Entanglement detection via entropies in spinor Bose gases

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Theory

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Tobi
Haas

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Experiments

Philipp
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Maximilian
Prüfer

Stefan
Lannig

Robin
Strohmaier

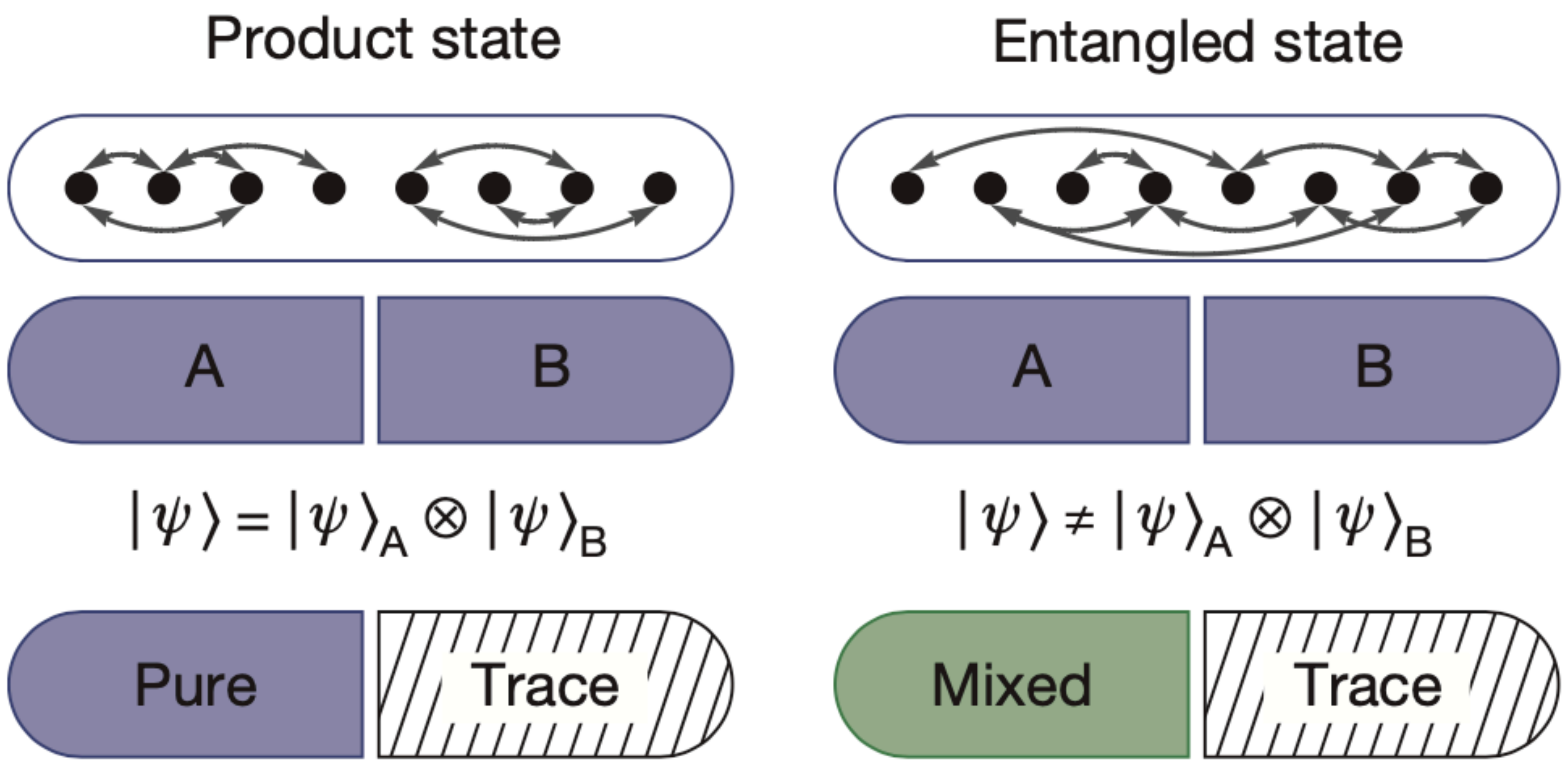
Helmut
Strobel

Markus
Oberthaler

How to measure many-body entanglement?

Partition system, make measurements

<u>With access to **full state** ρ</u>



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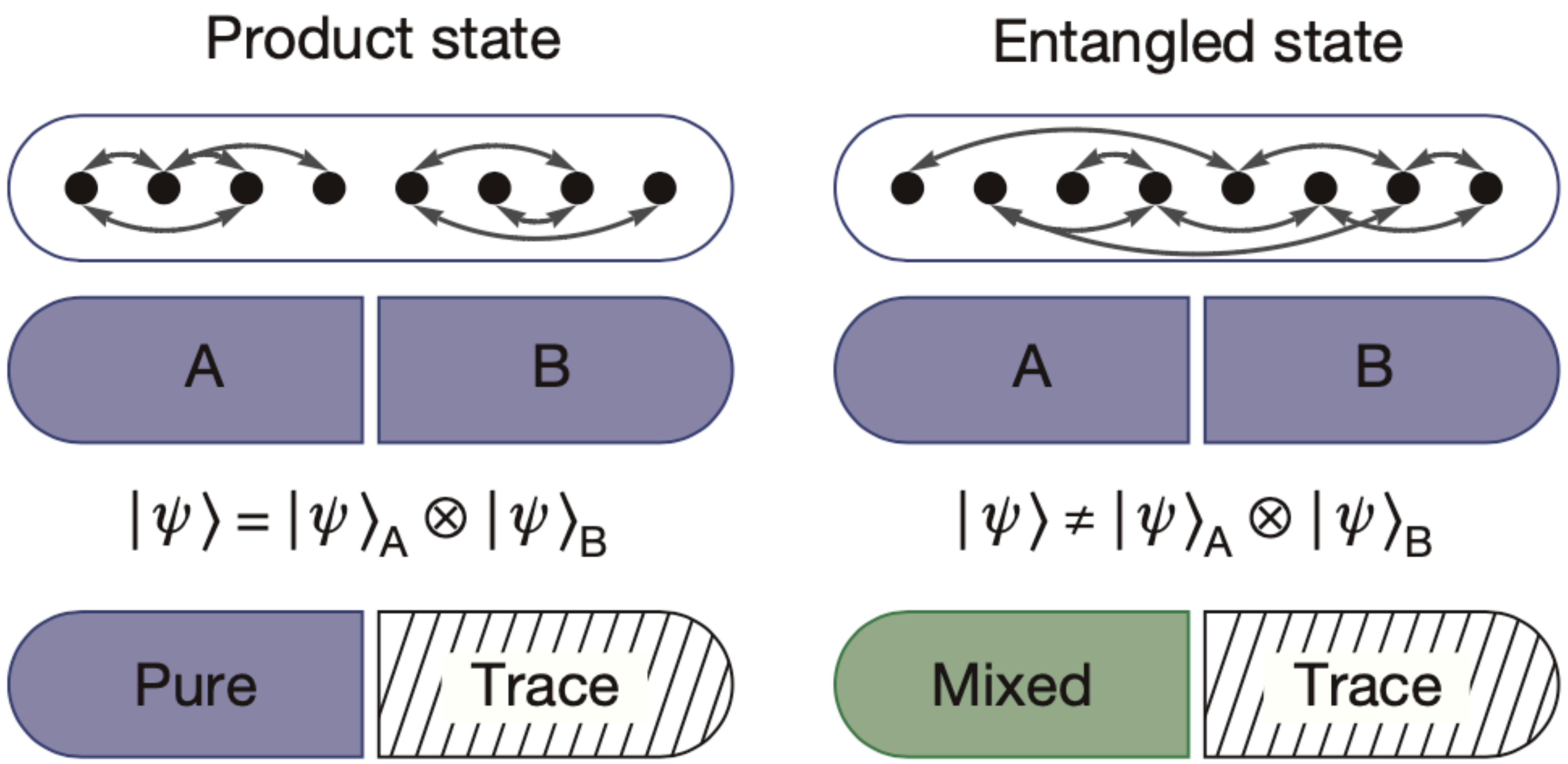

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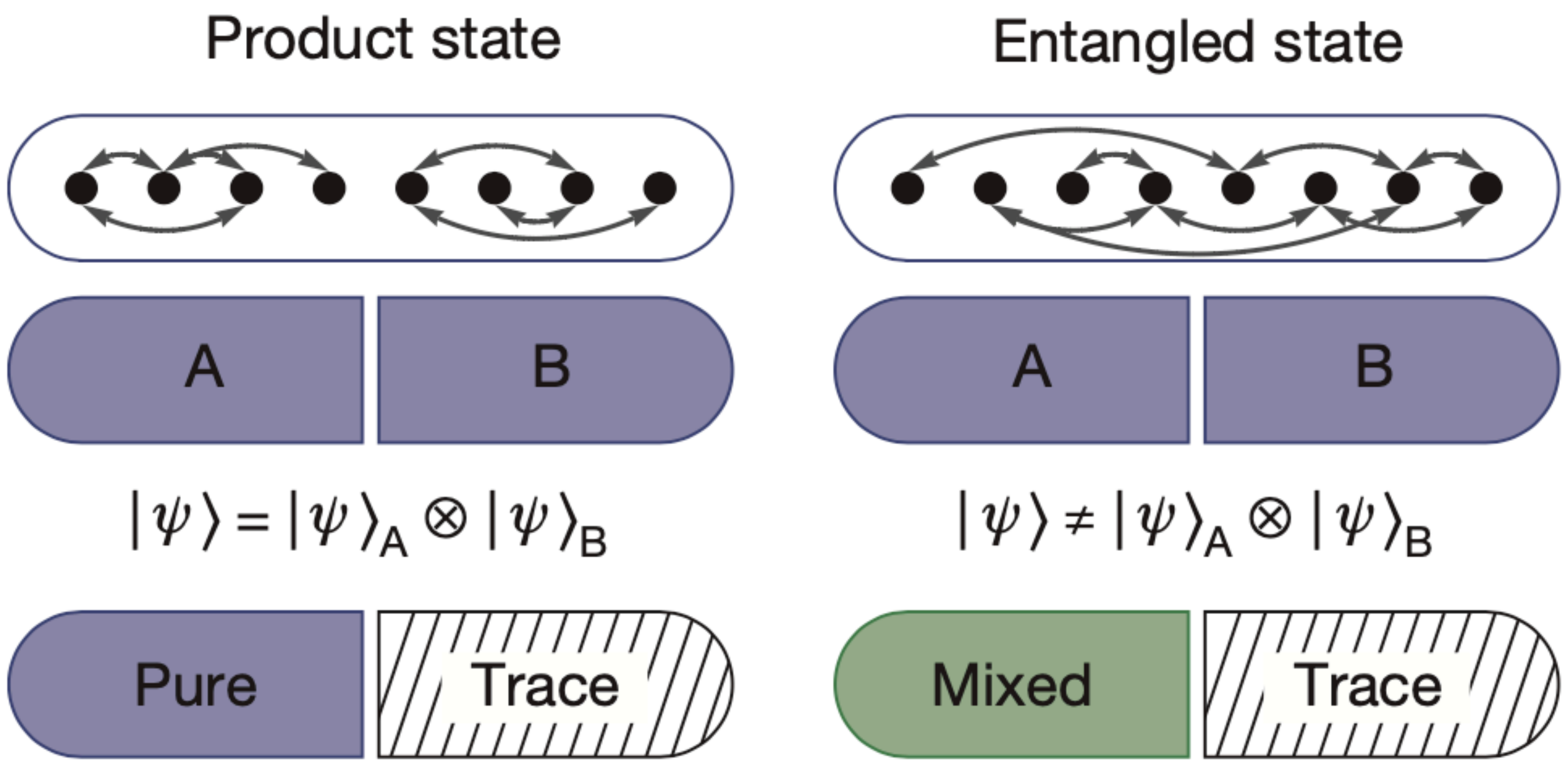

<u>With access to **uncertainties**</u>

$$\langle(\Delta\hat{u})^2\rangle_\rho + \langle(\Delta\hat{v})^2\rangle_\rho \geq a^2 + \frac{1}{a^2}$$

Violation flags entanglement

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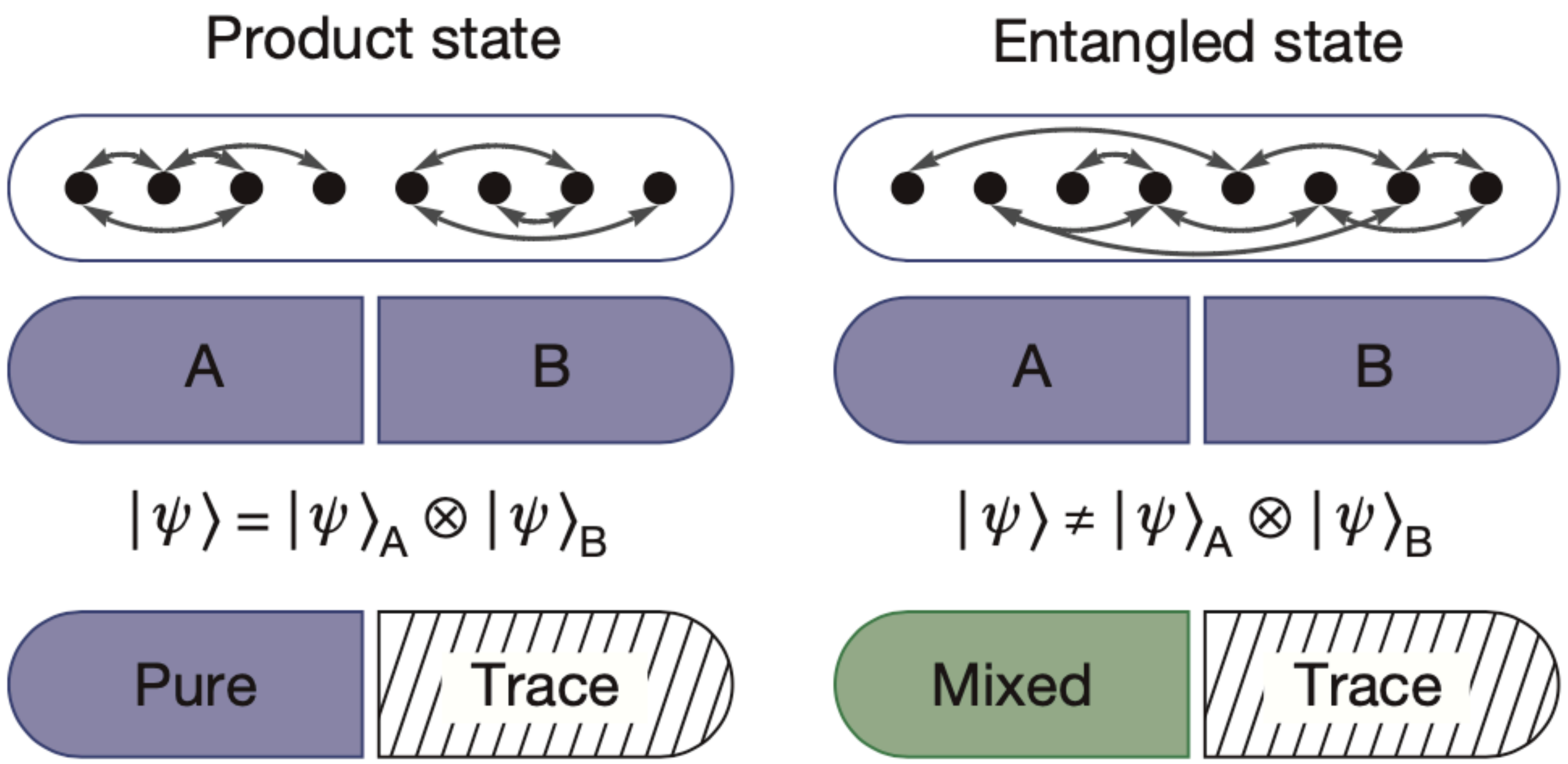

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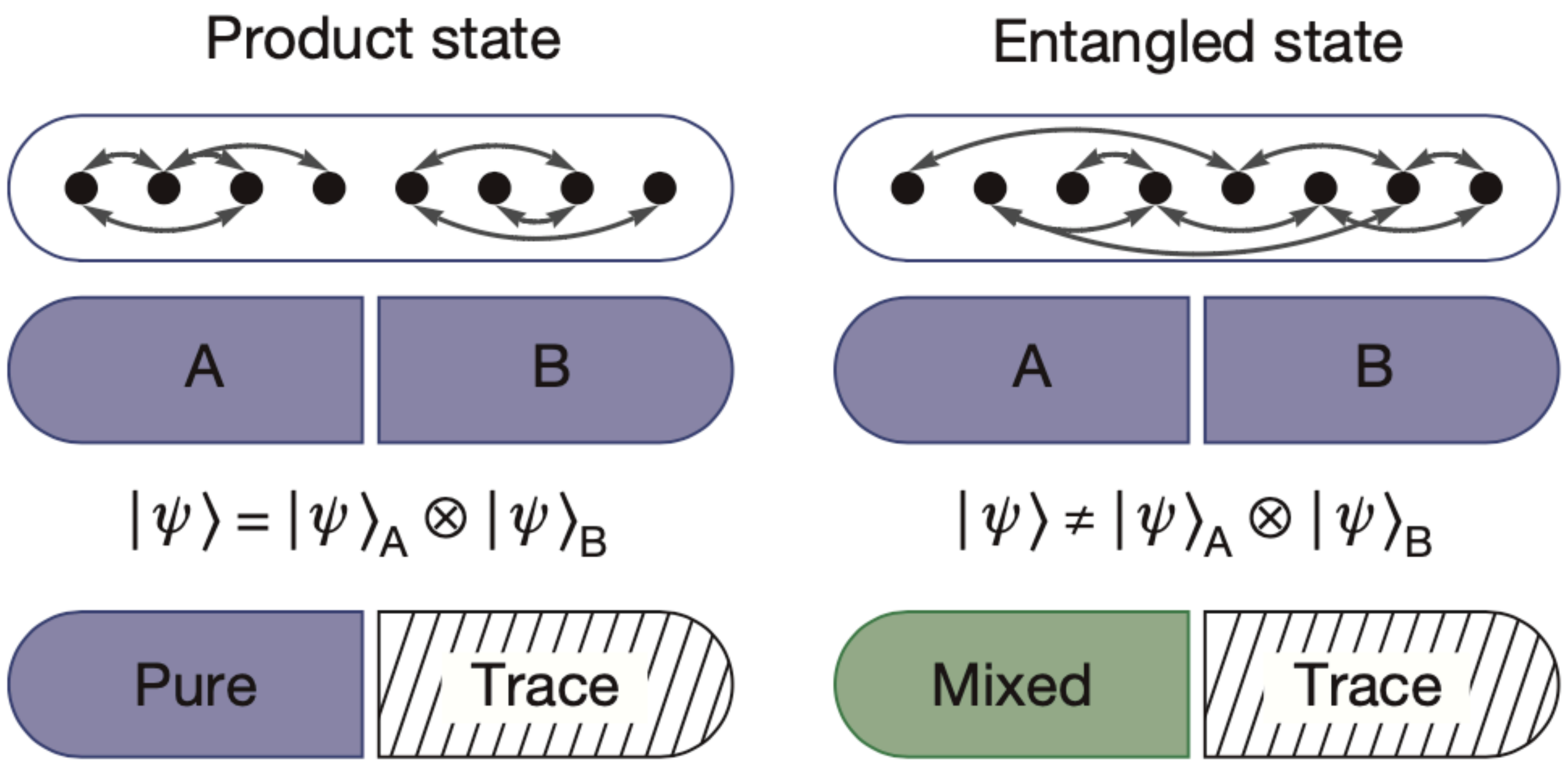

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Outline

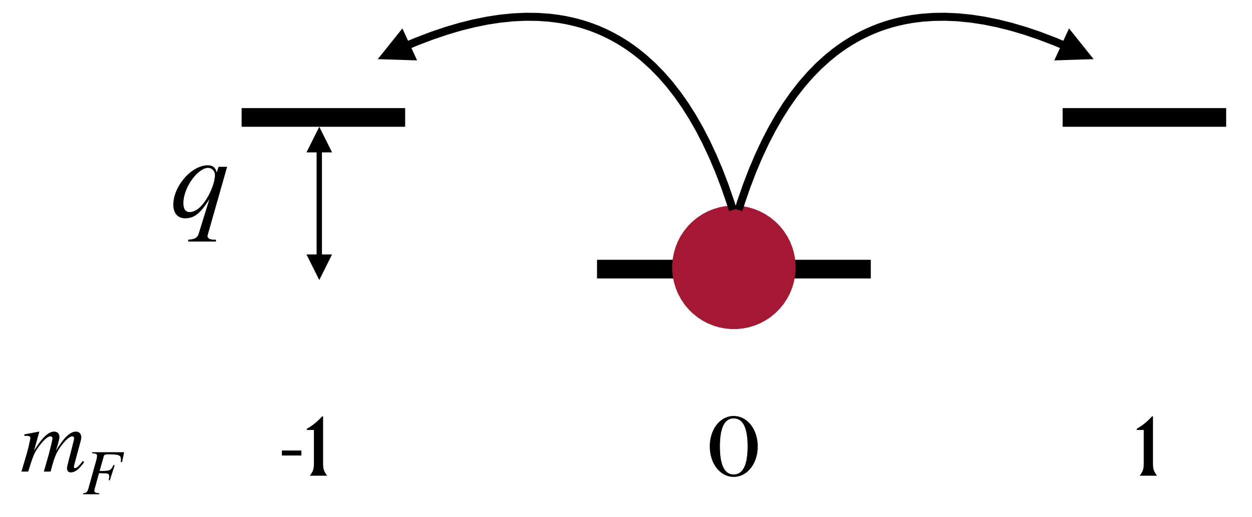
- Introduce the system: ^{87}Rb spin-1 gas
- Experimental readout: how do we measure entropy?
- An entropic entanglement measurement
- Numerical modelling
- Experimental data

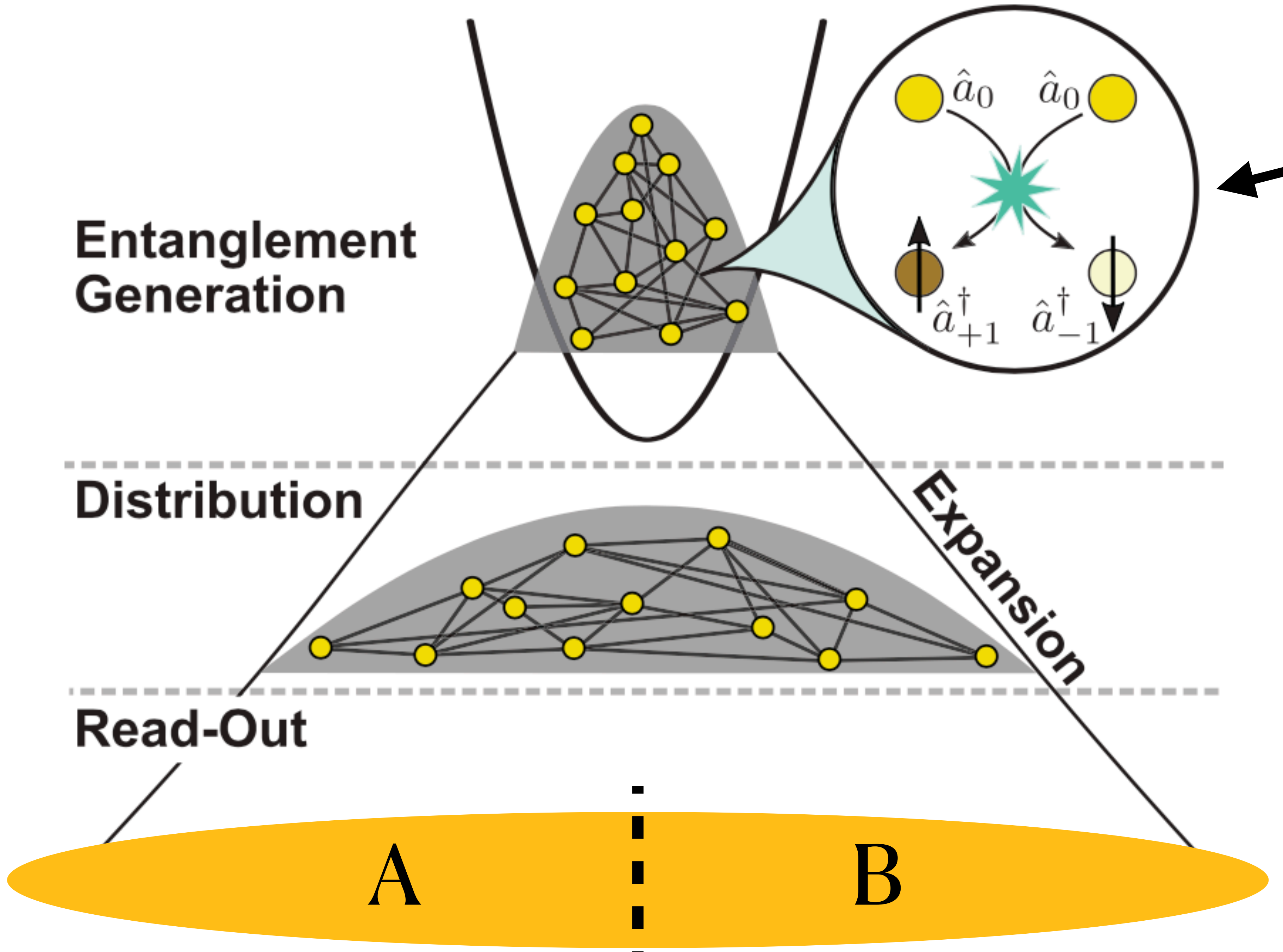
The system

The system

Rubidium-87 BEC in $F = 1$ hyperfine manifold

Characterised by
$|N_{-1}, N_0, N_{+1}\rangle, \quad \hat{a}_{-1}, \hat{a}_0, \hat{a}_{+1}$





Spin-changing collisions

q represents the quadratic Zeeman shift

$$H = g[(2\hat{N}_0 - 1)(\hat{N}_{+1} + \hat{N}_{-1}) + 2\hat{a}^\dagger_0\hat{a}^\dagger_0\hat{a}_1\hat{a}_{-1} + 2\hat{a}^\dagger_1\hat{a}^\dagger_{-1}\hat{a}_0\hat{a}_0] + q(\hat{N}_{+1} + \hat{N}_{-1})$$

Consider only a single spatial mode (for now)

P. Kunkel, *et al.*, Science **360**, 413 (2018).

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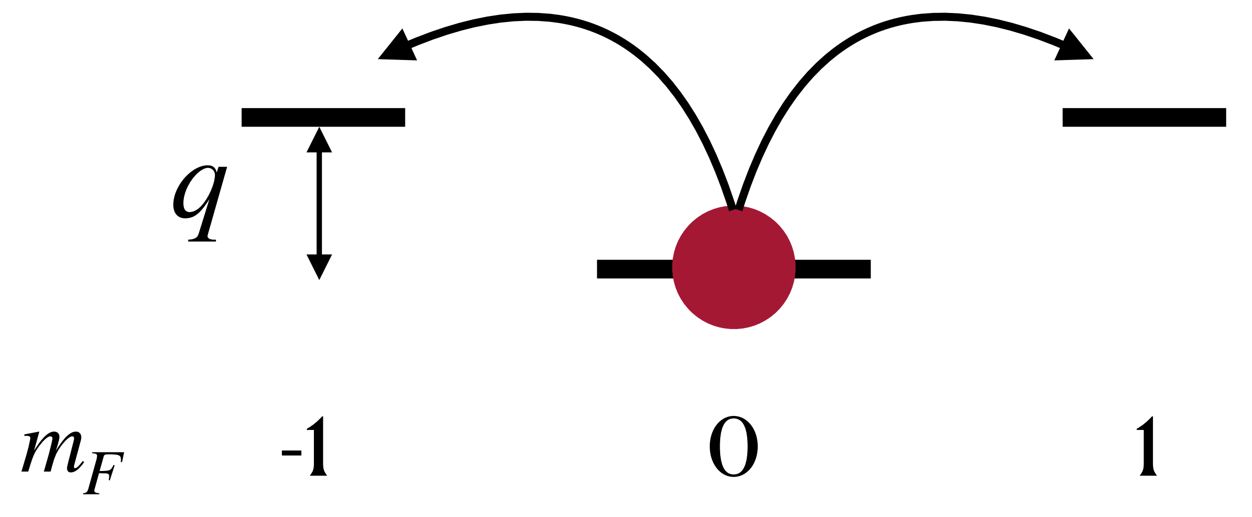

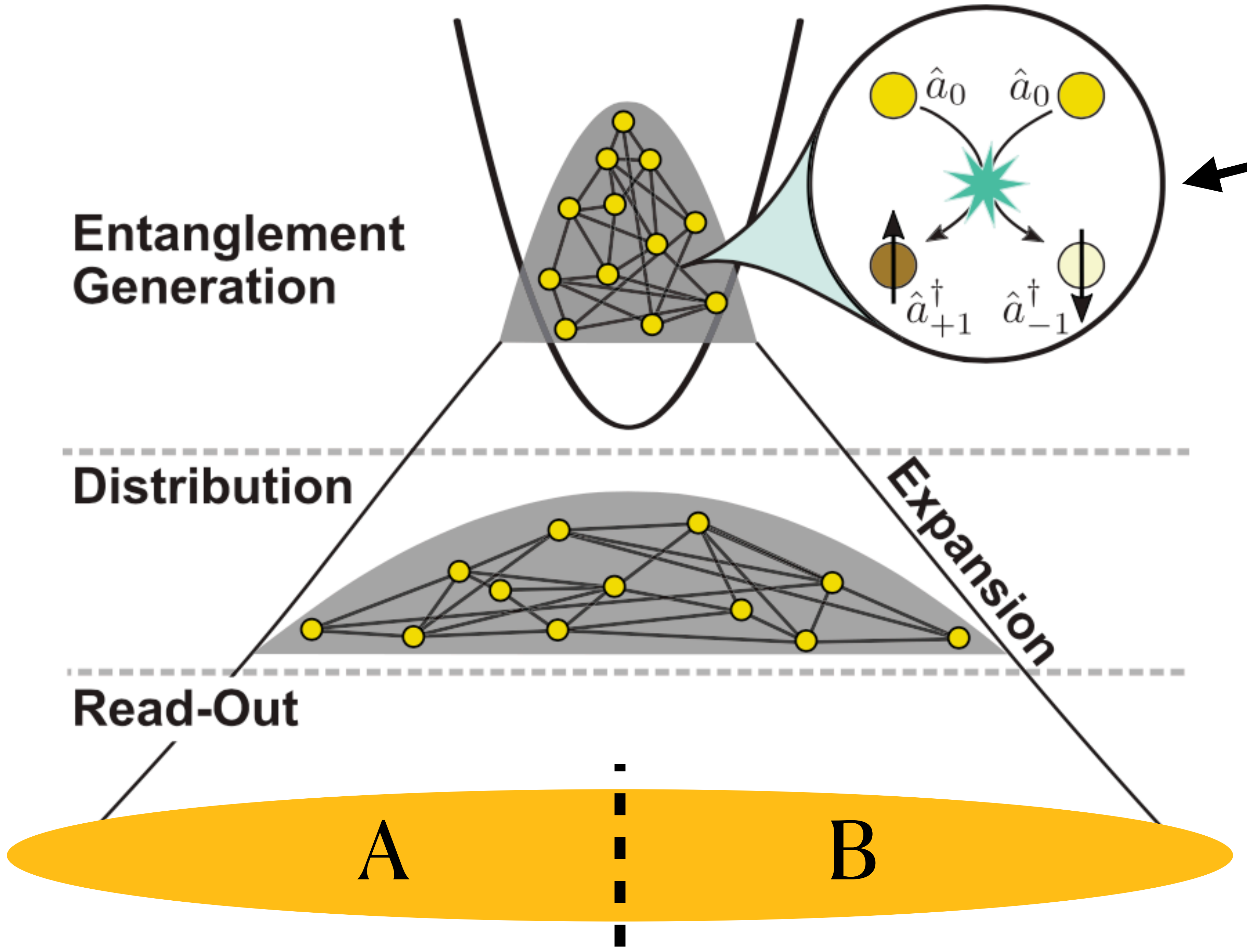

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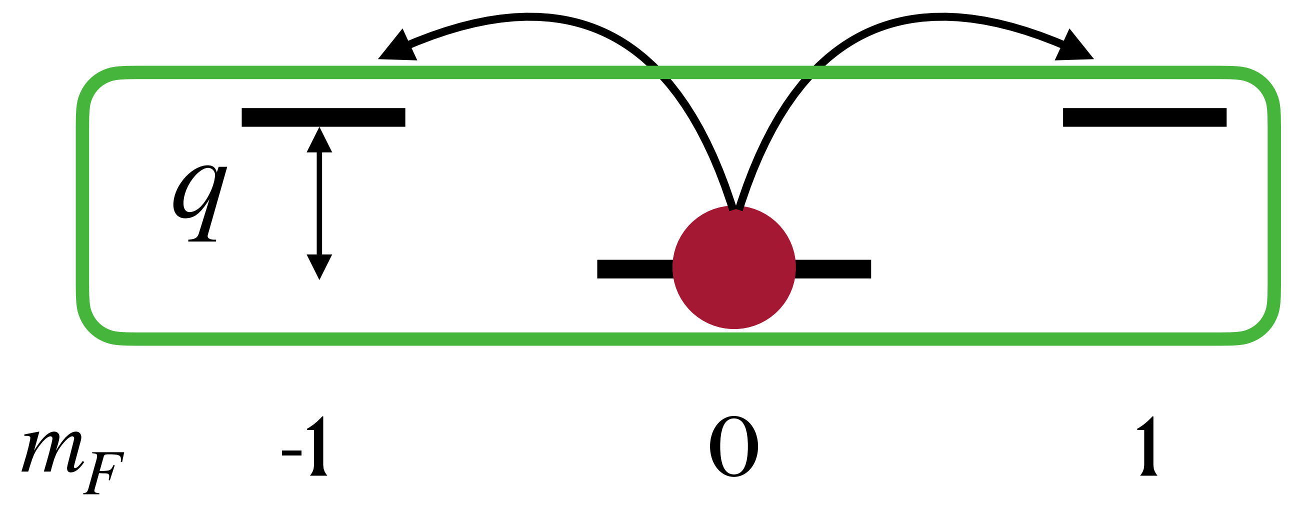

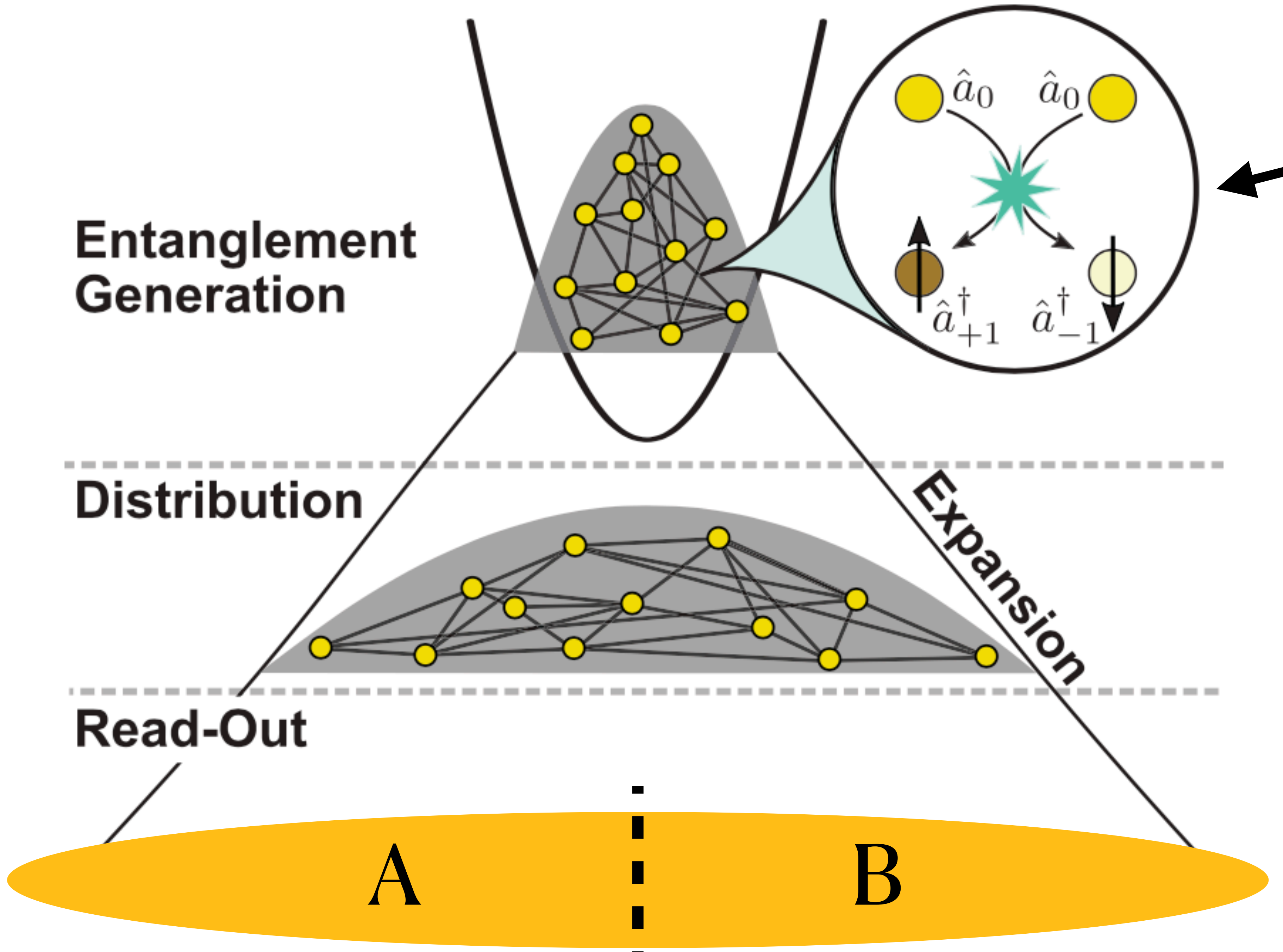

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Spin-changing collision dynamics

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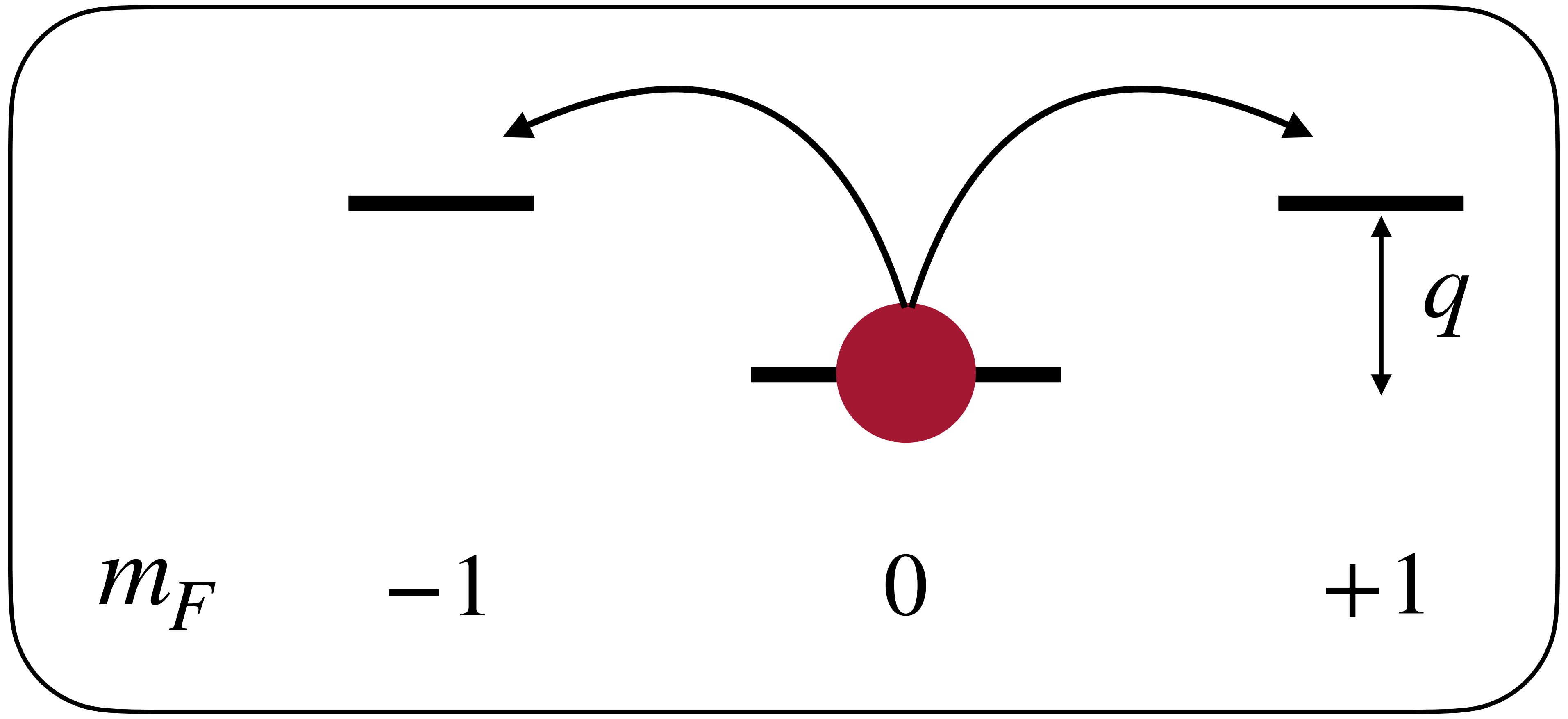
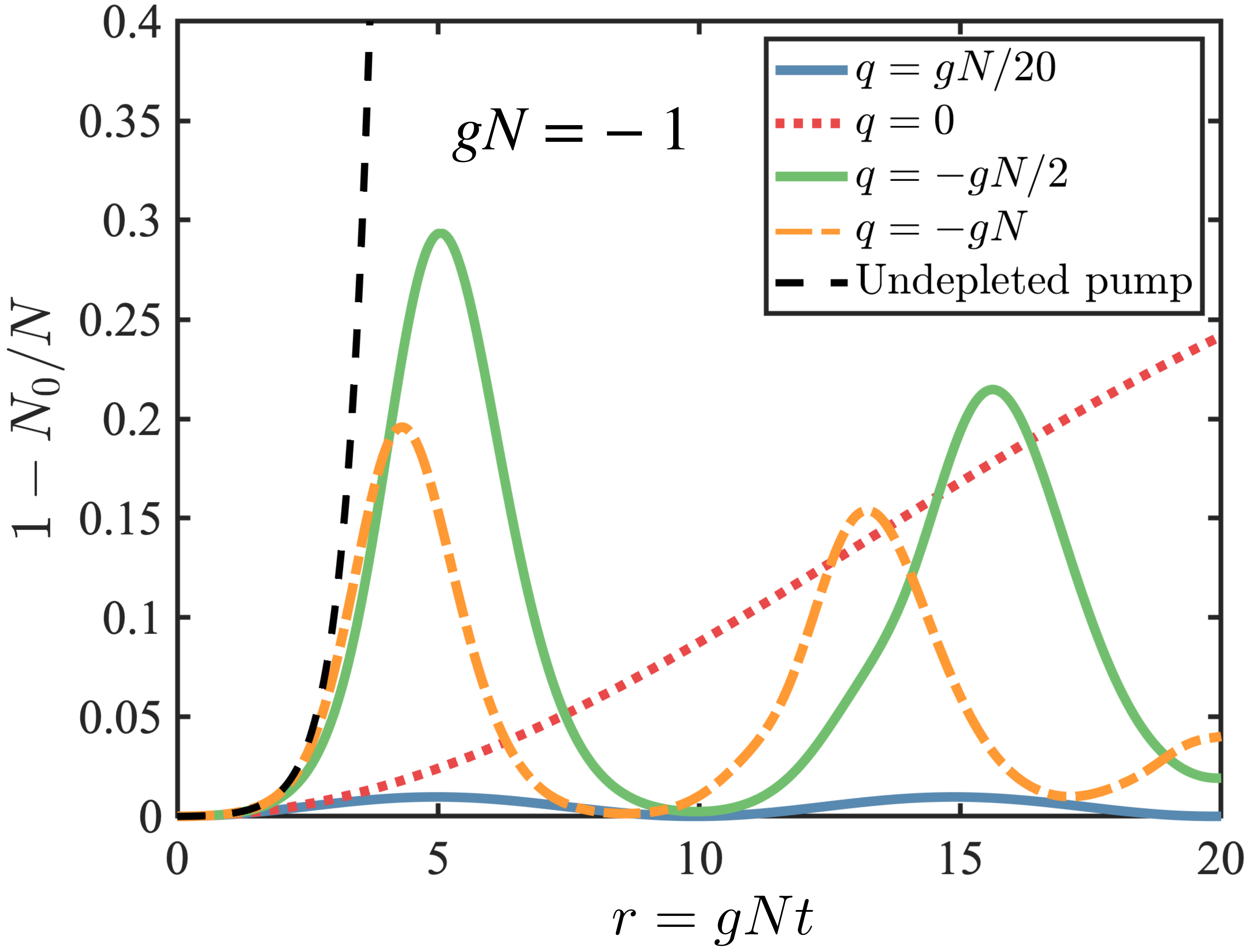


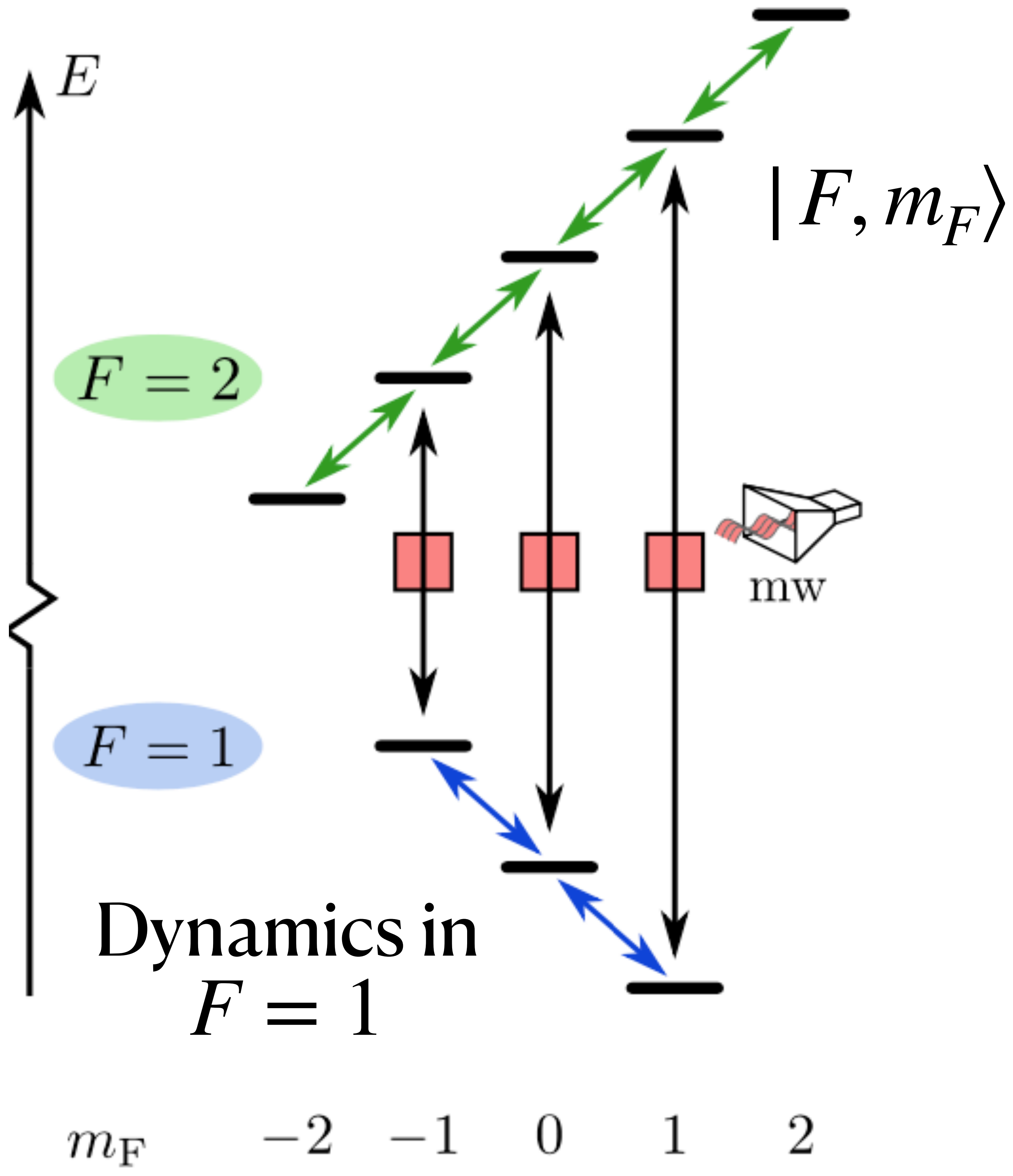
$$|\psi(t=0)\rangle = |0, N, 0\rangle$$

Propagate in time via exact diagonalisation



Measuring entropy: Experimental readout

Rubidium-87 has two hyperfine manifolds

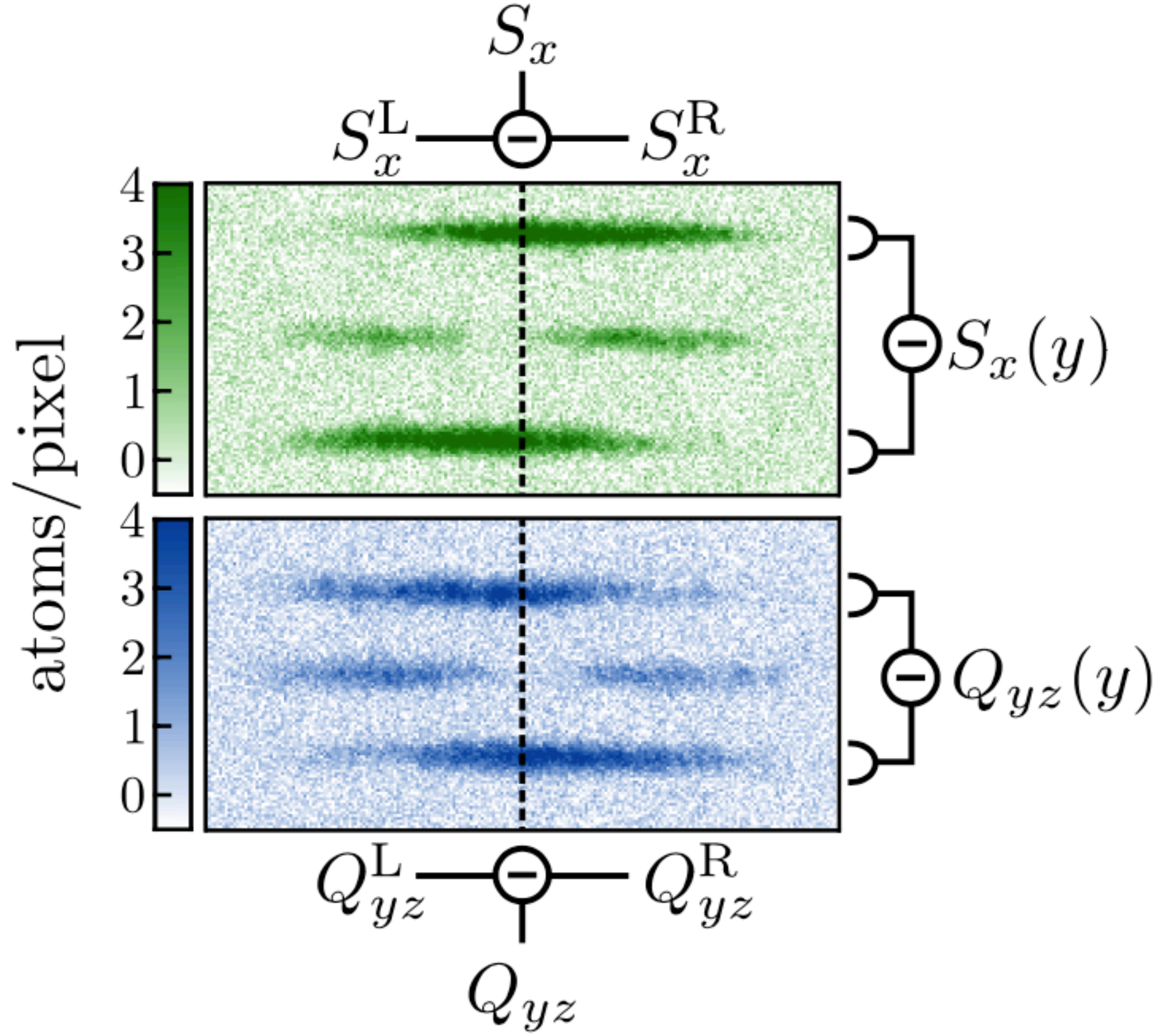


P. Kunkel, *et al.*, PRL **123**, 063603 (2019).

No need for state tomography!

fewer measurements

Transfer ~50% of atoms to $F = 2$,
simultaneously measure spin-1 observables



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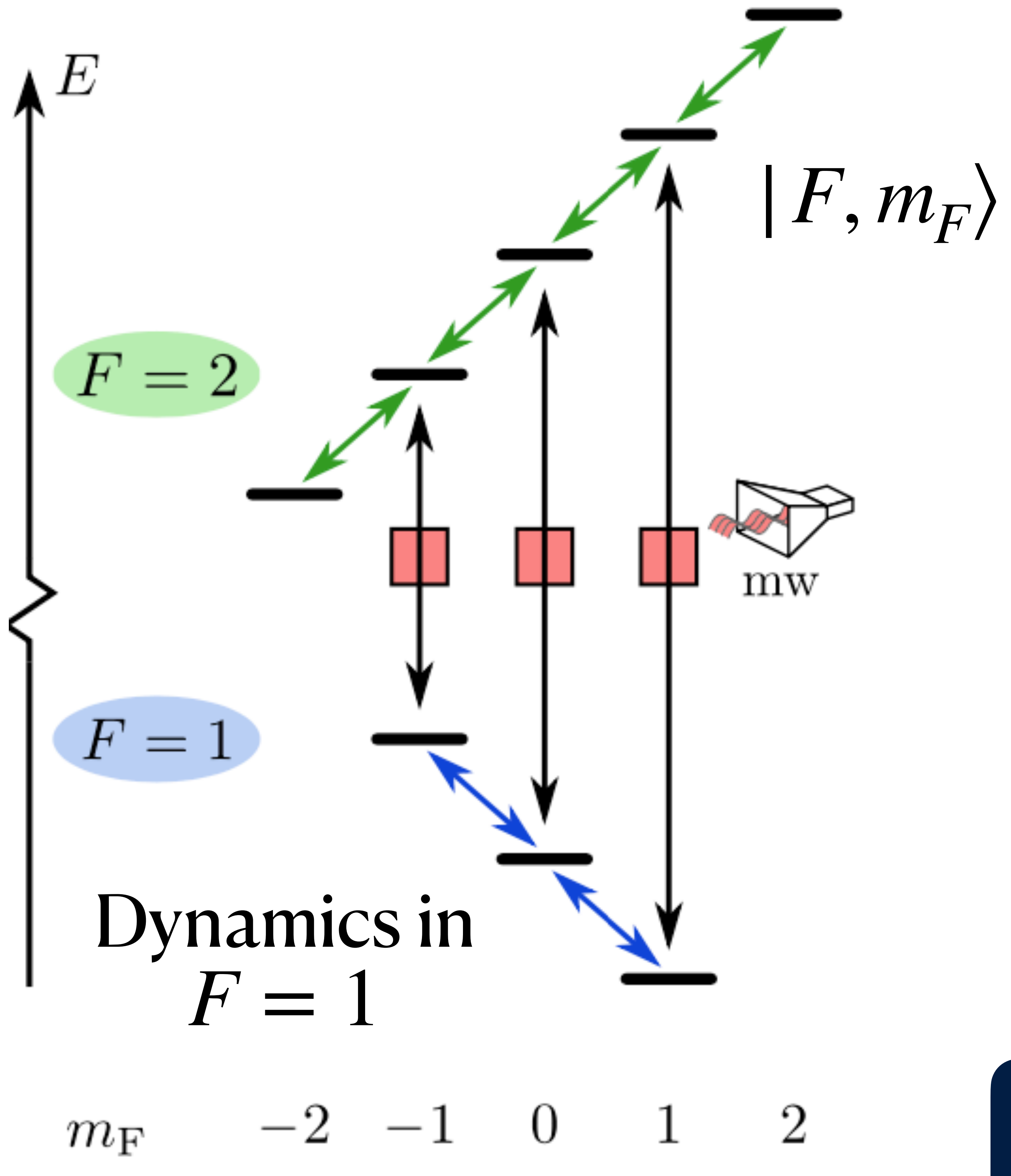

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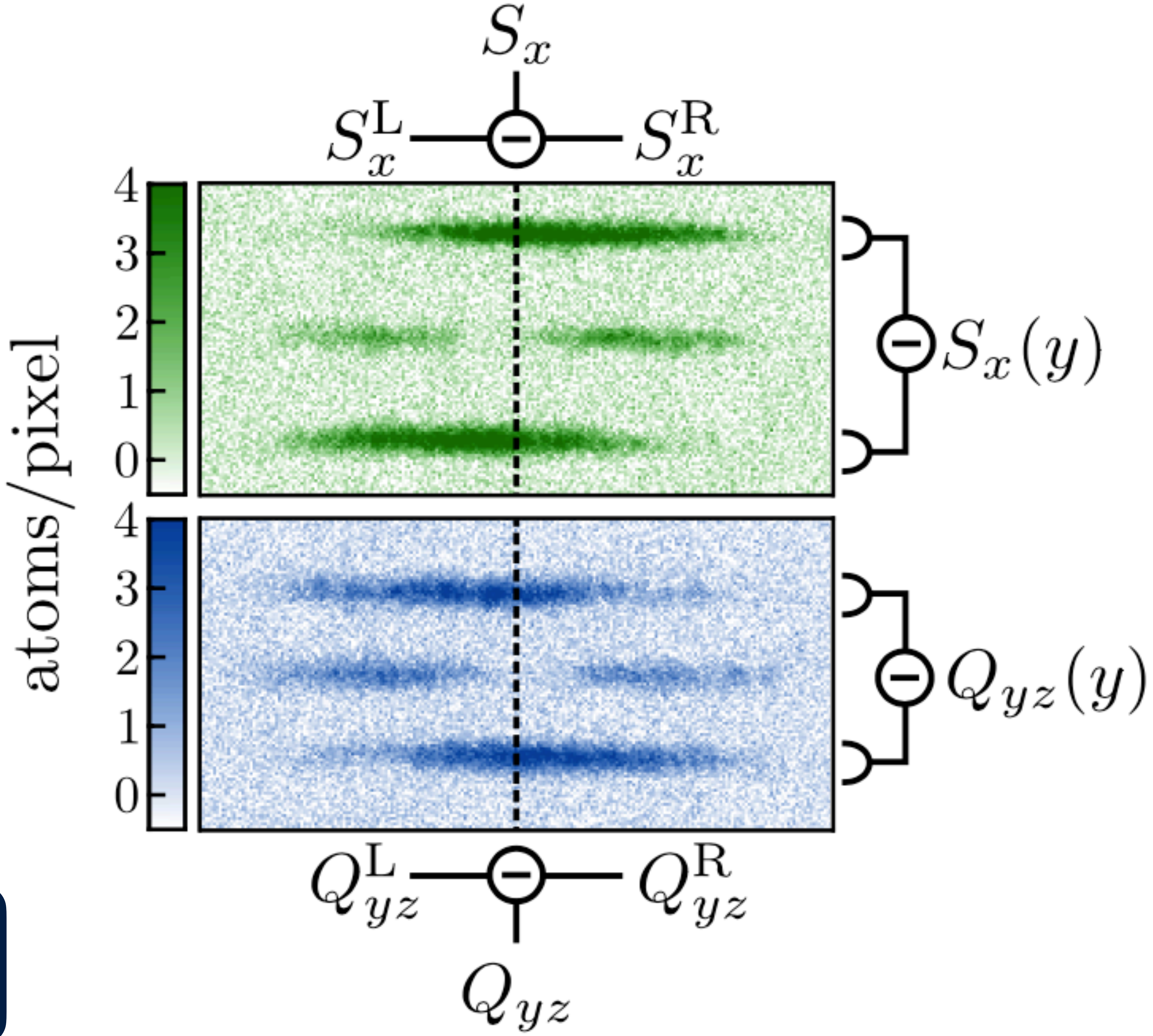

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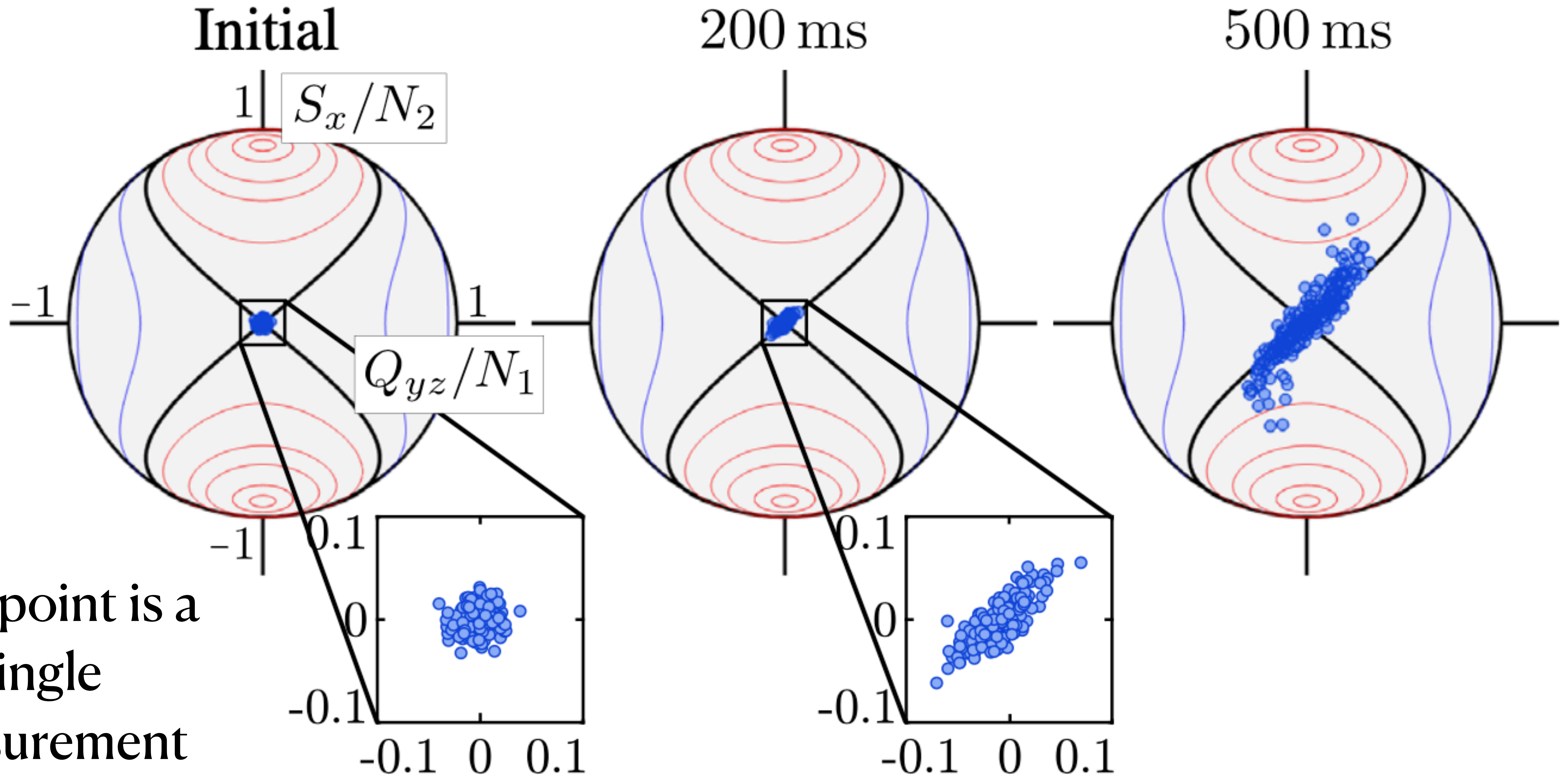
POVM measurement

P. Kunkel, *et al.*, PRL **123**, 063603 (2019).

Quasiprobability distribution

Our spin observables: $\hat{S}_x = \hat{a}_0^\dagger(\hat{a}_{+1} + \hat{a}_{-1})/\sqrt{2} + \text{h.c.}$ $\hat{Q}_{yz} = i\hat{a}_0^\dagger(\hat{a}_{+1} + \hat{a}_{-1})/\sqrt{2} + \text{h.c.}$



Each point is a single measurement

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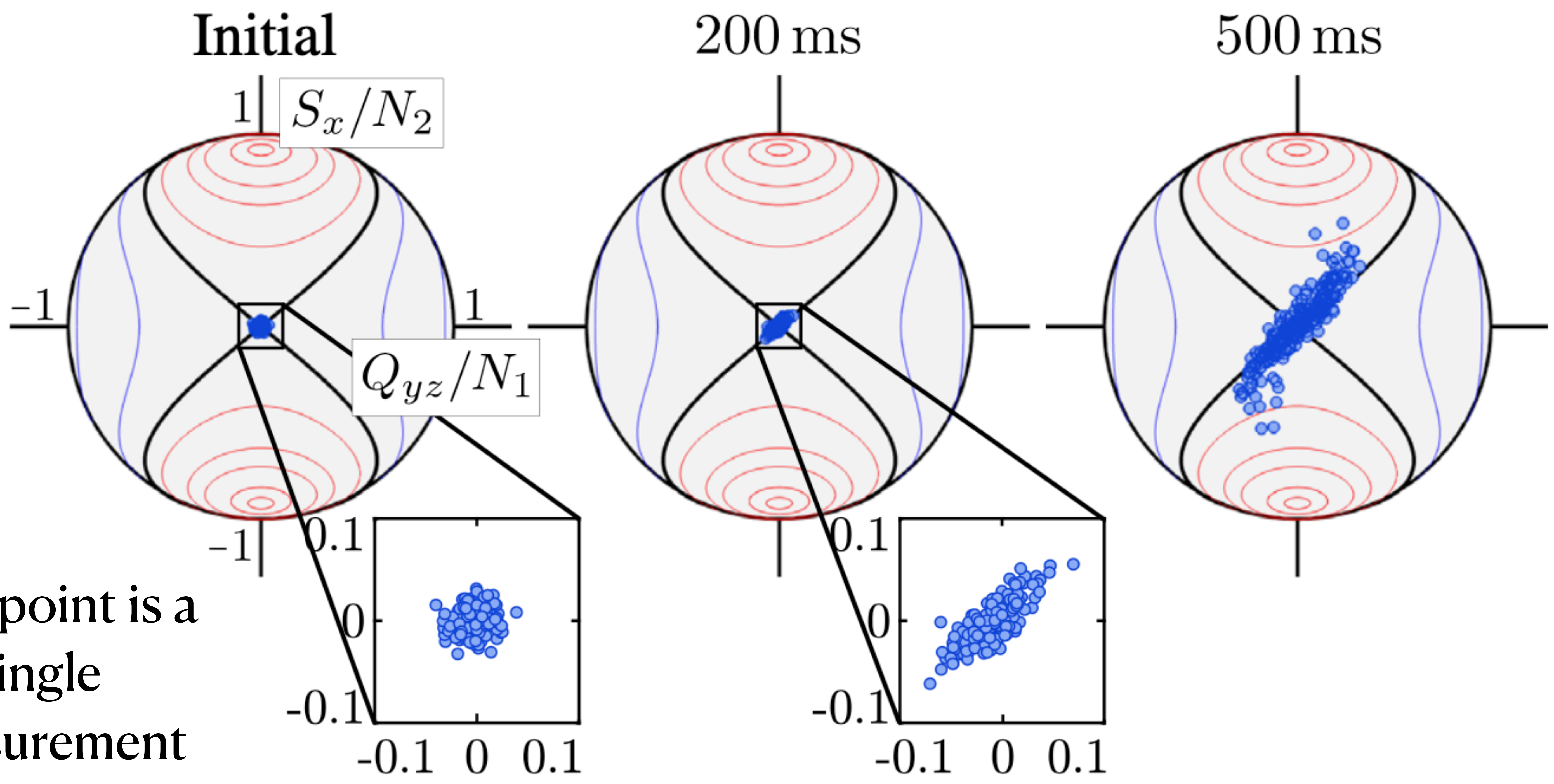

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What **information** can we **extract** from this distribution?

Distribution is:
- Non-negative
- Normalisable

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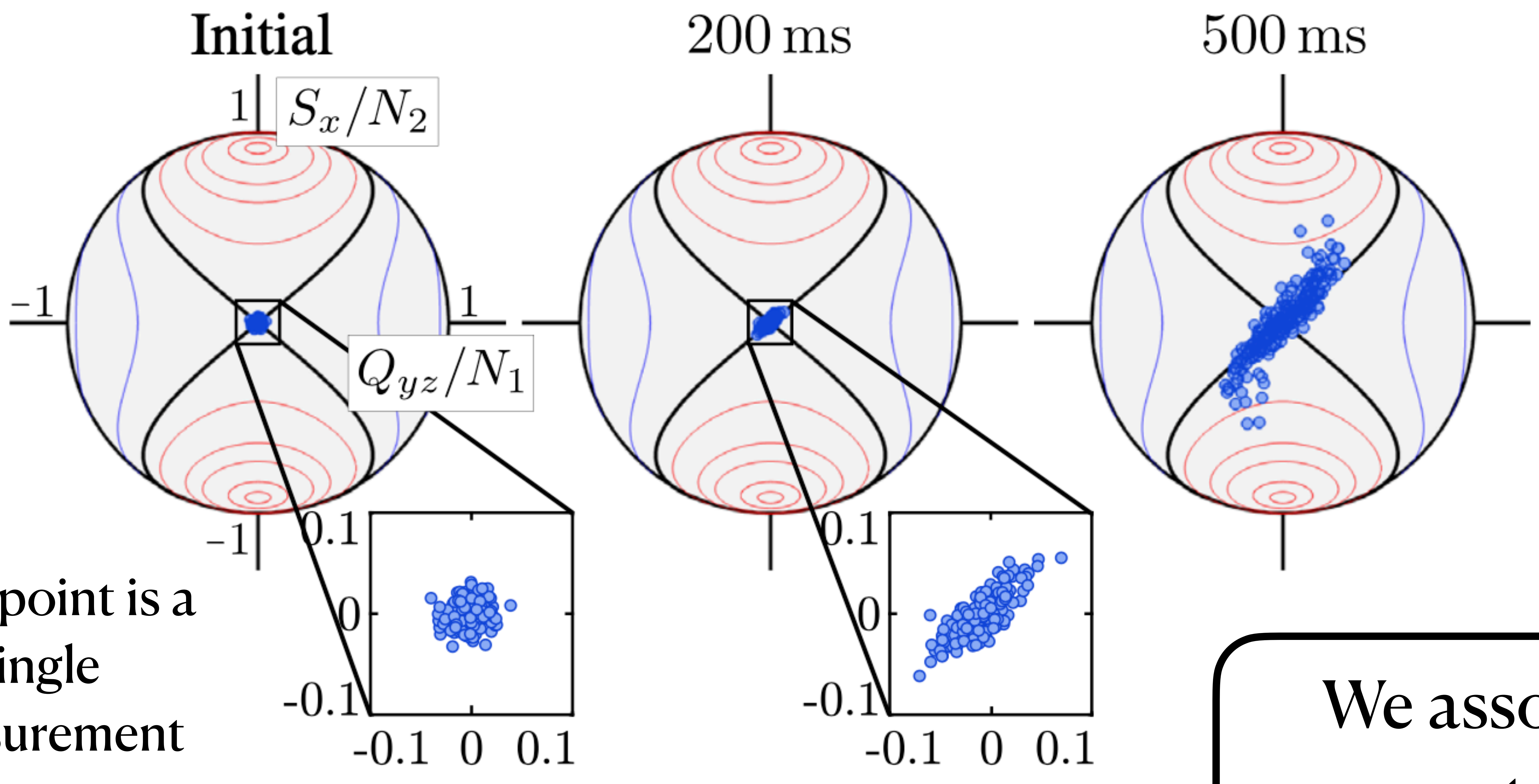

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$$S_W(\rho) = -\int \frac{\mathrm{d}S_x \mathrm{d}Q_{yz}}{2\pi} \; Q_\rho(S_x, Q_{yz}) \ln Q_\rho(S_x, Q_{yz})$$

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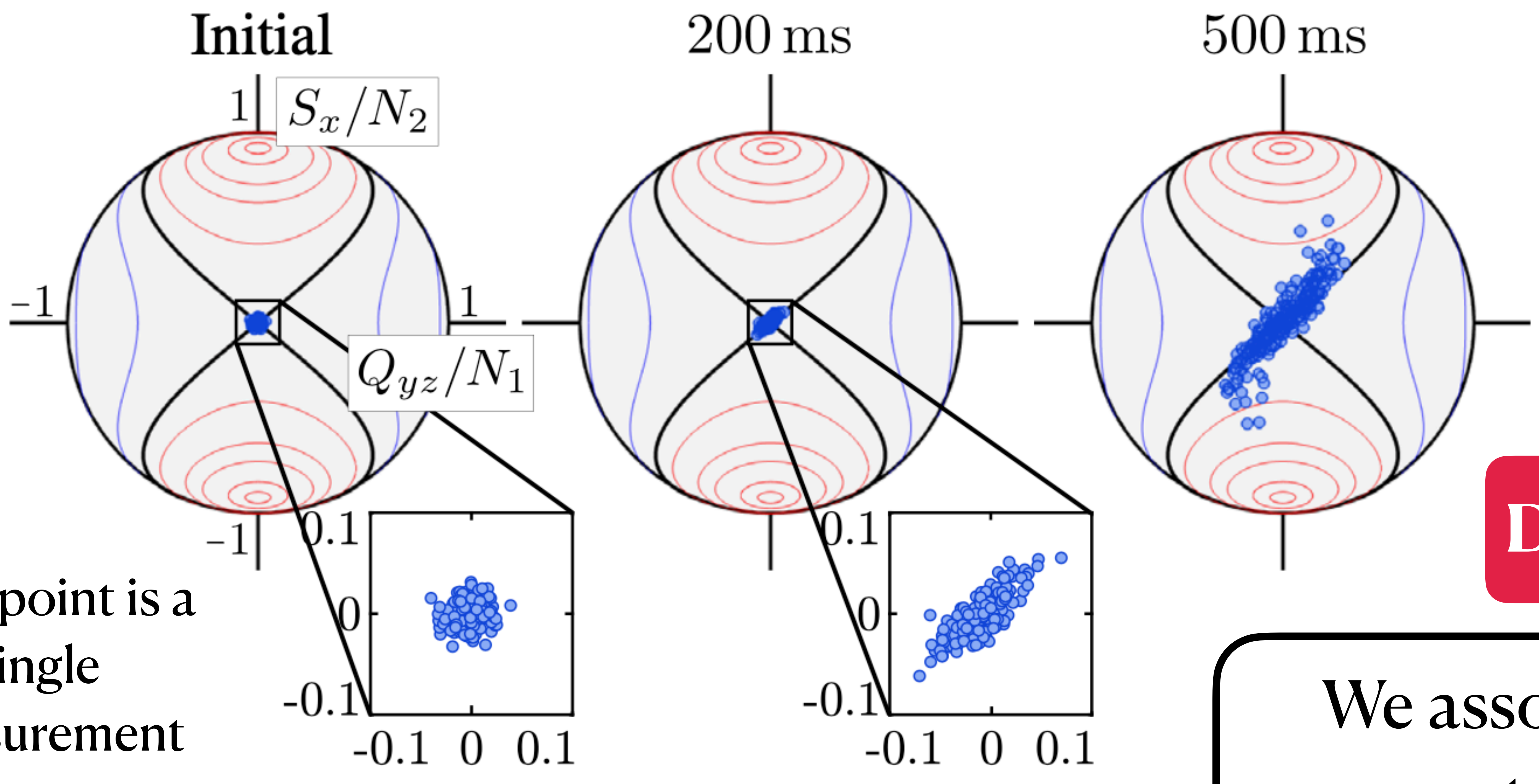

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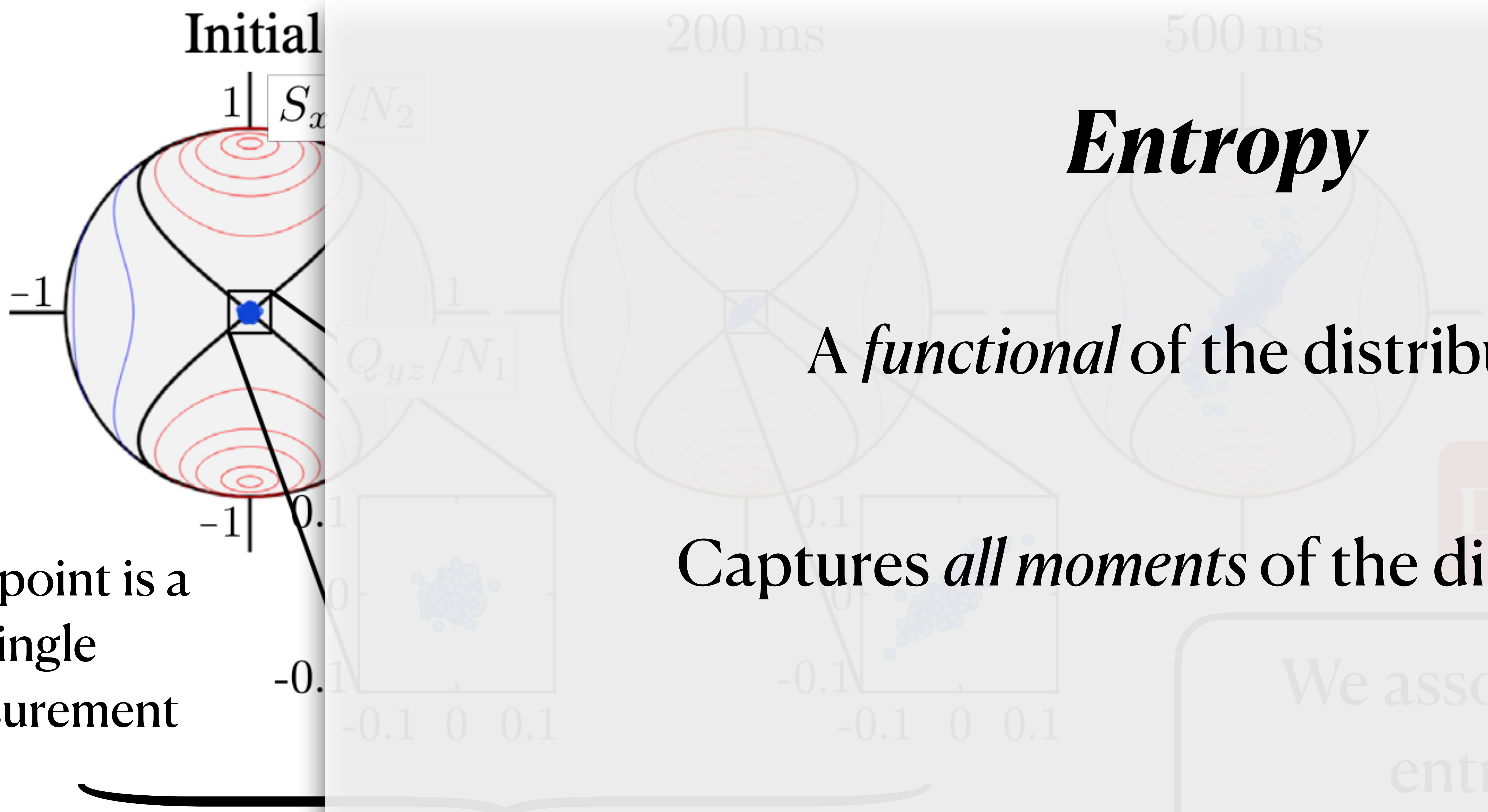

Each point is a single measurement

Entropy

A *functional* of the distribution

Captures *all moments* of the distribution

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Witnessing correlations

$$S_W = -\int \frac{dS_x\, dQ_{yz}}{2\pi N}\; Q_\rho(S_x, Q_{yz}) \log Q_\rho(S_x, Q_{yz})$$

Introduce **Wehrl mutual information**

$$I_W(\rho_A : \rho_B) = S_W(\rho_A) - S_W(\rho_A \,|\, \rho_B)$$

Measures *total* correlations, inc. classical!

Wehrl mutual information is a *perfect* witness for pure states

S. Floerchinger, *et. al*, PRA **103**, 062222 (2021).

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Lower bound on quantum MI

$$I_W(\rho_A : \rho_B) \le I(\rho_A : \rho_B)$$

Regardless of squeezing parameter, POVM measurement **bounds quantum MI**

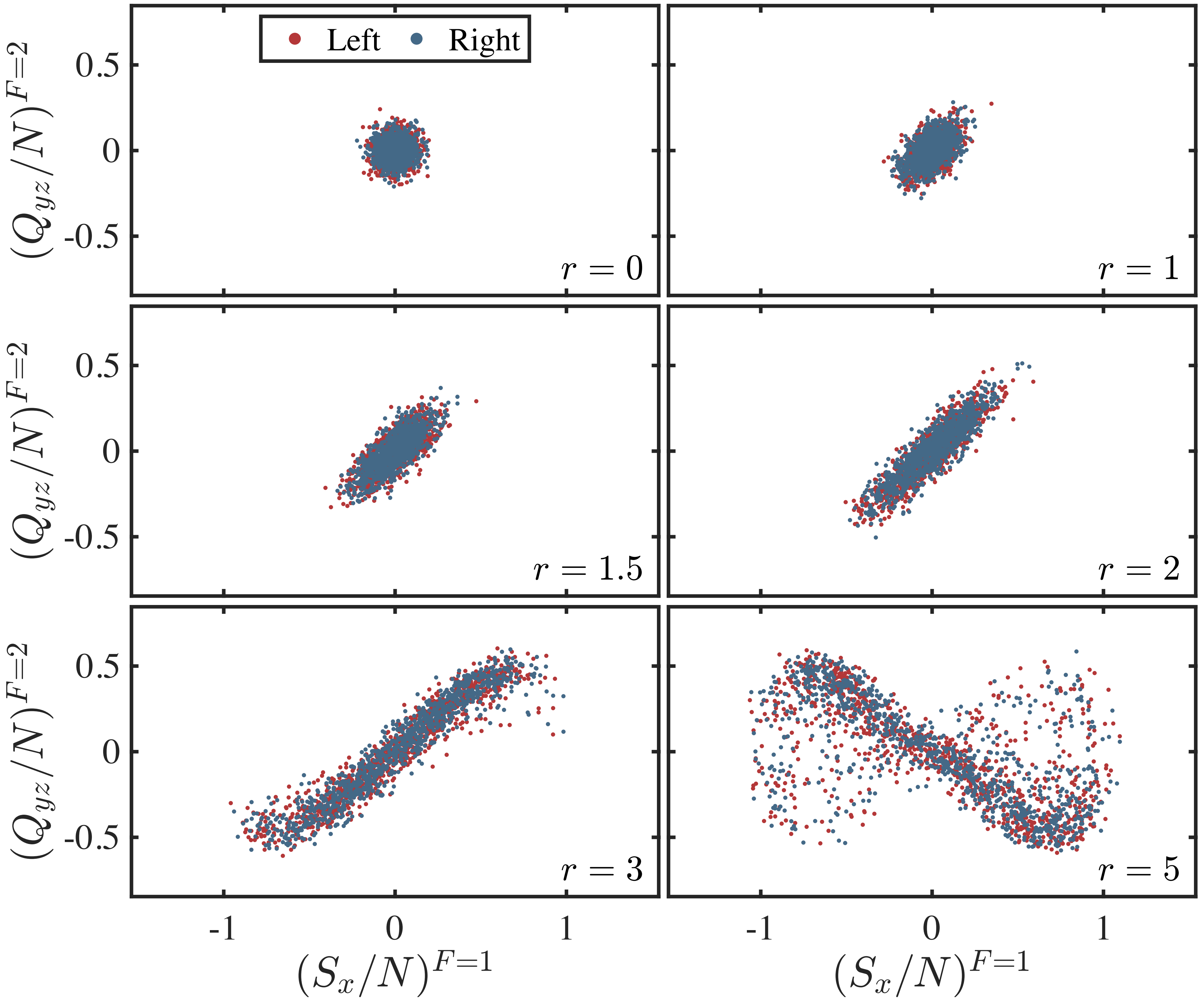
$$I_{\mathrm{POVM}}(\rho_A : \rho_B) \le I_Q(\rho_A : \rho_B)$$

E. H. Lieb and R. Seiringer, PRA **71**, 062329 (2005).

Theoretical modelling

Time dynamics of correlations

Truncated Wigner result



Numerical simulation of experimental readout

Time dynamics of correlations

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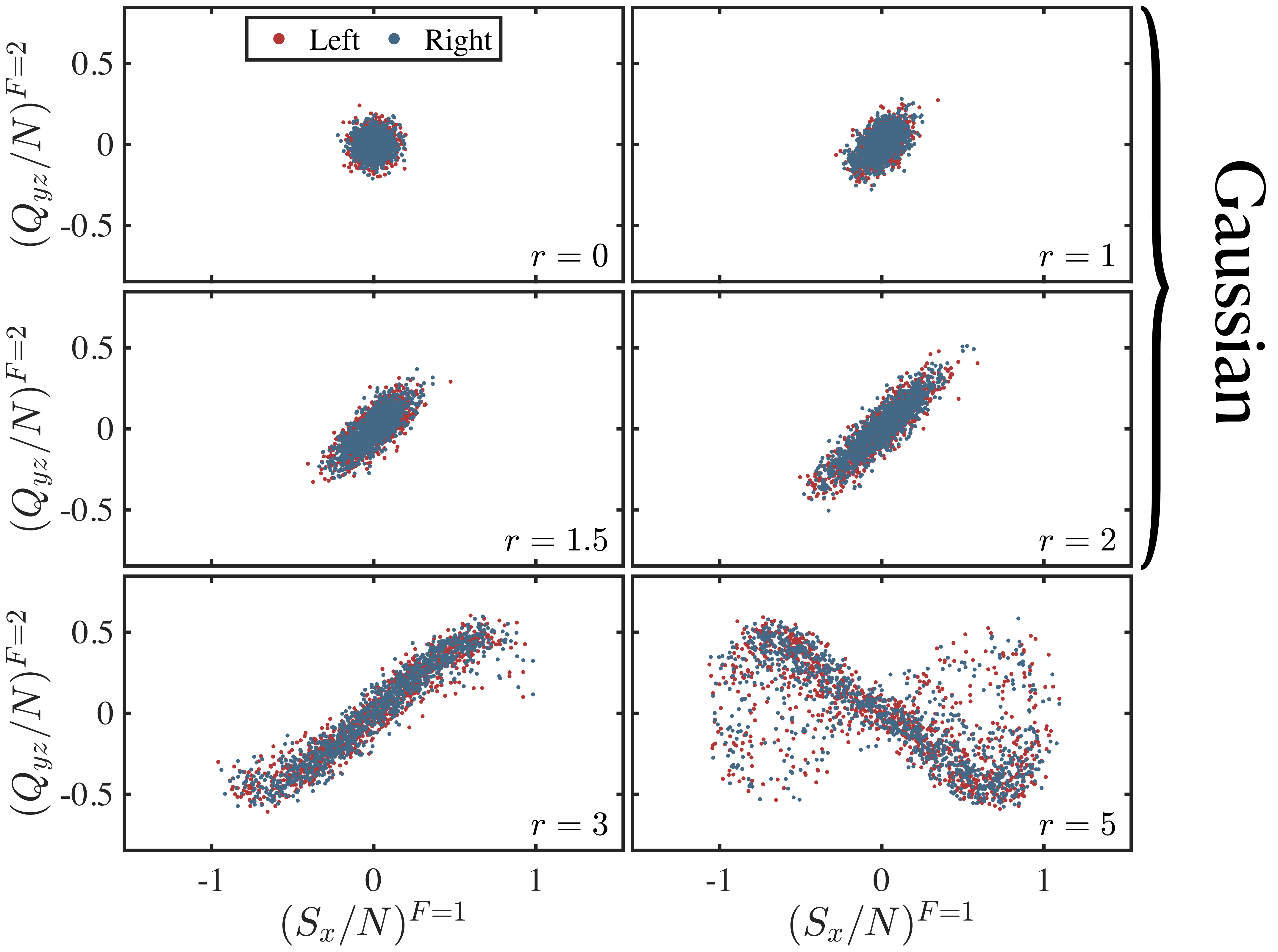

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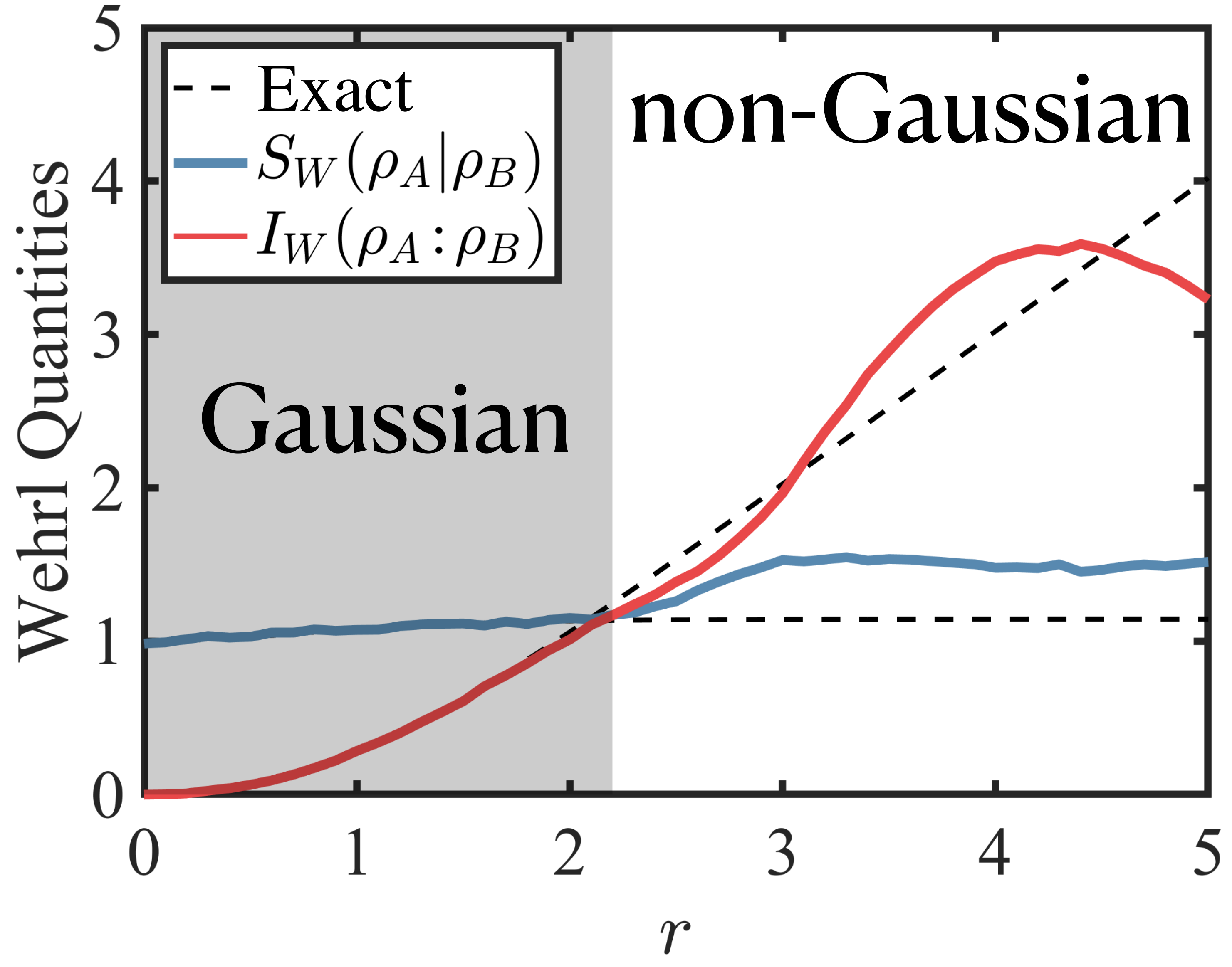
Measuring variances



$$I_W(\rho_A : \rho_B) = \frac{1}{2} \frac{\det C_A \det C_B}{\det C}$$

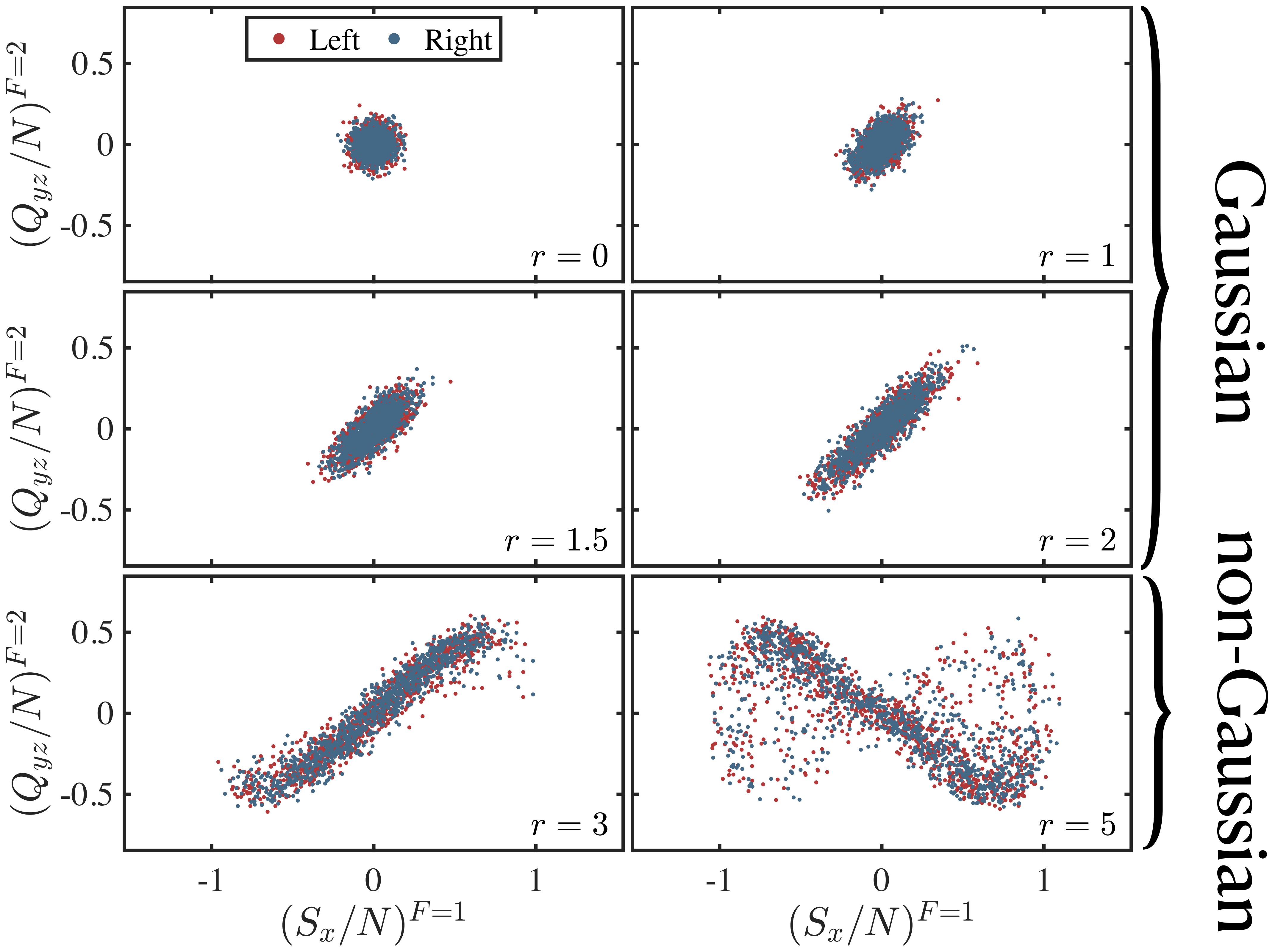
C = inverse cov. matrix

Recall, entanglement witnessed if

$$I_W > 0$$

Time dynamics of correlations

Truncated Wigner result



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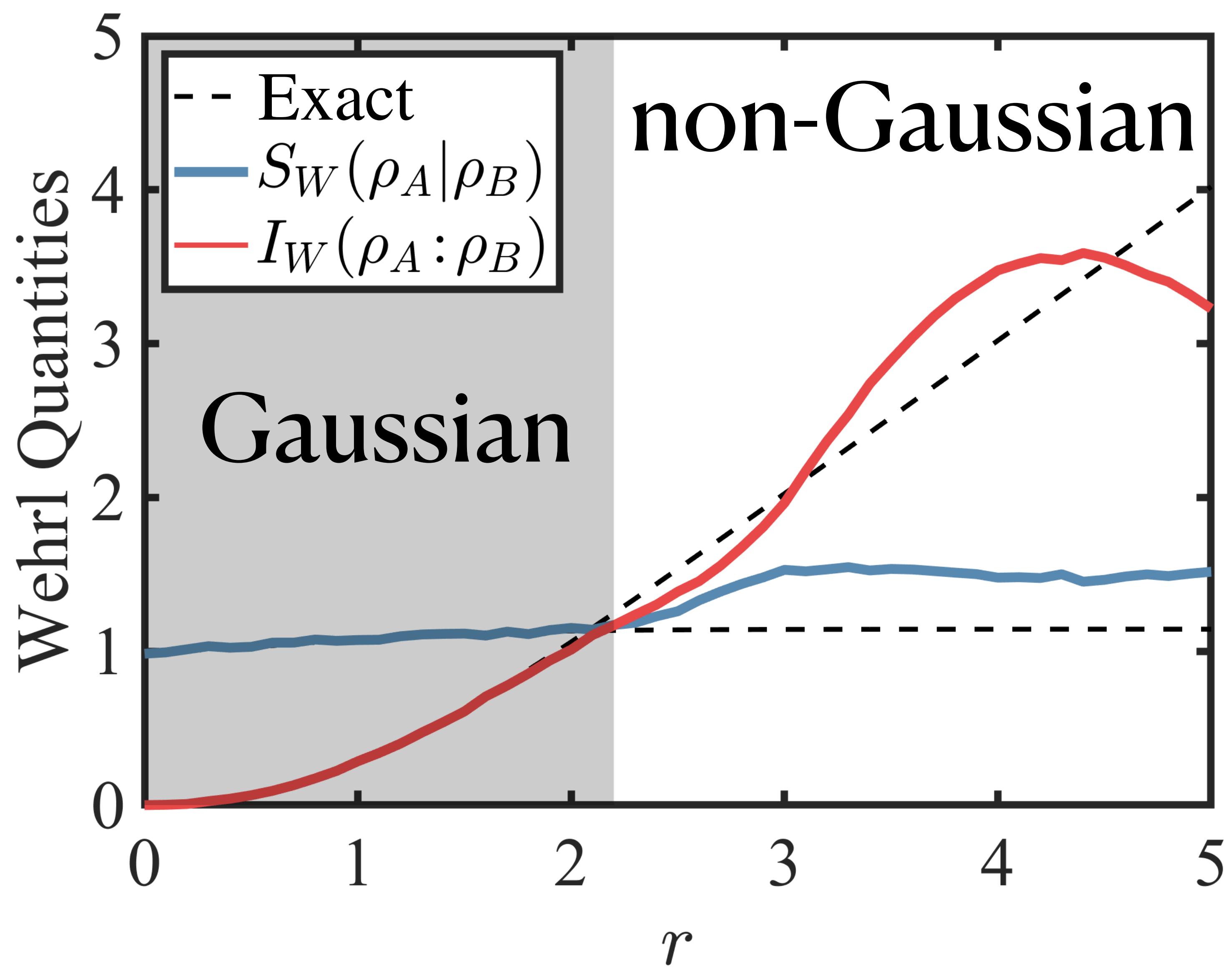

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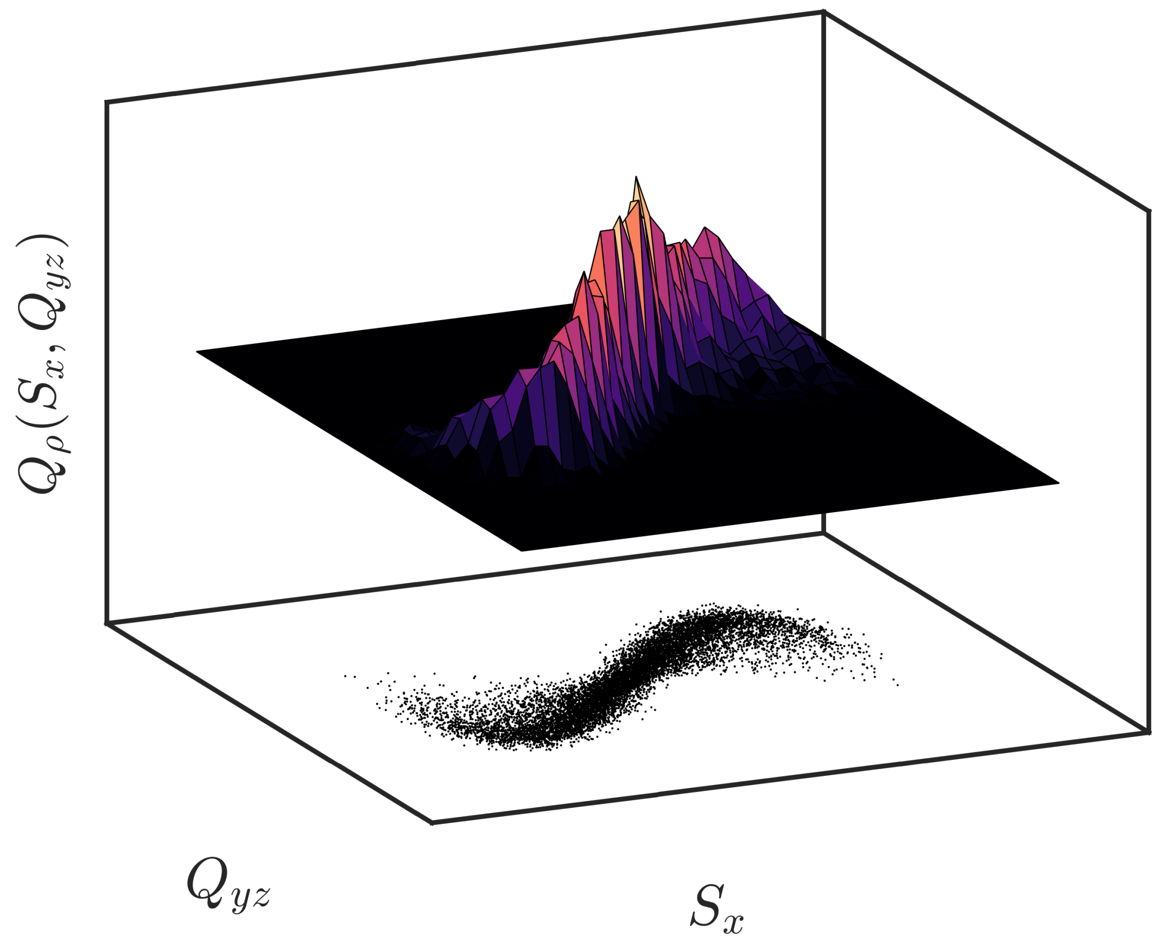
Density estimation

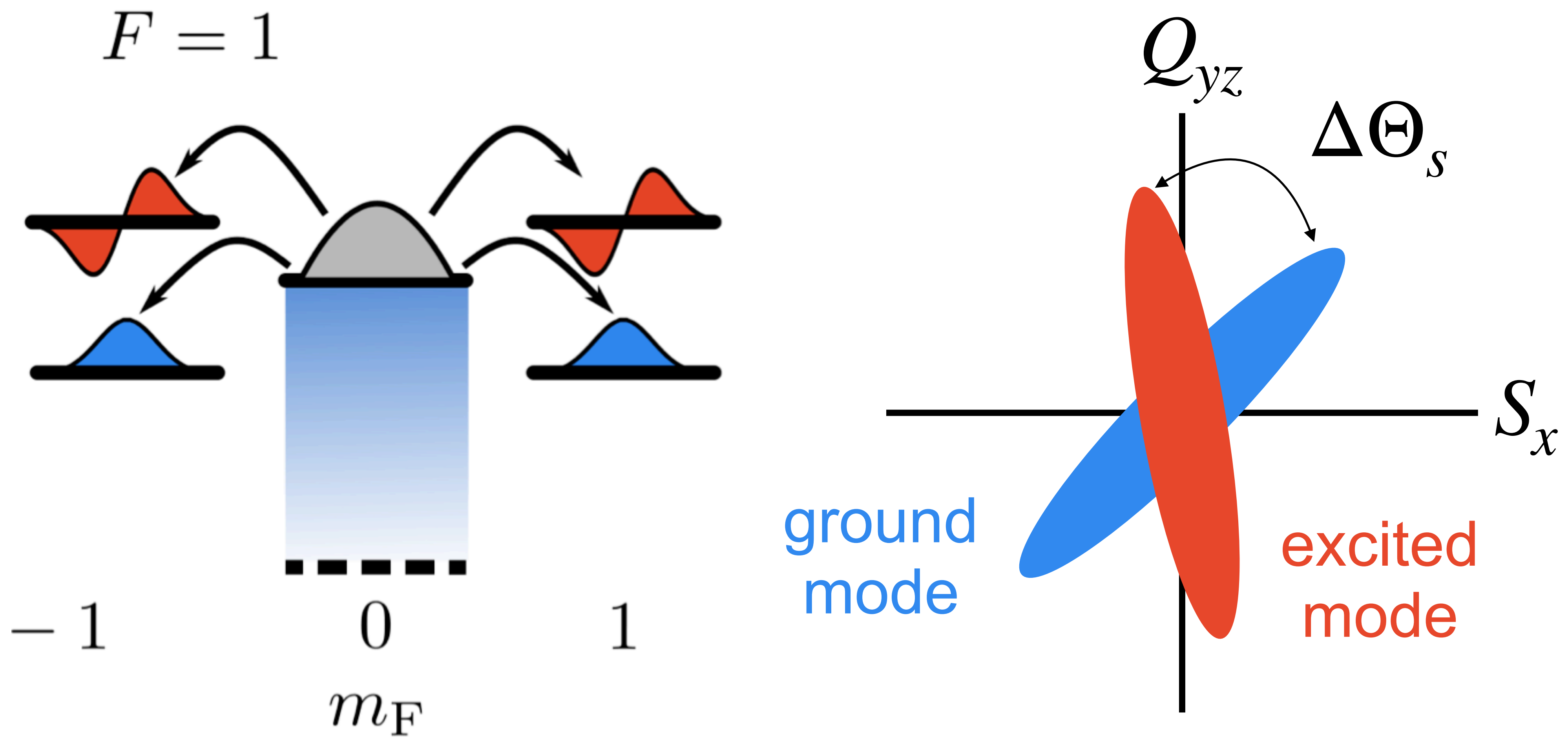
How to accurately measure probability density function?



Experimental analysis

Experimental results

Two spatial modes are excited



Different behaviour with spatially excited modes?

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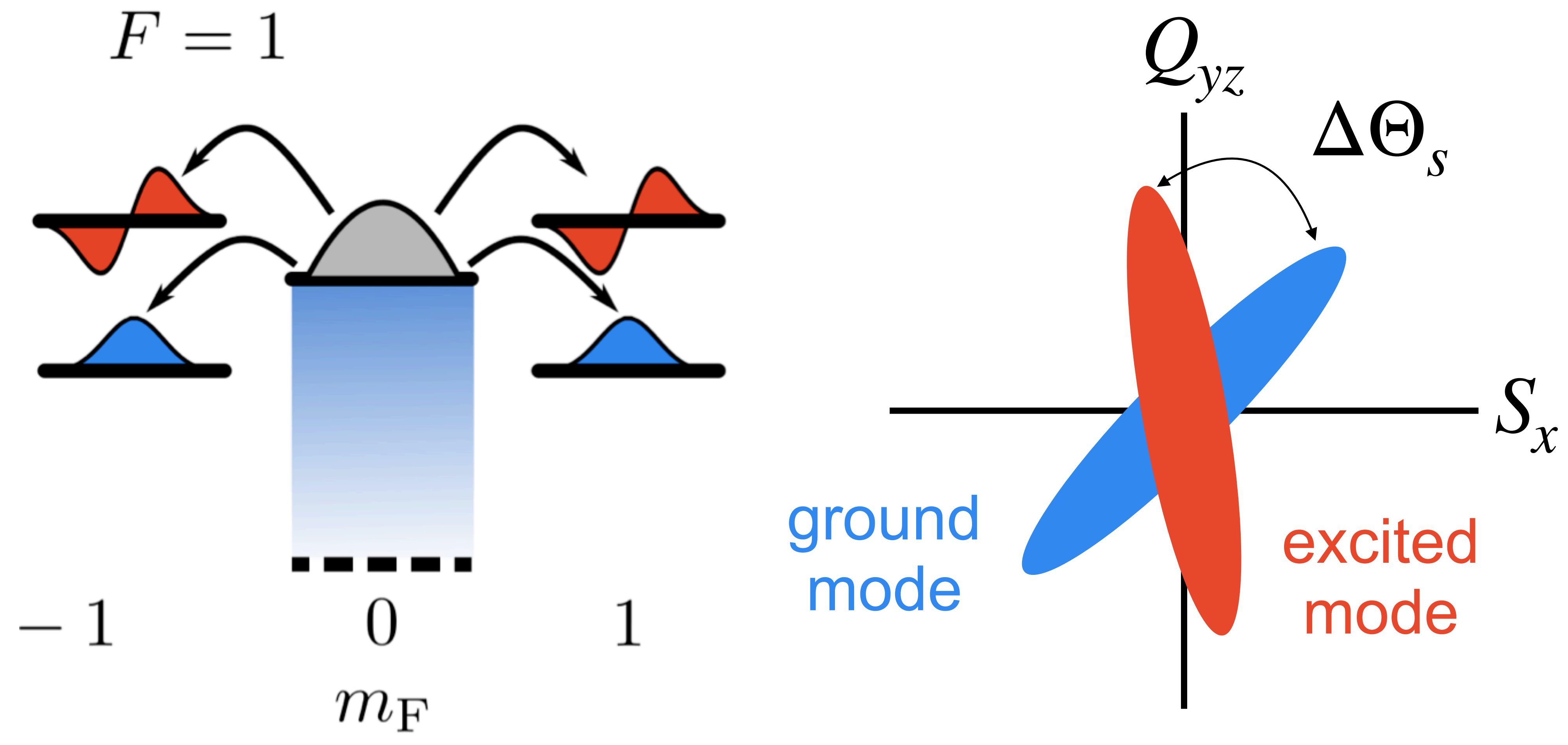
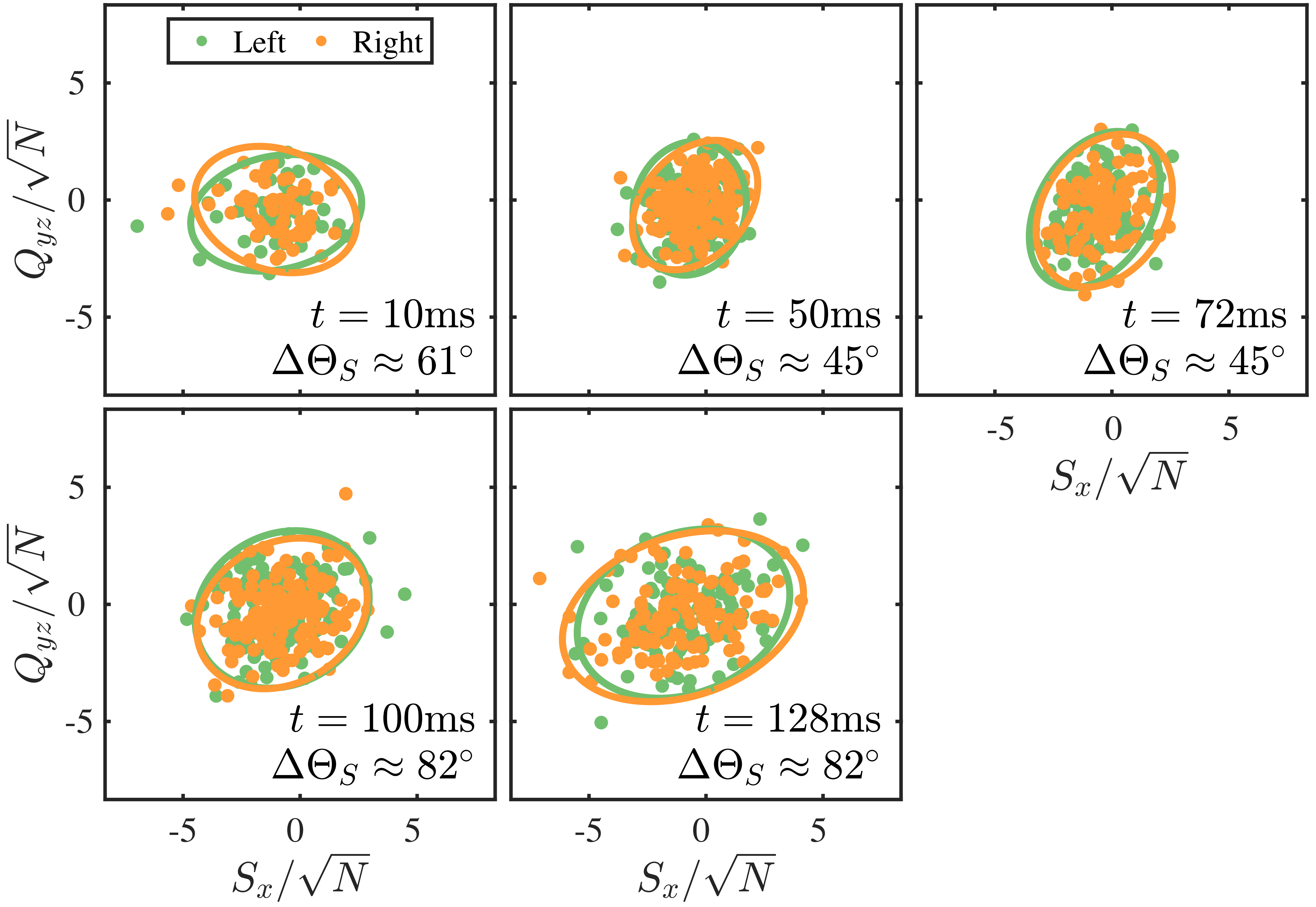


Different behaviour with spatially excited modes?

Total signal, i.e., both modes together



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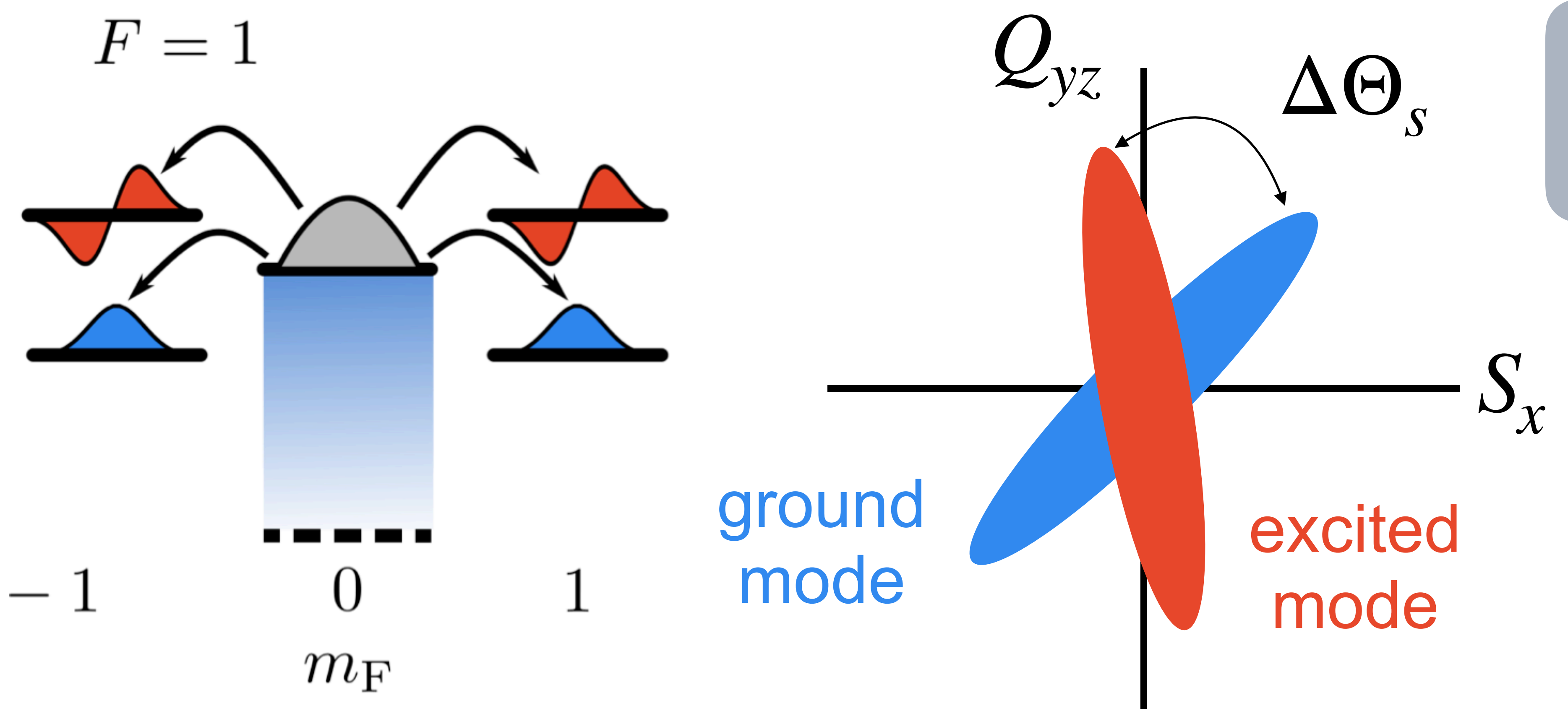
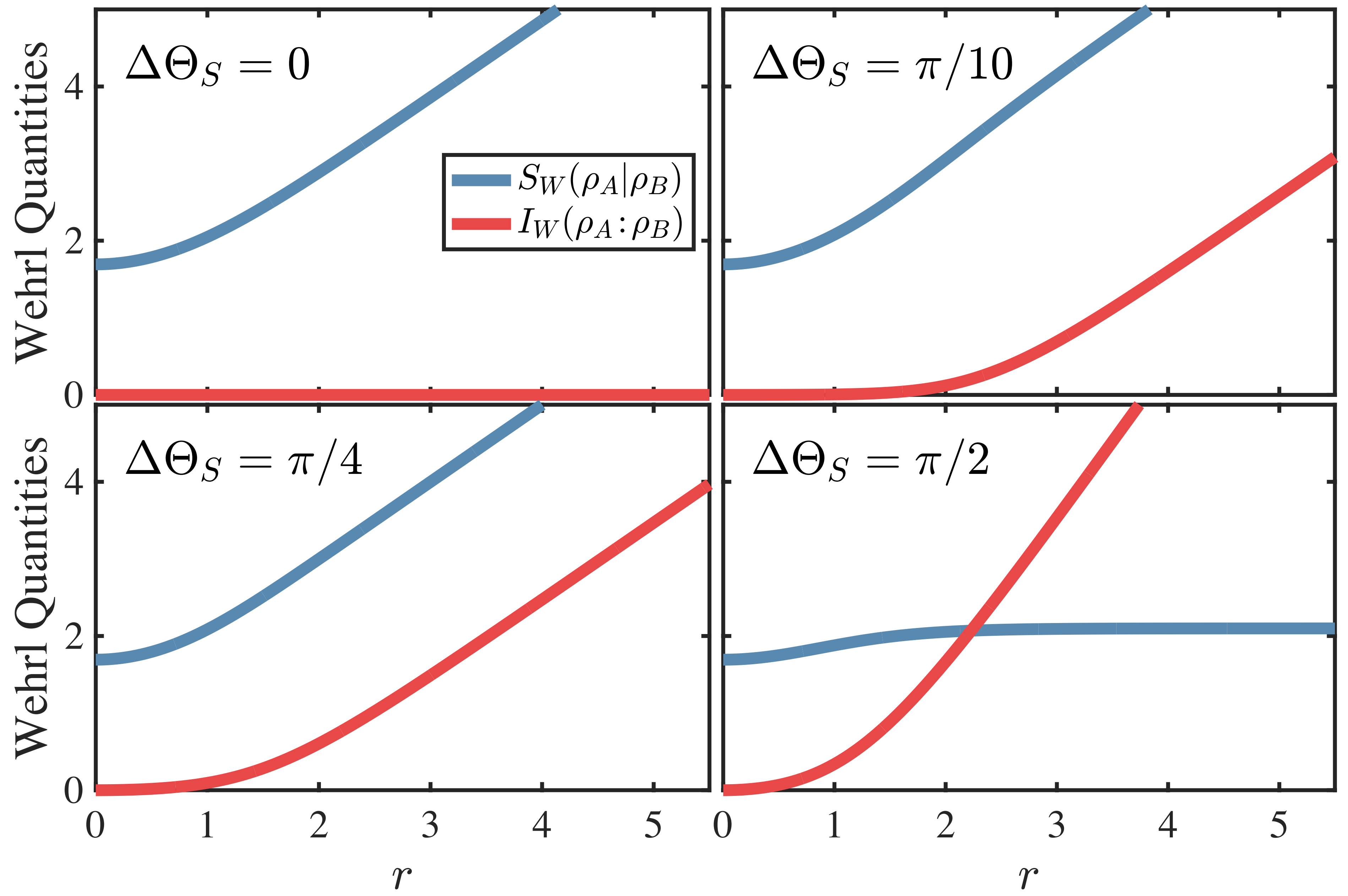


$I_W > 0$

Exact calculation of mutual information



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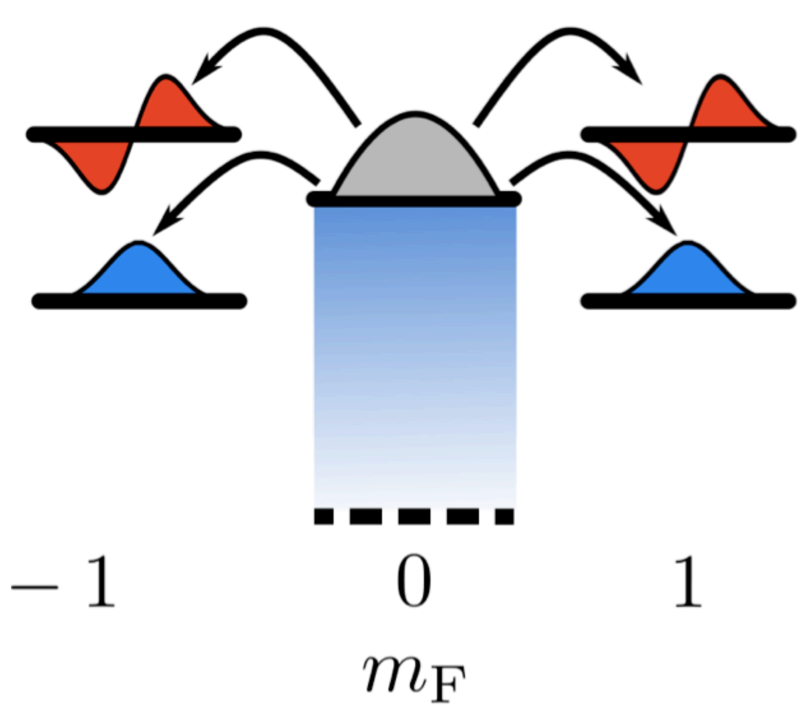

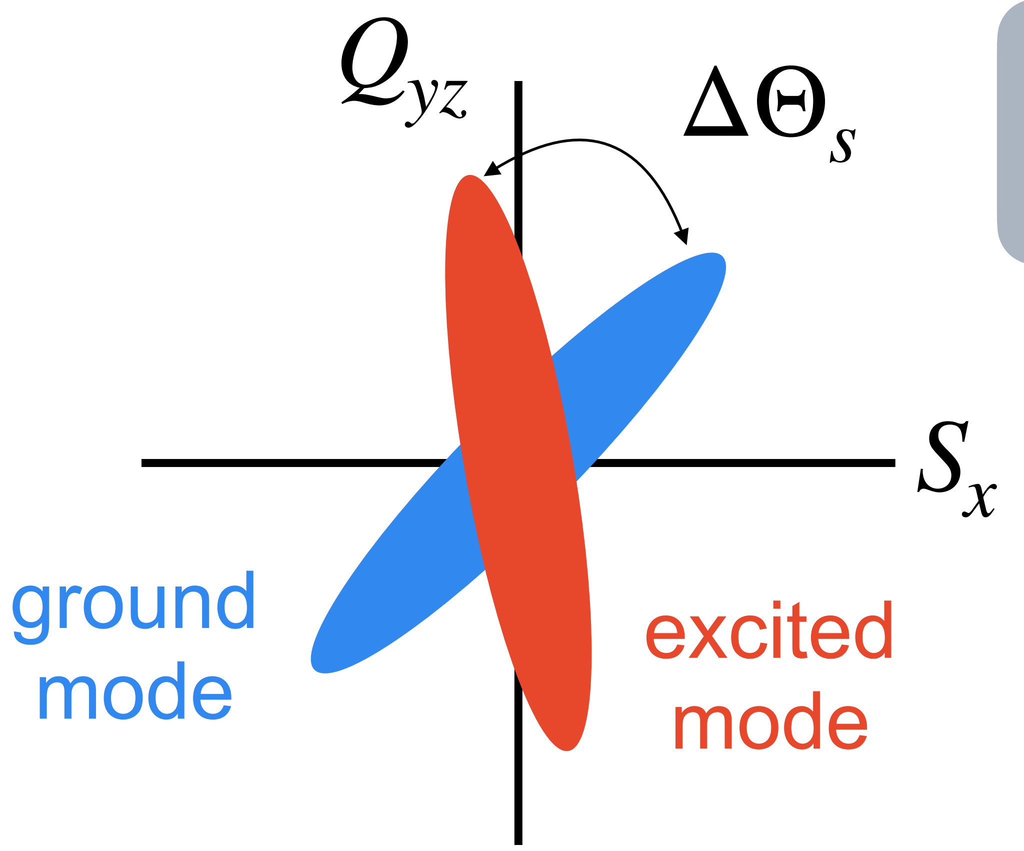

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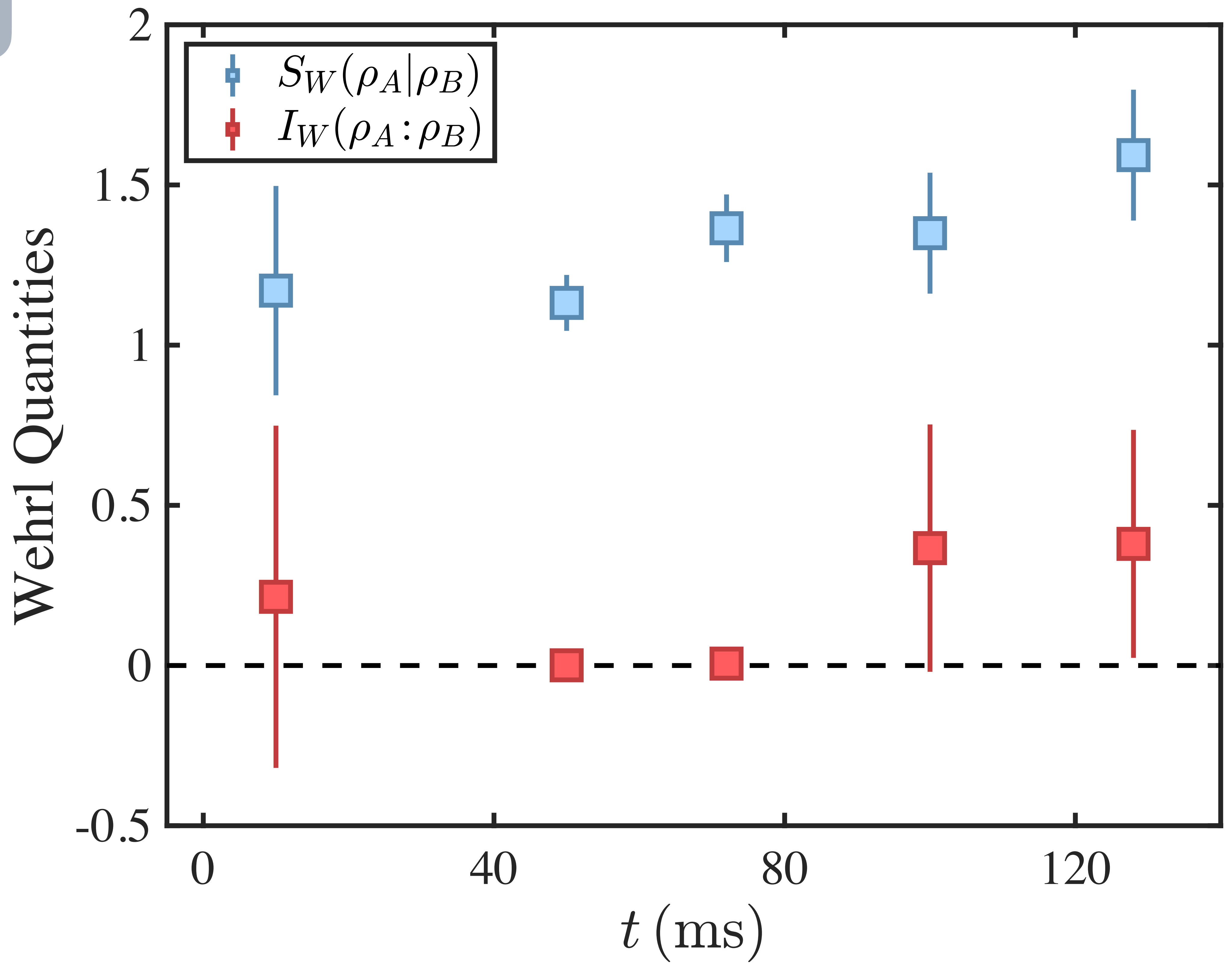

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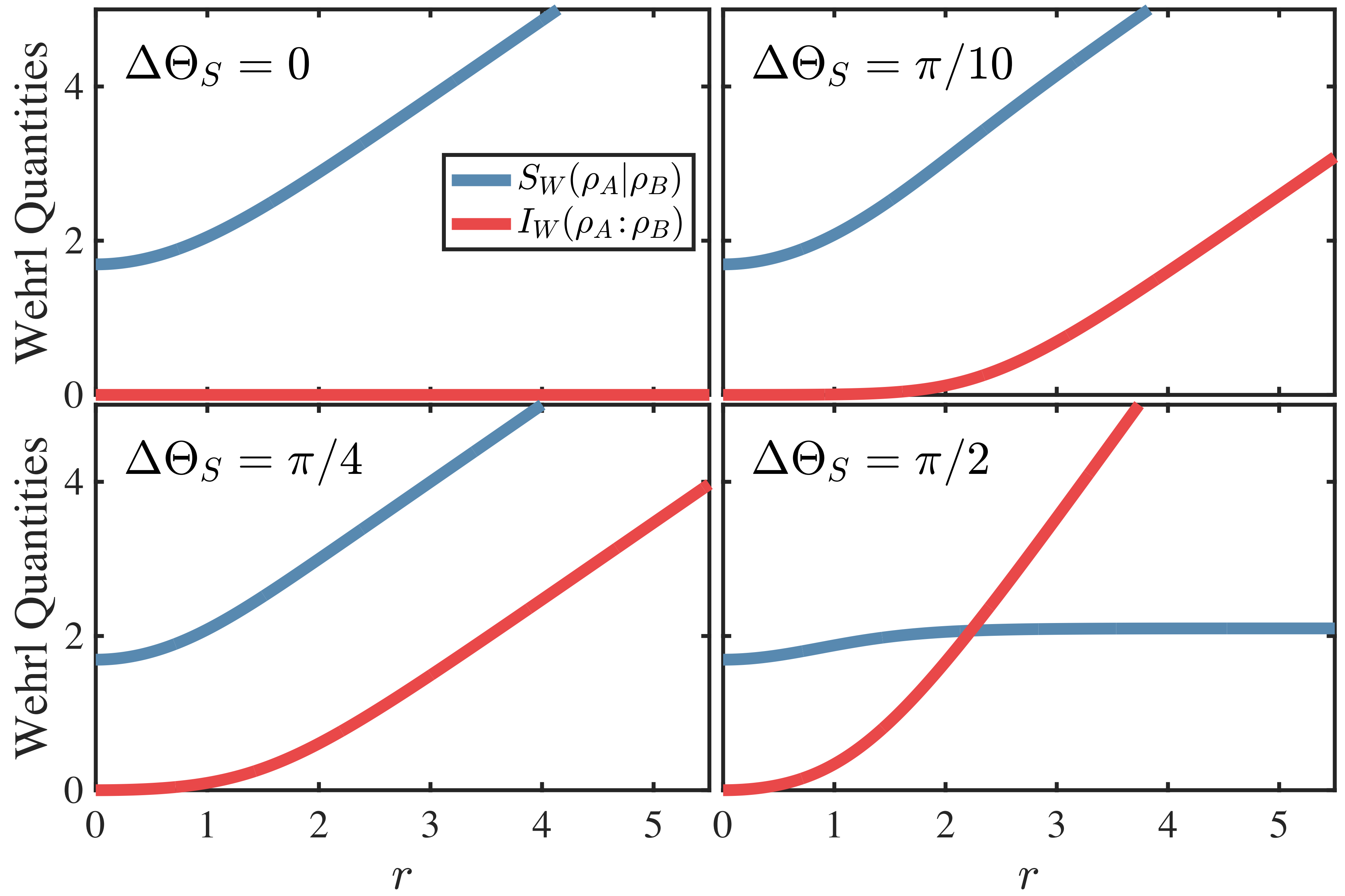
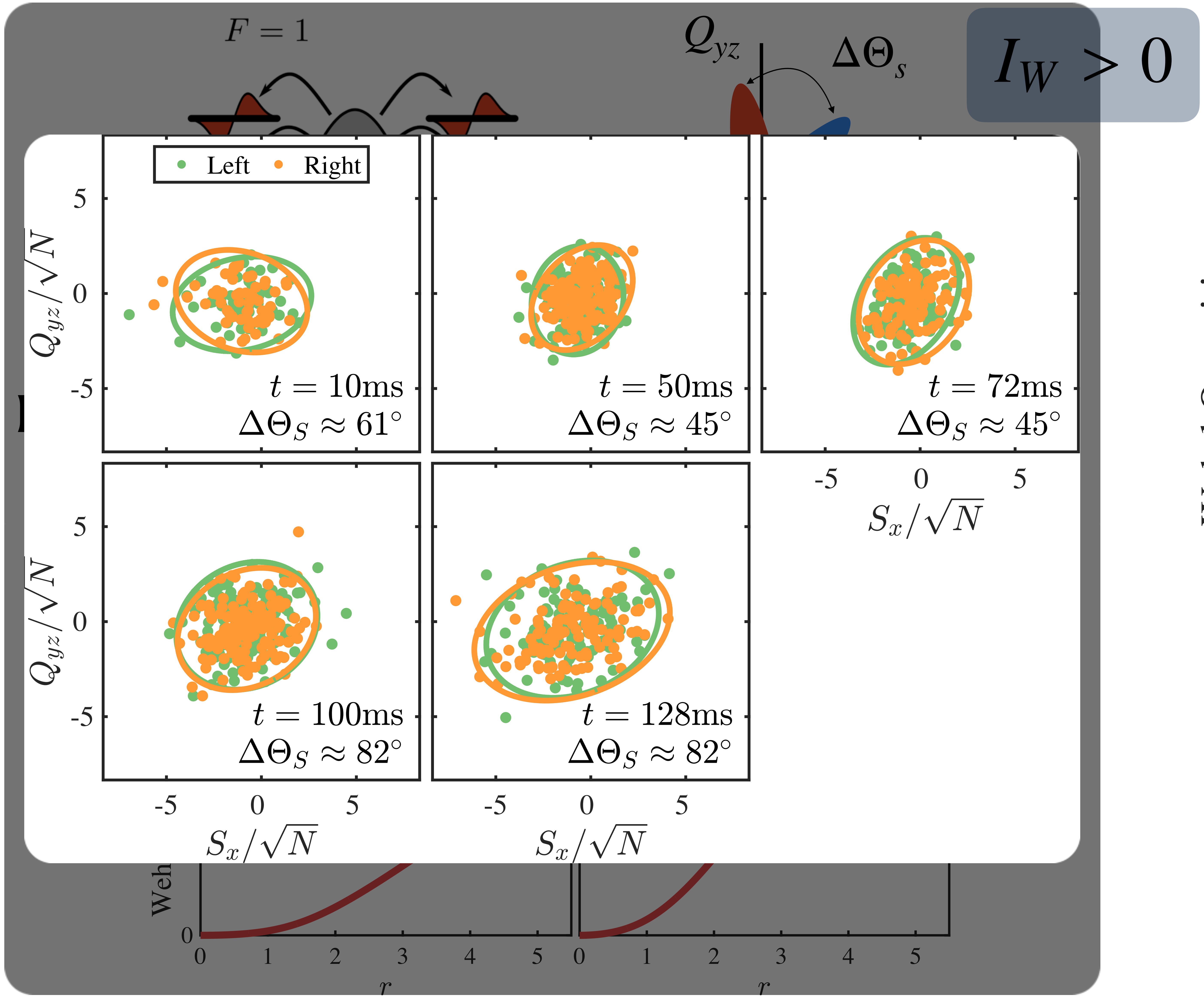
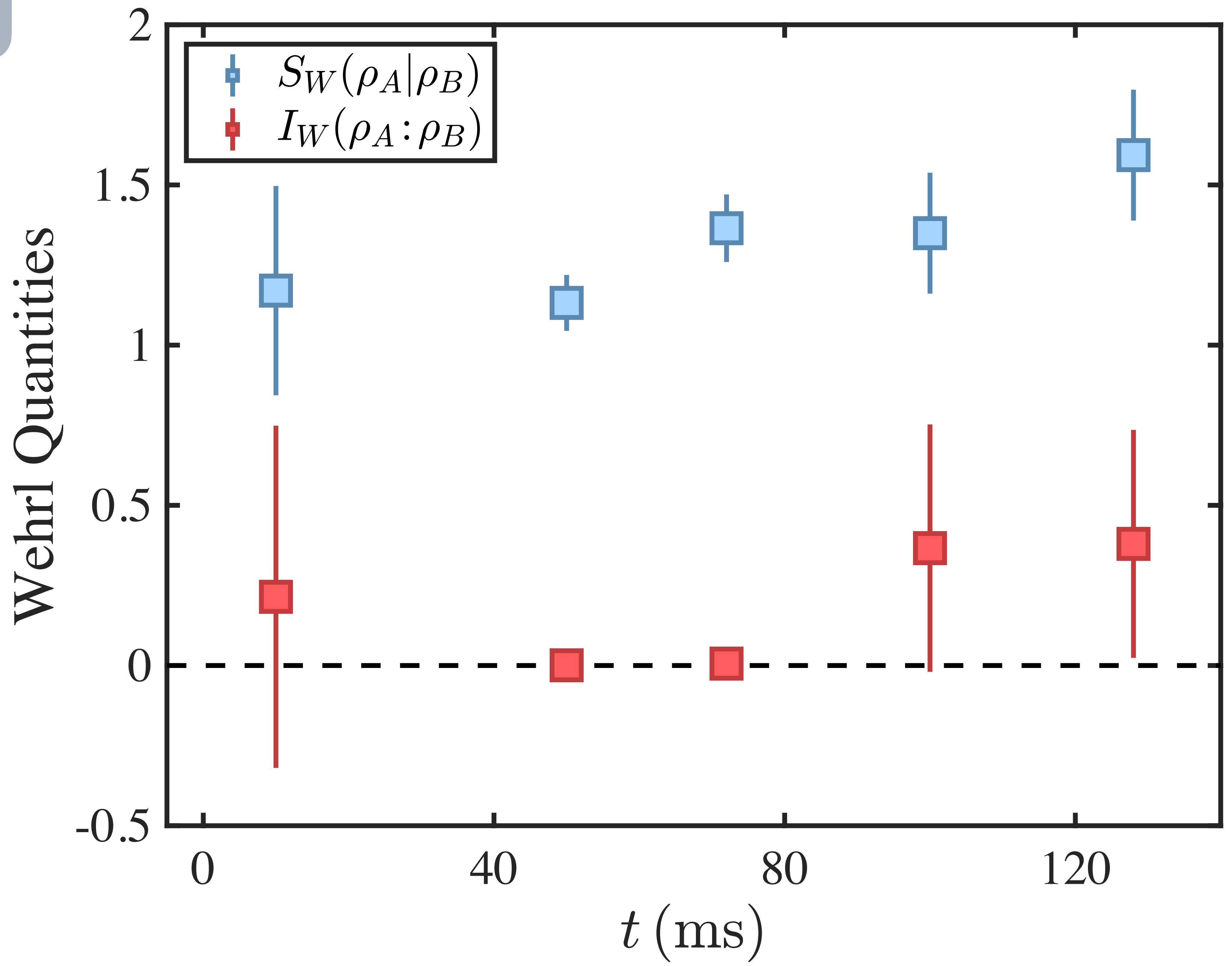


Error via jackknife resampling then naive error propagation

Experimental results





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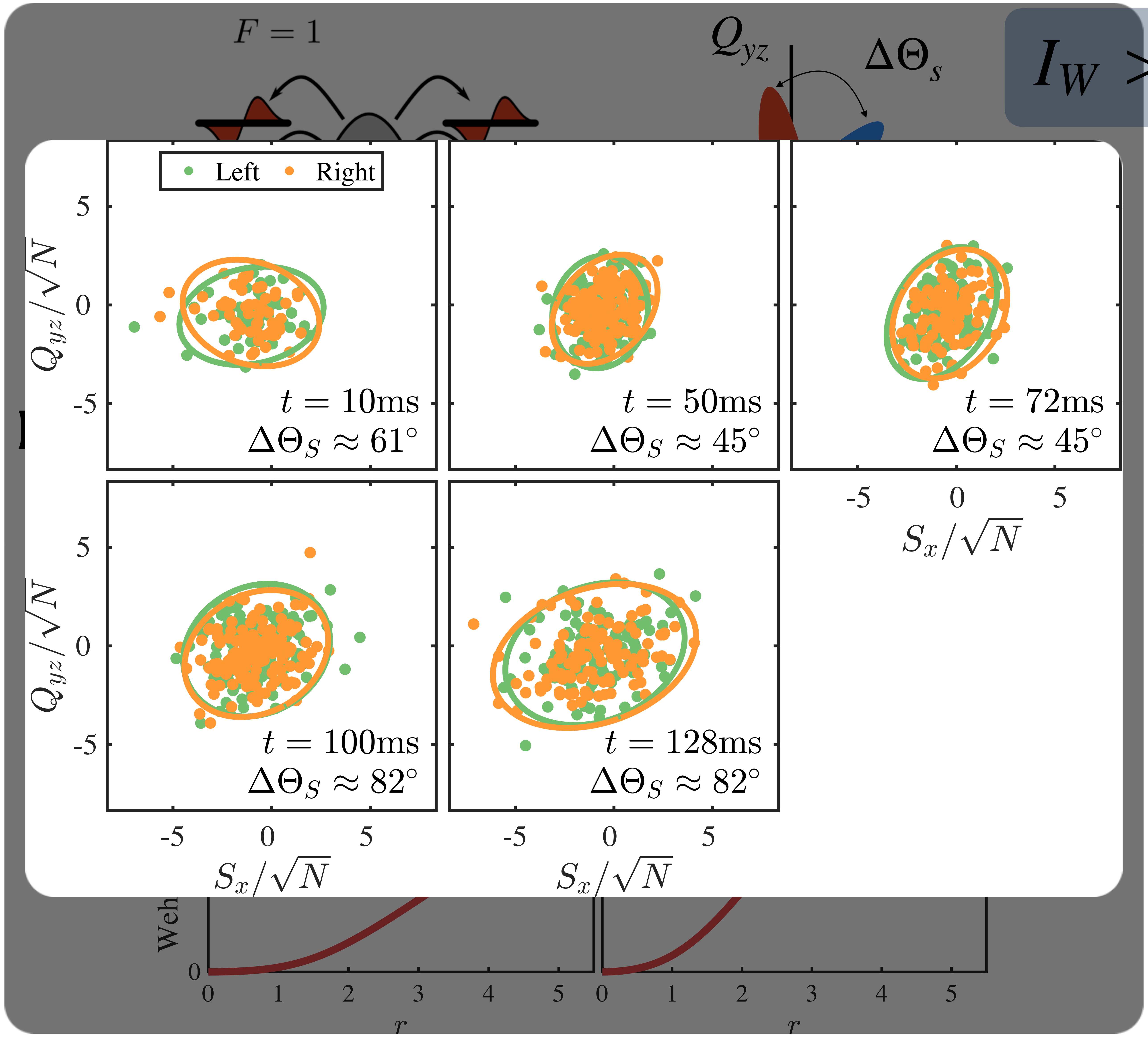

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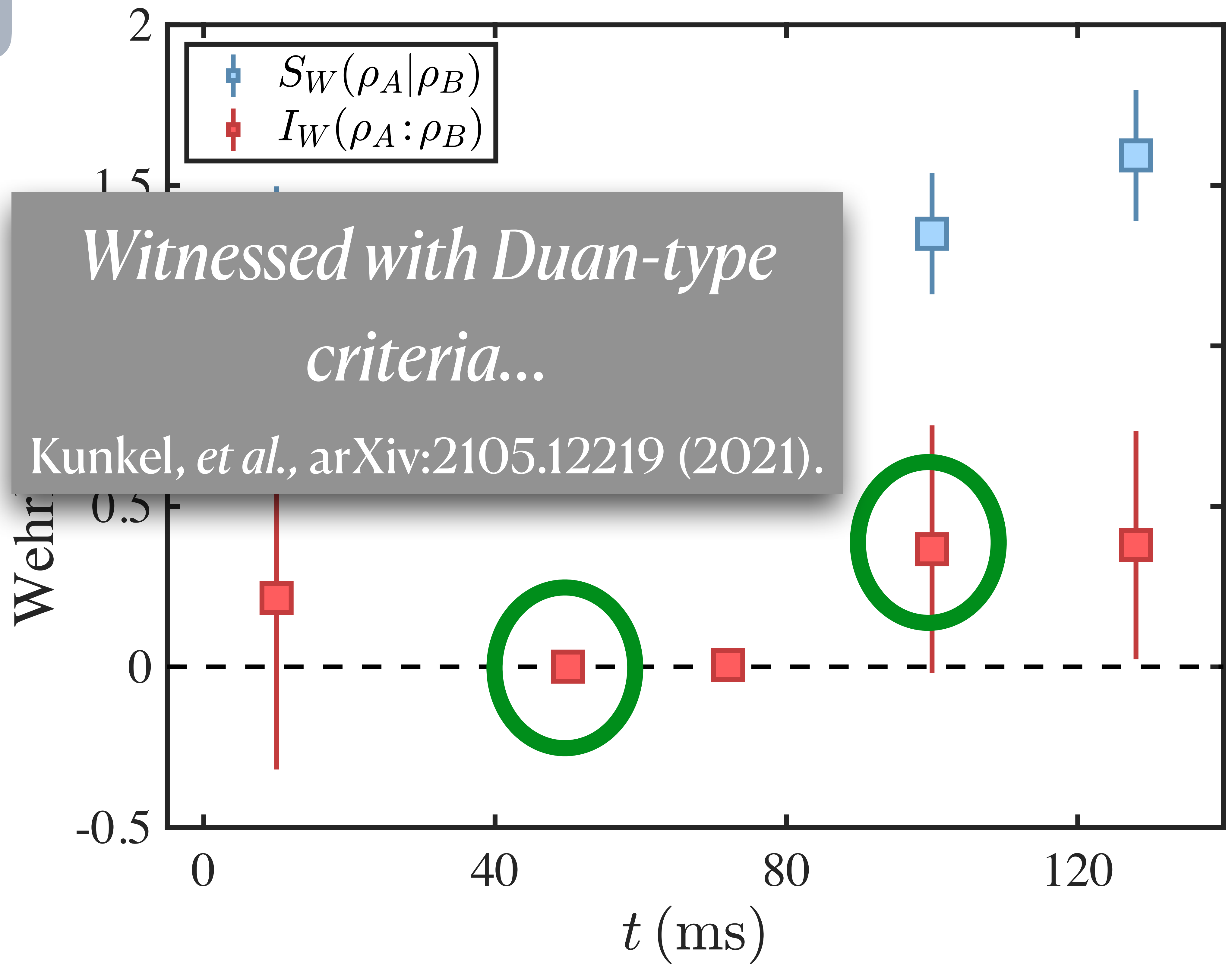

Witnessed with Duan-type criteria...

Kunkel, *et al.*, arXiv:2105.12219 (2021).

Error via jackknife resampling then naive error propagation

Conclusions & Future Direction

Entropies offer benefits to witnessing entanglement

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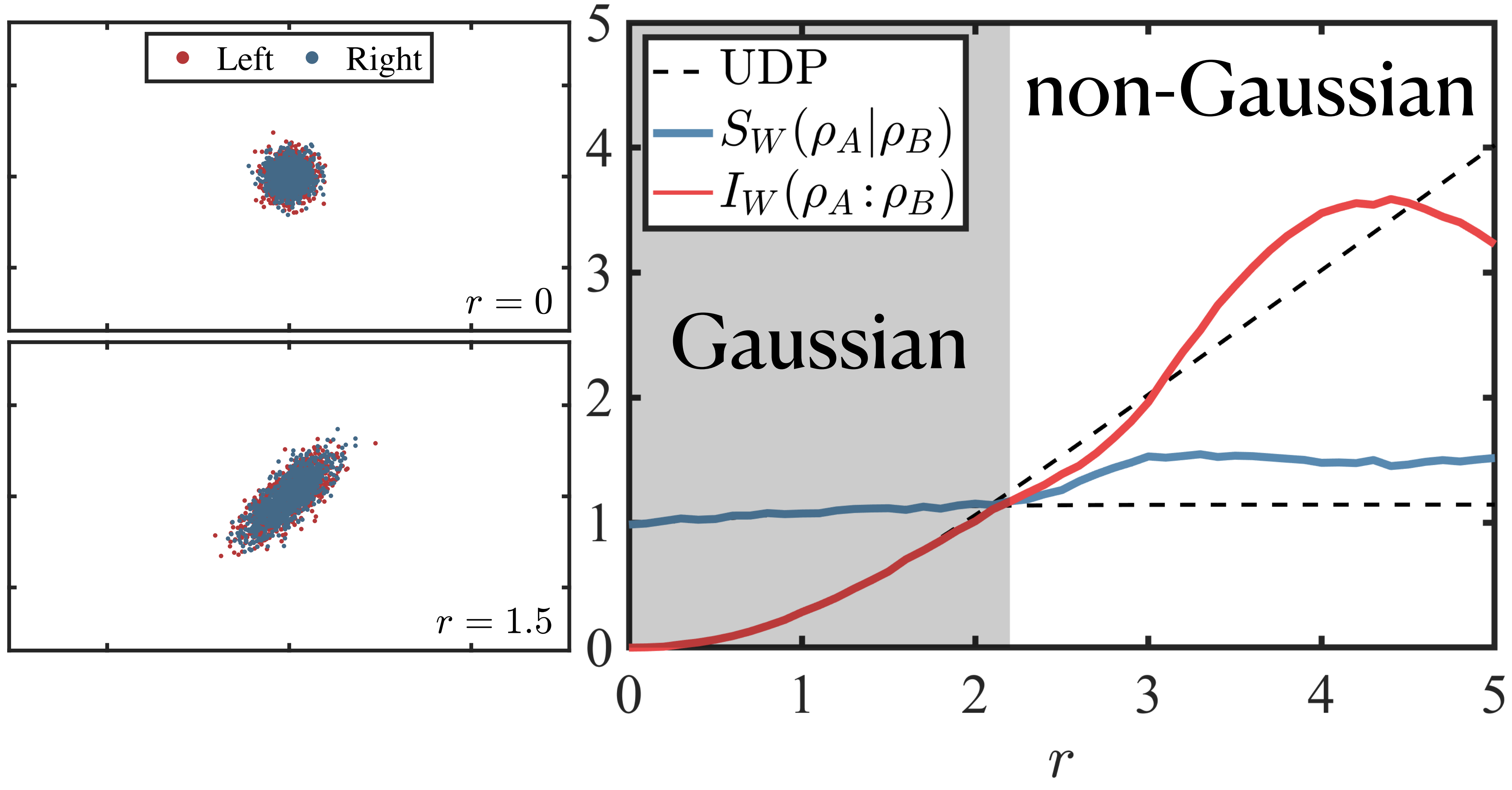

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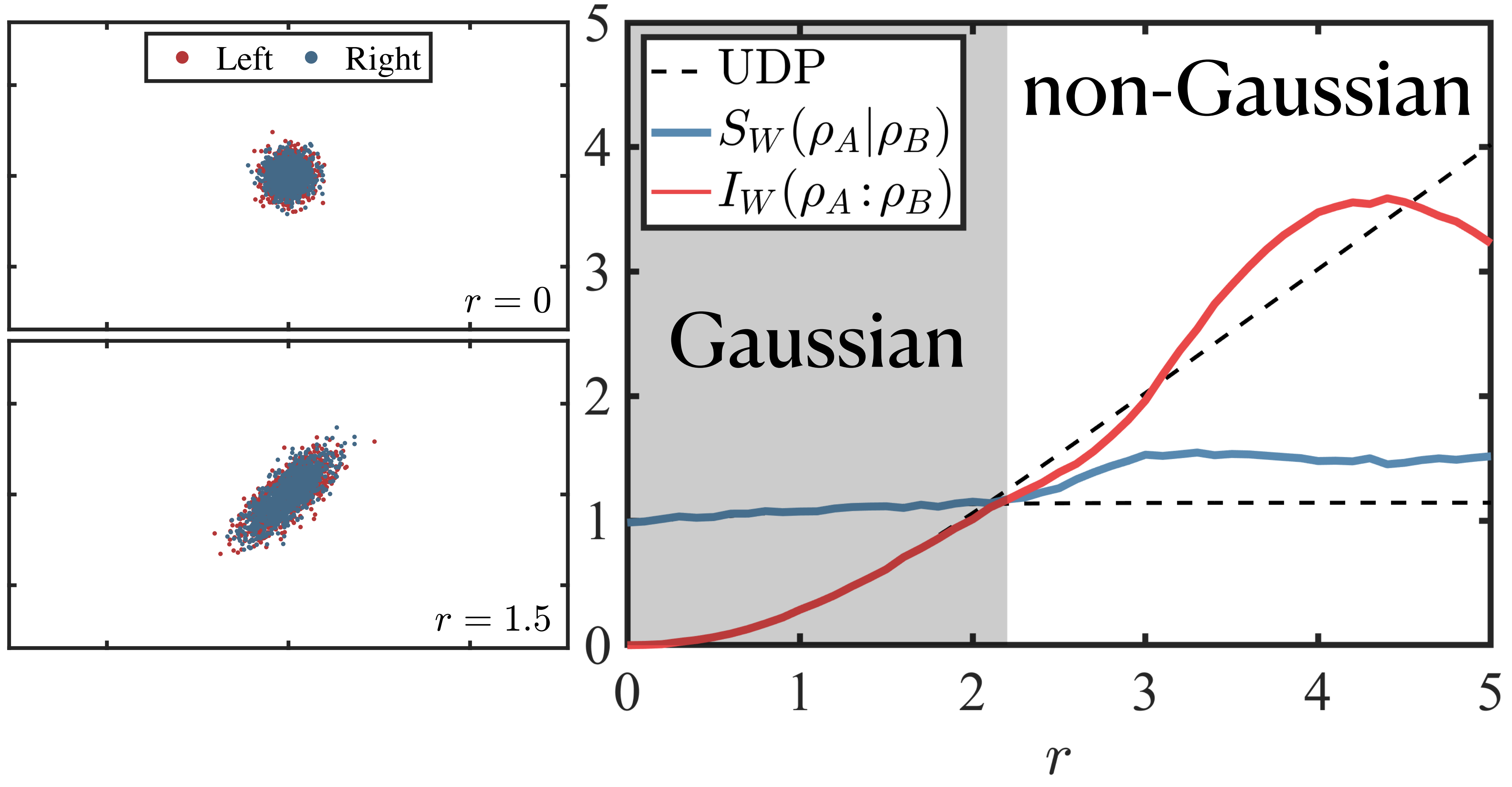
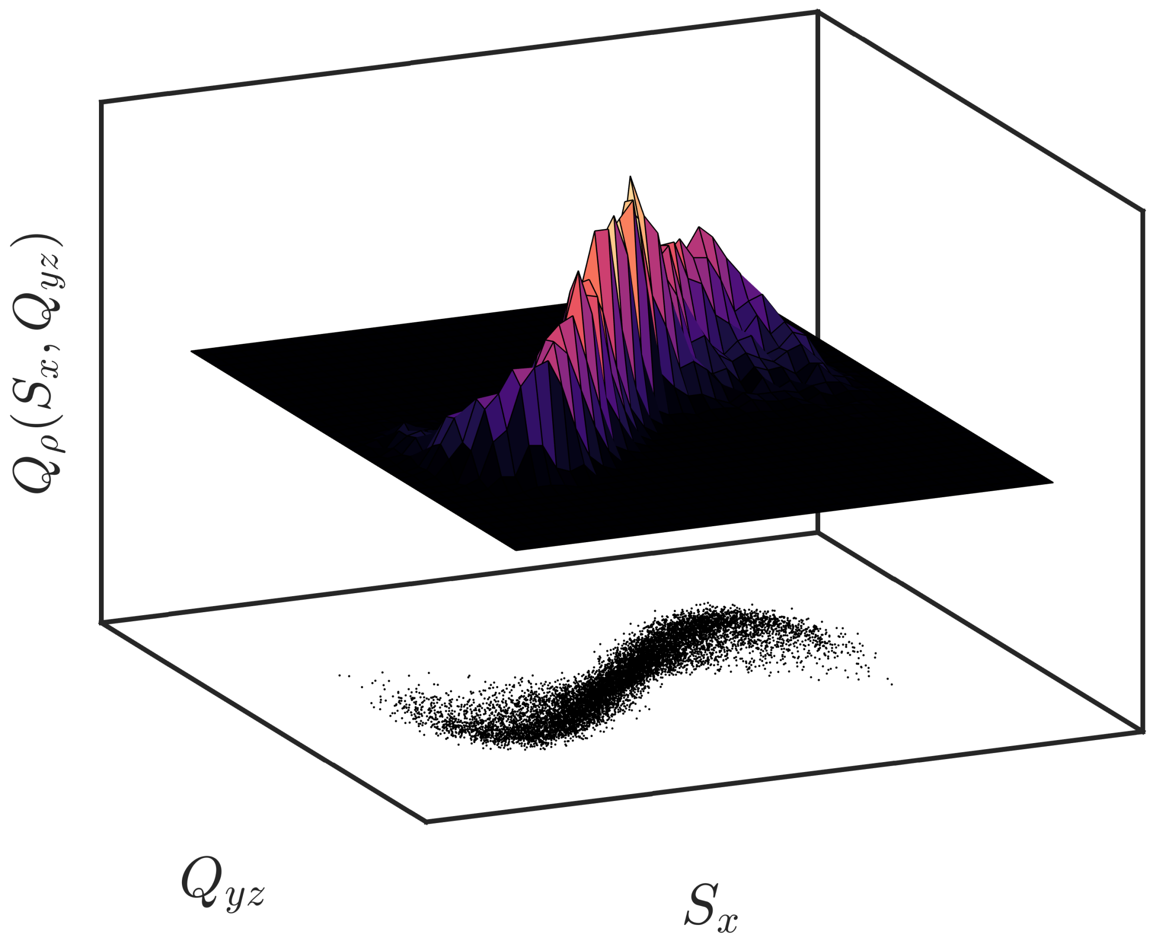
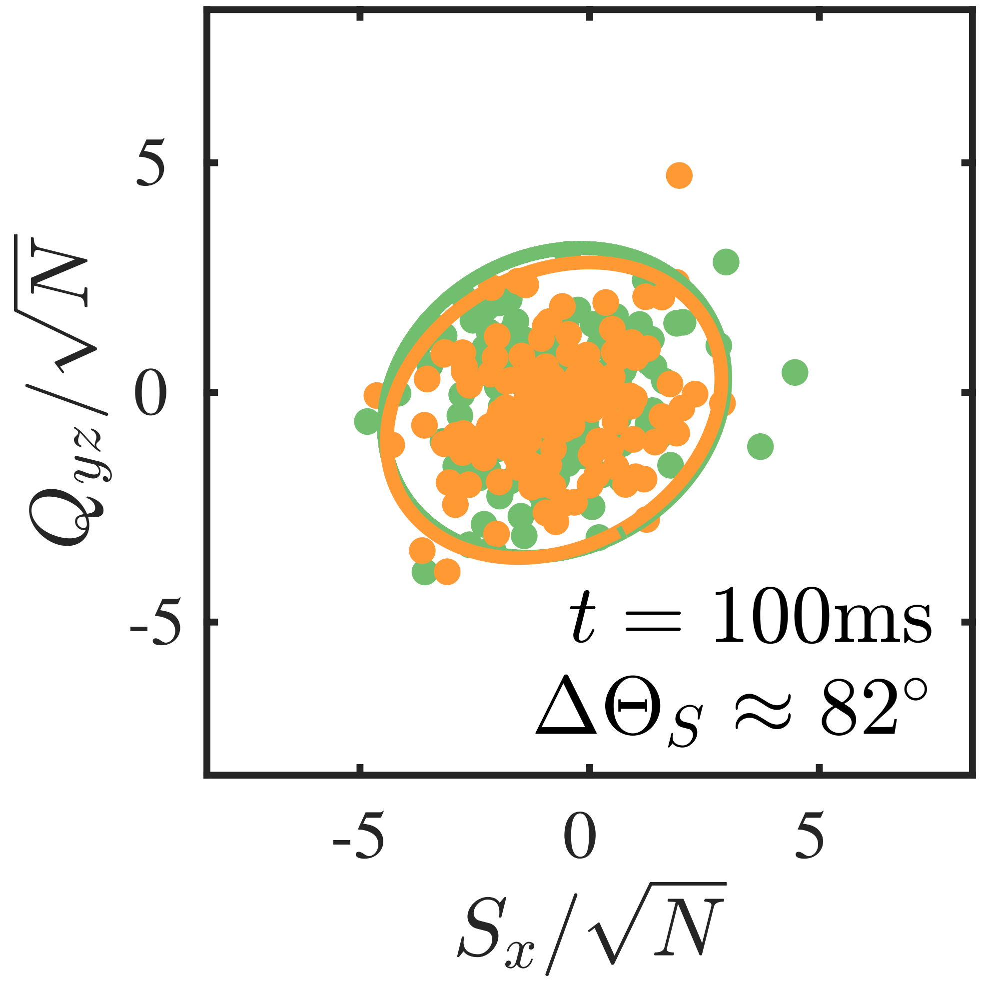


To **reap benefits**, we need to **approximate pdf**





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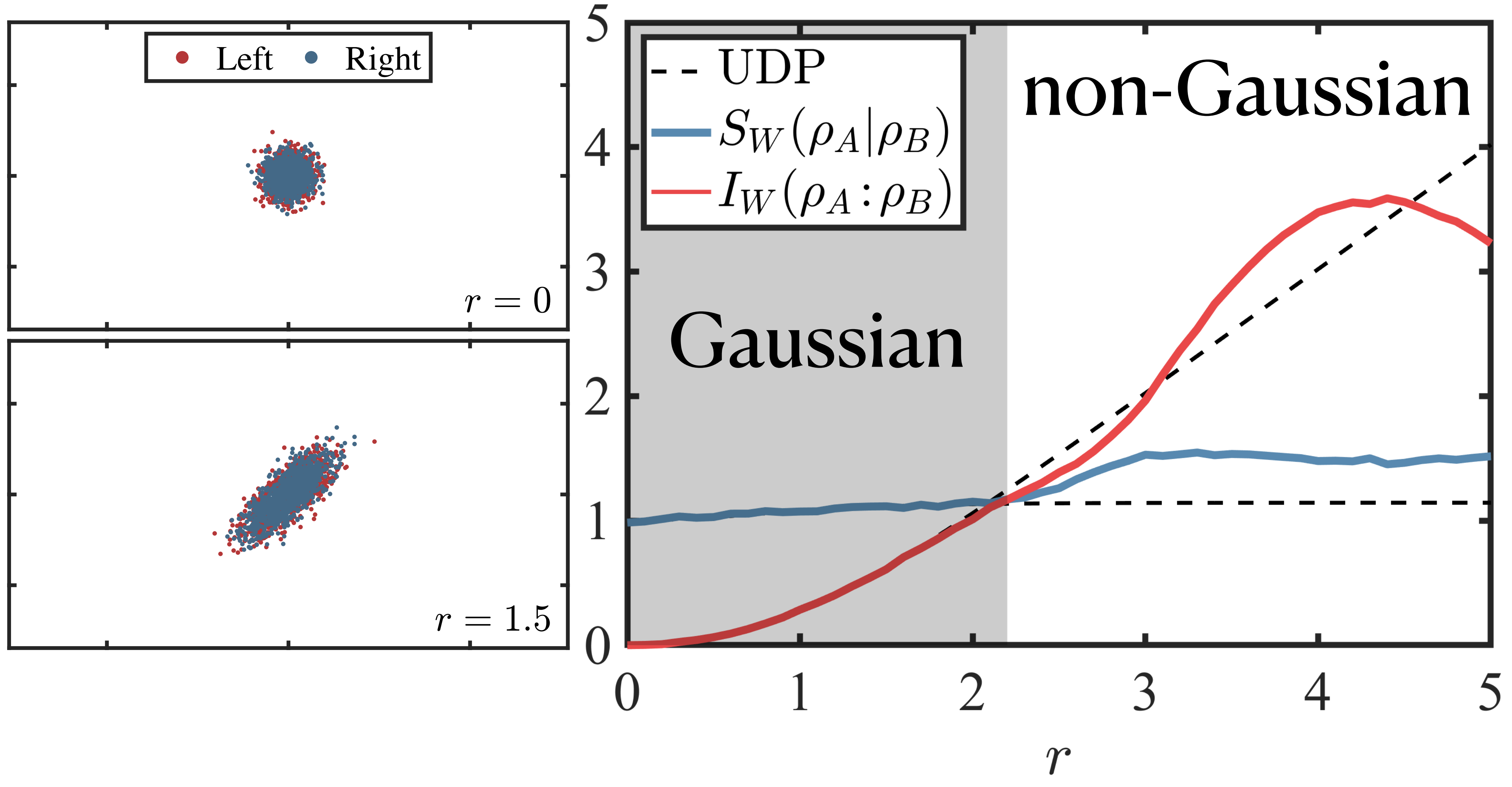

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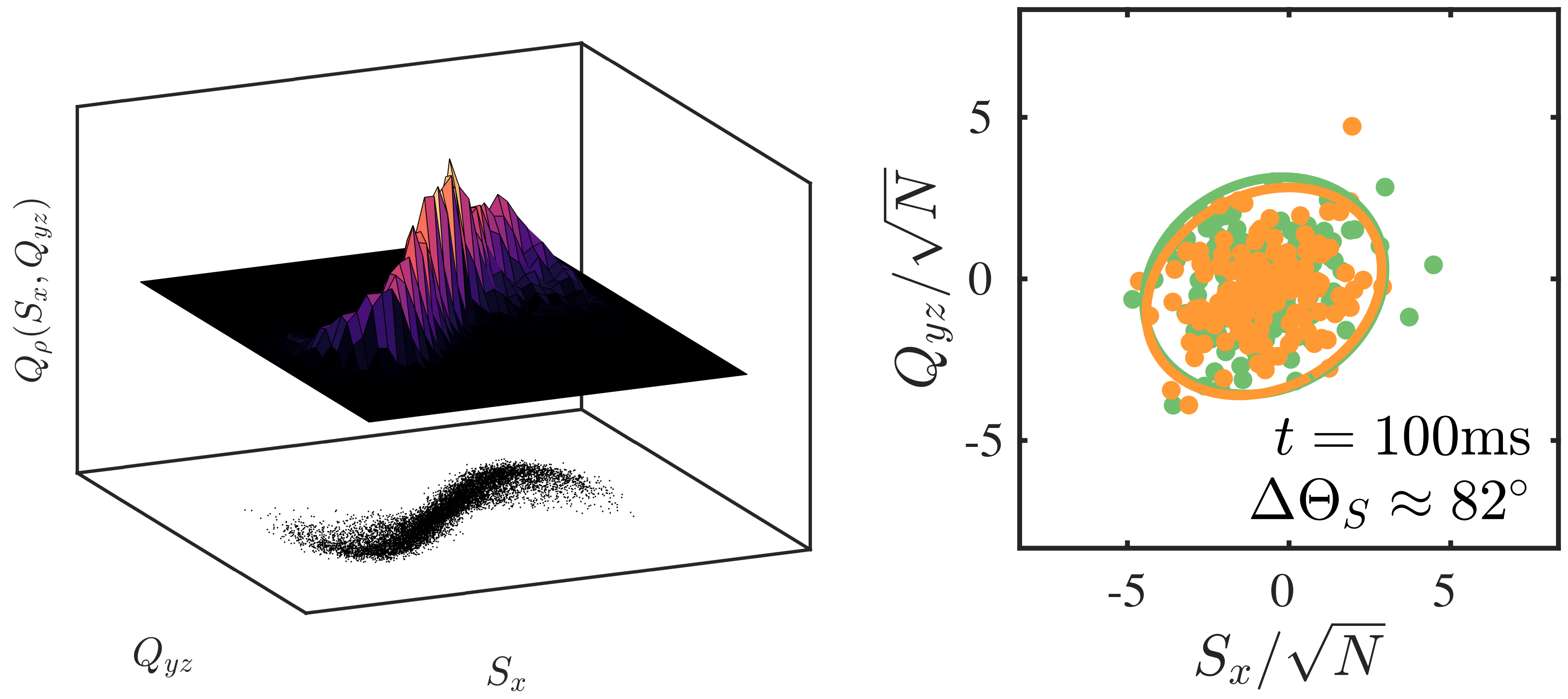

Test the ***true*** entanglement witness

$$S_M(Q_\pm) \geq 1 + \ln 2$$

Floerchinger, *et al.* arXiv:2106.08788 (2021).

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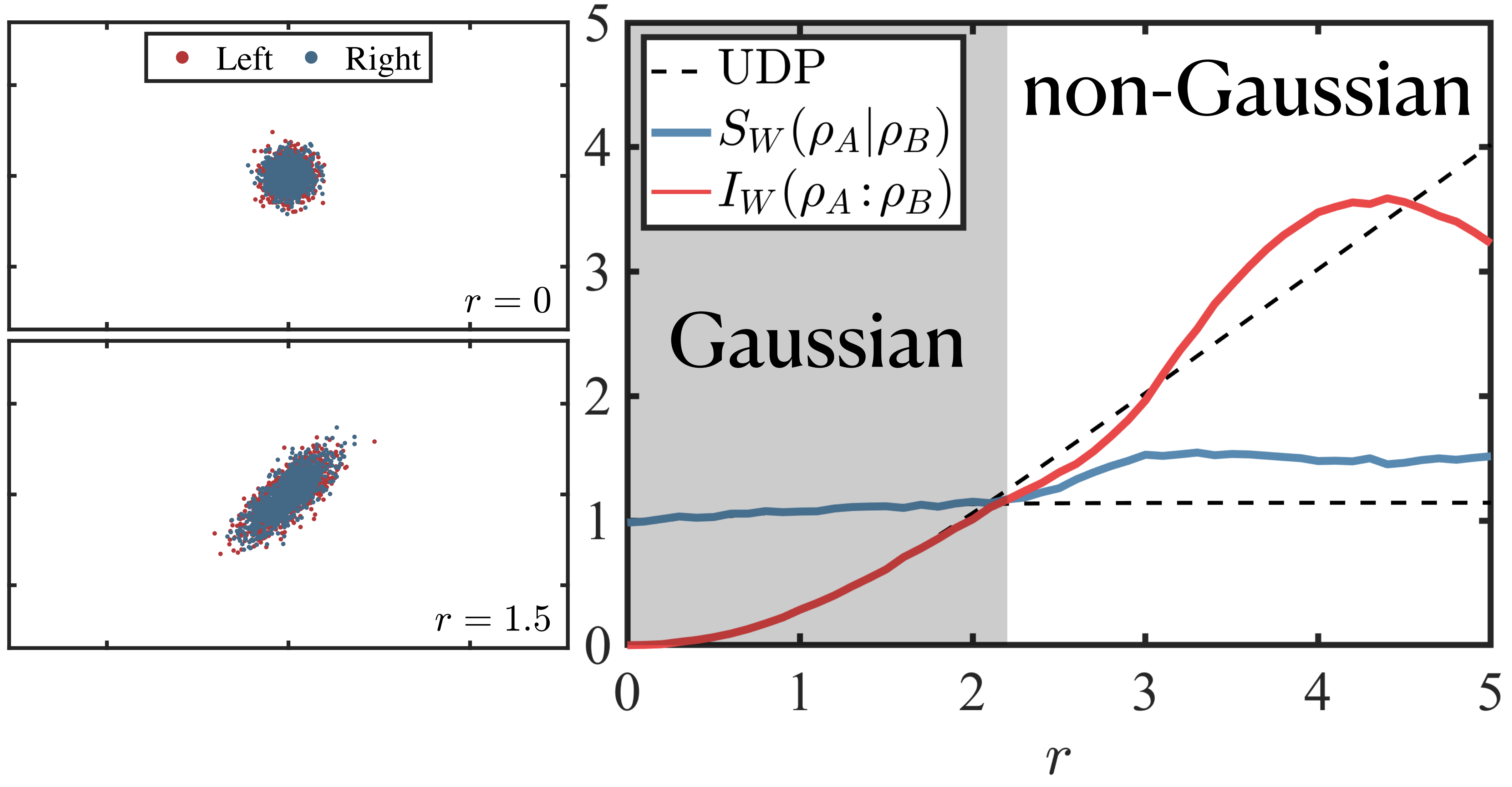

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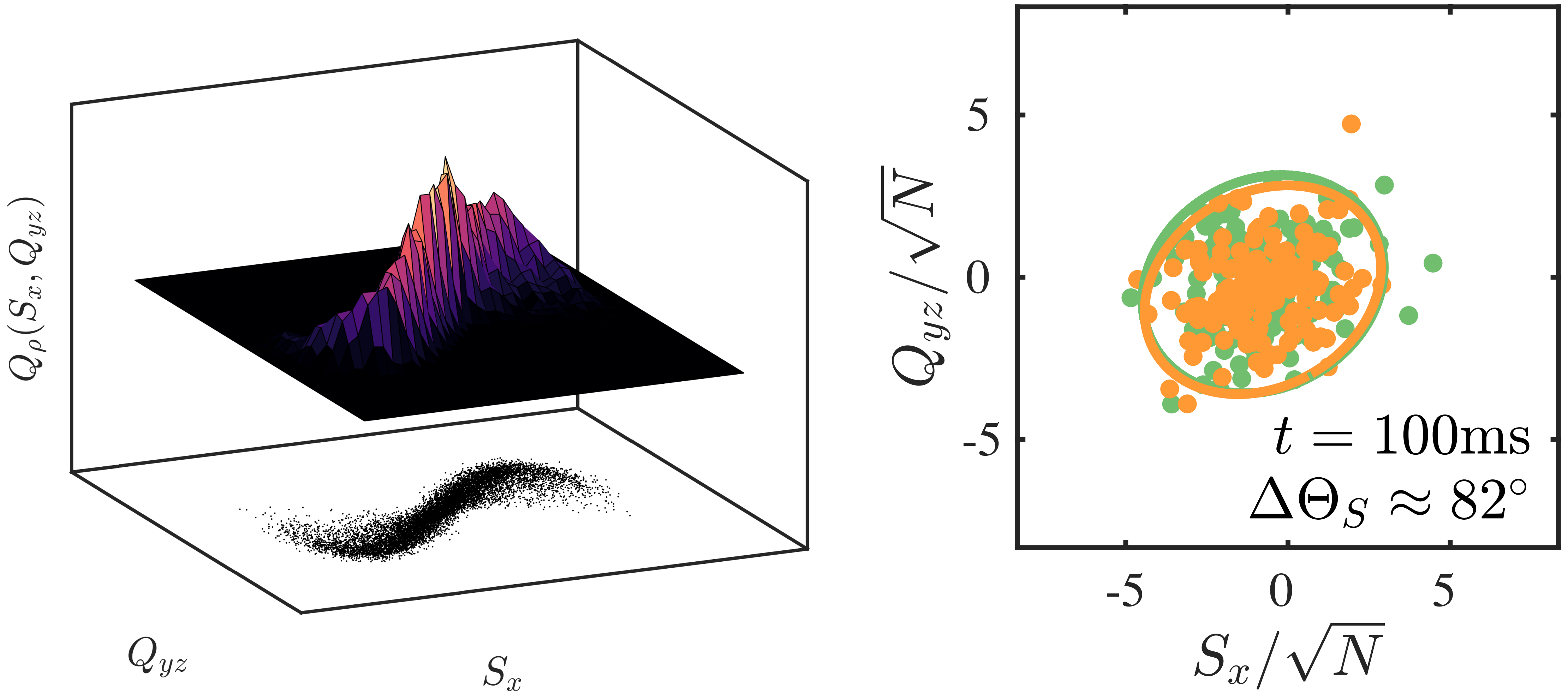

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More details! → @GaerrtnerGroup mbqd.de

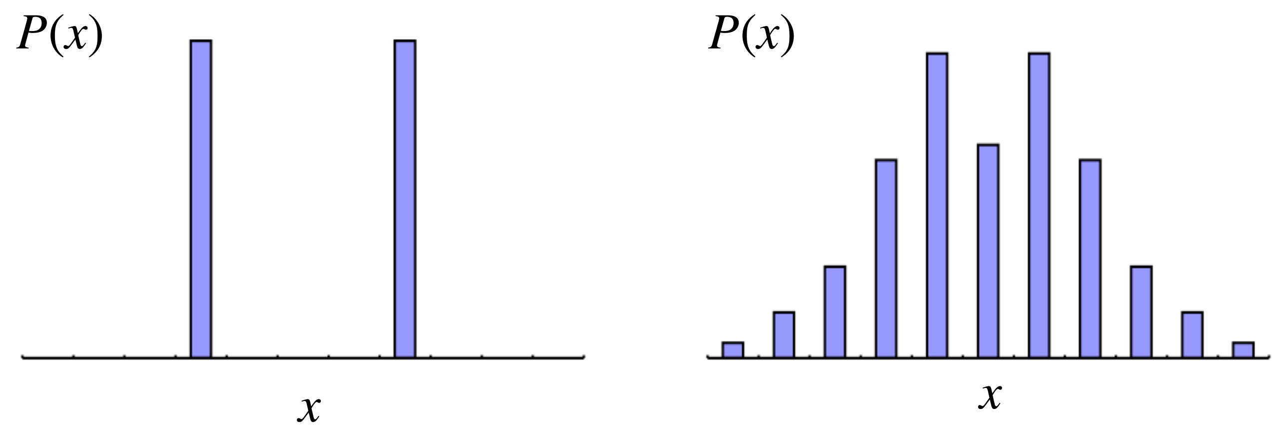
Thank you! Questions?

Entropic uncertainty relations

Single system:

AB

$$H(X) + H(Z) \geq -\log_2 c + S(\rho)$$



Not state dependent!

Measure two non-commuting observables

$$c = \max_{x,z} \left(|\langle X_x^A | Z_z^A \rangle|^2 \right)$$

Bipartite system:

A B

$$H(X_A|X_B) + H(Z_A|Z_B) \geq -\log_2 c + S(\rho_A|\rho_B)$$

For a separable state (not entangled)

$$S(\rho_A|\rho_B) \geq 0$$

Lower bound on distillable entanglement: $-\log_2 c - H(X_A|X_B) - H(Z_A|Z_B)$

Theoretical modelling

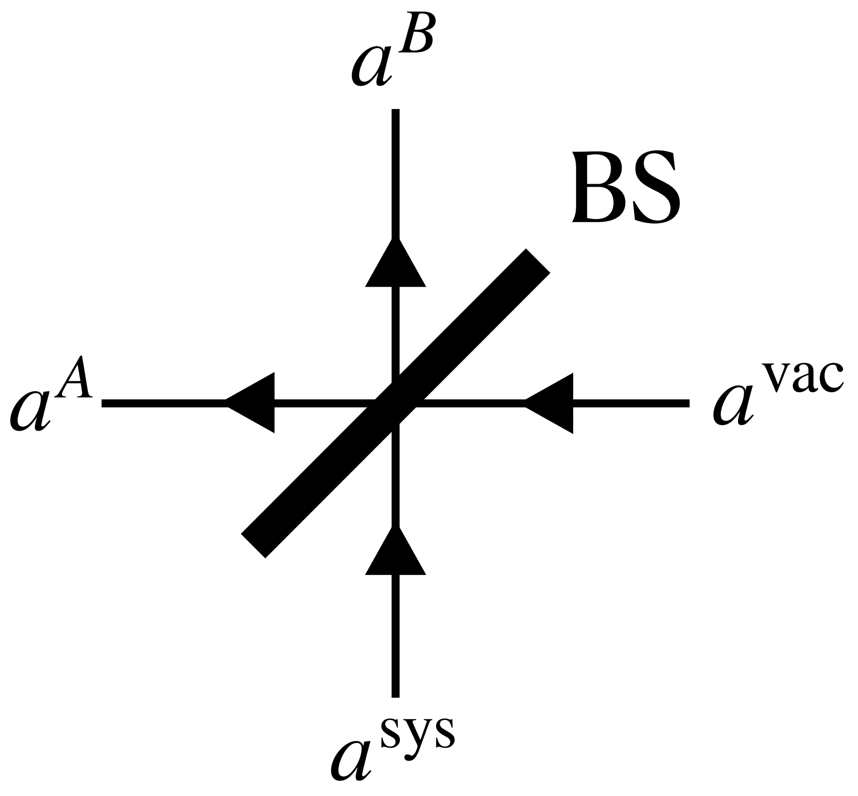
When state is **Gaussian**, covariance matrix holds all information: $I_W(\rho_A : \rho_B) = \dfrac{1}{2}\dfrac{\det C_A \det C_B}{\det C}$

Analytic description

Two-mode squeezed state

$$|\psi\rangle = \sqrt{1-\lambda^2}\sum_n (-\lambda)^n |n\rangle$$

Approximation for **very short *t*** when state is **Gaussian**



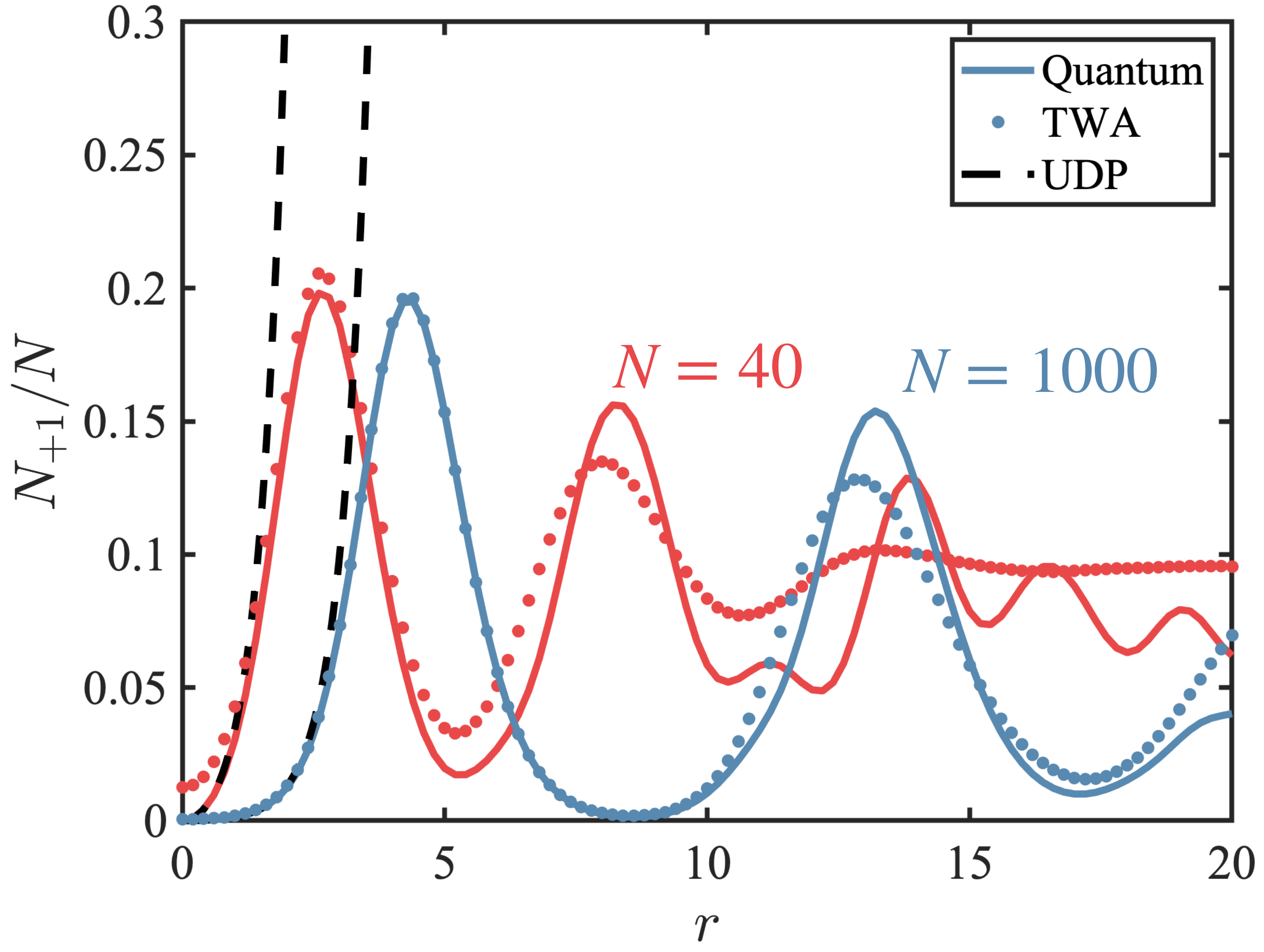
Calculate **covariance matrix** elements

Numerical description

Truncated Wigner: evolve operators semi-classically

$$i\hbar\frac{\partial a_i}{\partial t} = \frac{\partial H_W}{\partial a_i^*}$$

Approximate for **short *t***



Simulating the experiment

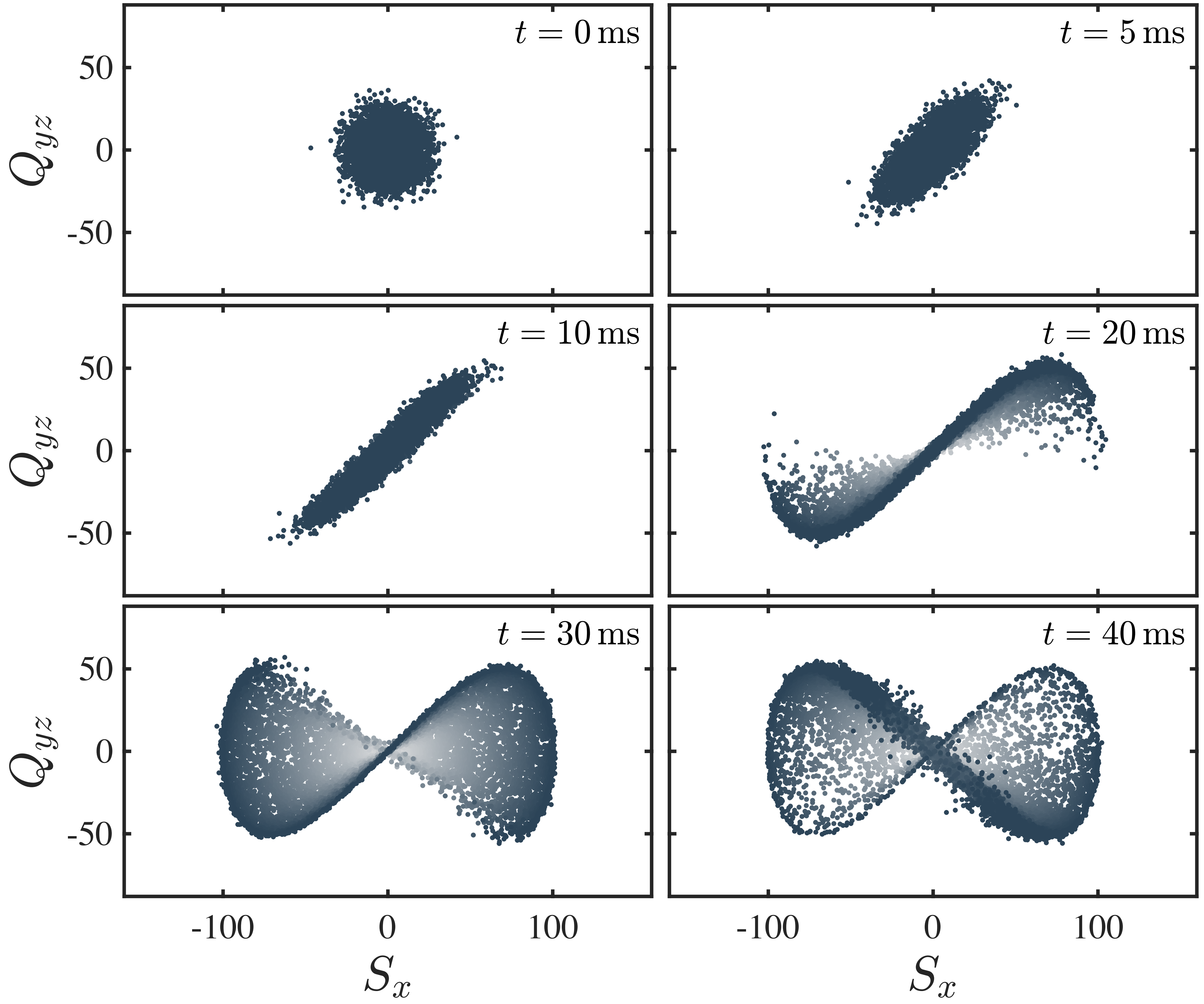
Truncated Wigner approximation

Each mode is evolved via the equation

$$i\hbar\frac{\partial a_i}{\partial t} = \frac{\partial H_W}{\partial a_i^*} \qquad \begin{pmatrix} a_{-1} \\ a_0 \\ a_{+1} \end{pmatrix} = \begin{pmatrix} X_1 \\ \sqrt{N} \\ X_2 \end{pmatrix}$$

and beam split via

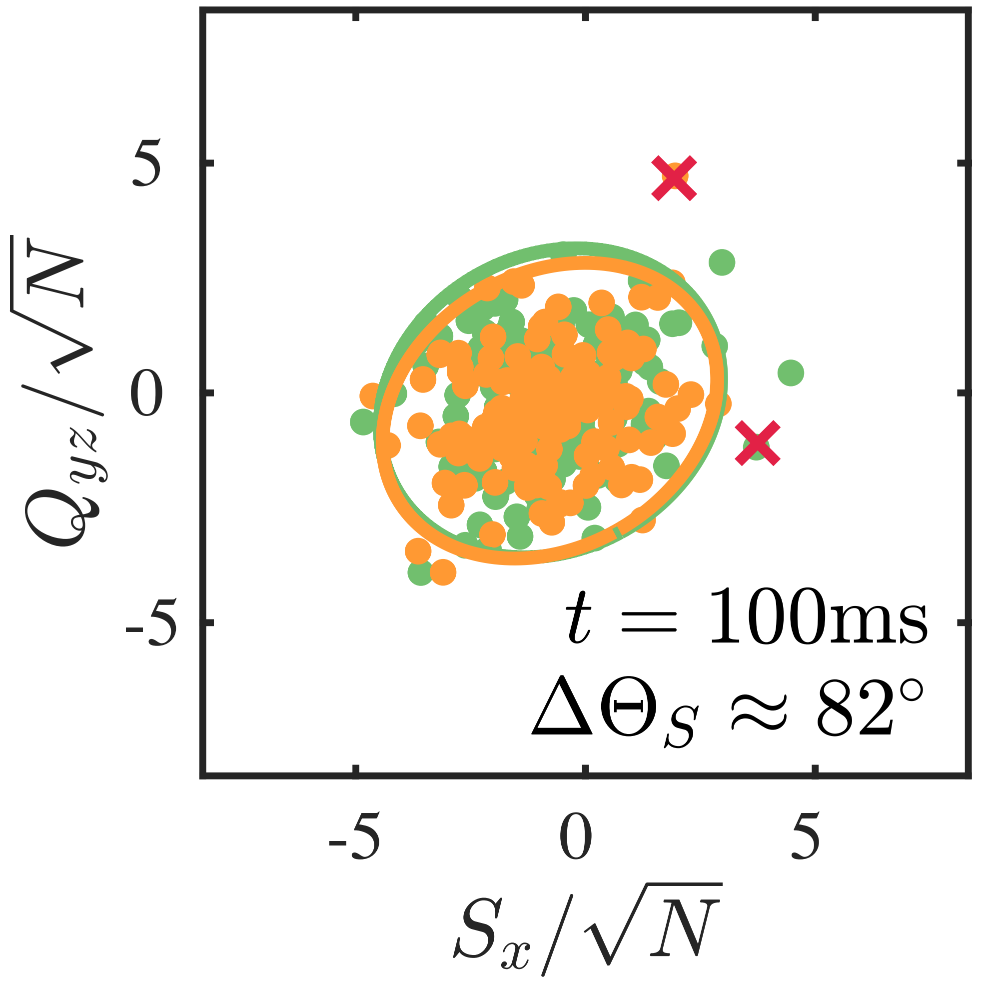
$$\begin{pmatrix} a_i^A \\ a_i^B \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} a_i^{\text{sys}} \\ a_i^{\text{vac}} \end{pmatrix}$$



where X_i are complex random numbers with $\langle X_i \rangle = 0$ and $\Delta^2 X_i = 1/2$

Error calculation

Jackknife resampling



Calculate variance of jackknife set *i*: $\mathrm{Var}(X)_i$

Repeat over *i* data points

Error in variance:
$\sqrt{n-1}\,\mathrm{std}(\{\mathrm{Var}(X)_i\})$

Variance error propagation

$$I_W(\rho_A : \rho_B) = \frac{1}{2} \frac{\det C_A \det C_B}{\det C}$$