

Entanglement in spinor Bose gases via entropic uncertainty relations

Oliver Stockdale¹, Tobi Haas², Martin Gärttner^{1,2,3}

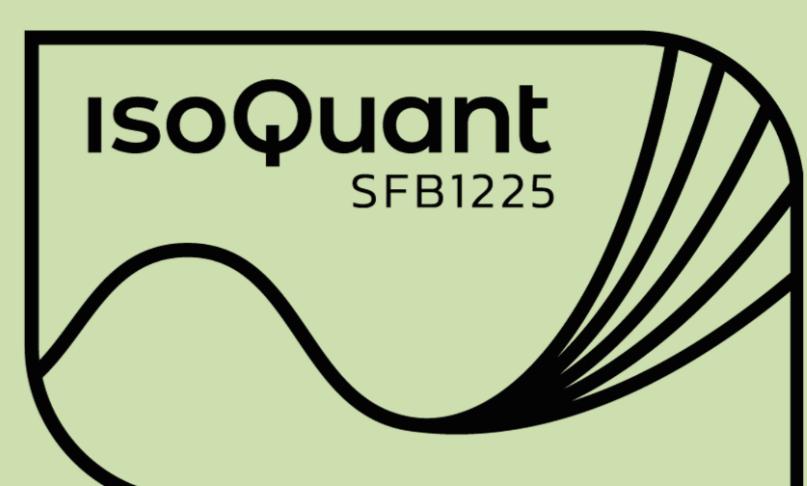
¹Kirchhoff-Institut für Physik, Universität Heidelberg, Im Neuenheimer Feld 227, 69120 Heidelberg, Germany

²Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany

³Physikalisches Institut, Universität Heidelberg, Im Neuenheimer Feld 226, 69120 Heidelberg, Germany



UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386



Summary

Motivation:

- Entanglement will be a key resource in future quantum technologies
- Entropy is a tool in quantum information theory for witnessing entanglement
- Bose-Einstein condensates offer a pristine playground for understanding quantum many-body systems

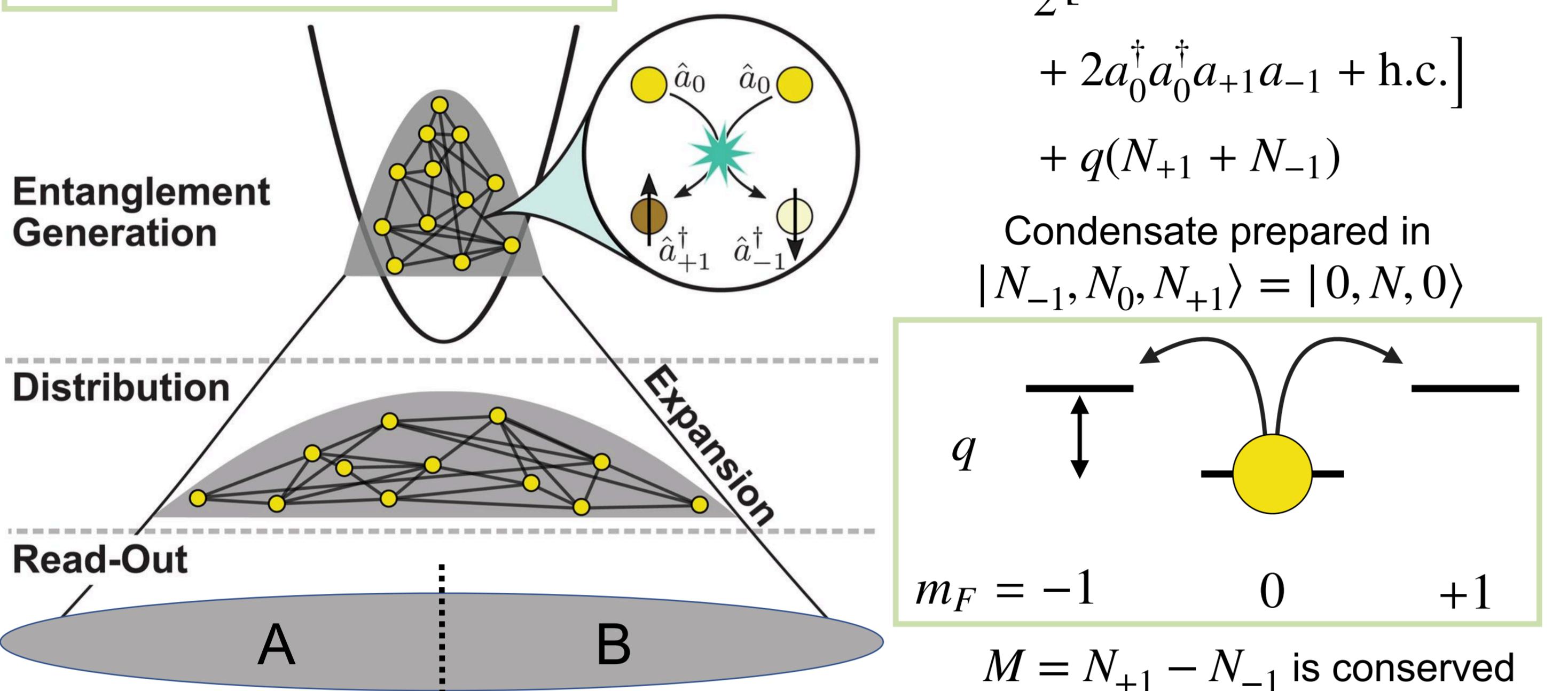
Central goals:

- Develop entropic uncertainty relations for spinor gas systems beyond variance based measurements
- Understand the temporal dynamics of entanglement in Bose gases

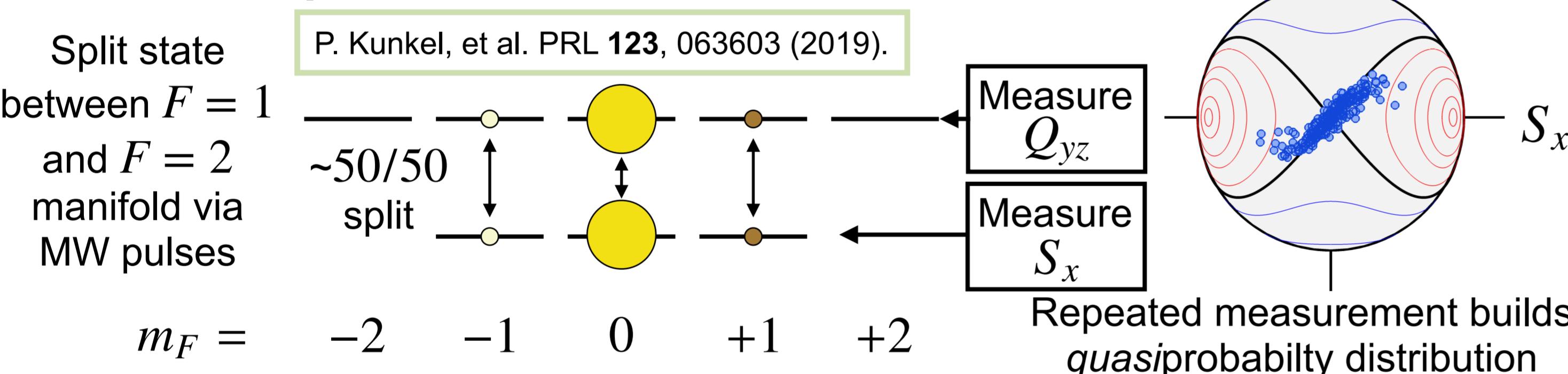
Experimental system: ^{87}Rb spin-1 Bose gas

Entanglement generation

P. Kunkel, et al. Science 360, 413 (2018).



Read-out sequence: simultaneous measurement

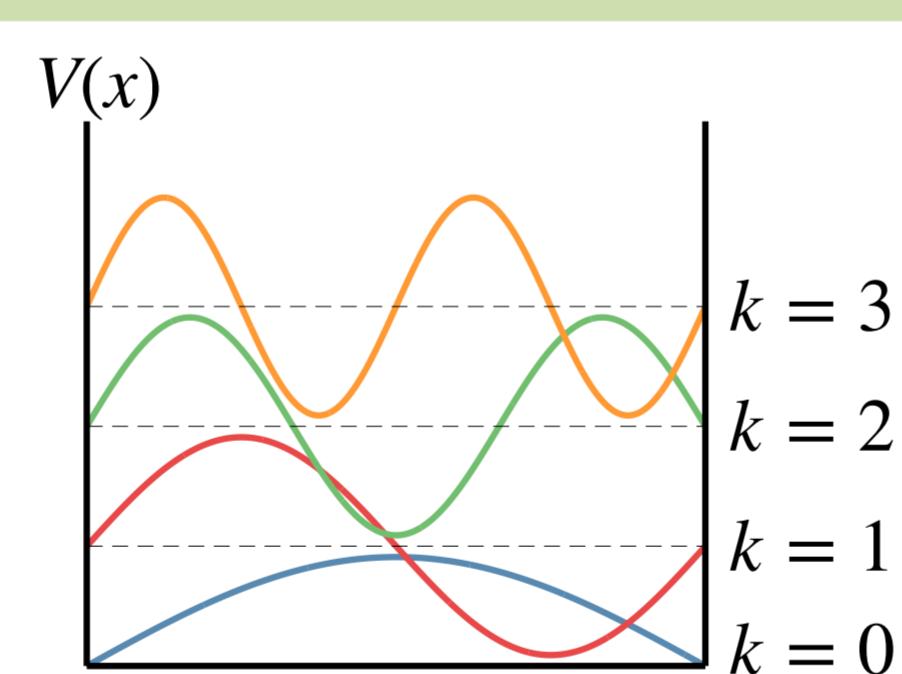


Numerical simulation

Truncated Wigner approximation (TWA)

We consider a system with k spatially excited modes

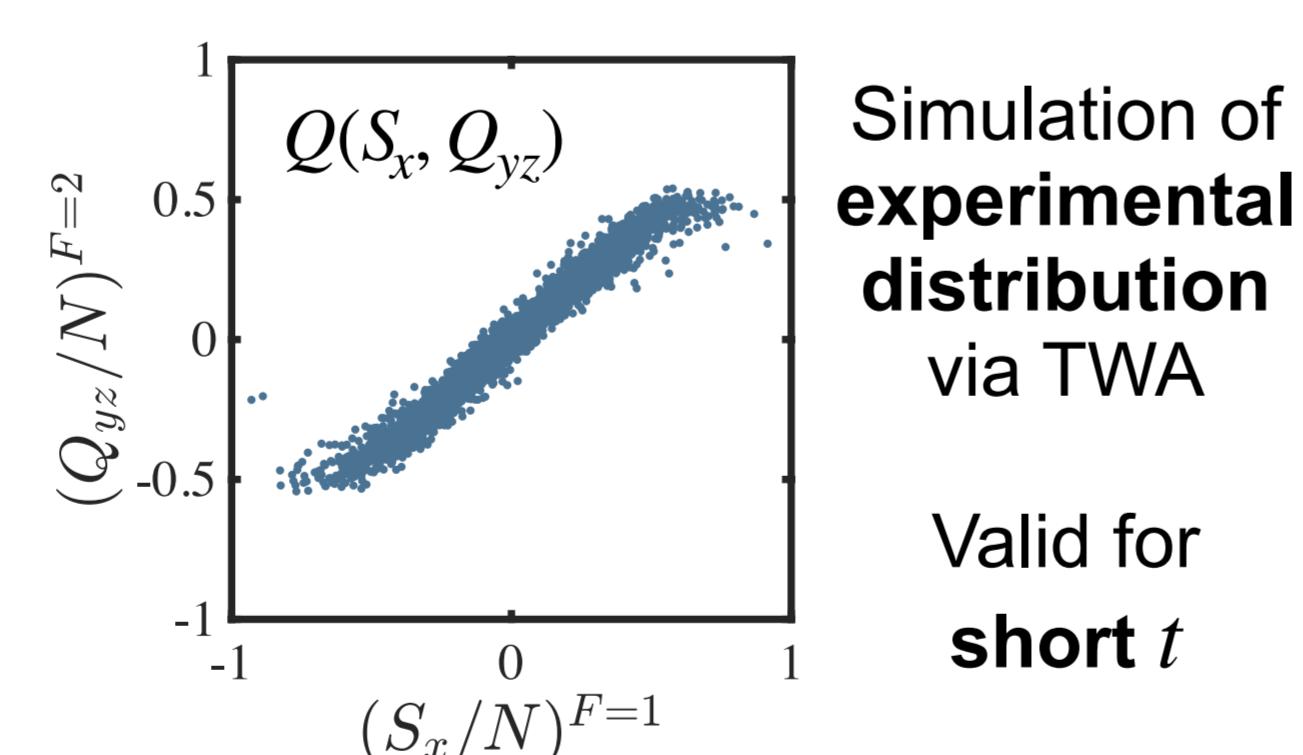
$$i\hbar \frac{\partial a_{i,k}}{\partial t} = \frac{\partial H_W}{\partial a_{i,k}^*}$$



The two beam splitters (once for A-B partition, once for $F=1 - F=2$ partition) are implemented via

$$\begin{pmatrix} a_{i,k}^A \\ a_{i,k}^B \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a_{i,k}^{\text{sys}} \\ a_{i,k}^{\text{vac}} \end{pmatrix}, \quad a^A \xrightarrow{\text{BS}} a^{\text{vac}}$$

'vac' represents vacuum contribution



Entropic uncertainty relations

From the phase space distributions $Q(S_x, Q_yz)$, we can calculate its entropy

$$S_W(\rho) = - \int dS_x dQ_{yz} Q_\rho(S_x, Q_{yz}) \ln Q_\rho(S_x, Q_{yz})$$

Typically, the conditional entropy flags entanglement, here it is the mutual information

$$I_W = S_W(\rho_A) + S_W(\rho_B) - S_W(\rho_{AB})$$

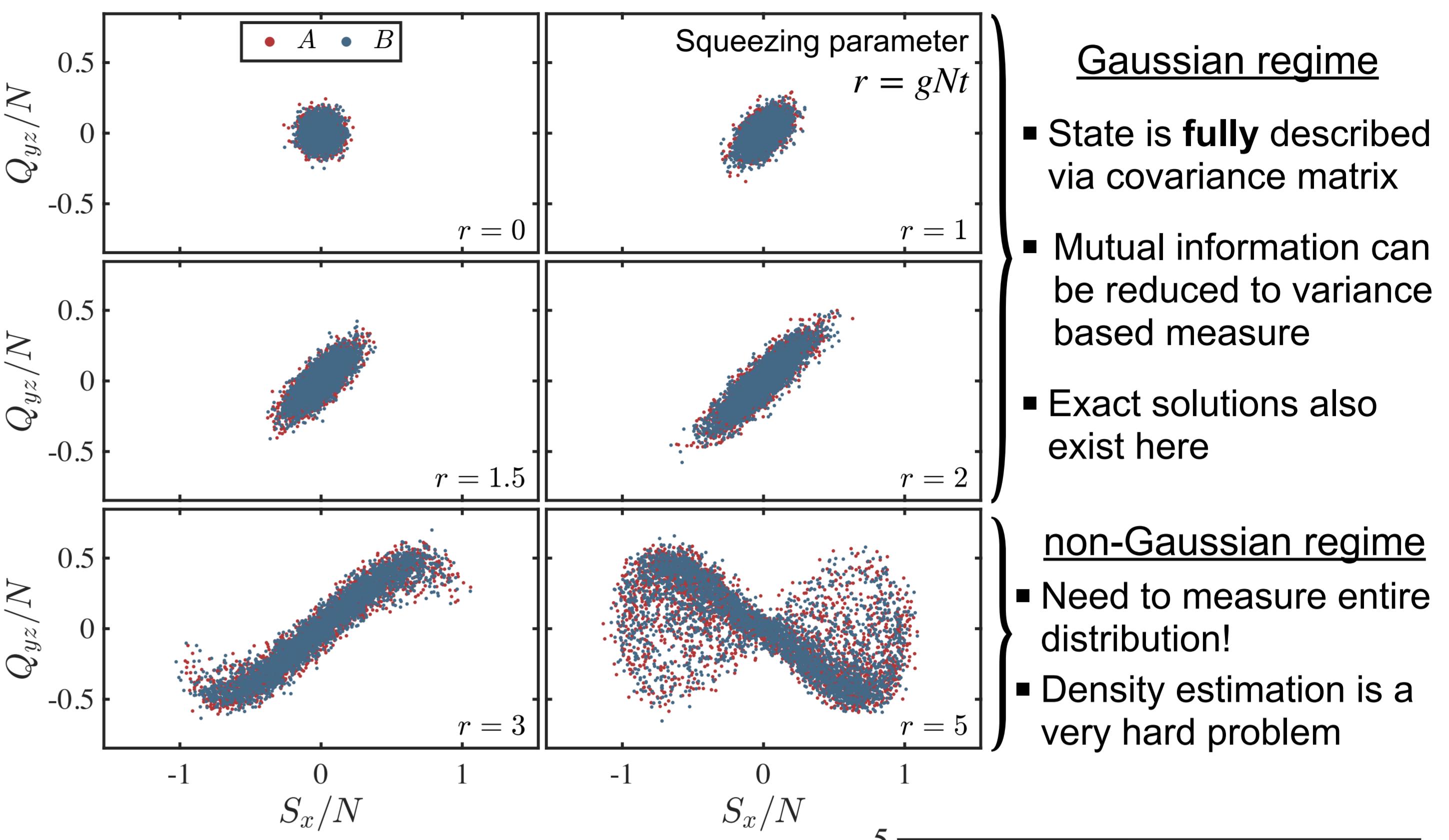
Mutual information measures **total correlations** between two subsystems A and B . Let's assume a pure state ρ_{AB} , therefore the state is entangled if

$$I_W > 0$$

Floerchinger, et al.
arXiv:2103.07229 (2021).

Witnessing entanglement in simulation

Time dynamics of the phase space distribution - single mode



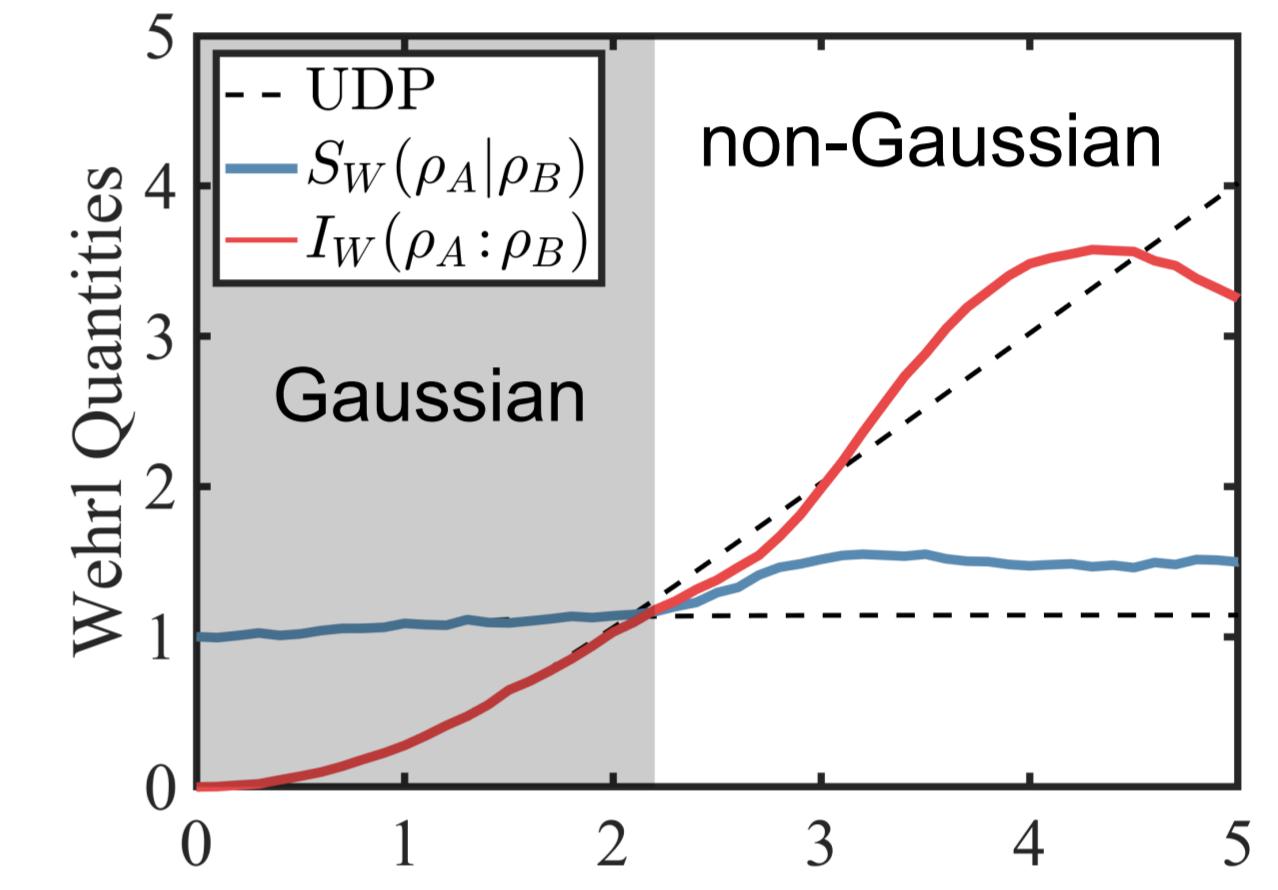
In the undepleted pump approximation (UDP), $N_{\pm 1} \ll N$ and the two-mode squeezed state is

$$|\psi\rangle = \sqrt{1 - \lambda^2} \sum_n (-\lambda)^n |nn\rangle$$

where $\lambda = \tanh(r)$. One can analytically calculate the inverse covariance matrix C and show

$$I_W(\rho_A : \rho_B) = \frac{1}{2} \frac{\det C_A \det C_B}{\det C}$$

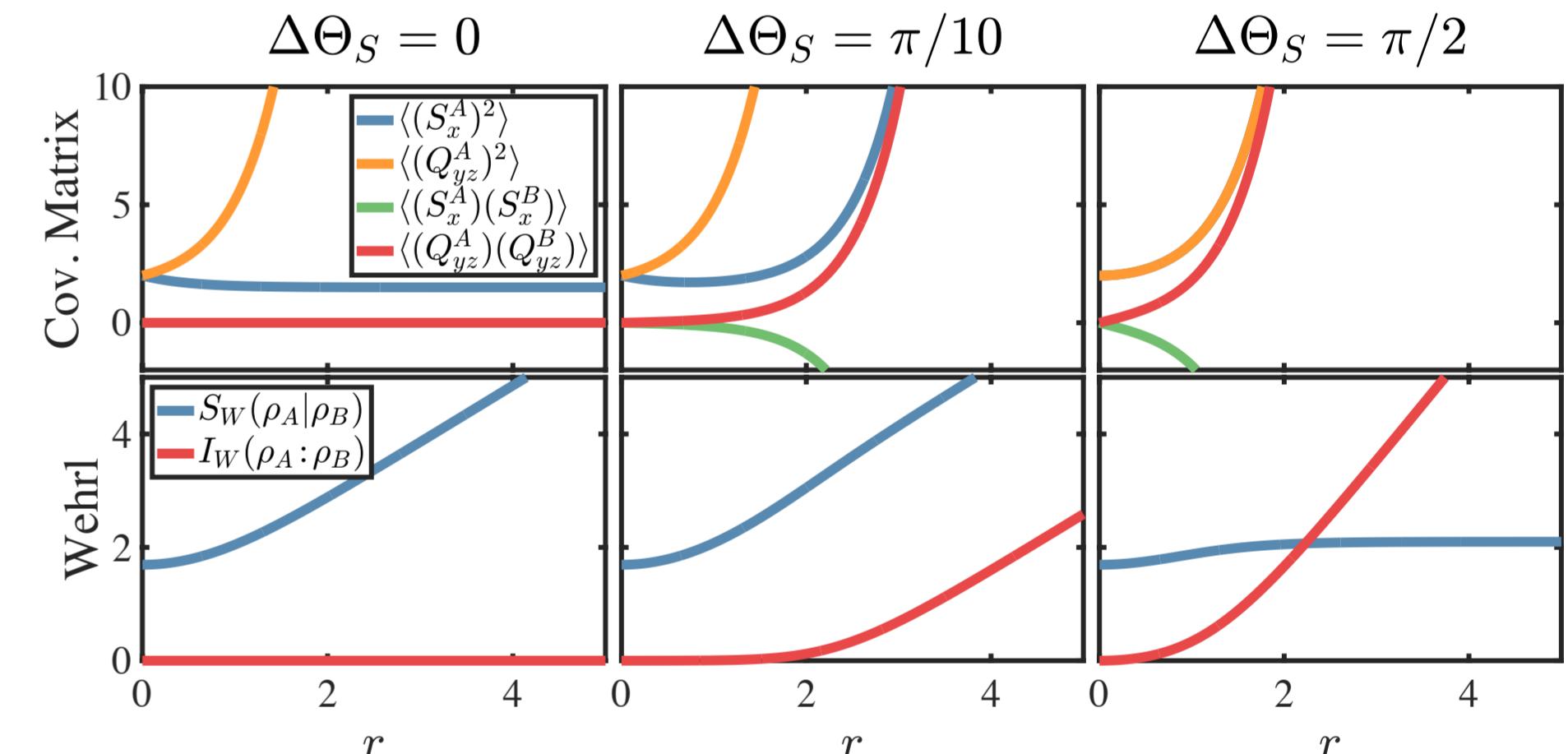
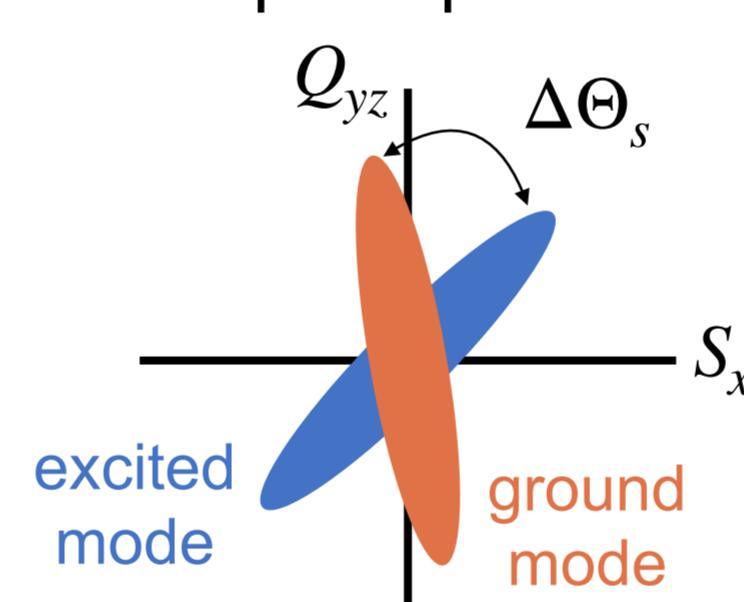
where C_A etc. are the block elements.



We measure $I_W > 0$ as the state squeezes!

Double mode dynamics

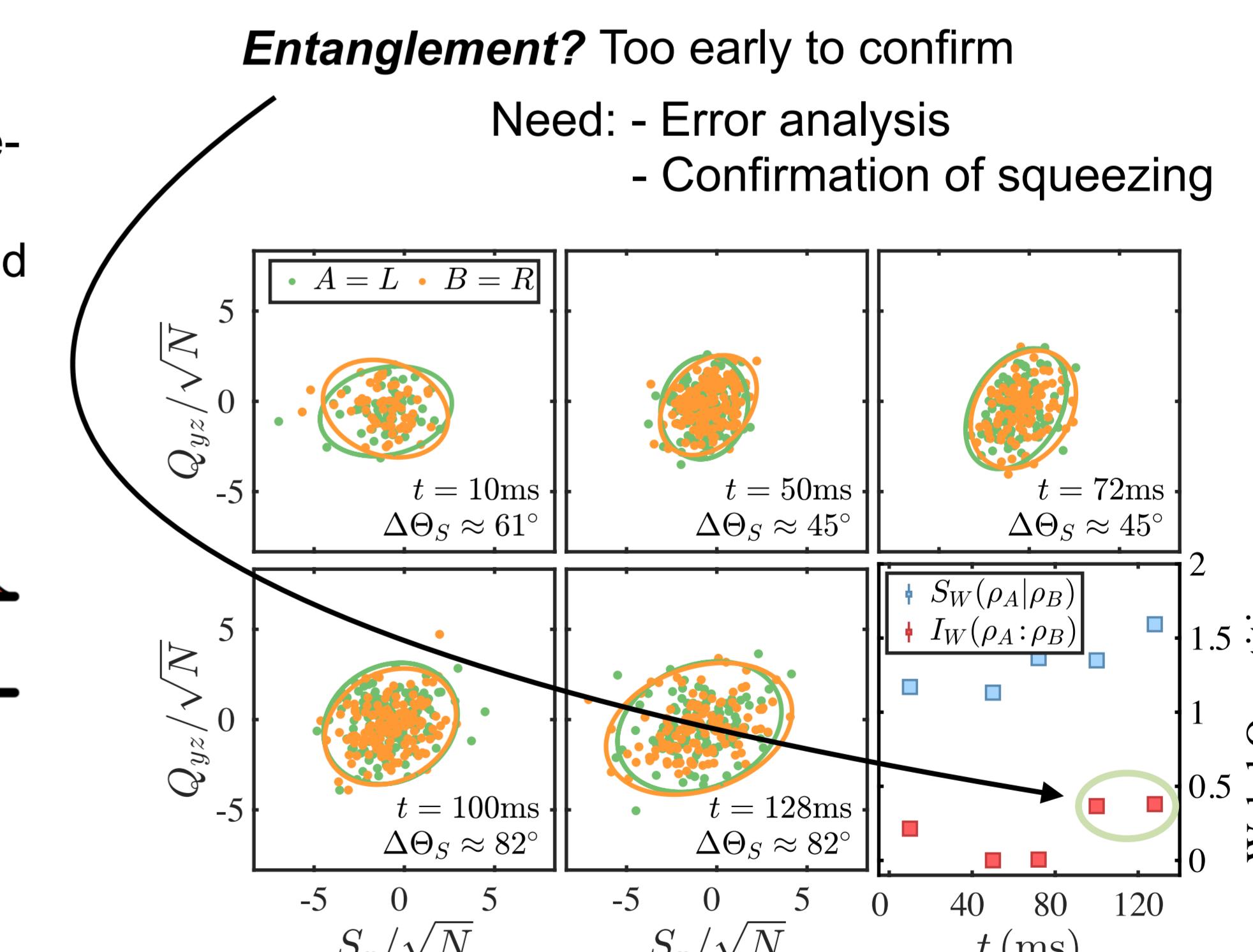
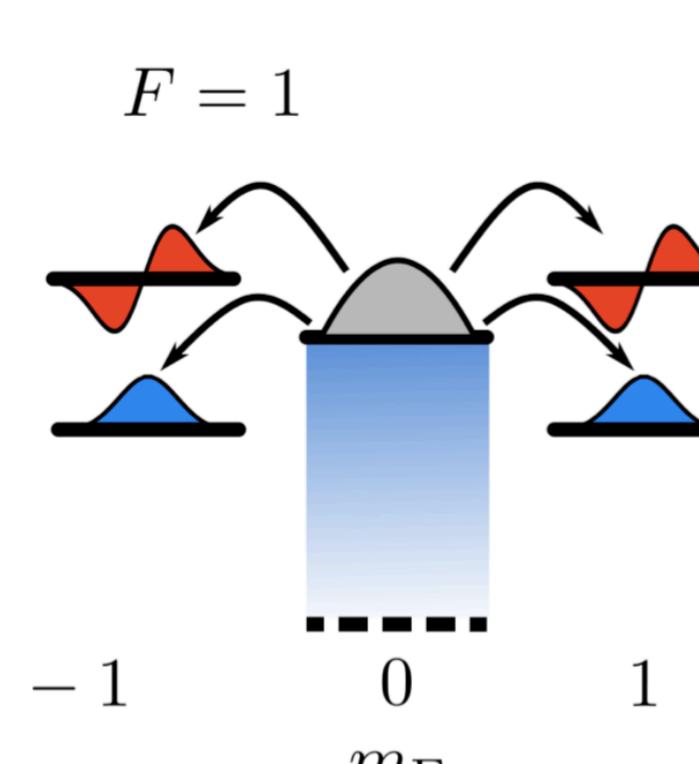
With two modes, we need to take into account the relative spinor phase $\Delta\Theta_s$



Experimental data

Details of experiment

- Rubidium-87 spinor Bose-Einstein condensate
- Two spatial modes excited
- t represents time of spin-changing collisions



Conclusions and future direction

Conclusions:

- Developed analytical and numerical methods to simulate spinor Bose gas experiments
- Showed that mutual information flags entanglement in these systems
- Presented experimental analysis that shows signs of correlations via information theoretic concepts

Future direction:

- Derive tight entanglement witnesses via the entropic measurements
- Better understand experimental data to robustly identify entanglement
- Develop entropy estimation methods to analyse non-Gaussian regime
- Understand long-time entanglement dynamics