Expansion of vortex clusters in a dissipative two-dimensional superfluid

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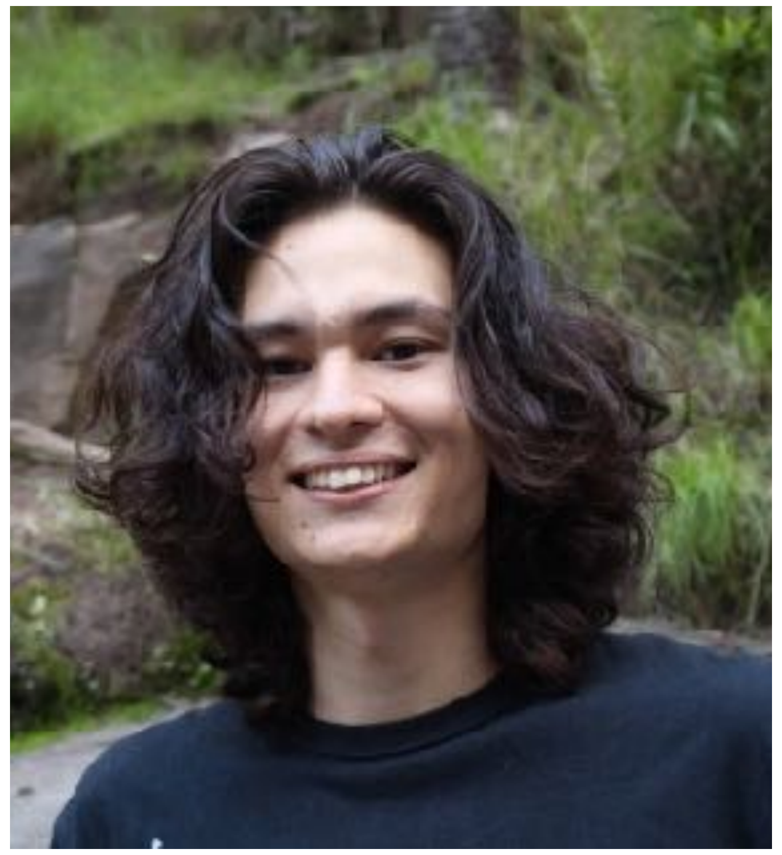

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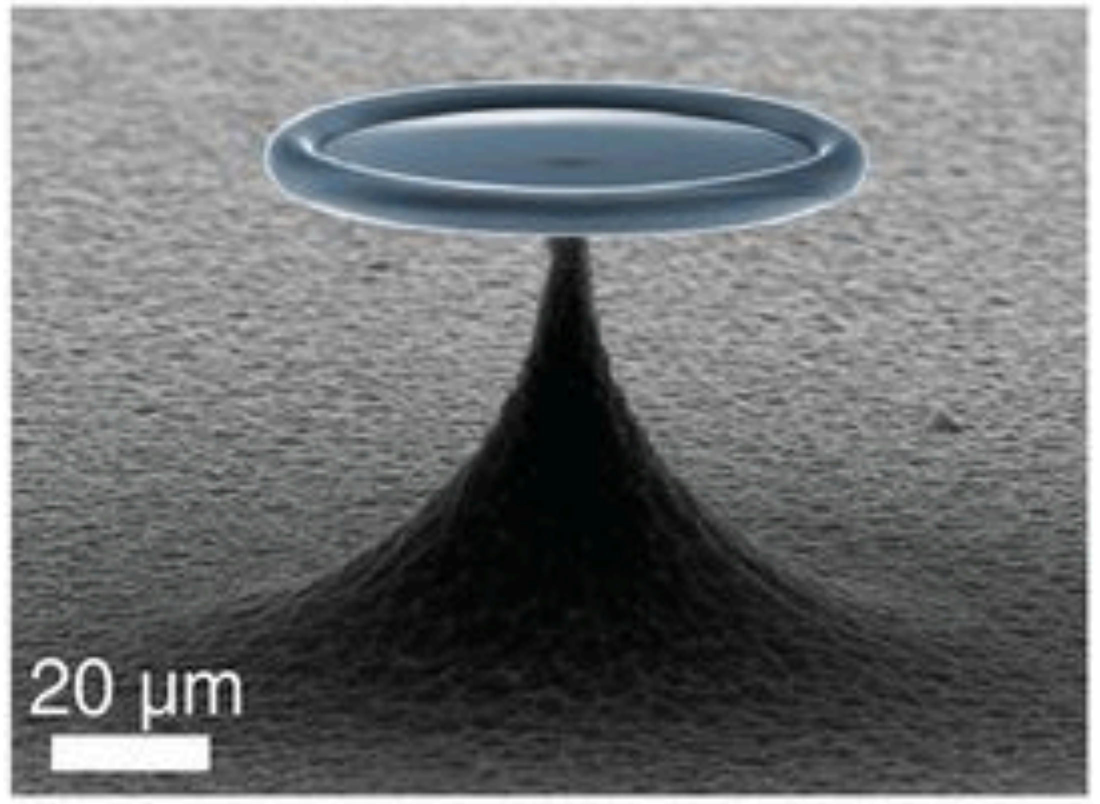
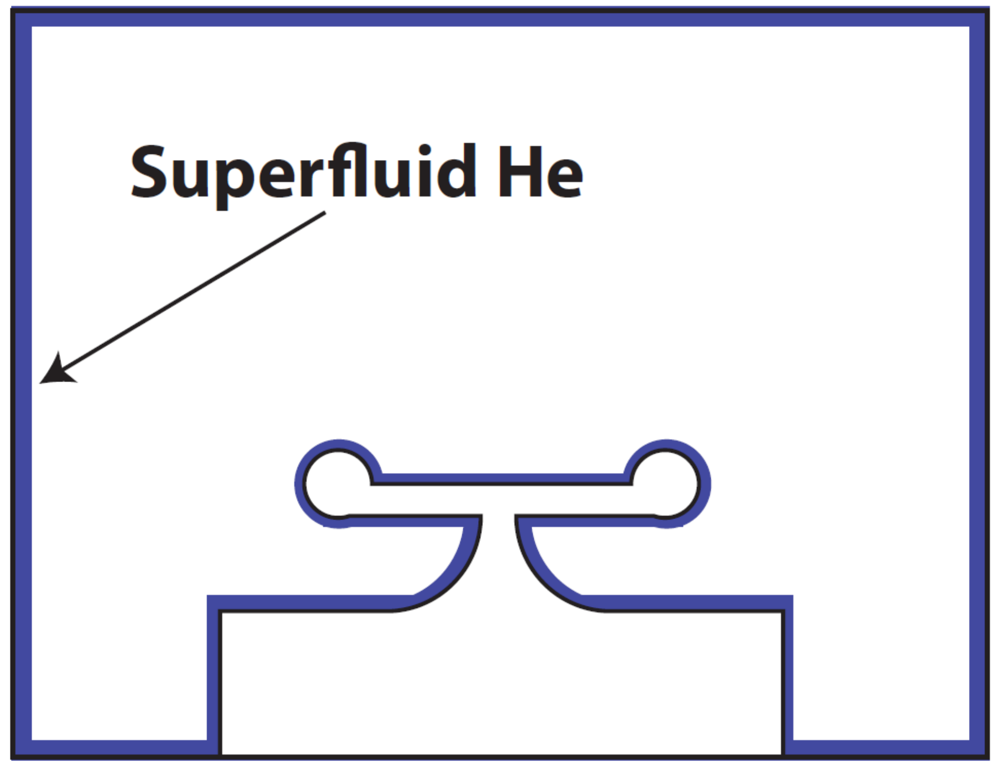
Motivation: superfluid optomechanics at UQ

A disk of **thin-film superfluid helium**



Direct imaging **not possible**

Superfluid covers every surface



➔Self-assembling film

Y. P. Sachkou, *et al.*, Science **366**, 1480 (2019).

Observation: Superfluid is **rotating** and **rotation is decreasing**

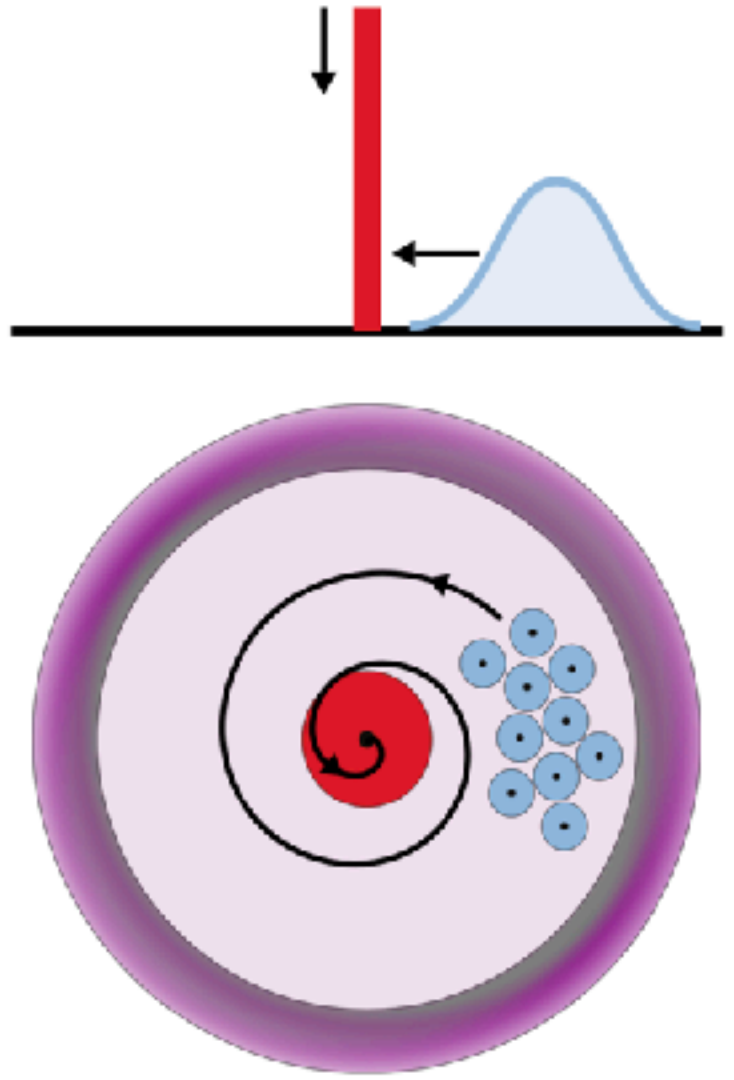
Possible explanations

positive vortex

negative vortex

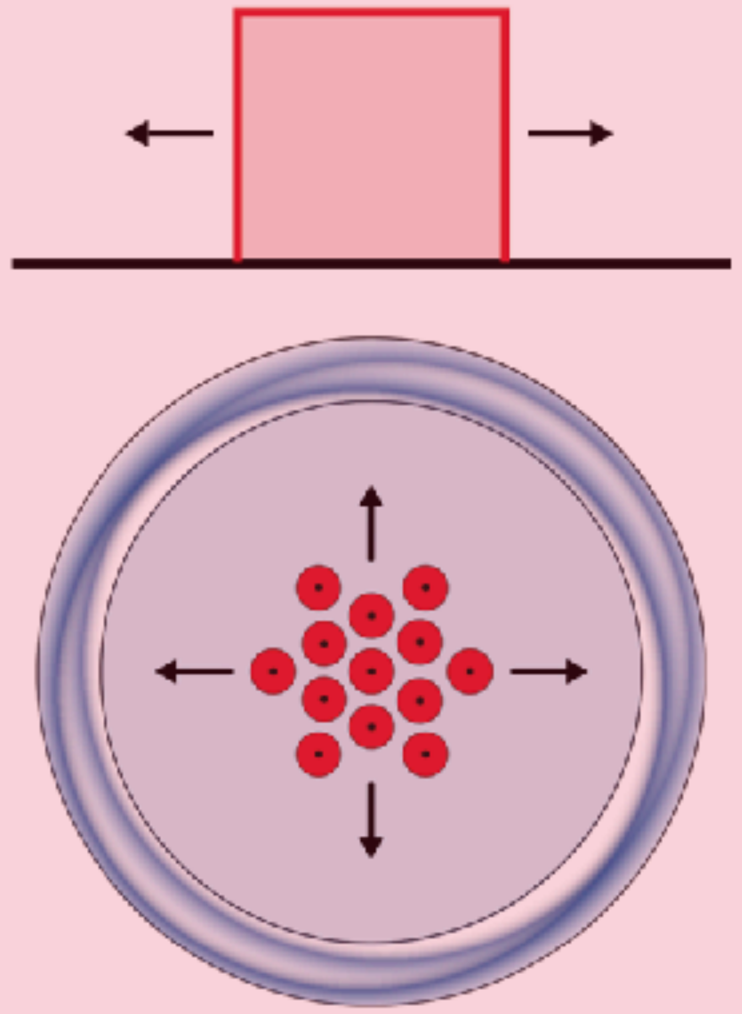
Superfluid vortices' circulation is quantised

$$\Gamma = \kappa \frac{h}{m}$$



Pinned cluster with annihilation

This Talk



Expanding cluster

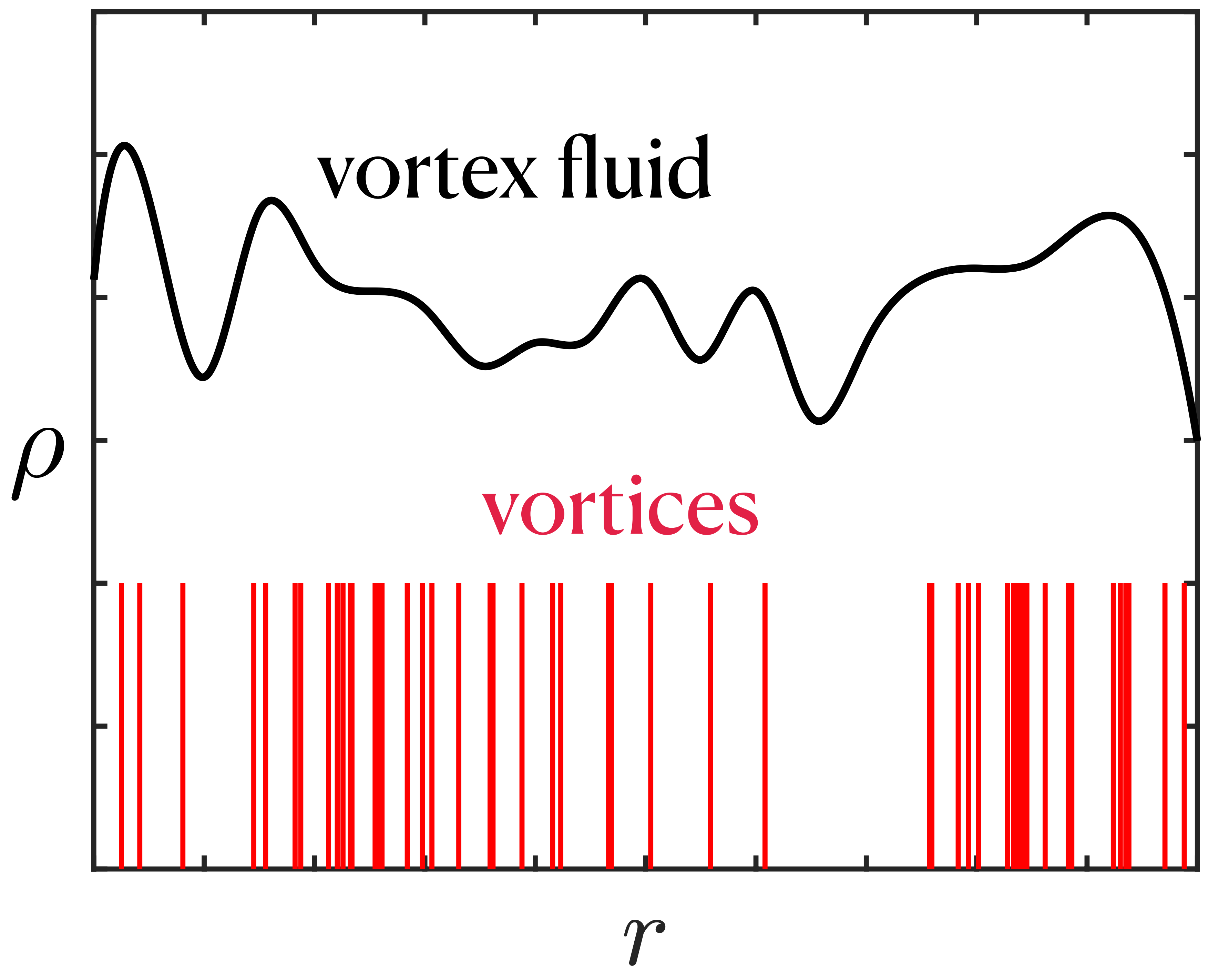
Modelling a chiral vortex cluster

Modelling the system: *anomalous* hydrodynamics

Large collections of vortices can be modelled as a **fluid**

$$\rho \equiv \sum_i \delta(\mathbf{r} - \mathbf{r}_i) \longrightarrow \rho \approx f(\mathbf{r})$$

(Where f is smooth)



'*Vortex fluid*' dynamics = Euler equation + anomalous stresses

<u>*Conservative* density evolution:</u>

$$(\partial_t + \mathbf{v} \cdot \nabla)\rho = 0$$

Modelling hydrodynamics of *vortex fluid*, <u>NOT</u> *superfluid* hydrodynamics

Wiegmann & Abanov, PRL **113**, 034501 (2014).
Yu & Bradley, PRL **119**, 18501 (2017).

Dissipative anomalous hydrodynamics

Real systems are **dissipative** — how do **dissipative vortex fluids** behave?

$$(\partial_t + \mathbf{v} \cdot \nabla)\rho = -\gamma \left[\Gamma \rho^2 + \frac{\Gamma}{8\pi} \nabla^2 \rho - \mathbf{v} \times \nabla \rho - \frac{\Gamma}{8\pi} \frac{|\nabla^2 \rho|}{\rho} \right]$$

$\gamma =$ dissipation $\rho =$ vortex density $\mathbf{v} =$ vortex fluid velocity field $\Gamma =$ vortex circulation

Yu & Bradley, PRL **119**, 18501 (2017).
ORS, *et al.*, PRR **2**, 033138 (2020).

Dissipative anomalous hydrodynamics

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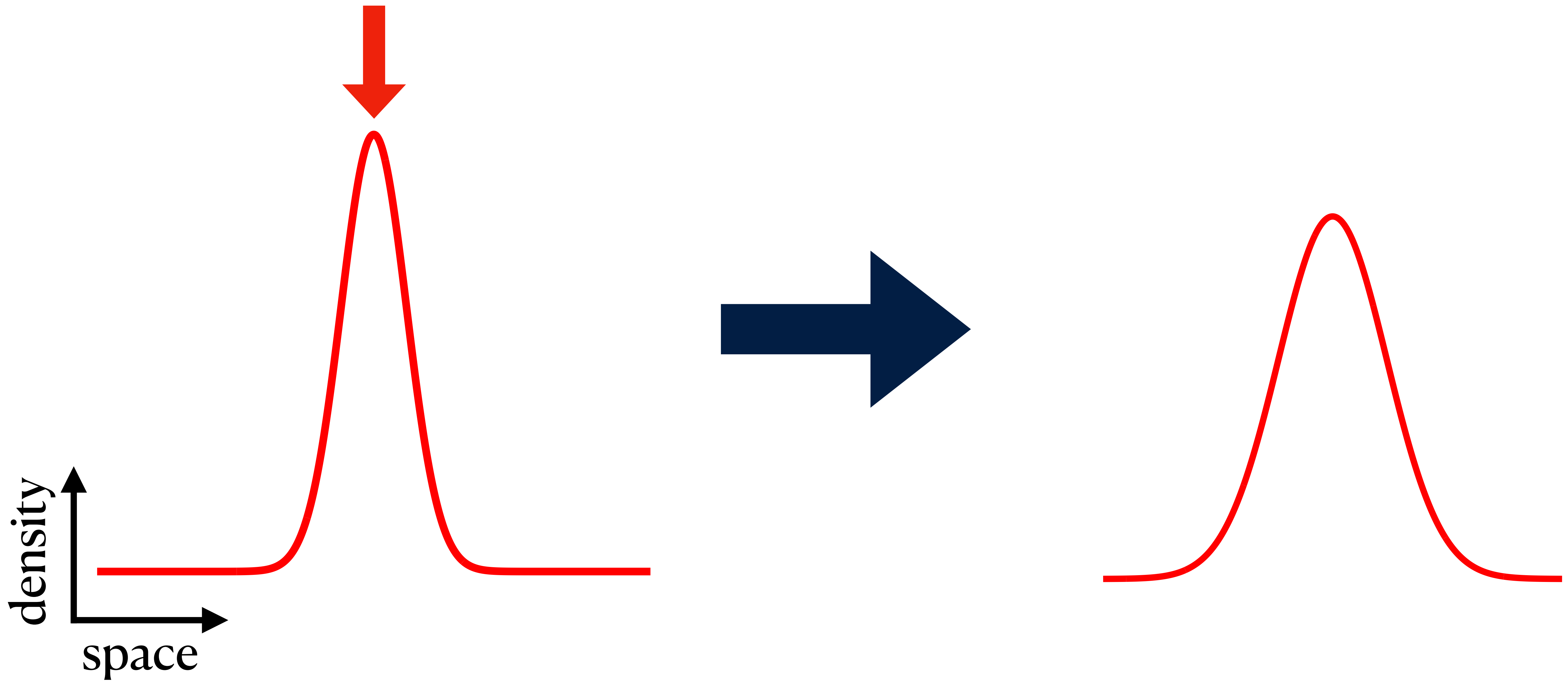
$$(\partial_t + \mathbf{v}\cdot\nabla)\rho = -\gamma\left[\boxed{\Gamma\rho^2} + \frac{\Gamma}{8\pi}\nabla^2\rho - \mathbf{v}\times\nabla\rho - \frac{\Gamma}{8\pi}\frac{|\nabla^2\rho|}{\rho}\right]$$



Suppresses high density regions → flattens distribution

Yu & Bradley, PRL **119**, 18501 (2017).
ORS, *et al.*, PRR **2**, 033138 (2020).

Dissipative anomalous hydrodynamics

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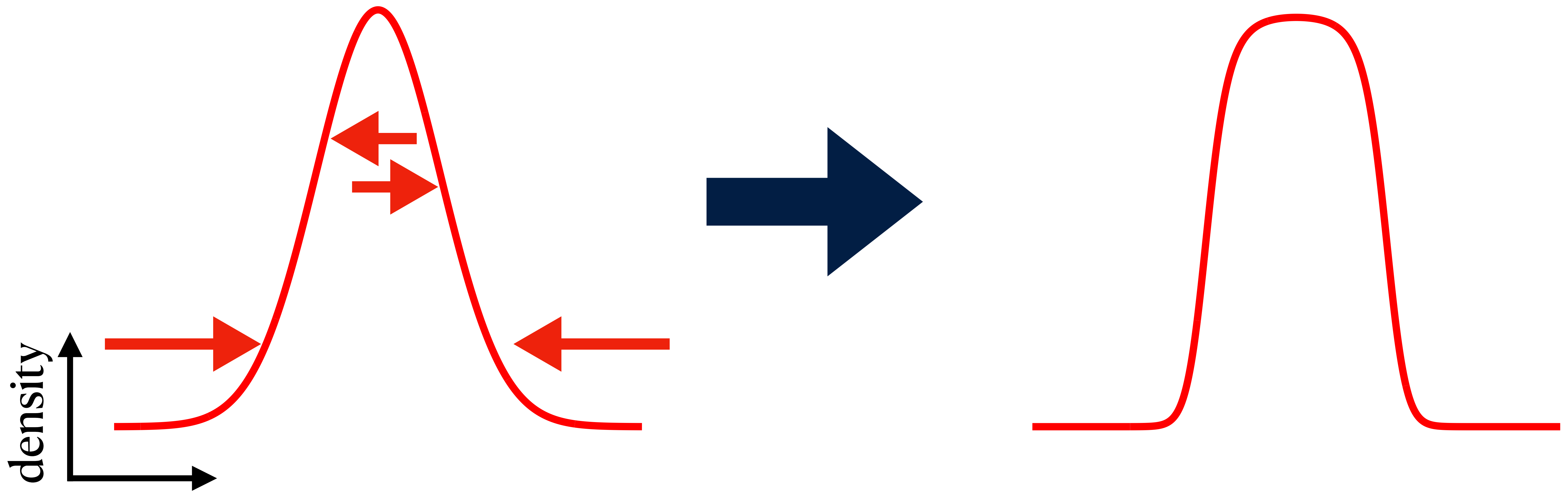
Negative viscosity, i.e., **steepens** density gradients



Yu & Bradley, PRL **119**, 18501 (2017).
ORS, *et al.*, PRR **2**, 033138 (2020).

Dissipative anomalous hydrodynamics

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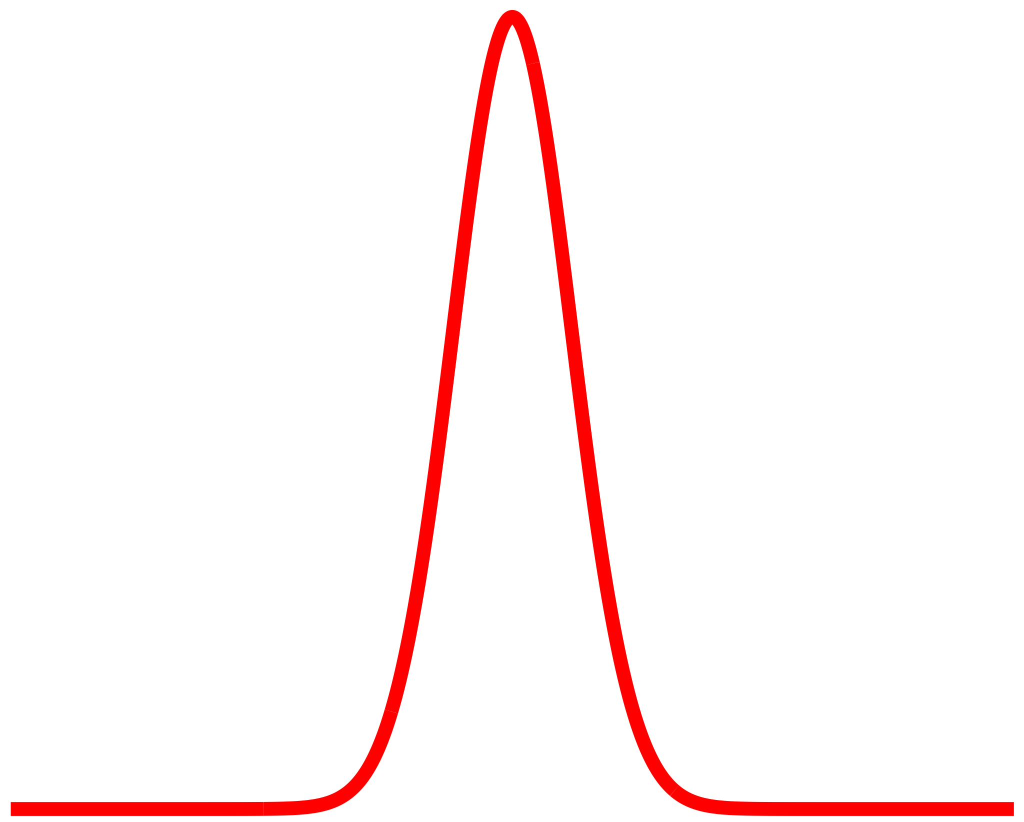
Let's (perhaps naively) assume:

Dissipation (γ) is essential!



Expansion

The '*Rankine vortex*'

Yu & Bradley, PRL **119**, 18501 (2017).
ORS, *et al.*, PRR **2**, 033138 (2020).

Dissipative anomalous hydrodynamics

Rankine vortex is a solution to dissipative vortex fluid theory

Assume $\nabla\rho = 0$ $\longrightarrow$ $\rho(t) = \dfrac{1}{\rho_0^{-1} + \gamma\Gamma t}$

$$(\partial_t + \cancel{\mathbf{v}\cdot\nabla})\rho = -\gamma\left[\Gamma\rho^2 + \cancel{\frac{\Gamma}{8\pi}\nabla^2\rho} - \cancel{\mathbf{v}\times\nabla\rho} - \cancel{\frac{\Gamma}{8\pi}\frac{|\nabla^2\rho|}{\rho}}\right]$$

ORS, *et al.*, PRR **2**, 033138 (2020).

Dissipative anomalous hydrodynamics

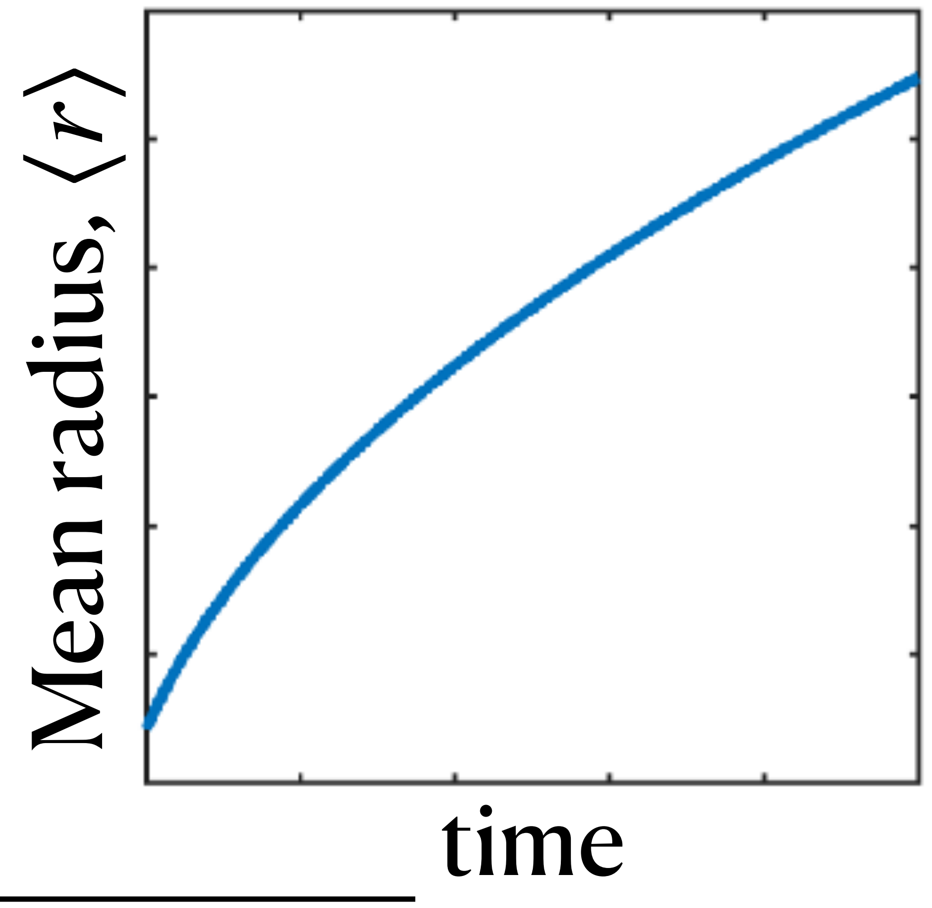
Rankine vortex is a solution to dissipative vortex fluid theory

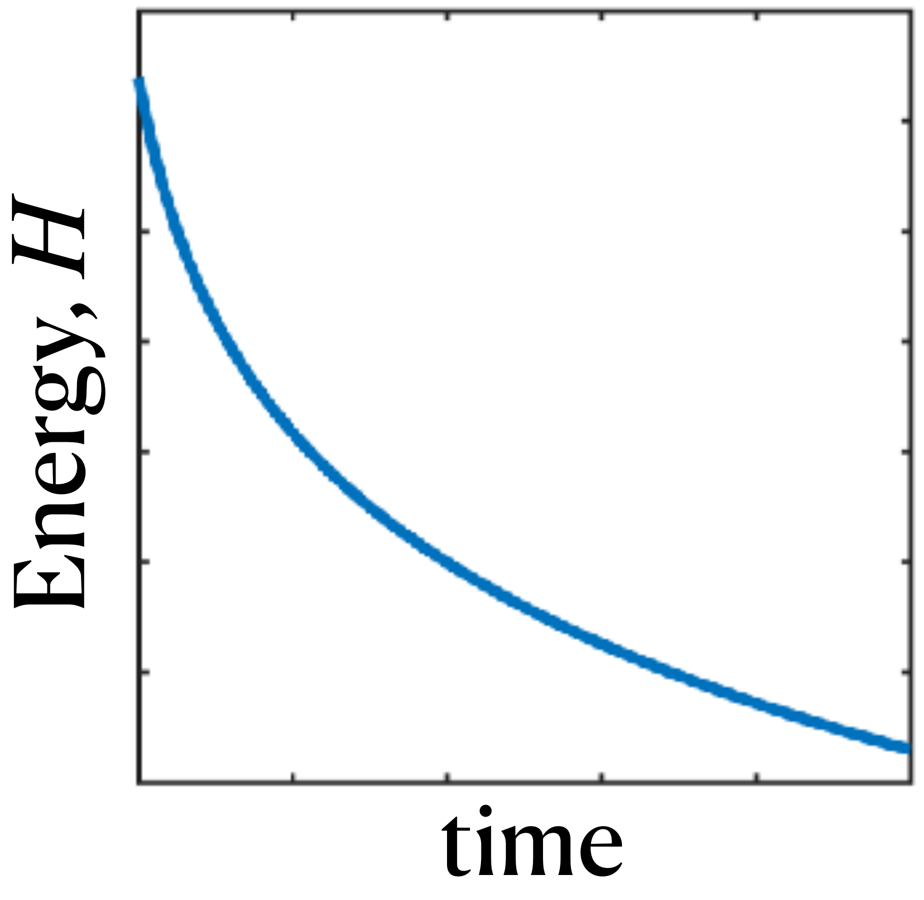
Assume $\nabla\rho = 0$ → $\rho(t) = \dfrac{1}{\rho_0^{-1} + \gamma\Gamma t}$

Enter a **universal scaling** regime

$\langle r\rangle \propto \sqrt{\rho_0^{-1} + \gamma\Gamma t}$

Diffusive growth





$H \propto -\log\left[\rho_0^{-1} + \gamma\Gamma t\right]$

Logarithmic decay

ORS, *et al.*, PRR **2**, 033138 (2020).

Dissipative anomalous hydrodynamics

Rankine vortex is a solution to dissipative vortex fluid theory

Perturbation analysis (with all terms) in universal regime

$$\rho + \delta\rho \to \rho$$

Rankine vortex is a ***stable attractor*** for ***every*** initial condition

$$\langle r \rangle \propto \sqrt{\rho_0^{-1} + \gamma\Gamma t}$$

Diffusive growth

Numerical modelling of vortex dynamics in *excellent agreement*

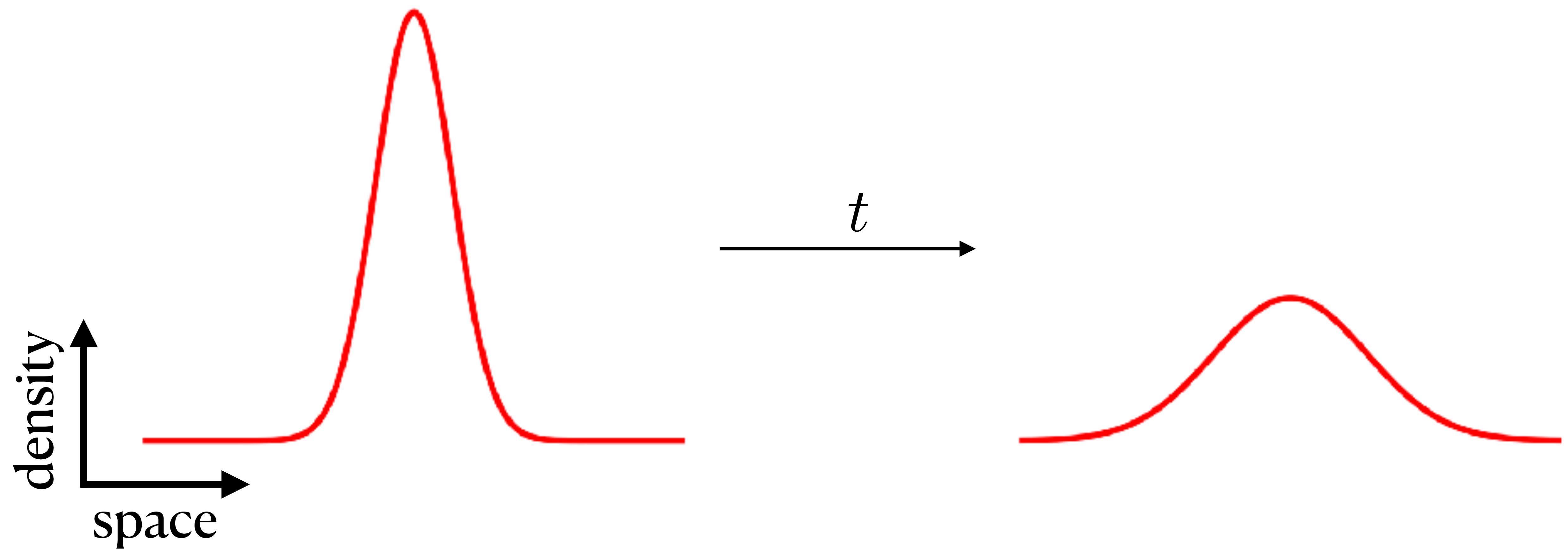
$$H \propto -\log\left[\rho_0^{-1} + \gamma\Gamma t\right]$$

Logarithmic decay

ORS, *et al.*, PRR **2**, 033138 (2020).

Why is this significant?

Classical fluids are ***viscous***, and so they **dissipate energy differently**



Symmetric vortices in classical fluids expand to form a *Lamb-Oseen* vortex (Gaussian profile)

Rankine vortex is forbidden in classical fluids!

Experimental comparison

Experiment

We use a quasi-2D ^{87}Rb Bose-Einstein condensate

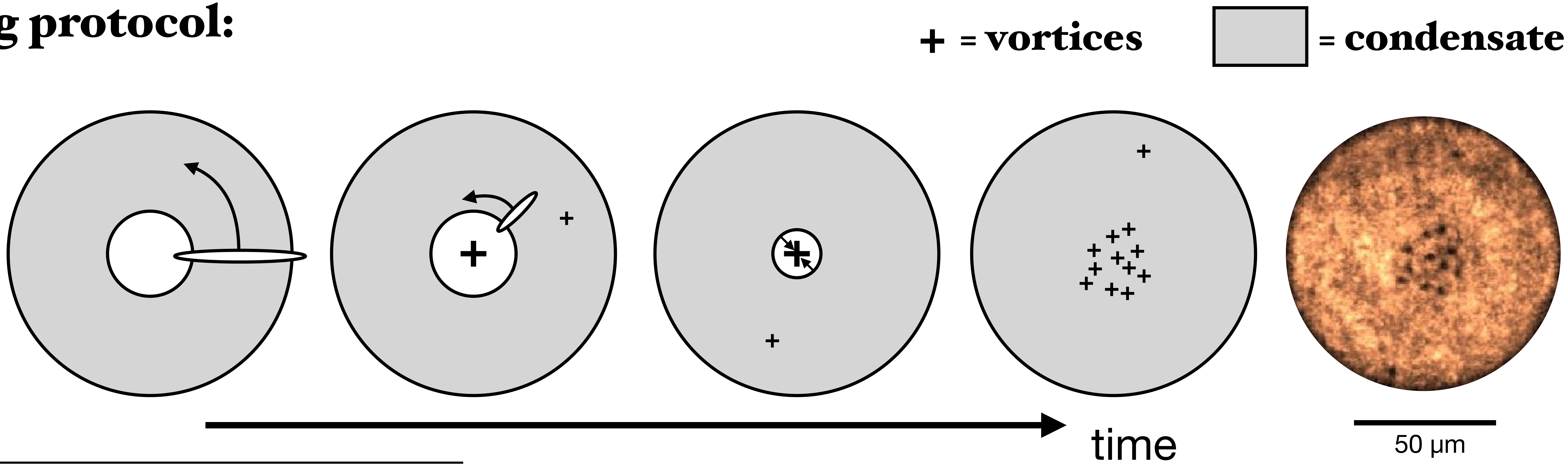
Why?

- High degree of control and precision measurements
- Routine injection/imaging of vortices can be achieved

On average, the vortex cluster has N~11 vortices

We take ~40 samples for each hold time

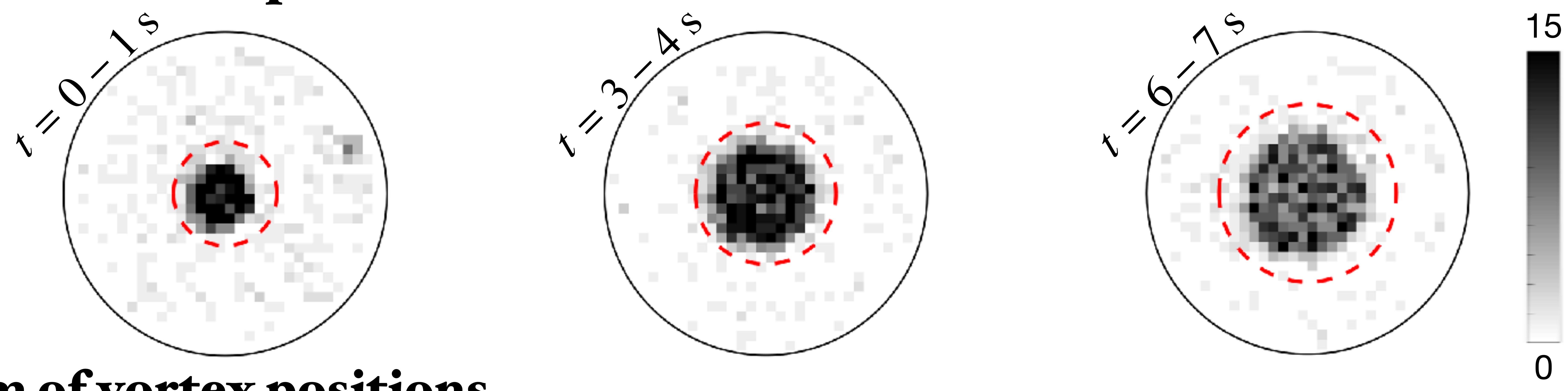
Stirring protocol:



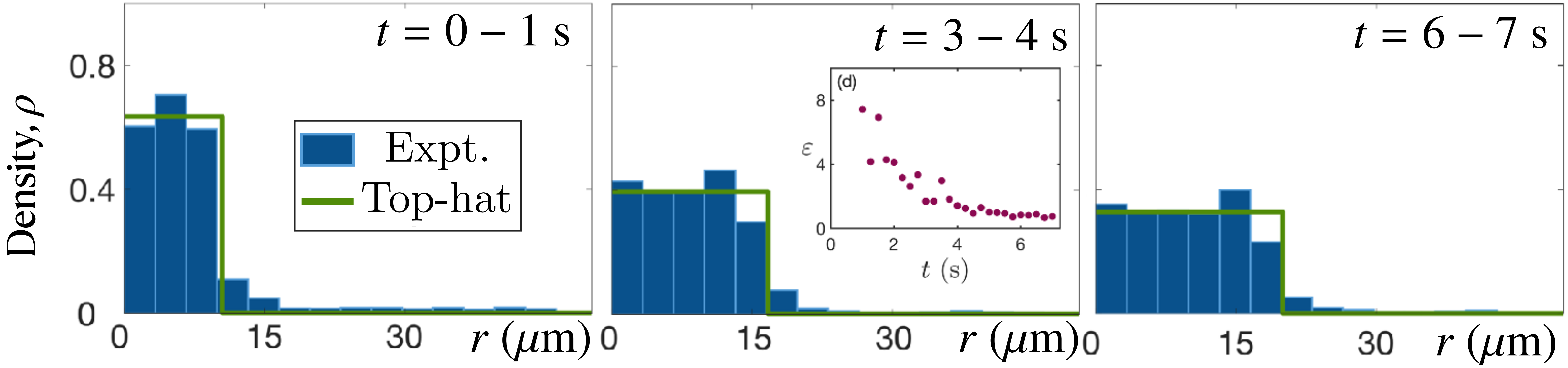
ORS, *et al.*, PRR **2**, 033138 (2020).

Experimental expansion

2D histogram of vortex positions



1D histogram of vortex positions



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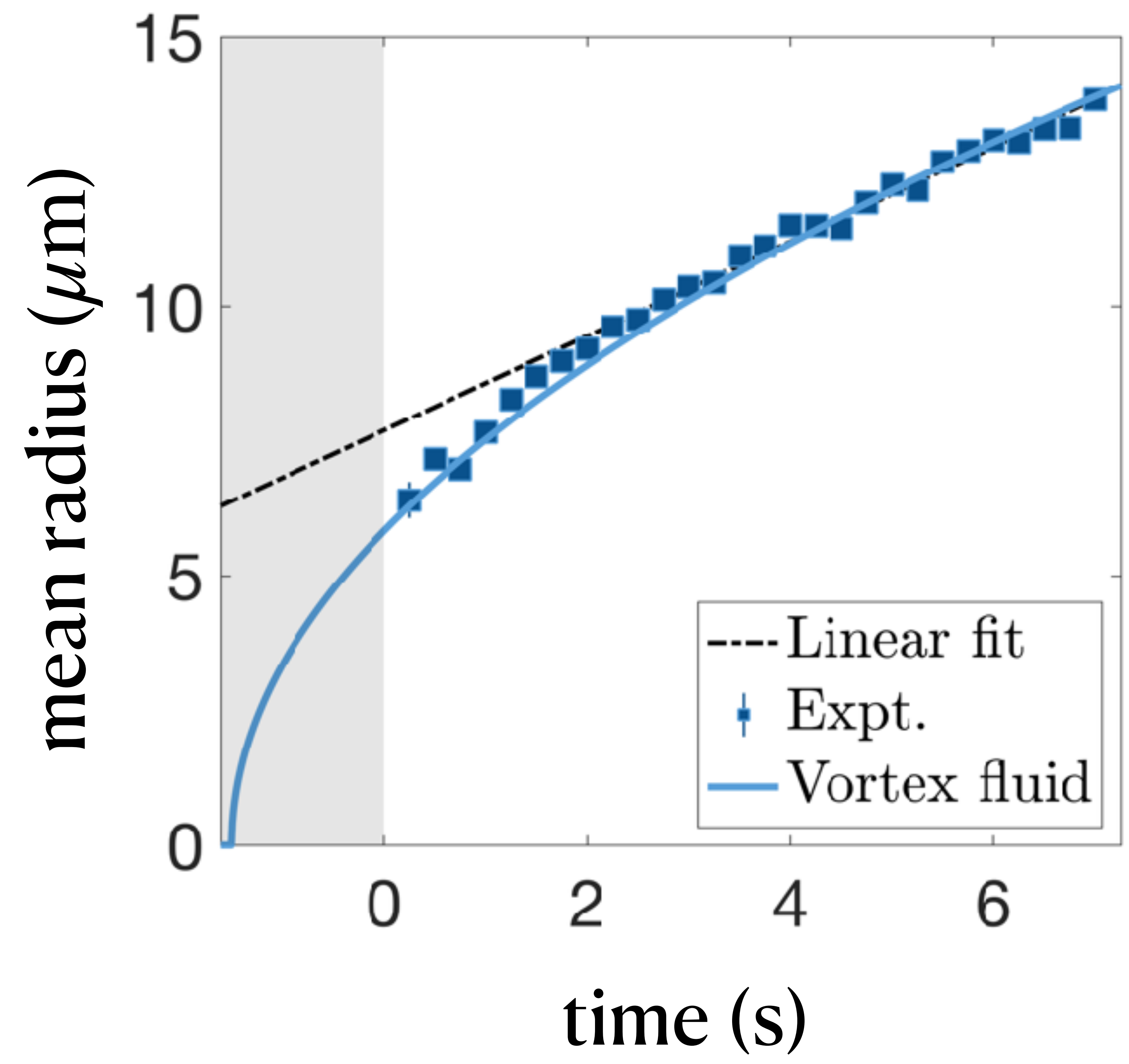
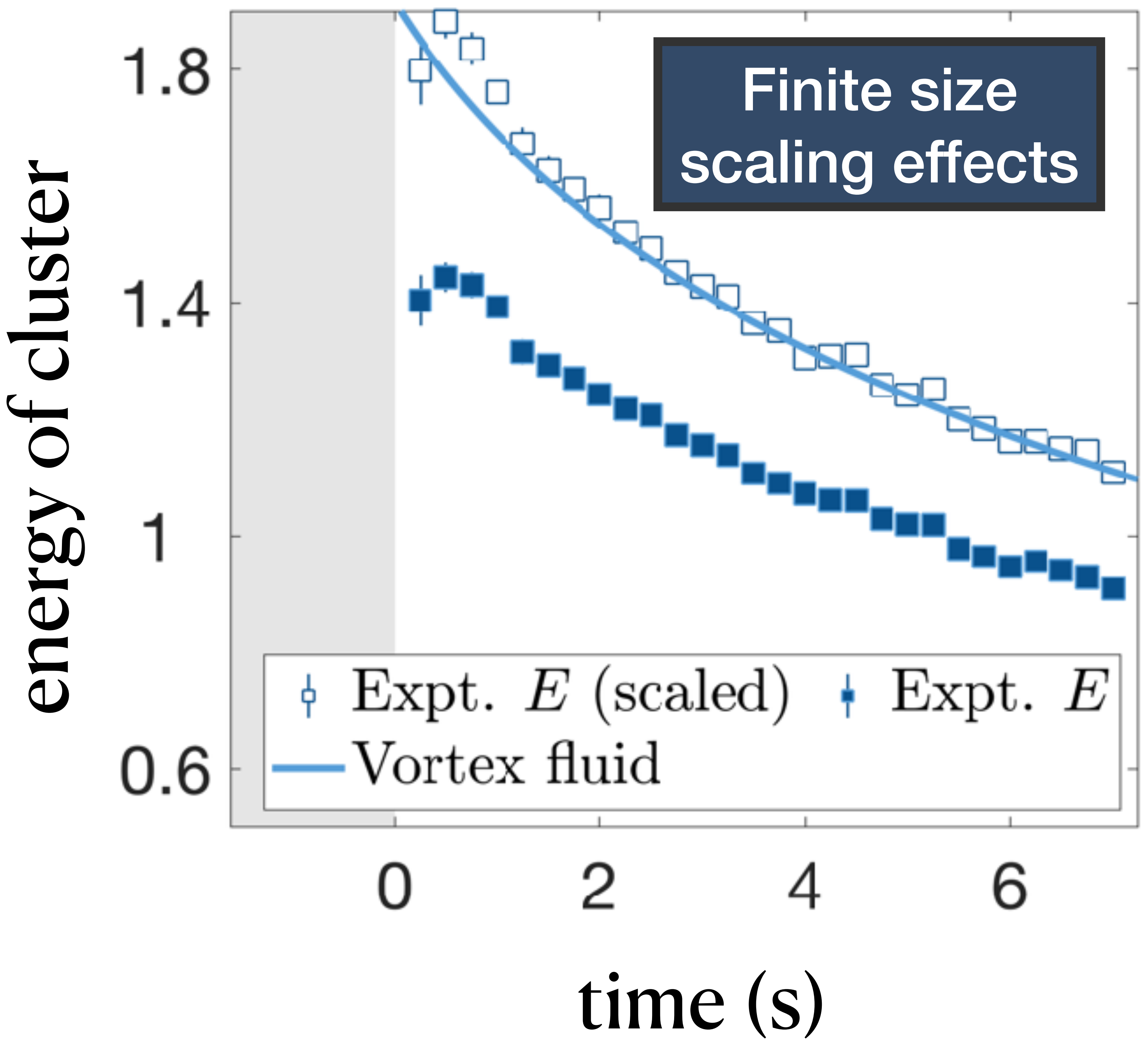
Experimental parameters





ORS, *et al.*, PRR **2**, 033138 (2020).

Conclusions

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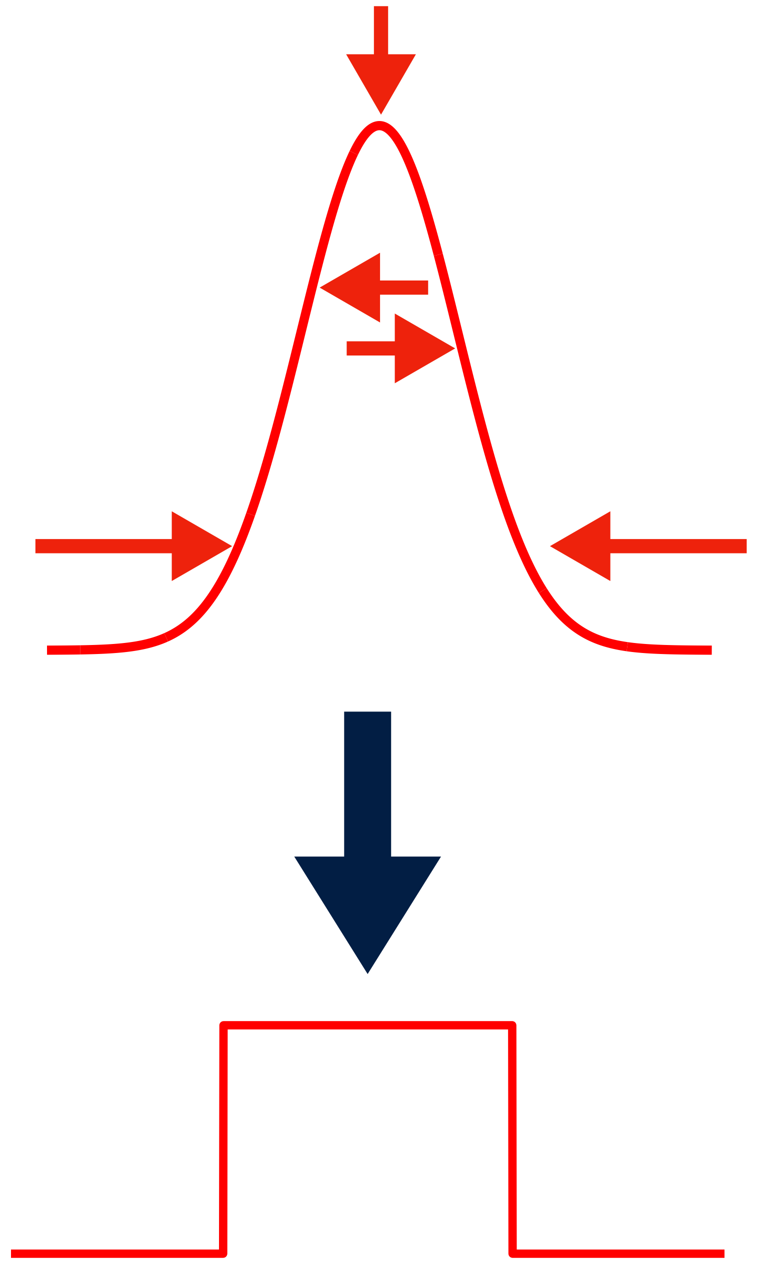
Theory predicts universal expanding regime

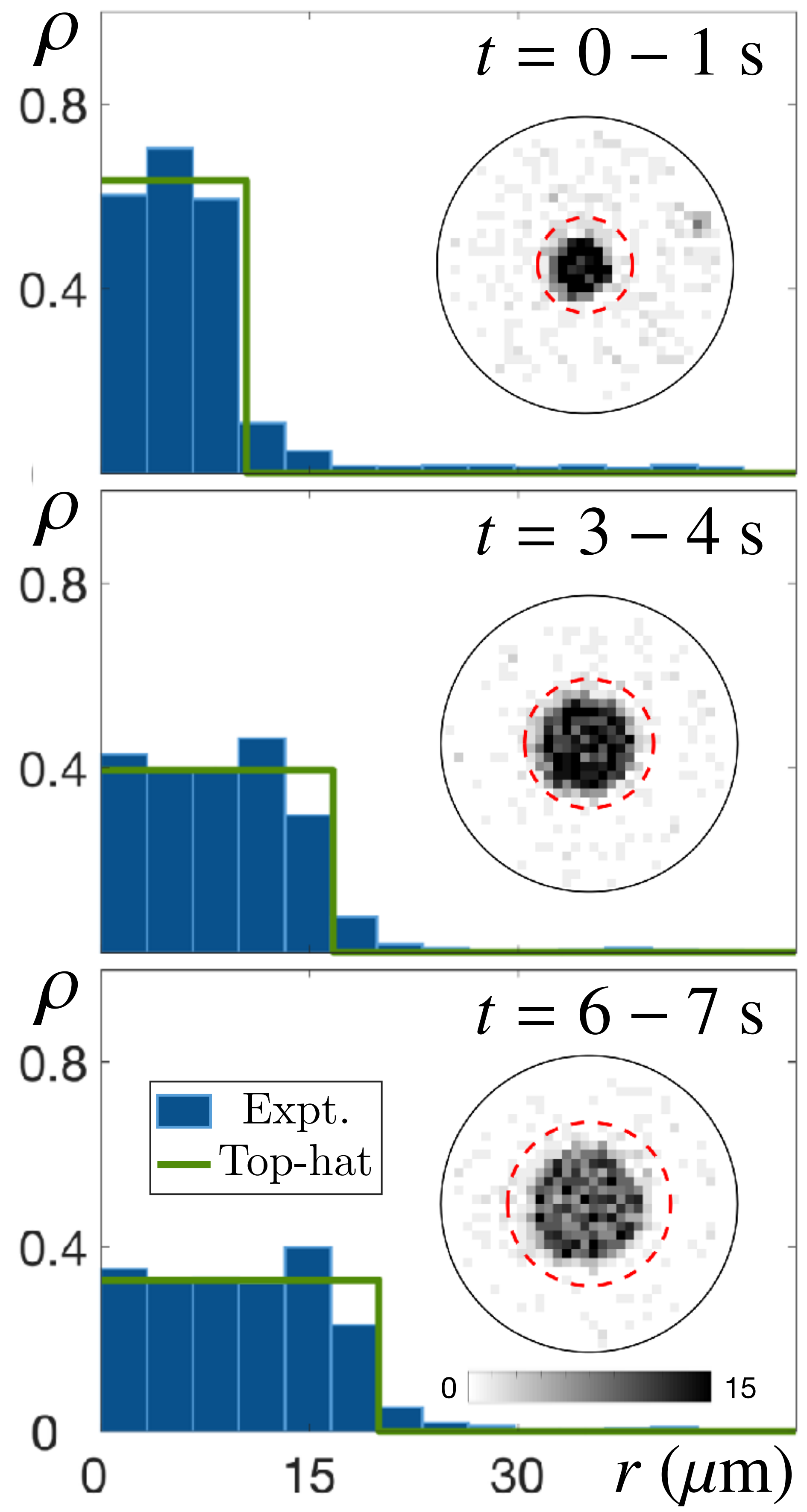
$$\mathcal{D}_t^v \rho = -\gamma\left(\Gamma\rho^2 + \frac{\Gamma}{8\pi}\nabla^2\rho - \mathbf{v}\times\nabla\rho\right)$$

$$\mathcal{D}_t^v = \partial_t + \left(\mathbf{v} - \frac{\gamma\Gamma}{8\pi}\nabla\rho\right)\cdot\nabla$$

$$\partial_t \rho = -\gamma\Gamma\rho^2$$



Experimental evidence to support



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Conclusions

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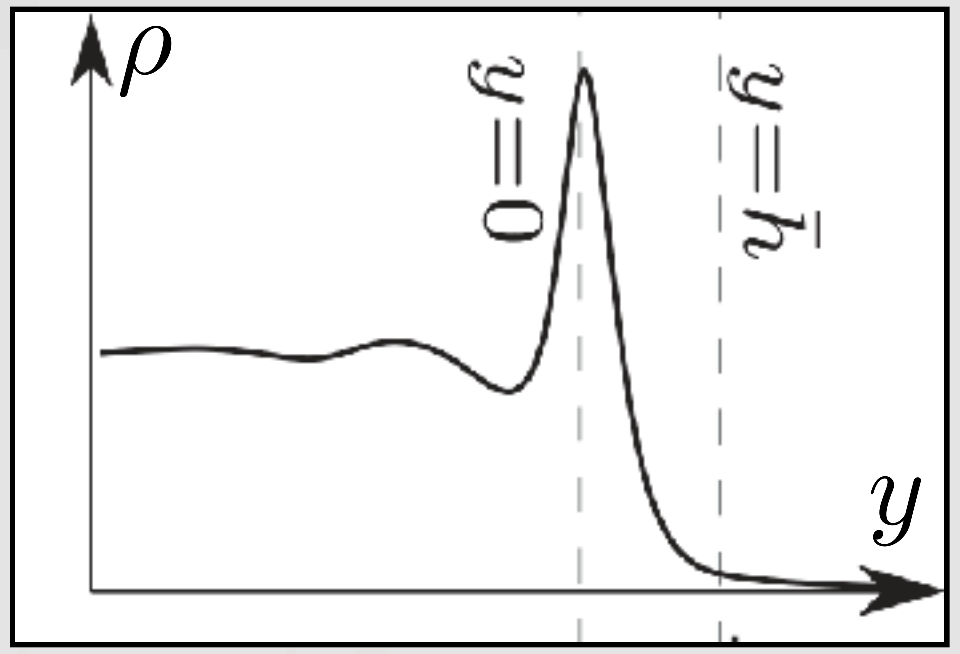
Outlook

Edge waves



Edge solitons in Rankine vortex

Bogatskiy & Wiegmann PRL **122**, 214505 (2019).

Further experiments

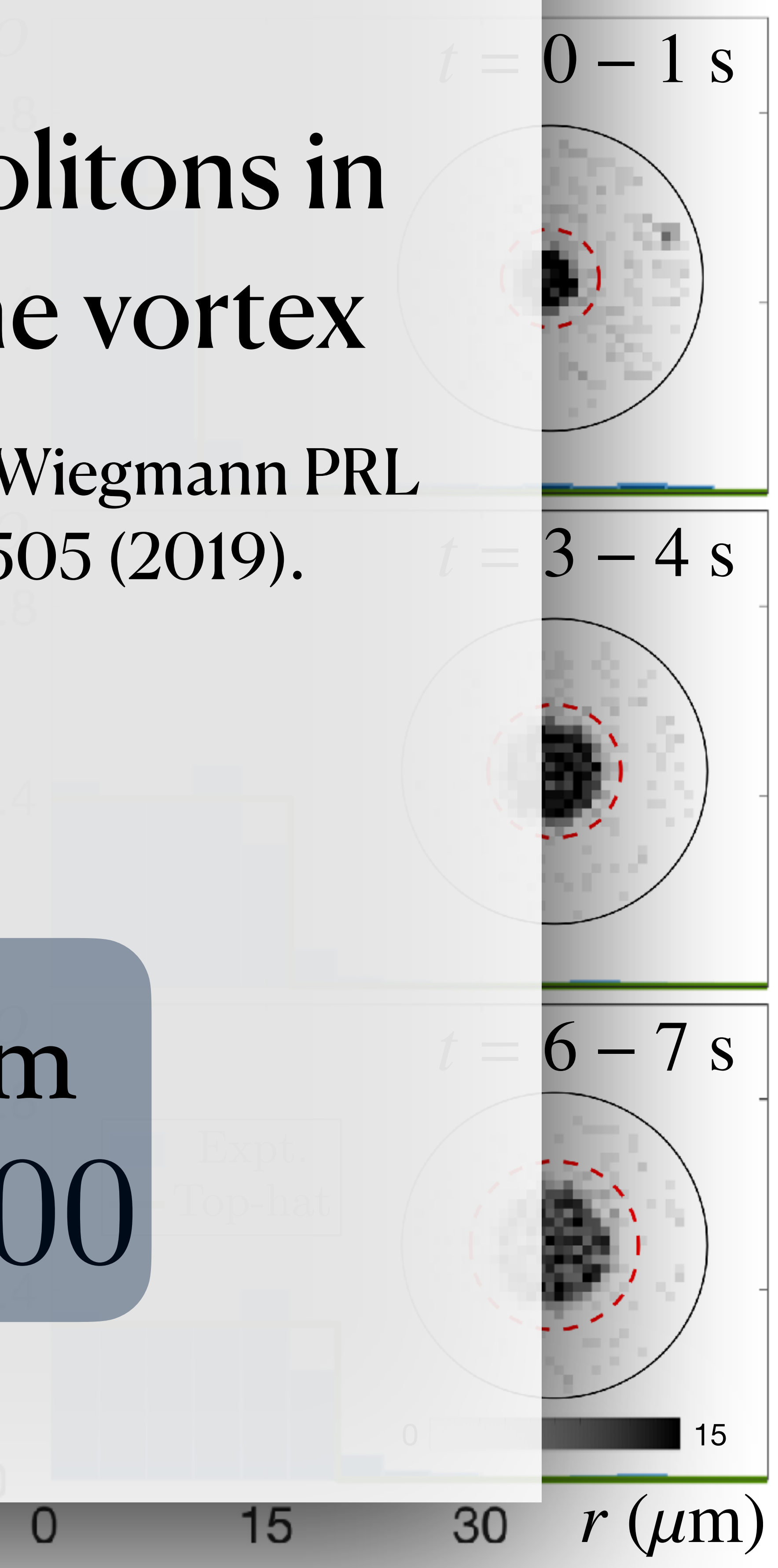
Fermi Gas	SF Helium
$N \sim 50$	$N \sim 1000$

Experimental evidence to support

$$\mathcal{D}_t^v \rho = -\gamma \left(\Gamma \rho^2 + \ldots \right.$$

$$\mathcal{D}_t^v = \partial_t + \left(\mathbf{v} \ldots \right.$$

$$\partial_t \rho = \ldots$$



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