Entanglement detection via entropies in spinor Bose gases

Oliver Stockdale

Kirchhoff Institut für Physik, Universität Heidelberg

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

Entanglement in Quantum Fields Workshop

29 June 2021

isoQuant SFB1225

@GaerrtnerGroup mbqd.de

Acknowledgements

Theory

Martin
Gärttner

Stefan
Floerchinger

Tobi
Haas

Experiments

Philipp
Kunkel

Maximilian
Prüfer

Stefan
Lannig

Robin
Strohmaier

Helmut
Strobel

Markus
Oberthaler

Funding

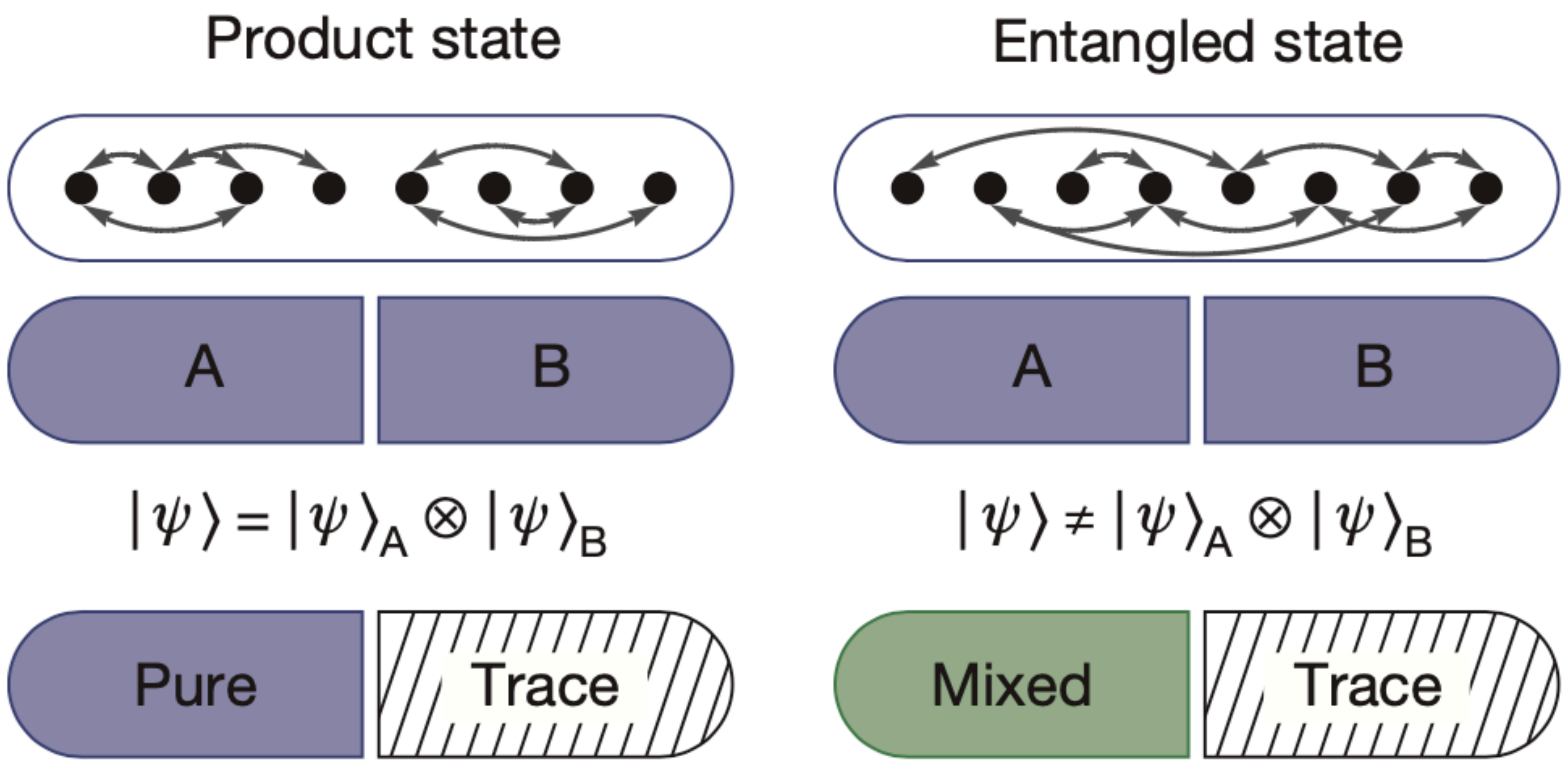
IsoQuant SFB1225

DFG Deutsche Forschungsgemeinschaft

How to measure many-body entanglement?

Partition system, make measurements

With access to **full state** ρ ... not likely



With access to **uncertainties**

$$\langle(\Delta\hat{u})^2\rangle_\rho + \langle(\Delta\hat{v})^2\rangle_\rho \geq a^2 + \frac{1}{a^2}$$

Violation flags entanglement

With access to **entropies**

$$H(X_A\,|\,X_B) + H(Z_A\,|\,Z_B) \geq -\log c + H(A\,|\,B)$$

R. Islam, *et al.*, Nature **528**, 77 (2015).
L. M. Duan, *et al.*, PRL **84**, 2722 (2000).
M. Berta, *et al.*, Nature Physics **6**, 659 (2012)

Outline

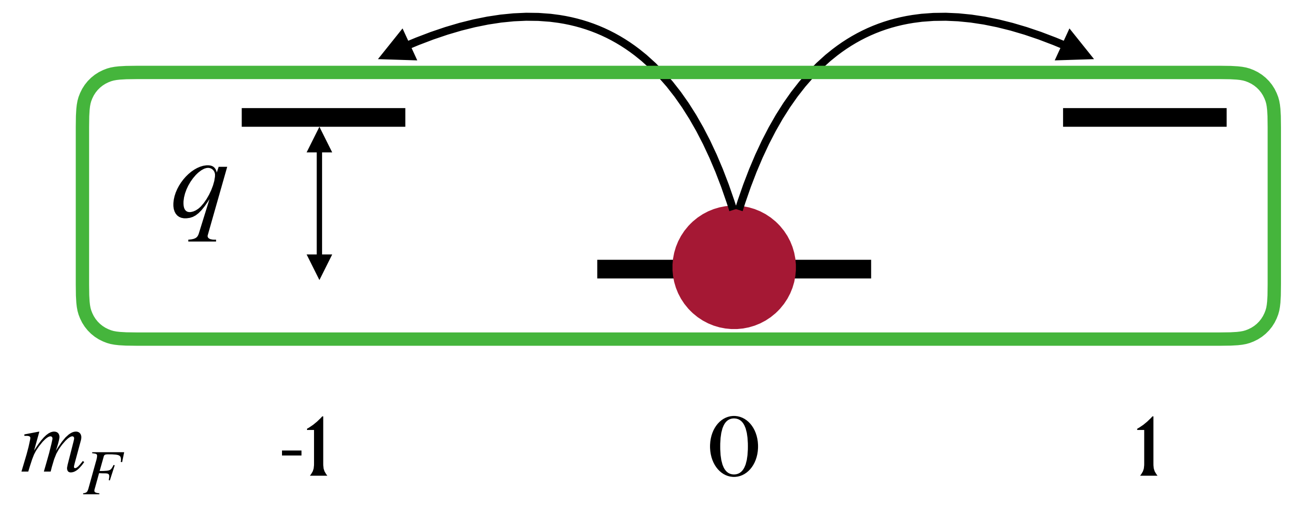
- Introduce the system: ^{87}Rb spin-1 gas
- Experimental readout: how do we measure entropy?
- An entropic entanglement measurement
- Numerical modelling
- Experimental data

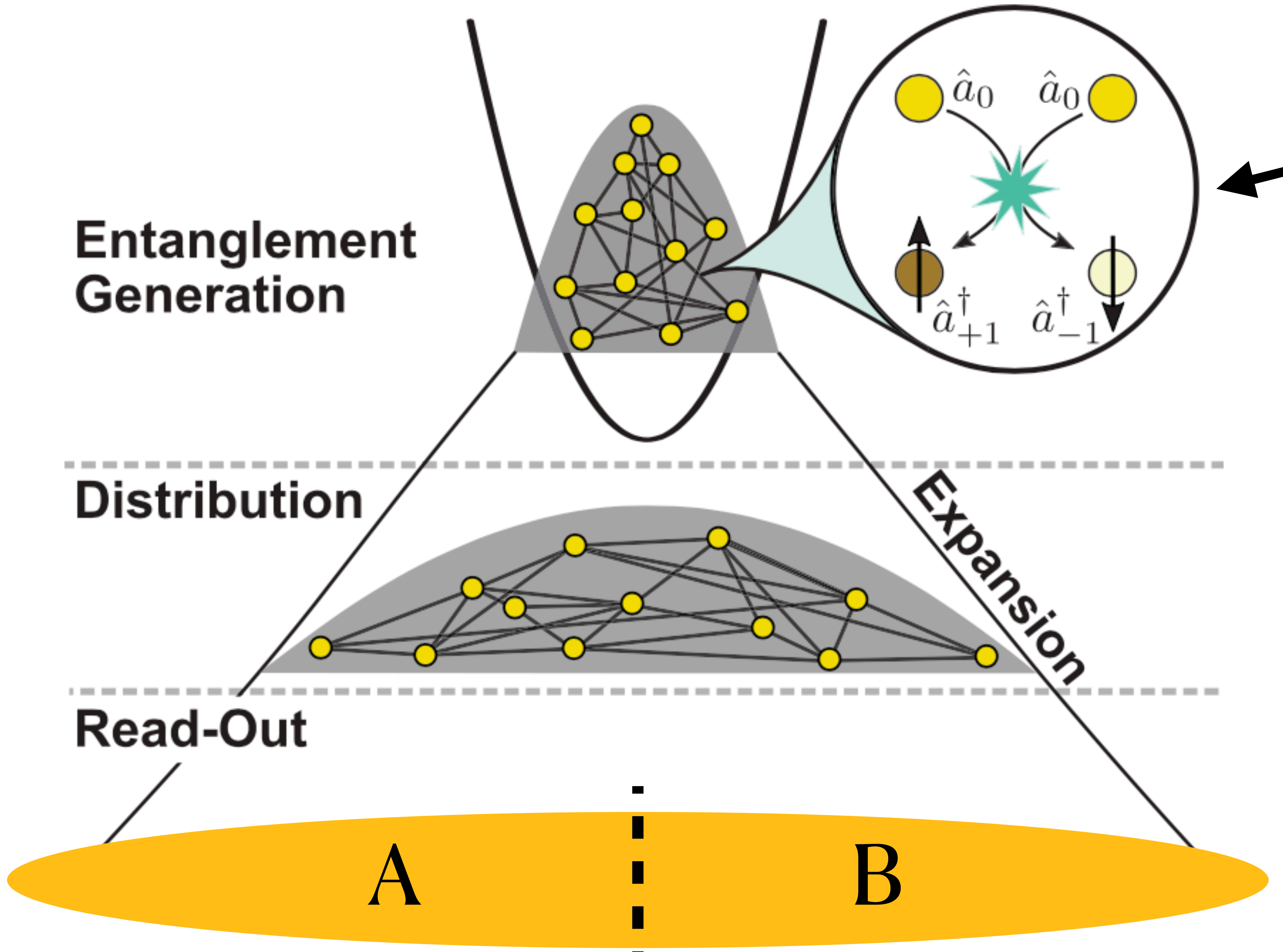
The system

The system

Rubidium-87 BEC in $F = 1$ hyperfine manifold

Characterised by
$|N_{-1}, N_0, N_{+1}\rangle, \quad \hat{a}_{-1}, \hat{a}_0, \hat{a}_{+1}$





Spin-changing collisions

q represents the quadratic Zeeman shift

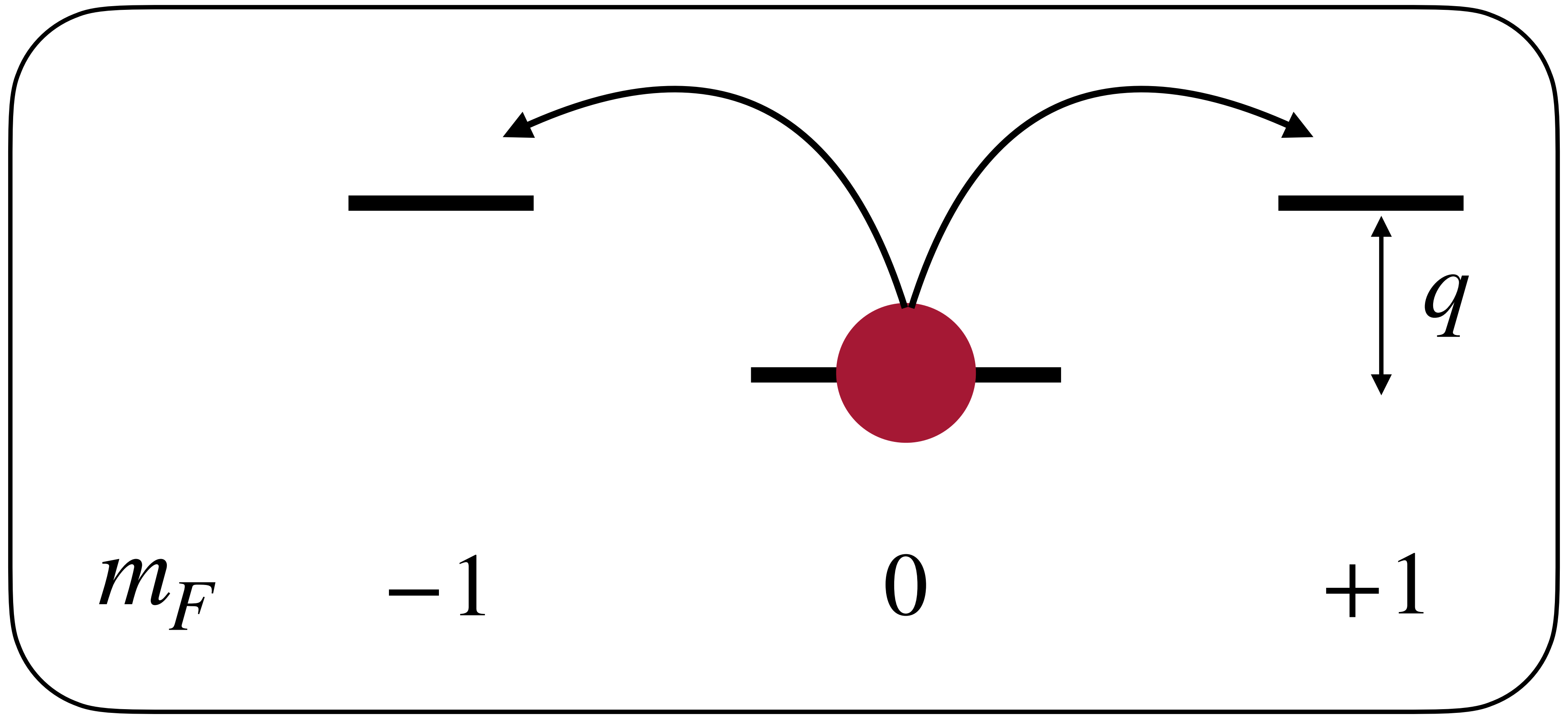
$$H = g[(2\hat{N}_0 - 1)(\hat{N}_{+1} + \hat{N}_{-1}) + \boxed{2\hat{a}_0^\dagger\hat{a}_0^\dagger\hat{a}_1\hat{a}_{-1} + 2\hat{a}_1^\dagger\hat{a}_{-1}^\dagger\hat{a}_0\hat{a}_0}] + \boxed{q(\hat{N}_{+1} + \hat{N}_{-1})}$$

Consider only a single spatial mode (for now)

P. Kunkel, *et al.*, Science **360**, 413 (2018).

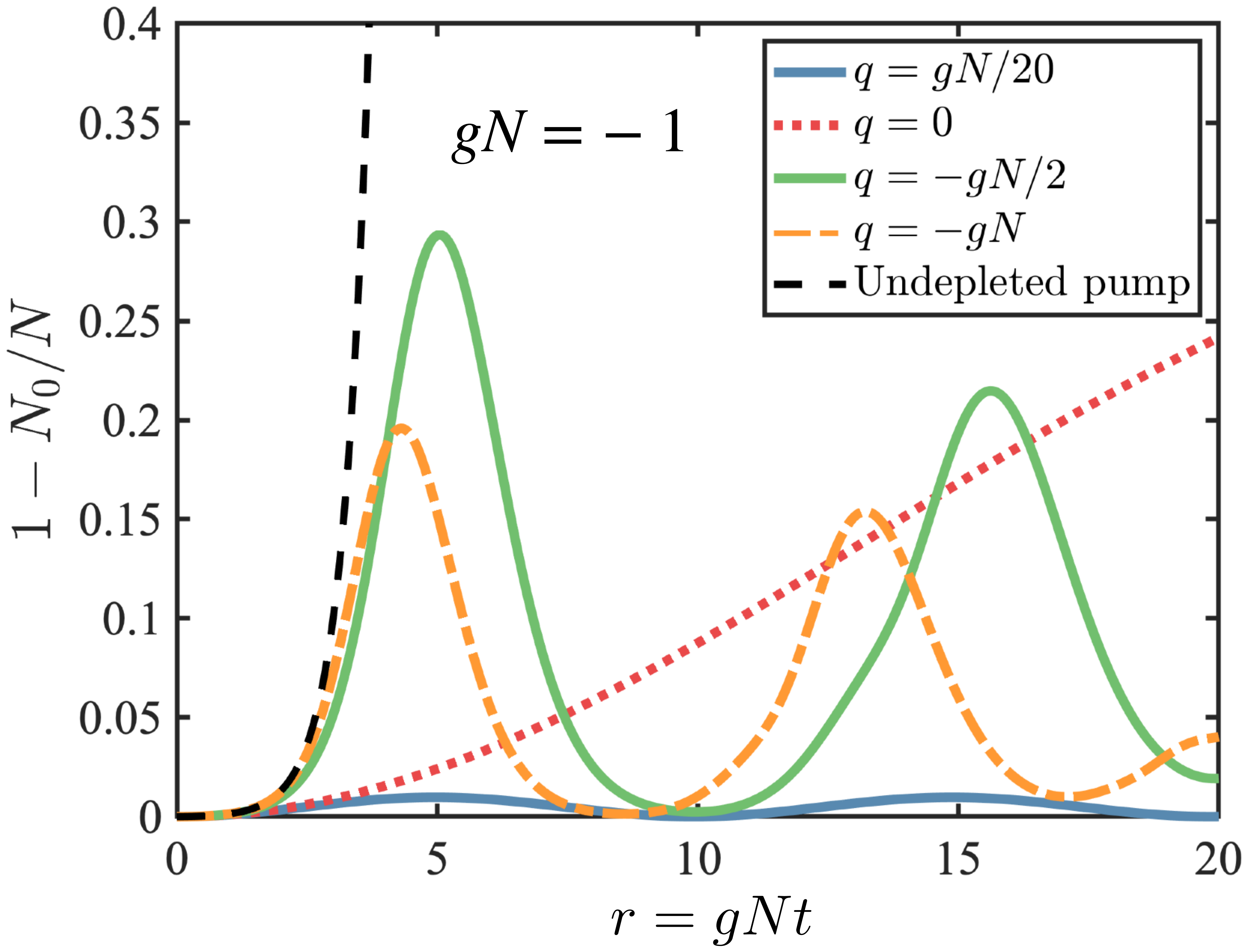
Spin-changing collision dynamics

$$H = g[(2\hat{N}_0 - 1)(\hat{N}_{+1} + \hat{N}_{-1}) + 2\hat{a}_0^\dagger\hat{a}_0^\dagger\hat{a}_1\hat{a}_{-1} + 2\hat{a}_1^\dagger\hat{a}_{-1}^\dagger\hat{a}_0\hat{a}_0] + q(\hat{N}_{+1} + \hat{N}_{-1})$$



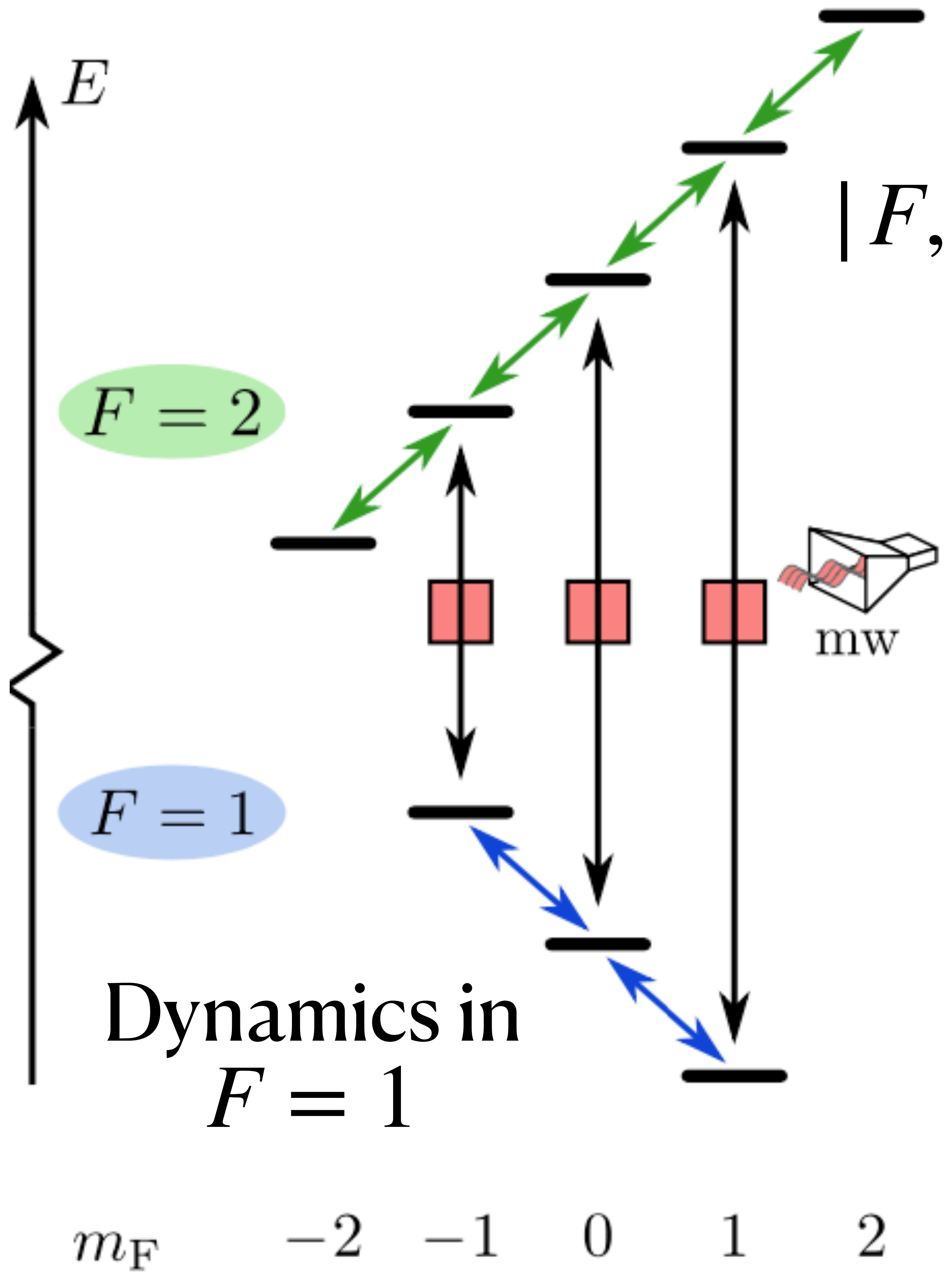
$$|\psi(t=0)\rangle = |0, N, 0\rangle$$

Propagate in time via exact diagonalisation



Measuring entropy: Experimental readout

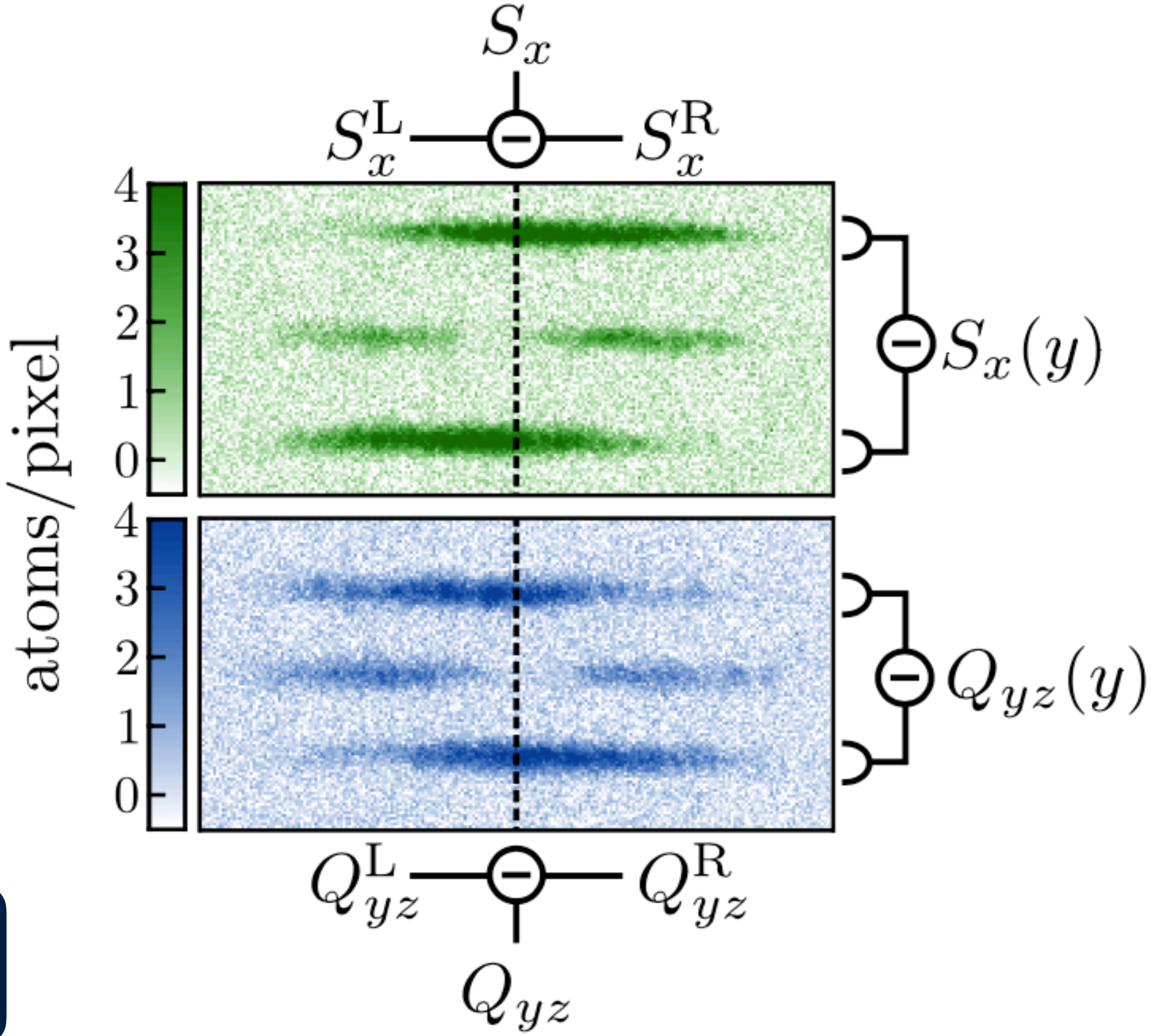
Rubidium-87 has two hyperfine manifolds



Transfer ~50% of atoms to $F = 2$,
simultaneously measure spin-1 observables

No need for state tomography!

fewer measurements

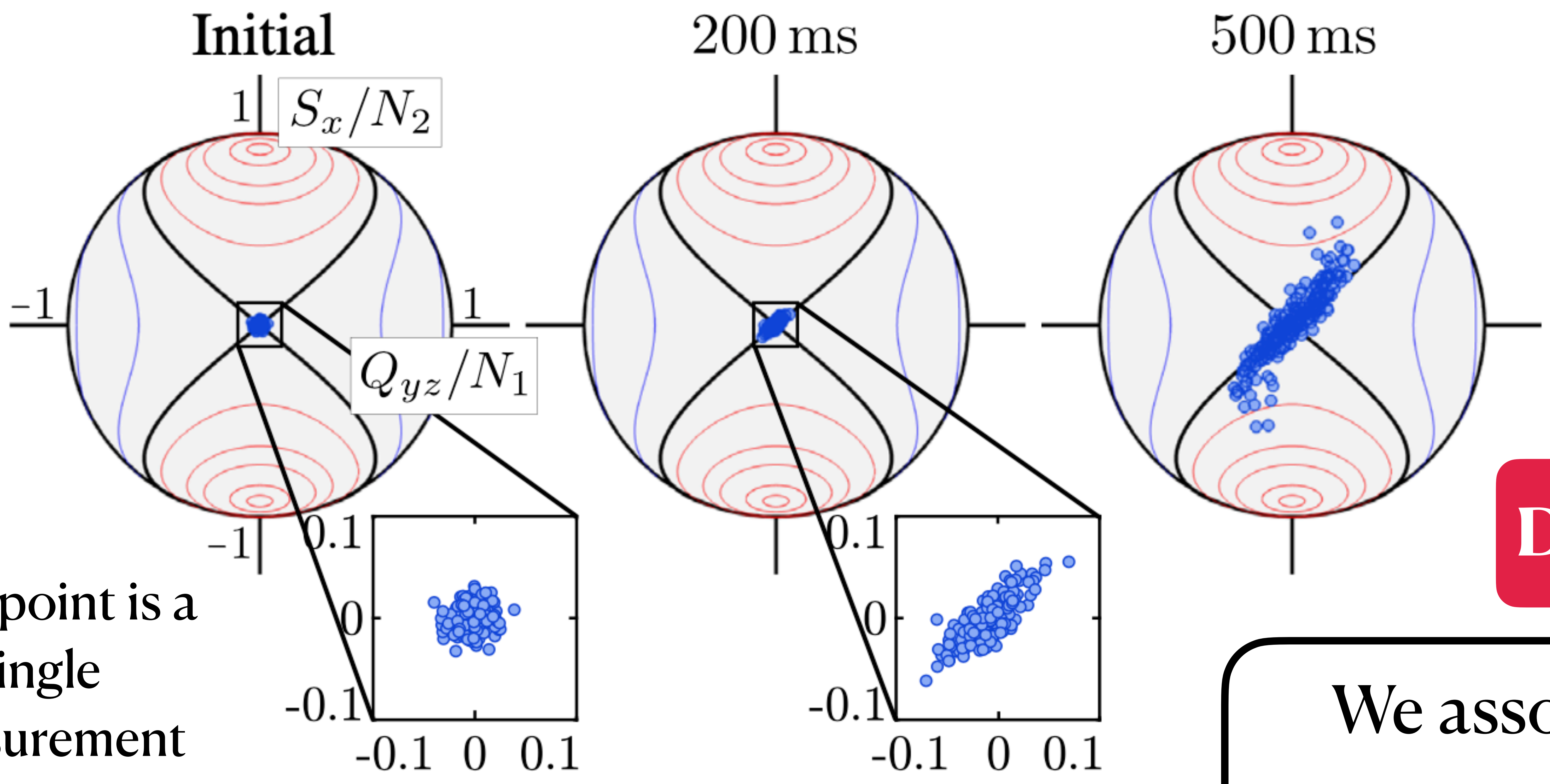


POVM measurement

P. Kunkel, *et al.*, PRL **123**, 063603 (2019).

Quasiprobability distribution

Our spin observables: $\hat{S}_x = \hat{a}_0^\dagger(\hat{a}_{+1} + \hat{a}_{-1})/\sqrt{2} + \text{h.c.}$ $\hat{Q}_{yz} = i\hat{a}_0^\dagger(\hat{a}_{+1} + \hat{a}_{-1})/\sqrt{2} + \text{h.c.}$



Each point is a single measurement

What **information** can we **extract** from this distribution?

Distribution is:
- Non-negative
- Normalisable

DISCLAIMER

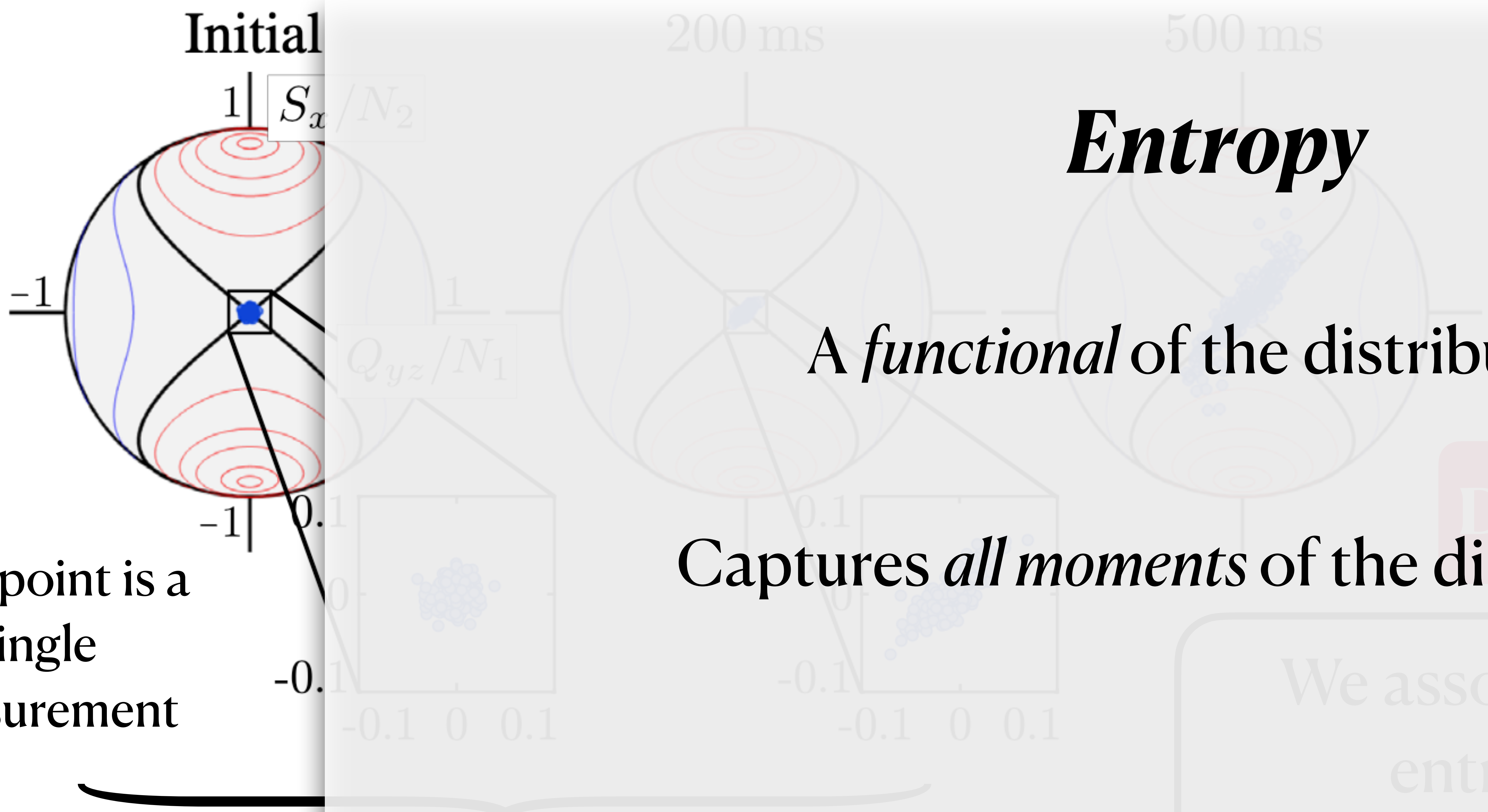
We associate an **entropy** S_W (called Wehrl entropy) to the distribution Q_ρ via

$$S_W(\rho) = -\int \frac{dS_x dQ_{yz}}{2\pi} Q_\rho(S_x, Q_{yz}) \ln Q_\rho(S_x, Q_{yz})$$

P. Kunkel, *et al.*, PRL **123**, 063603 (2019).

Quasiprobability distribution

Our spin observables: $\hat{S}_x = \hat{a}_0^\dagger(\hat{a}_{+1} + \hat{a}_{-1})/\sqrt{2} + \text{h.c.}$ $\hat{Q}_{yz} = i\hat{a}_0^\dagger(\hat{a}_{+1} + \hat{a}_{-1})/\sqrt{2} + \text{h.c.}$



Each point is a single measurement

Entropy

A *functional* of the distribution

Captures *all moments* of the distribution

What **information** can we **extract** from this distribution?

Distribution is:
- Non-negative
- Normalisable

DISCLAIMER

We associate an **entropy** S_W (called Wehrl entropy) to the distribution Q_ρ via

$$S_W(\rho) = -\int \frac{\mathrm{d}S_x \mathrm{d}Q_{yz}}{2\pi} Q_\rho(S_x, Q_{yz}) \ln Q_\rho(S_x, Q_{yz})$$

P. Kunkel, *et al.*, PRL **123**, 063603 (2019).

Witnessing correlations

$$S_W = -\int \frac{\mathrm{d}S_x\,\mathrm{d}Q_{yz}}{2\pi N}\; Q_\rho(S_x, Q_{yz}) \log Q_\rho(S_x, Q_{yz})$$

Introduce **Wehrl mutual information**

$$I_W(\rho_A : \rho_B) = S_W(\rho_A) - S_W(\rho_A \,|\, \rho_B)$$

Measures *total* correlations, inc. classical!

Wehrl mutual information is a *perfect* witness for pure states

S. Floerchinger, *et. al*, PRA **103**, 062222 (2021).

Lower bound on quantum MI

$$I_W(\rho_A : \rho_B) \leq I(\rho_A : \rho_B)$$

Regardless of squeezing parameter, POVM measurement **bounds quantum MI**

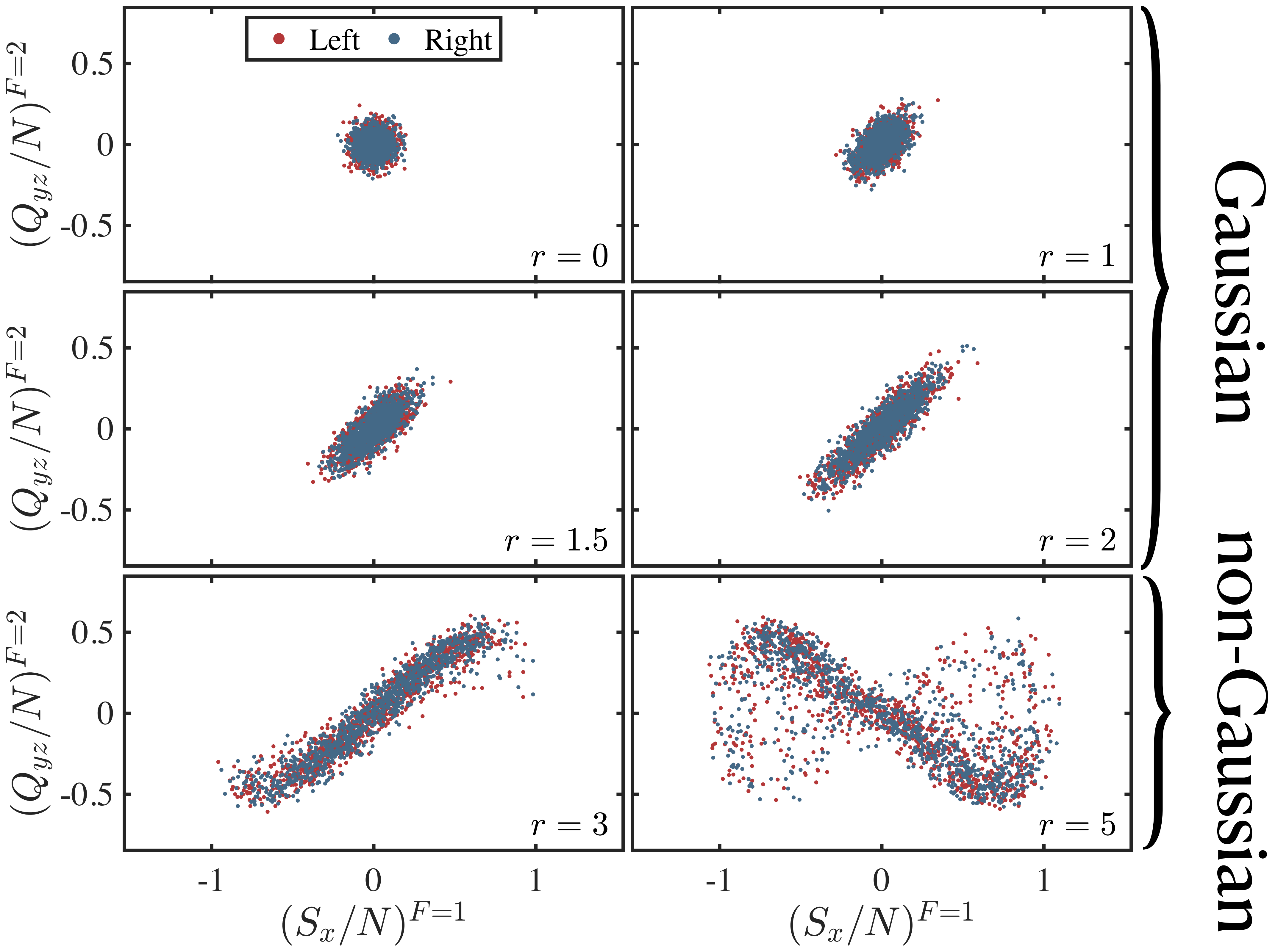
$$I_{\mathrm{POVM}}(\rho_A : \rho_B) \leq I_Q(\rho_A : \rho_B)$$

E. H. Lieb and R. Seiringer, PRA **71**, 062329 (2005).

Theoretical modelling

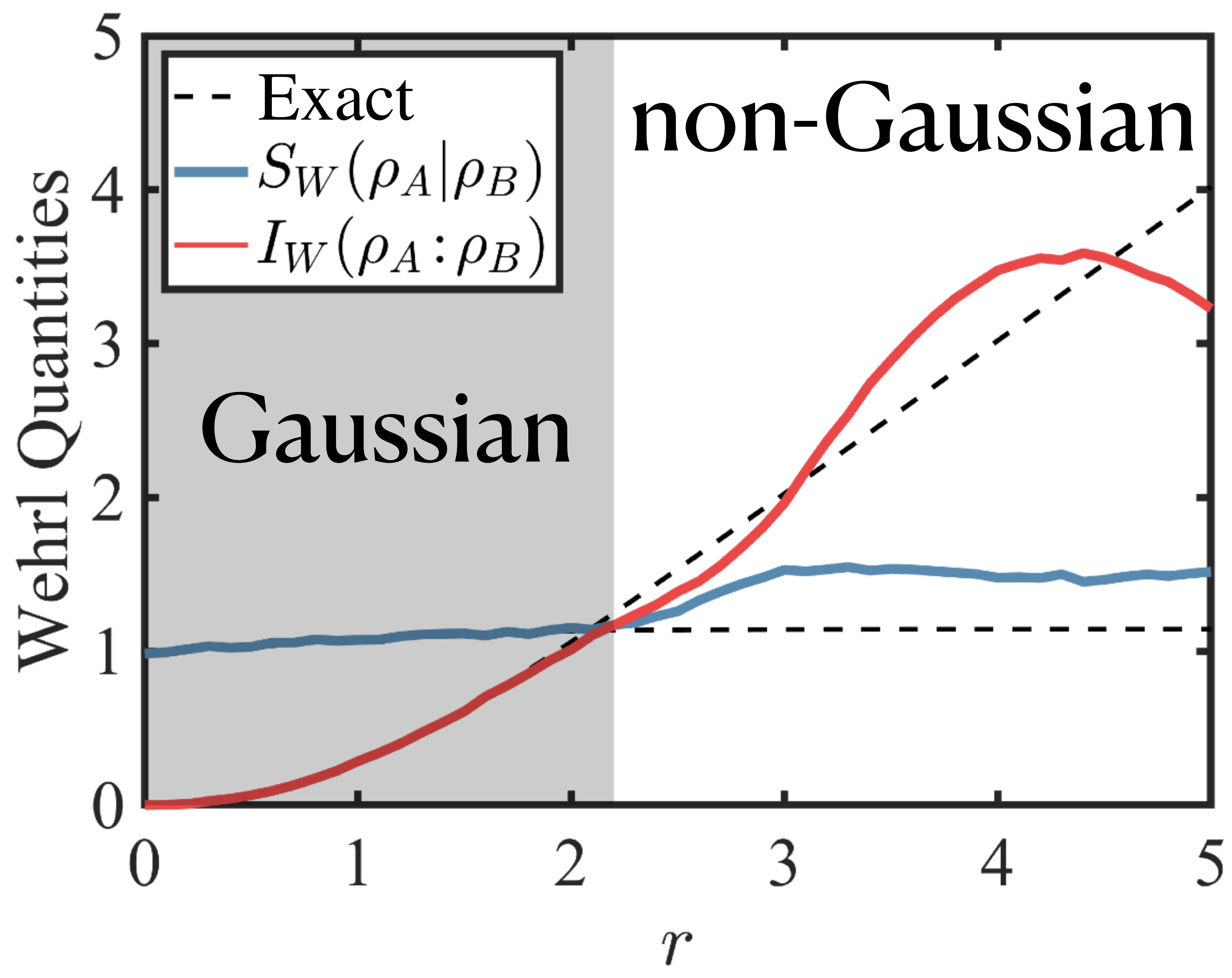
Time dynamics of correlations

Truncated Wigner result



Numerical simulation of experimental readout

Measuring variances



$$I_W(\rho_A:\rho_B) = \frac{1}{2}\frac{\det C_A \det C_B}{\det C}$$

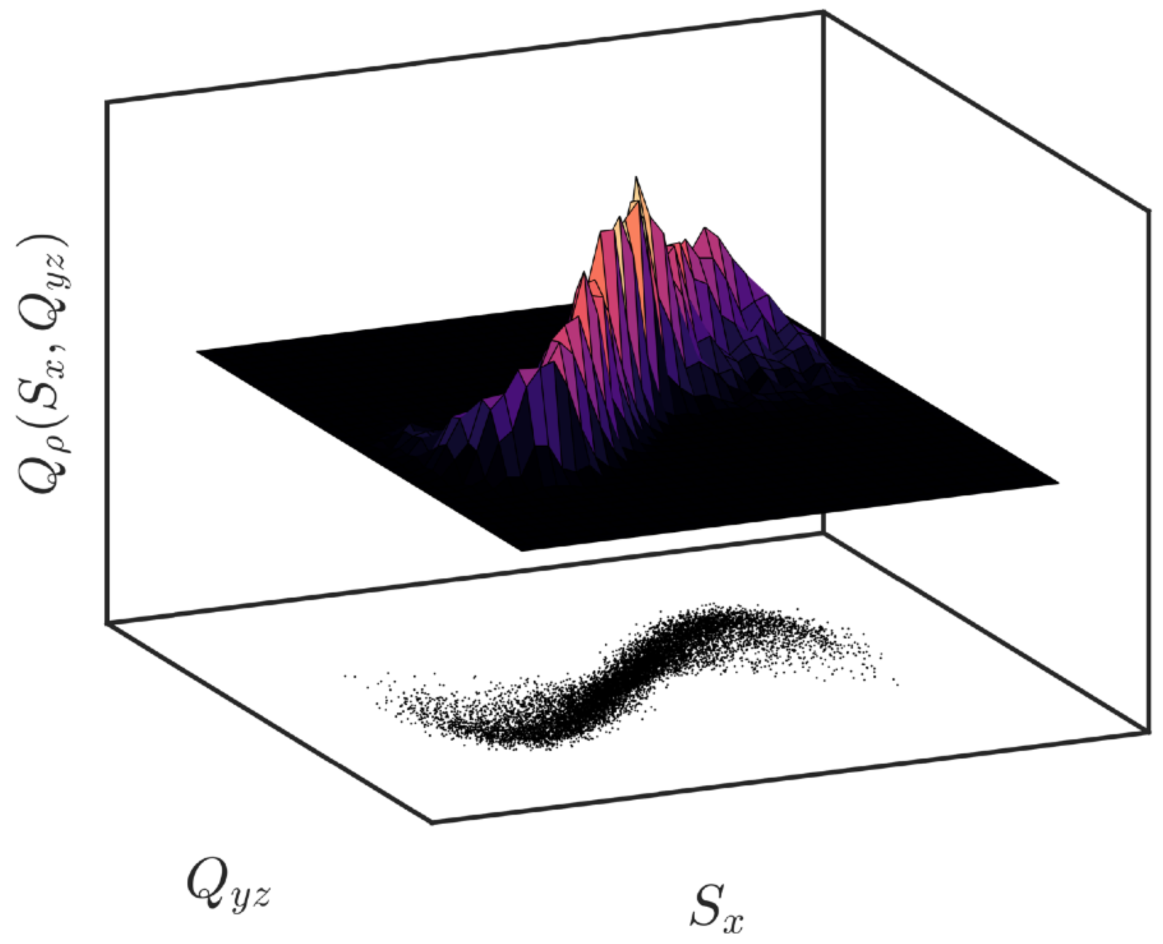
C = inverse cov. matrix

Recall, entanglement witnessed if

$$I_W > 0$$

Density estimation

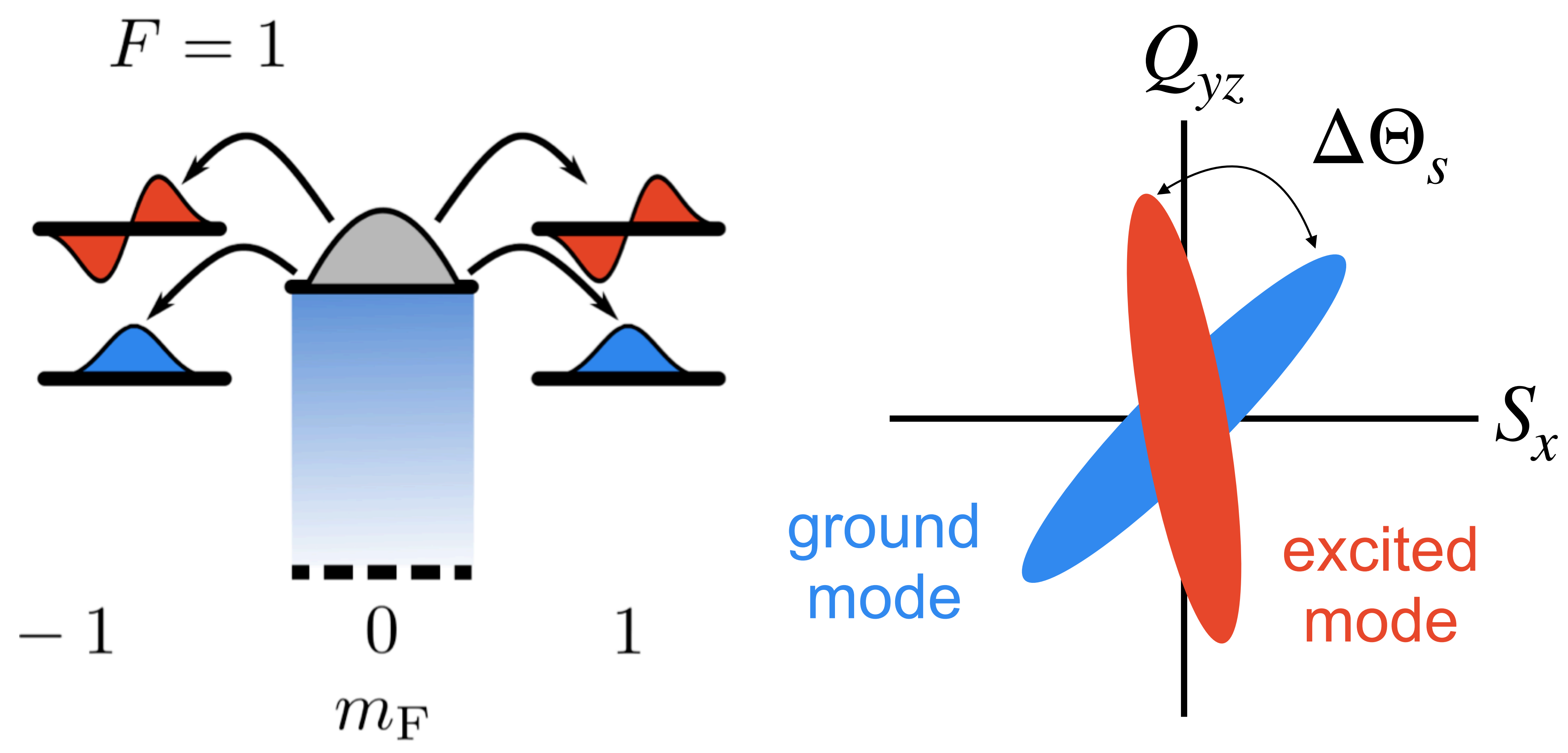
How to accurately measure probability density function?



Experimental analysis

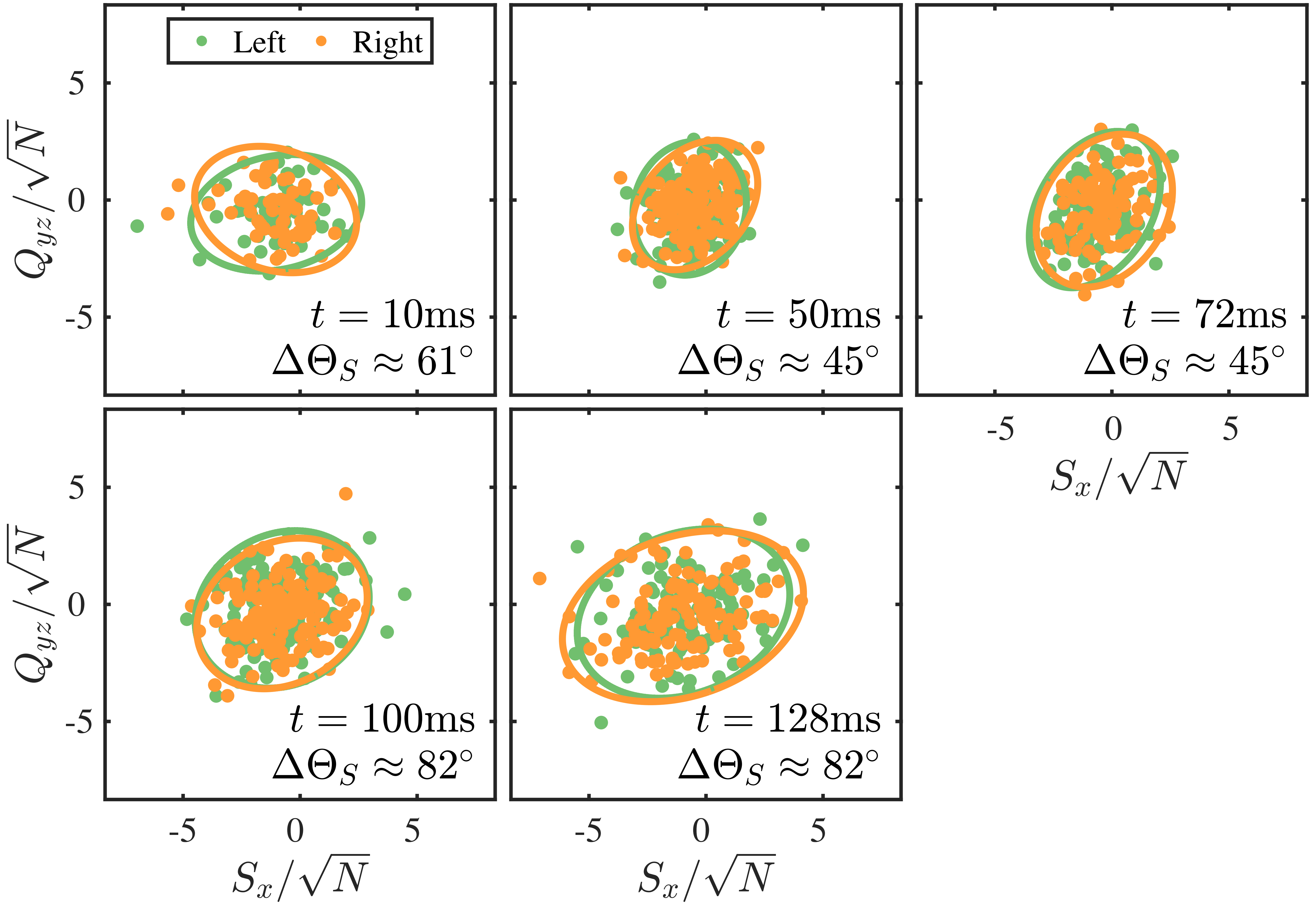
Experimental results

Two spatial modes are excited

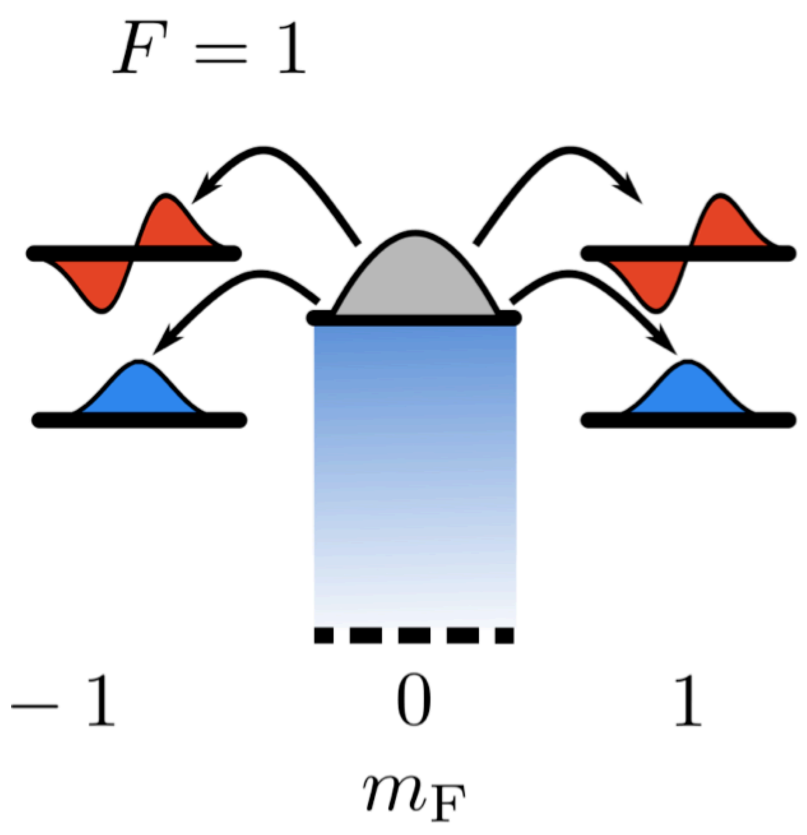


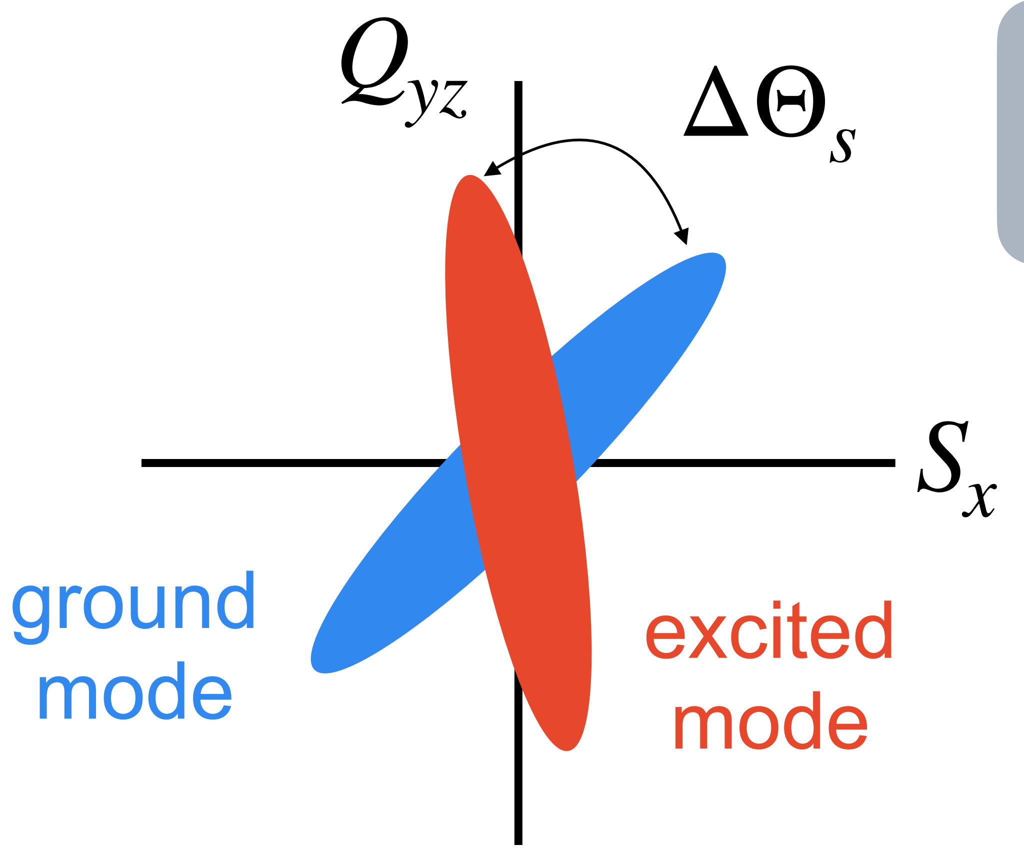
Different behaviour with spatially excited modes?

Total signal, i.e., both modes together



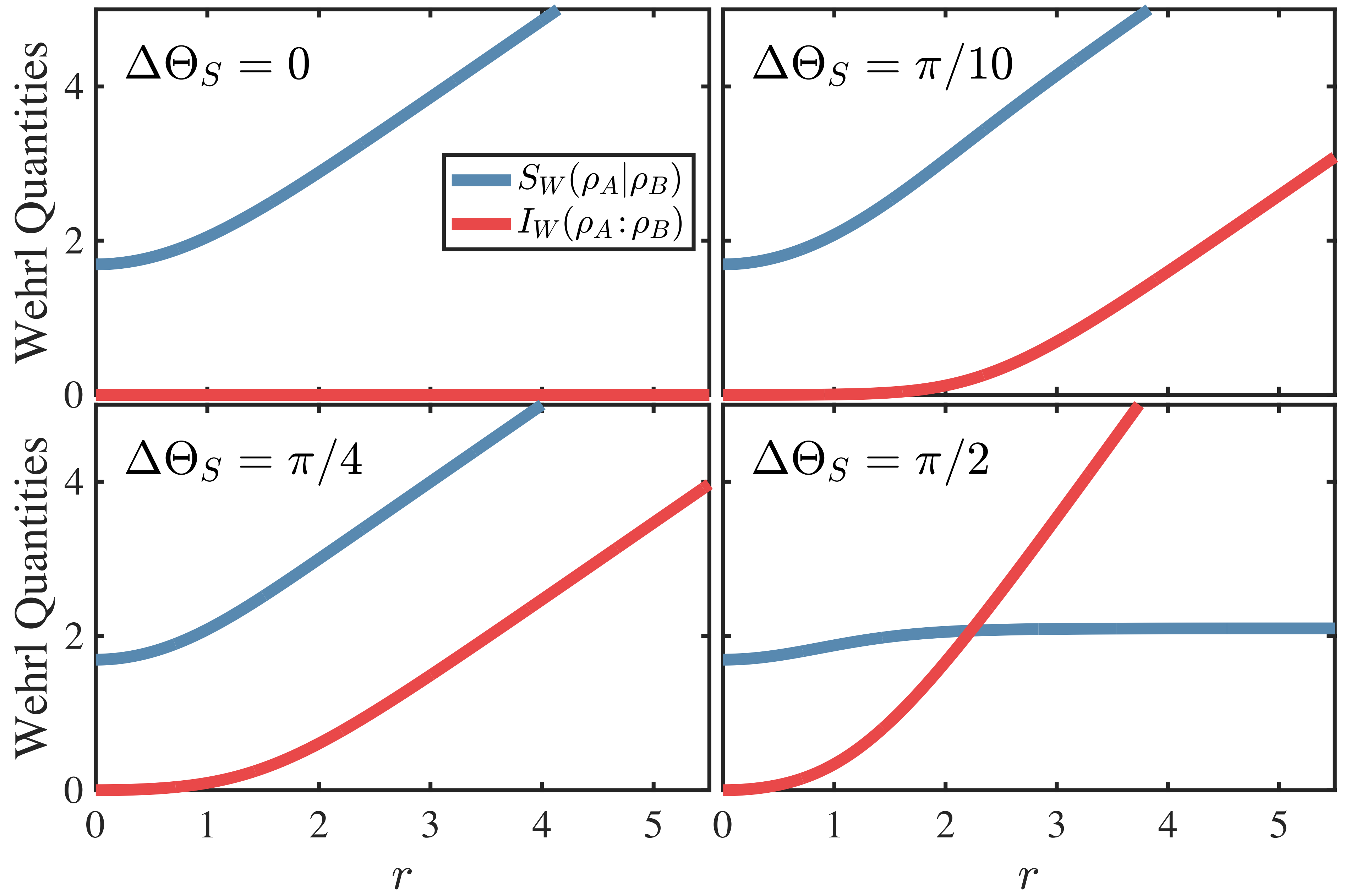
Experimental results

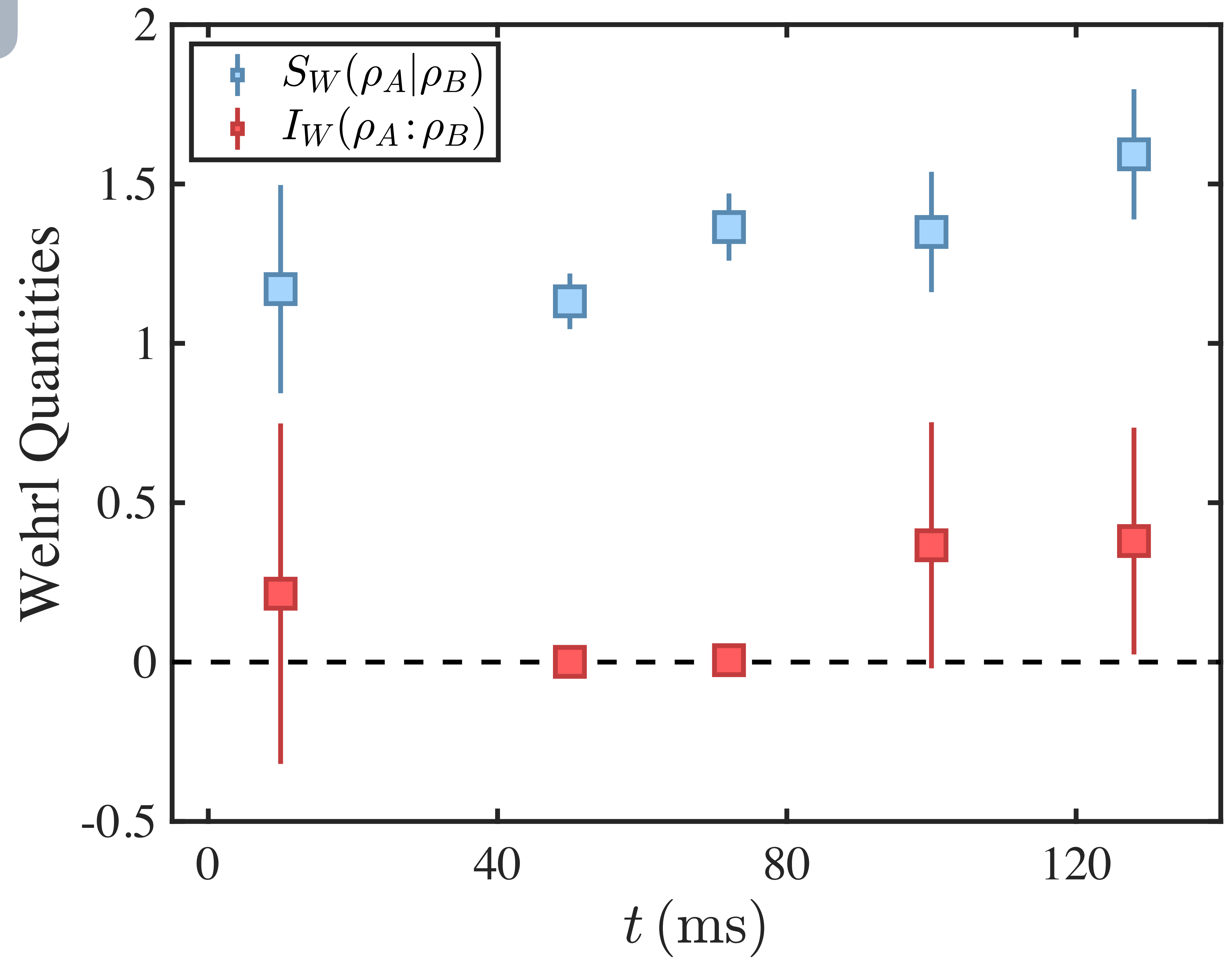




$I_W > 0$

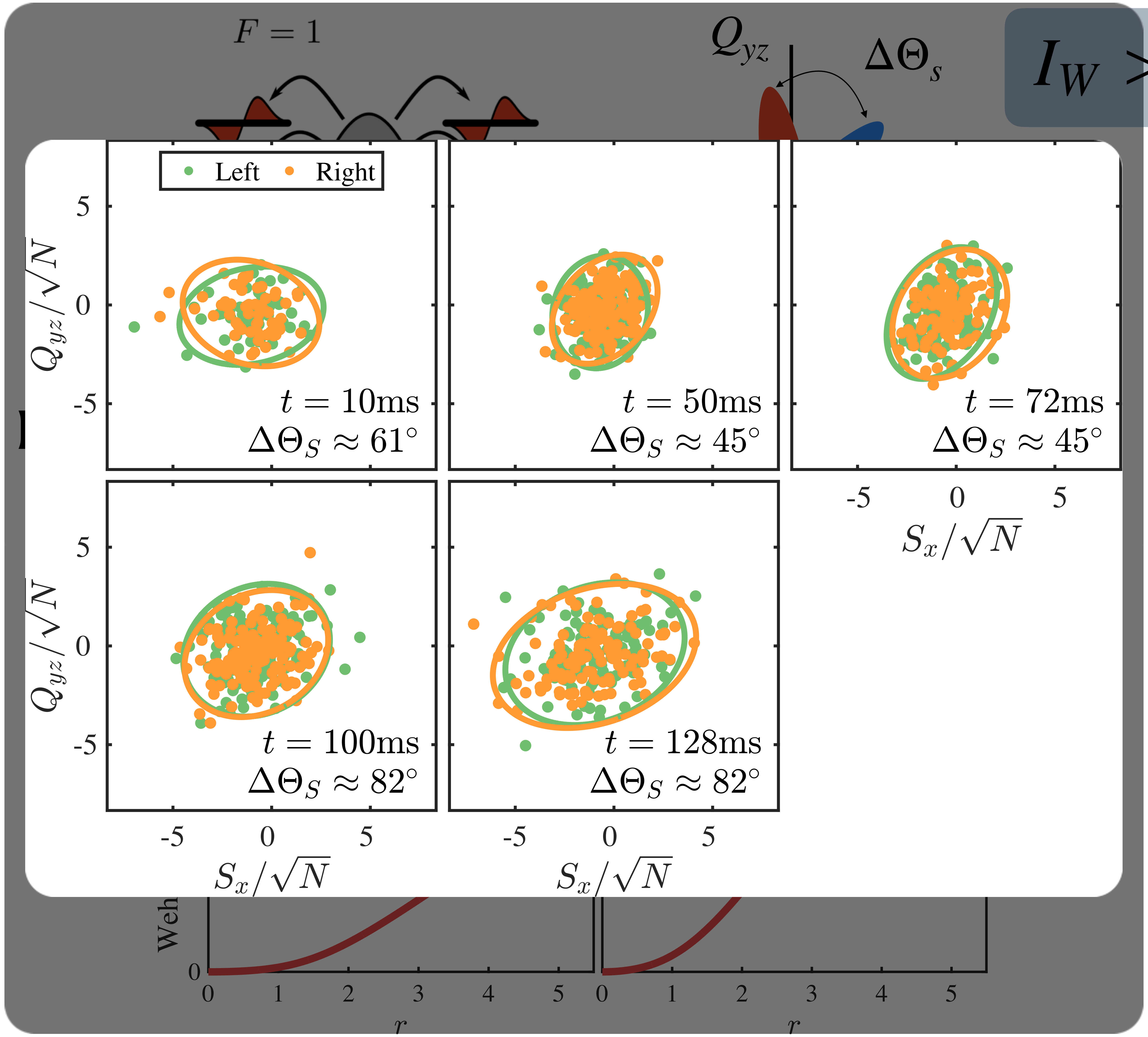
Exact calculation of mutual information



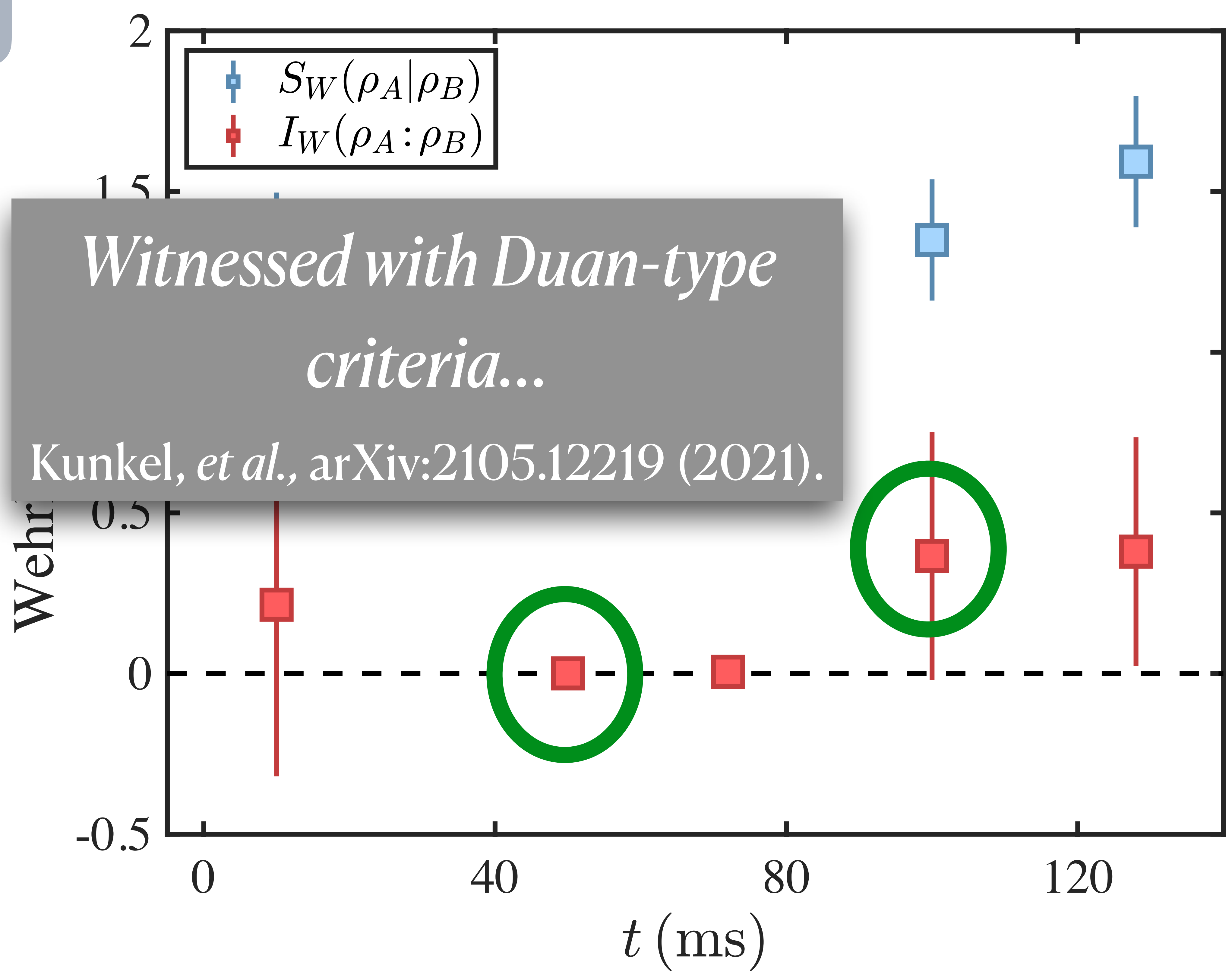


Error via jackknife resampling then naive error propagation

Experimental results



$I_W > 0$



Witnessed with Duan-type criteria…

Kunkel, *et al.*, arXiv:2105.12219 (2021).

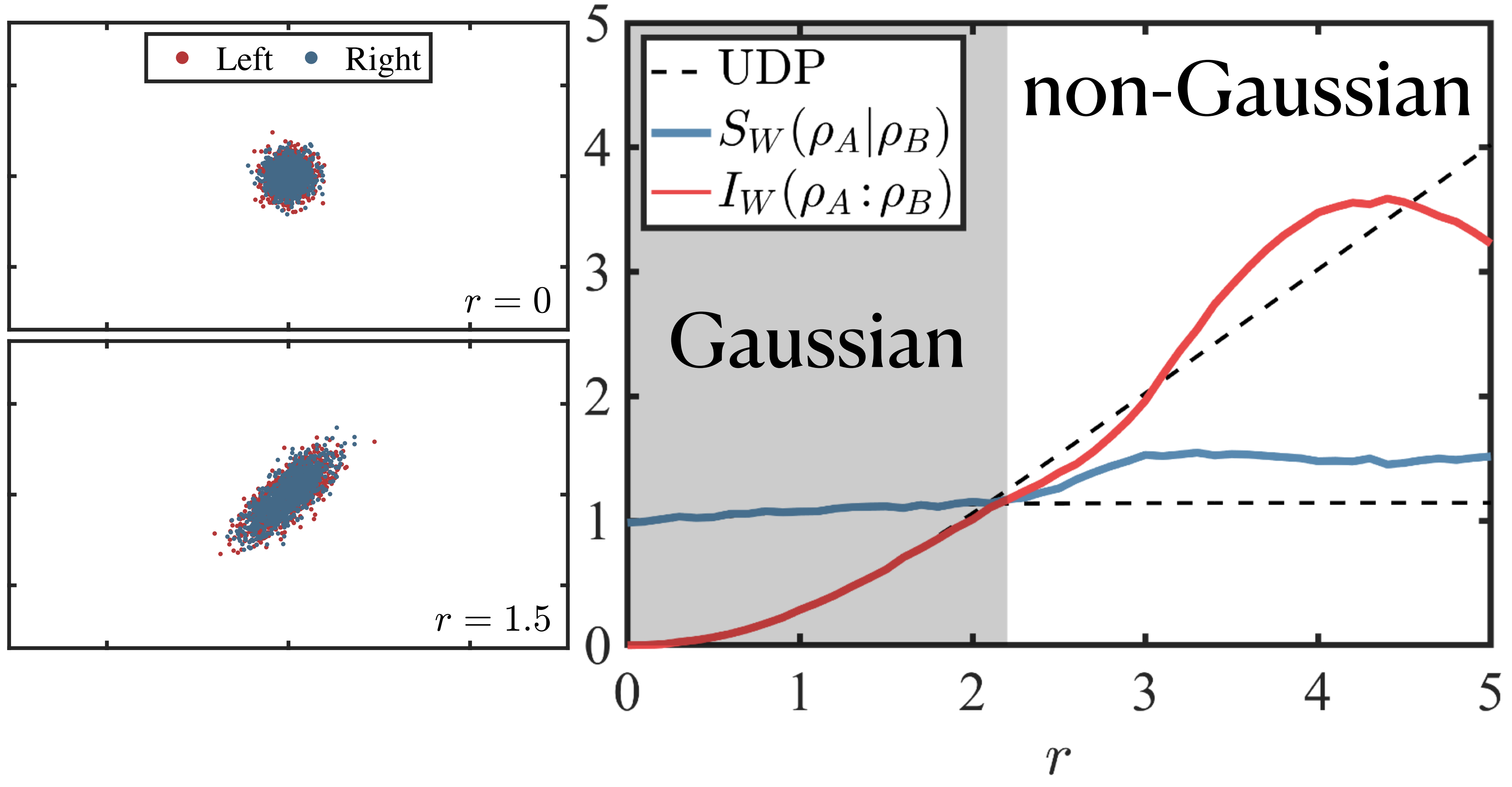
Error via jackknife resampling then naive error propagation

Conclusions & Future Direction

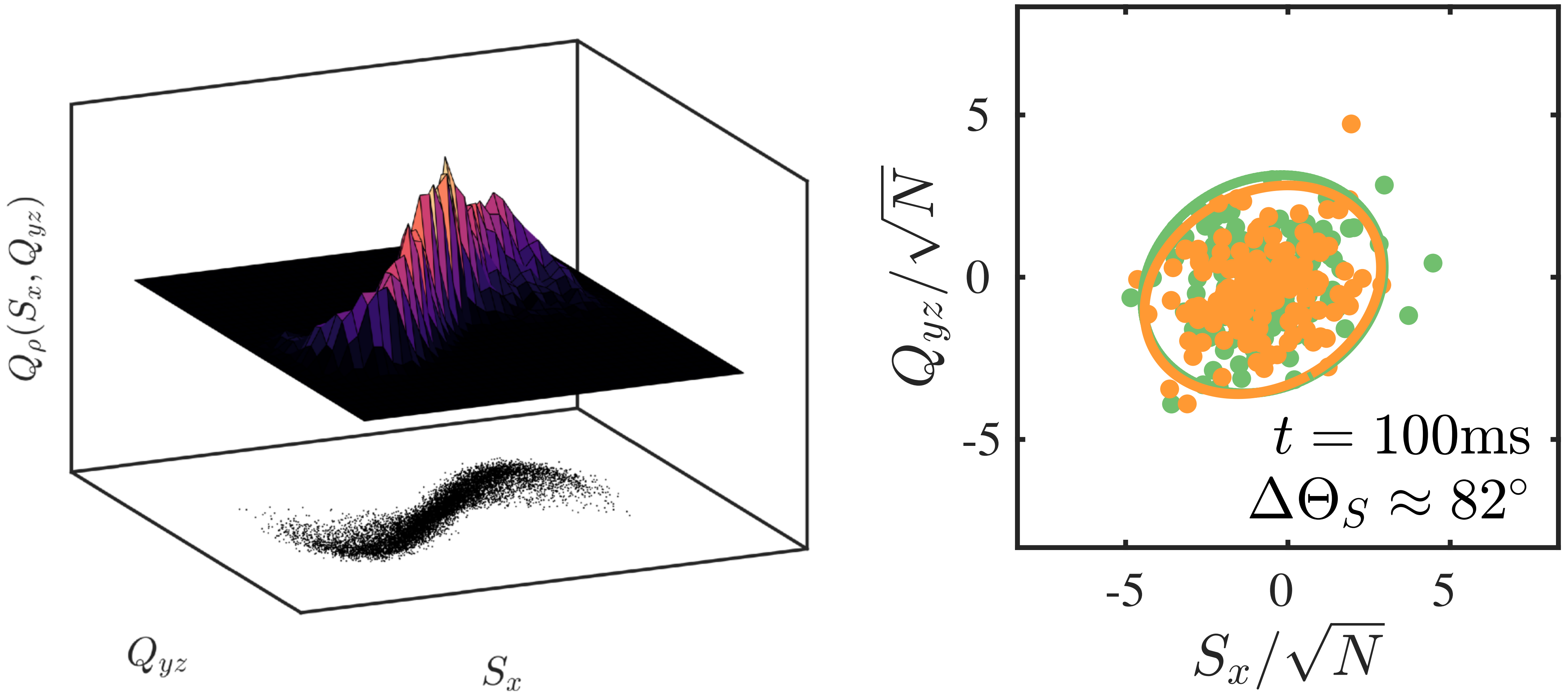
<u>Entropies offer benefits to witnessing entanglement</u>

$$S_W = -\int \frac{dS_x\, dQ_{yz}}{2\pi N}\, Q_\rho(S_x, Q_{yz}) \log Q_\rho(S_x, Q_{yz})$$

Benchmarked the method in **Gaussian** regime



To **reap benefits**, we need to **approximate pdf**



Test the ***true*** entanglement witness

$$S_M(Q_\pm) \geq 1 + \ln 2$$

Floerchinger, *et al.* arXiv:2106.08788 (2021).

More details! → @GaerrtnerGroup mbqd.de

Thank you! Questions?