Game Theory Analysis

For Brilliant professors:

$$\text{Let the cost of producing } N \text{ publications be } \$30,000N.$$
$$\text{The payoff for a tenured career at UChicago is } \$2,000,000.$$
$$\begin{aligned} \text{To ensure that only Brilliant professors aim for tenure:} \\ \$30,000N &< \$2,000,000 \\ N &< \frac{\$2,000,000}{\$30,000} \\ N &< \frac{200}{3} \end{aligned}$$
$$\text{Since } N \text{ must be an integer, we choose: } N \leq 66.$$

For Good professors:

$$\text{Let the cost of producing } N \text{ publications be } \$60,000N.$$
$$\text{The payoff for a career at Westnortherton University is } \$500,000.$$
$$\begin{aligned} \text{To prevent Good professors from aiming for tenure:} \\ \$60,000N &< \$500,000 \\ N &< \frac{\$500,000}{\$60,000} \\ N &< \frac{25}{3} \end{aligned}$$
$$\text{Since } N \text{ must be an integer, we choose: } N \geq 9.$$

Therefore, to screen only the Brilliant professors for tenure, the Dean should set the number of required publications N such that:

$$9 \leq N \leq 66$$

This range of N will create a separating equilibrium where only the Brilliant professors are incentivized to meet the publication requirement for tenure.

Coup d'État Game Analysis

a. Military's Best Response at the Final Nodes

The military's best response is to choose the action that maximizes its payoff at the final nodes.

- If the officers launch a coup:
  - Join payoff: 10
  - Stay Out payoff: -5

– Best response: Join (since $10 > -5$)

- If the officers do not launch a coup:
  – No Coup payoff: 1
  – No decision to be made here.

b. Officers' Expected Utility

The officers' expected utility (EU) for launching a coup and not launching a coup is calculated as follows:

$$\begin{aligned} EU(\text{Coup}) &= 0.7 \times 10 + 0.3 \times (-20) \\ EU(\text{Coup}) &= 7 - 6 \\ EU(\text{Coup}) &= 1 \\ EU(\text{No Coup}) &= 0 \end{aligned}$$

Since $EU(\text{Coup}) > EU(\text{No Coup})$, the officers will choose to launch the coup.

c. Equilibrium Condition

Suppose the likelihood of military support for a coup is p. For (Coup, Join) to be an equilibrium, the following condition must hold:

$$\begin{aligned} p \times 10 + (1-p) \times (-20) &\geq 0 \\ 30p - 20 &\geq 0 \\ p &\geq \frac{20}{30} \\ p &\geq \frac{2}{3} \end{aligned}$$

Thus, for any $p \geq \frac{2}{3}$, the strategy (Coup, Join) is an equilibrium.

The Forgiving Prisoner's Dilemma

a. Infinite-horizon Payoff for Deviation

The infinite-horizon payoff for Player 1 if it deviates in the first round, then follows the TMP strategy, is given by:

$$\text{Payoff} = 4 + 2\delta + 2\delta^2 + \frac{3\delta^3}{1-\delta}$$

b. Non-profitable Deviation Condition

The range of δ for which deviation is not profitable is determined by solving the inequality:

$$4 + 2\delta + 2\delta^2 + \frac{3\delta^3}{1-\delta} < \frac{3}{1-\delta}$$

This simplifies to:

$$-\frac{1}{2} + \frac{\sqrt{5}}{2} < \delta < 1$$

For the Prisoner's Dilemma, the discount factor δ is between 0 and 1. Hence, the condition for δ becomes:

$$0.618 < \delta < 1$$

This range ensures that deviating from the cooperative strategy is not profitable for Player 1.

Lopsided Median Voter Theorem

a. Nash Equilibrium $(x_L, x_R) = (1, 1)$

Given that the median voter's ideal point is $x_m > 1$, both candidates positioning at 1 is a Nash Equilibrium because:

- Deviating to any point less than 1 would only increase the distance to x_m, thus reducing the candidate's payoff.
- If one candidate stays at 1 while the other deviates leftward, the candidate at 1 would be closer to x_m, attracting the median voter.
- Since neither candidate can improve their payoff unilaterally by choosing a different strategy, $(1, 1)$ constitutes a Nash Equilibrium by definition.

b. Uniqueness of the Nash Equilibrium

While technically any position (x, x) with $x \geq 1$ could be an equilibrium due to symmetry and non-decreasing payoffs to the right of 1, in practice the equilibrium $(1, 1)$ is unique for the following reasons:

- Moving further right from 1 does not yield a higher payoff because it would not attract the median voter.
- Since the candidates have identical payoff functions and because there is no advantage in moving rightward beyond 1, there is no incentive to choose a position other than 1.

Thus, the equilibrium is practically unique at $(1, 1)$ given the median voter's position.

Divergence with Valence

a. Preexisting Electoral Advantage

Candidate R has a preexisting electoral advantage due to the valence advantage ν, where $x_R - x_L > \nu$ implies that some voters who are ideologically closer to L will still vote for R.

b. Range of Ideal Points

The range of voter ideal points x for which a citizen would vote for L or R is determined by the inequality:

$$|x_i - x_R| - |x_i - x_L| > \nu$$

Solving this inequality gives us the range of x for which L is preferred over R and vice versa.

c. Probability of L Winning

The probability $\pi(x_L, x_R)$ that L wins can be calculated by integrating over the voter ideal points' distribution for which L is preferred, considering the valence advantage ν.

d. Equilibrium Policy Positions

The equilibrium policy positions x_L^* and x_R^* are where neither candidate can unilaterally change their position to increase their probability of winning. This would require solving for the positions given the distribution of voter ideal points and the valence advantage.