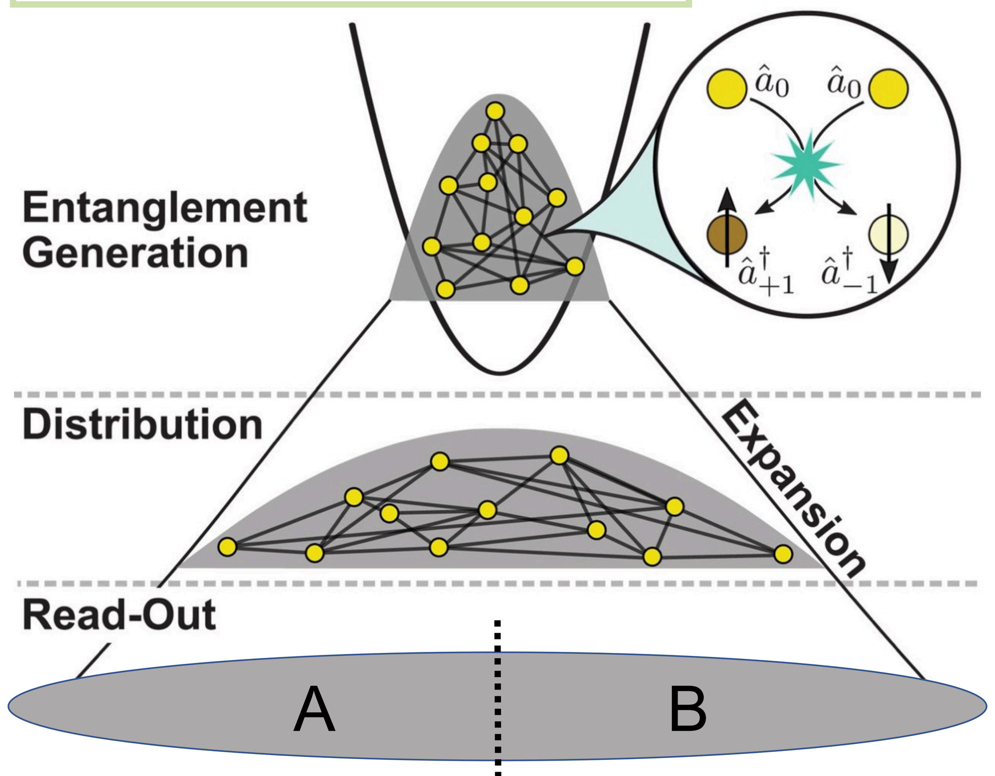
Central goals:

- Develop **entropic uncertainty relations** for spinor gas systems **beyond variance** bases measurements
- Understand the **temporal dynamics** of entanglement in Bose gases

Experimental system: ^{87}Rb spin-1 Bose gas

Entanglement generation

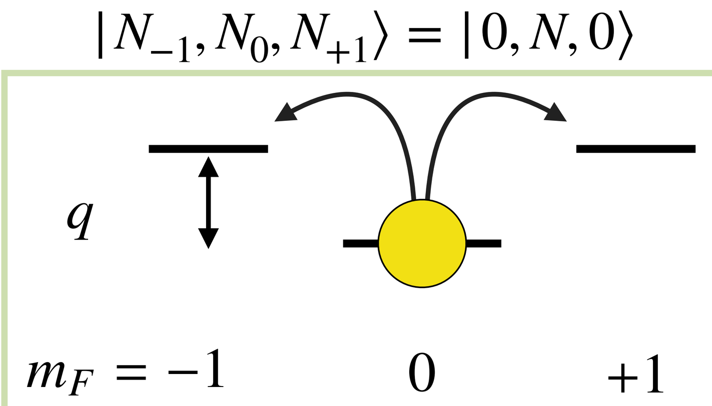
P. Kunkel, et al. Science **360**, 413 (2018).



Single (spatial) mode Hamiltonian

$$H = \frac{g}{2}\Big[(2N_0 - 1)(N_{+1} + N_{-1}) + 2a_0^\dagger a_0^\dagger a_{+1} a_{-1} + \text{h.c.}\Big] + q(N_{+1} + N_{-1})$$

Condensate prepared in $|N_{-1}, N_0, N_{+1}\rangle = |0, N, 0\rangle$

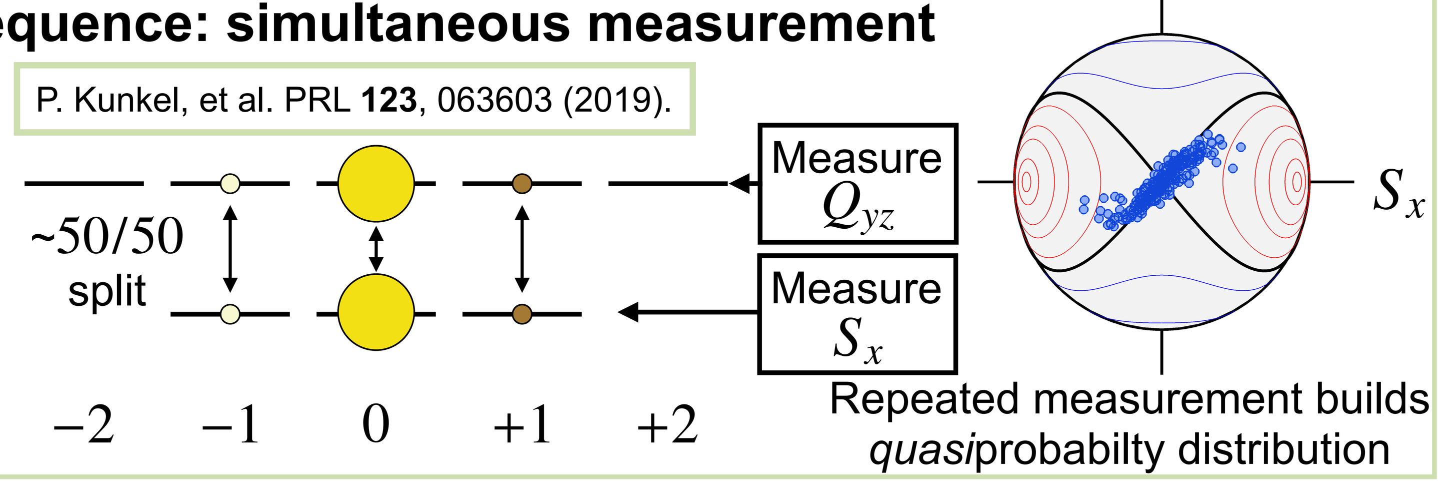


$M = N_{+1} - N_{-1}$ is conserved

Read-out sequence: simultaneous measurement

P. Kunkel, et al. PRL **123**, 063603 (2019).

Split state between $F = 1$ and $F = 2$ manifold via MW pulses



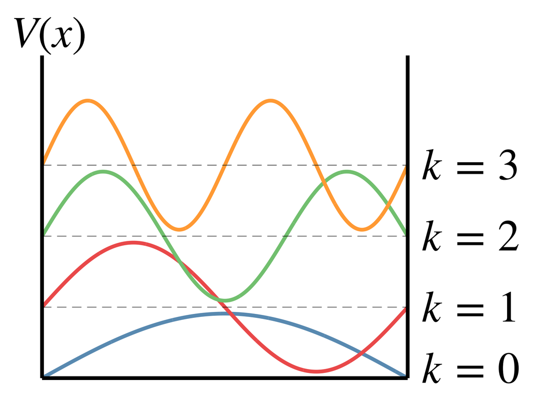
Repeated measurement builds *quasi*probabilty distribution

Numerical simulation

Truncated Wigner approximation (TWA)

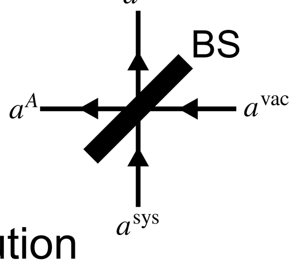
We consider a system with k spatially excited modes

$$\boxed{i\hbar\frac{\partial a_{i,k}}{\partial t} = \frac{\partial H_W}{\partial a_{i,k}^*}}$$

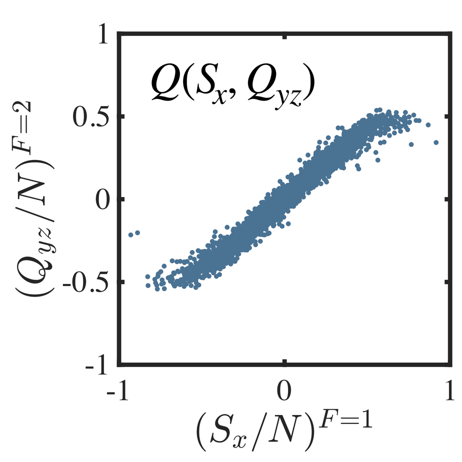


The two beam splitters (once for A-B partition, once for $F = 1$ - $F = 2$ partition) are implemented via

$$\begin{pmatrix} a_{i,k}^A \\ a_{i,k}^B \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} a_{i,k}^{\text{sys}} \\ a_{i,k}^{\text{vac}} \end{pmatrix},$$



'vac' represents vacuum contribution



Simulation of **experimental distribution** via TWA

Valid for **short** t

Entropic uncertainty relations

From the phase space distributions $Q(S_x, Q_{yz})$, we can calculate its **entropy**

$$S_W(\rho) = -\int \mathrm{d}S_x\, \mathrm{d}Q_{yz}\, Q_\rho(S_x, Q_{yz}) \ln Q_\rho(S_x, Q_{yz})$$

Typically, the conditional entropy flags entanglement, here it is the **mutual information**

$$I_W = S_W(\rho_A) + S_W(\rho_B) - S_W(\rho_{AB})$$

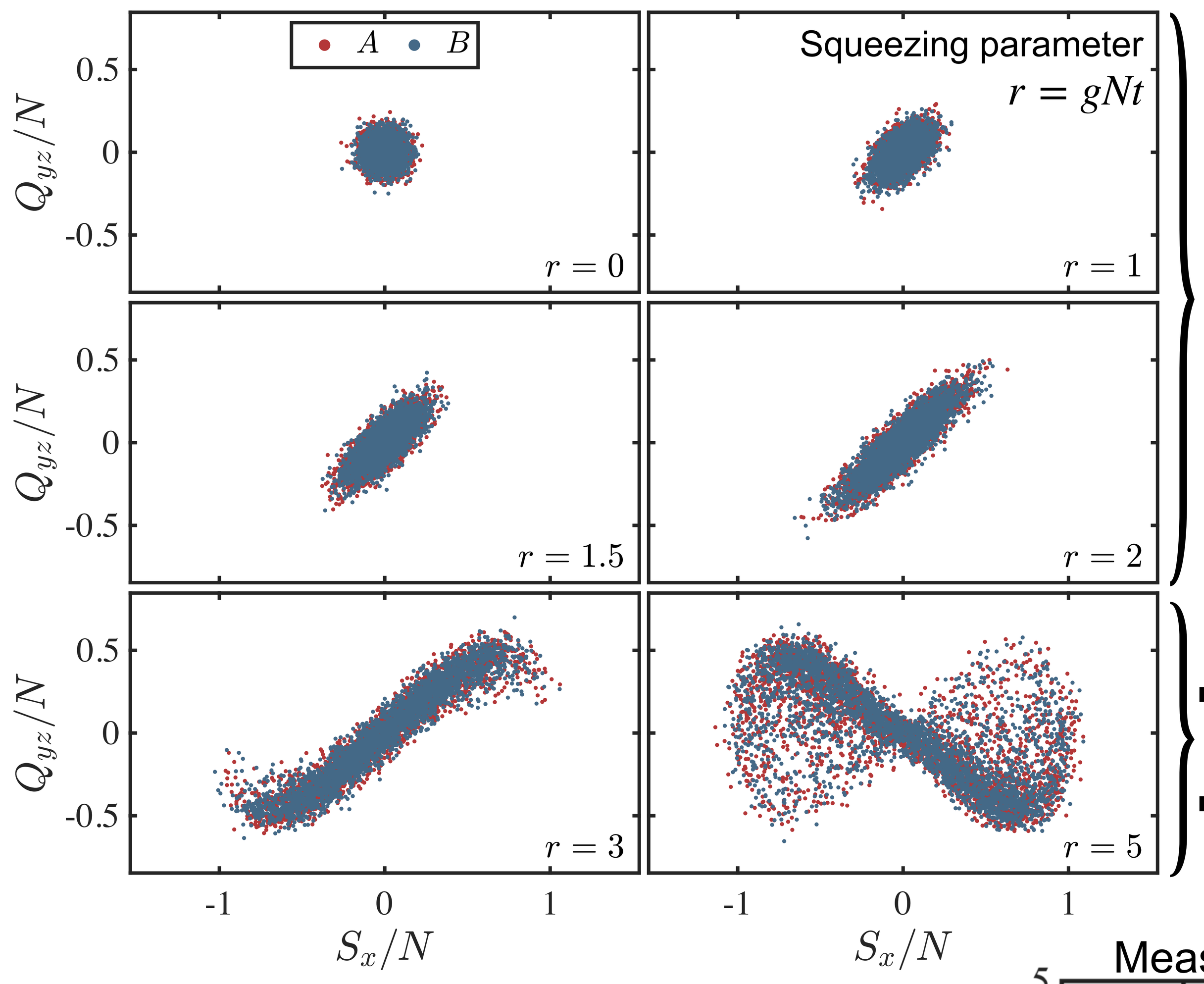
Mutual information measures ***total correlations*** between two subsystems A and B. Let's assume a pure state ρ_{AB}, therefore the state is entangled if

$$I_W > 0$$

Floerchinger, et al. arXiv:2103.07229 (2021).

Witnessing entanglement in simulation

Time dynamics of the phase space distribution - single mode



Gaussian regime

- State is **fully** described via covariance matrix
- Mutual information can be reduced to variance based measure
- Exact solutions also exist here

non-Gaussian regime

- Need to measure entire distribution!
- Density estimation is a very hard problem

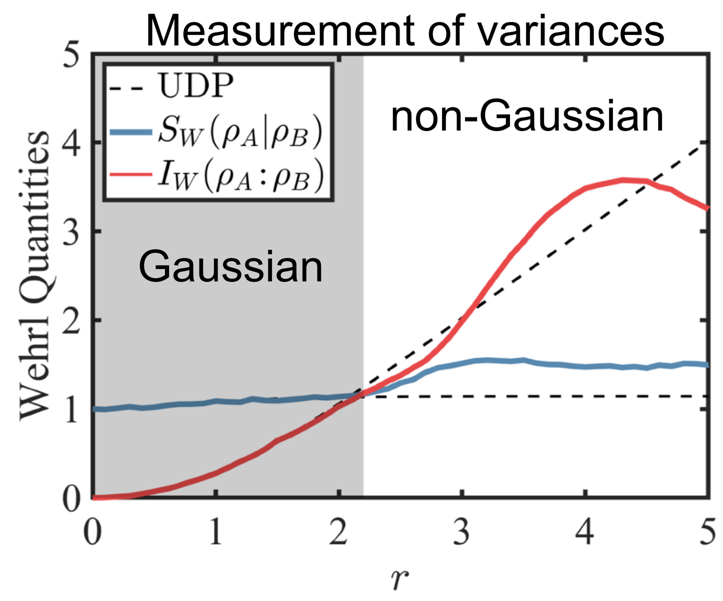
In the undepleted pump approximation (UDP), $N_{\pm 1} \ll N$ and the two-mode squeezed state is

$$|\psi\rangle = \sqrt{1-\lambda^2}\sum_n (-\lambda)^n |nn\rangle$$

where $\lambda = \tanh(r)$. One can analytically calculate the inverse covariance matrix C and show

$$I_W(\rho_A : \rho_B) = \frac{1}{2}\frac{\det C_A \det C_B}{\det C}$$

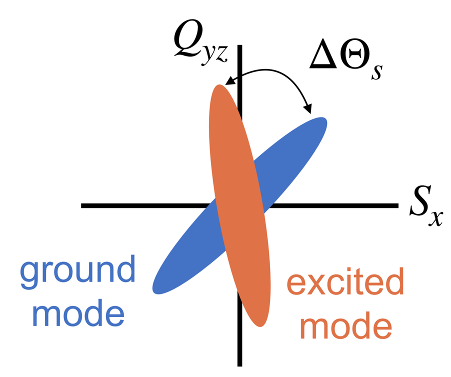
where C_A etc. are the block elements.

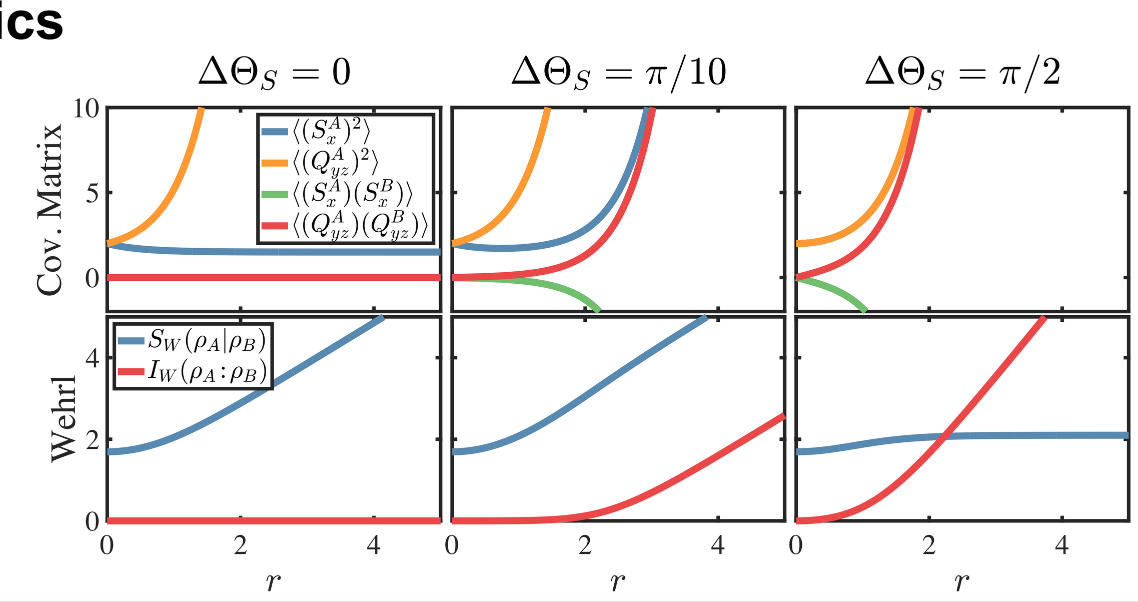


We measure $I_W > 0$ as the state squeezes!

Double mode dynamics

With two modes, we need to take into account the relative spinor phase $\Delta\Theta_s$

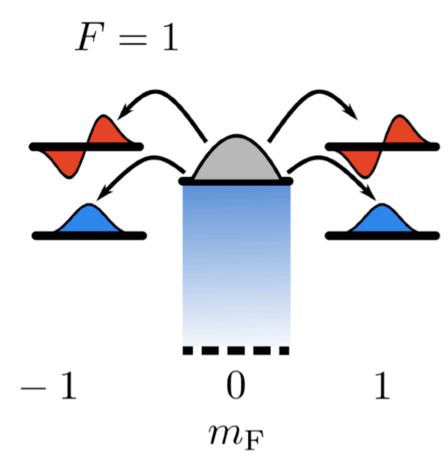




Experimental data

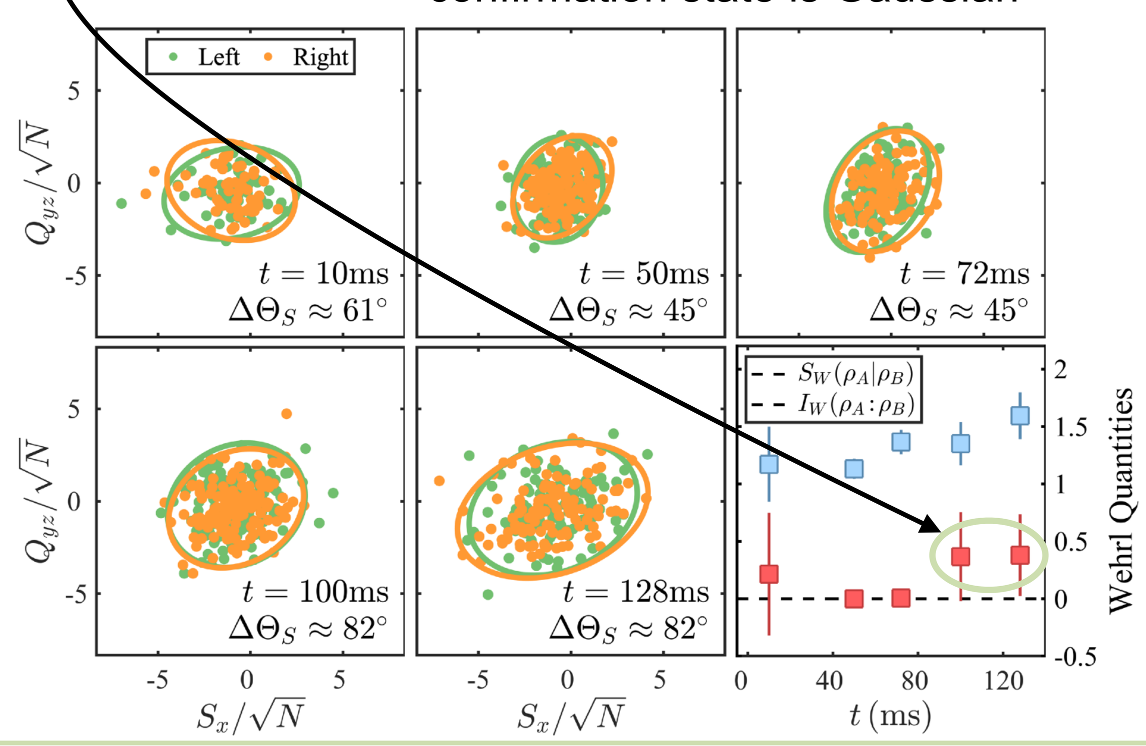
Details of experiment

- Rubidium-87 spinor Bose-Einstein condensate
- Two spatial modes excited
- t represents time of spin-changing collisions



Entanglement? Too early to confirm

We need: — better statistics
— confirmation state is Gaussian



Conclusions and future direction

Conclusions:

- Developed **analytical** and **numerical** methods to simulate spinor Bose gas experiments
- Shown that mutual information **flags entanglement** in these systems
- Presented **experimental analysis** that shows **signs of correlations** via information theoretic concepts

Future direction:

- Derive tight **entanglement witnesses** via the entropic measurements
- Better **understand experimental data** to robustly identify entanglement
- Develop **entropy estimation methods** to analyse non-Gaussian regime
- Understand **long-time** entanglement dynamics