Problem Set 2

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1 Problem 1

Assume that markets are complete. Since markets are complete and there are no additional externalities, the competitive equilibrium for this economy will be pareto efficient. We find the allocation with the planners problem and then back out the discount bond price. We solve

$$\max \sum_{t=0}^{\infty} \beta^t \left[\pi(e) \frac{c(e)^{1-\alpha} - 1}{1-\alpha} + (1-\pi(e)) \frac{c(u)^{1-\alpha} - 1}{1-\alpha} \right]$$
$$\text{s.t } \pi(e)c(e) + (1-\pi(e))c(u) \leq \pi(e)y(e) + (1-\pi(e))y(u)$$

First order conditions imply

$$\beta^t (1-\alpha) c(e)^{-\alpha} = \lambda_t$$
$$\beta^t (1-\alpha) c(u)^{-\alpha} = \lambda_t$$
$$\implies c(e) = c(u).$$

To find the invariant distribution , we solve

$$\begin{bmatrix} \pi(e) \\ \pi(u) \end{bmatrix} = \begin{bmatrix} 0.97 & 0.50 \\ 0.03 & 0.50 \end{bmatrix} \begin{bmatrix} \pi(e) \\ \pi(u) \end{bmatrix}$$

along with the restriction $\pi(e) + \pi(e) = 1$ to obtain $\pi(e) \approx 0.9434$ and $\pi(u) \approx 0.0566$.

To find the price of the arrow security $q_t(s)$, we solve

$$\sum_{t=0}^{\infty} \beta^t \left(u(c_t(e))\pi(e) + u(c_t(u))\pi(u) \right)$$

s.t.

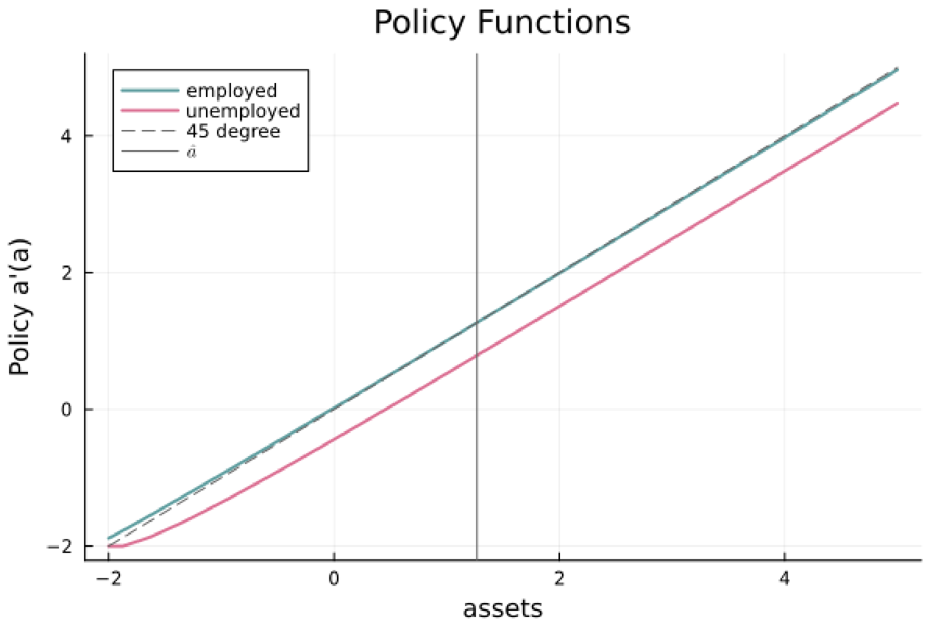
$$c_t(s^t) + \sum_{s^t} q_{t+1}(s^t) a_{t+1}(s^t) \leq y_t(s^t) + a_t(s^t)$$

so prices are $q(e) = \beta\pi(e)$ and $q(u) = \beta\pi(u)$

where we used the first/second welfare theorem to note that allocations will be equalized across all states.

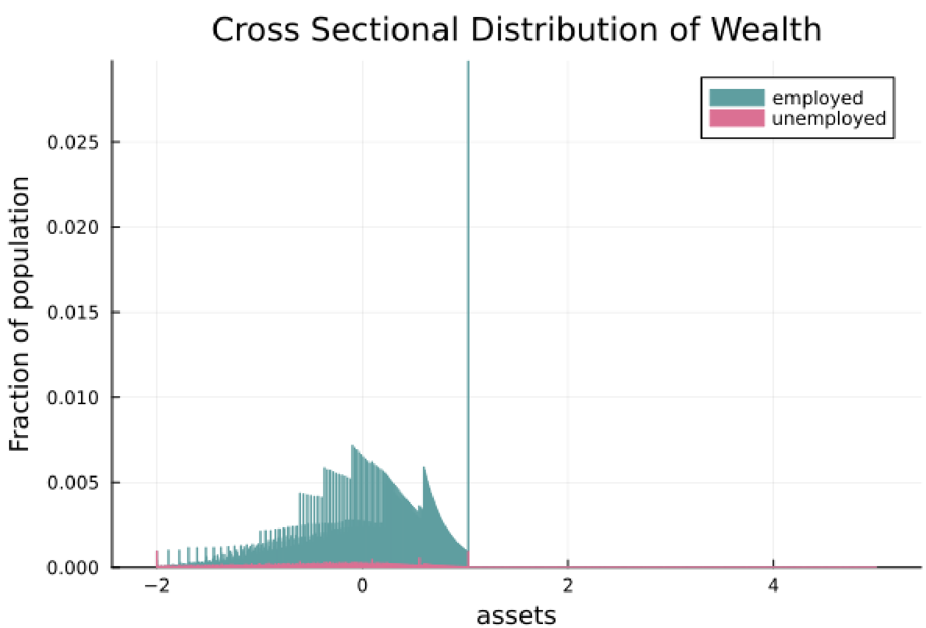
2 Problem 2

2.1 Policy Function



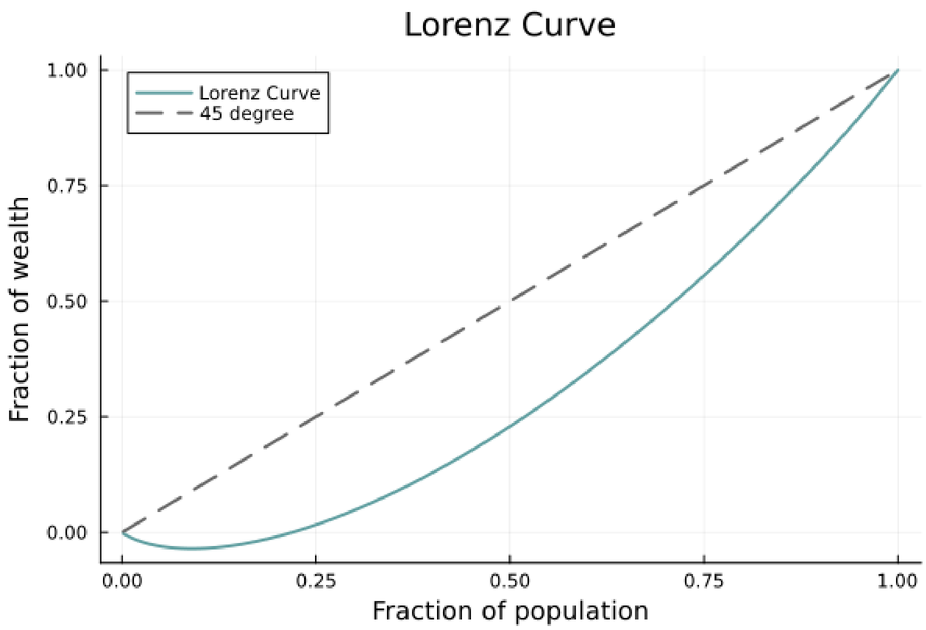
The policy function plotted above shows that $\hat{\alpha} \approx 1.28$.

2.2 Invariant Distribution and Bond Price



The wealth distribution is plotted above. The discount bond price that clears the market is $q \approx 0.9943$.

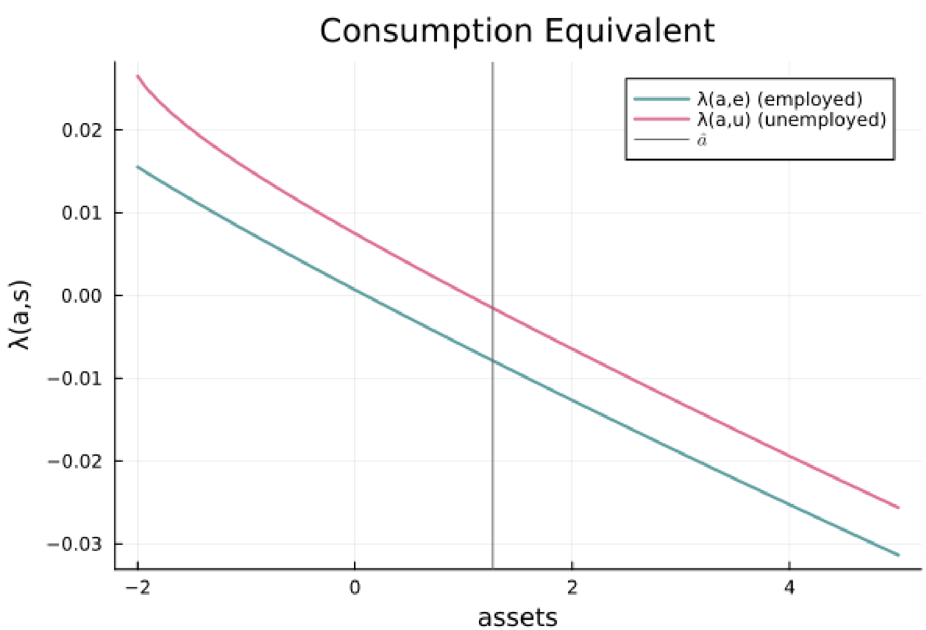
2.3 Lorenz Curve and Gini Coefficient



The lorenz curve is plotted above. The gini coefficient is ≈ 0.383.

3 Problem 3

3.1 Welfare Analysis



We plot the consumption equivalent above.

- Aggregate welfare with complete markets is -4.283
- Aggregate welfare with incomplete markets is -4.454.

- Aggregate welfare gain is 0.001156.

Finally, the fraction of the population who would pay for complete markets is 0.523.