

Problem Set 1

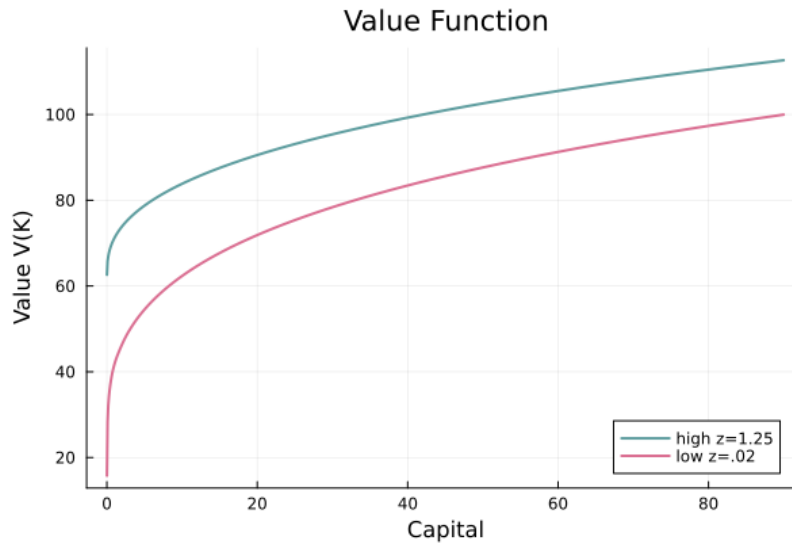
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1 Problem 1

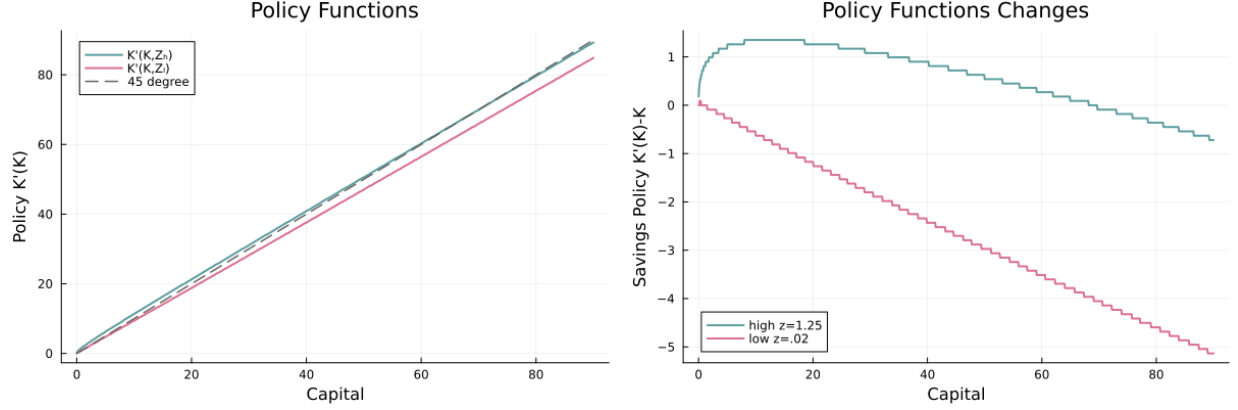
We set the error tolerance to $1e-6$ in both versions and did not exploit the monotonicity of the policy function in the non-parallelized version. **The execution time for the un-parallelized implementation takes about 63 seconds while the parallelized implementation finishes in about 20 seconds.** Note that in this example, it would be more efficient to exploit the monotonicity (finishes in a few seconds with far fewer iterations) of the policy function rather than use parallelization.

2 Problem 2



The value function is plotted for both states $z = 1.25$ and $z = 0.2$. The value function is clearly increasing and concave.

3 Problem 3



The policy function is clearly increasing in K . Since $K'(K, Z_h)$ lies above $K'(K, Z_l)$, it is also increasing in Z . Savings is increasing in Z . When productivity is high, savings is increasing in K for low values of K but then decreasing. When productivity is low, savings is monotonically decreasing.

The euler equation is given by

$$\underbrace{\frac{1}{c_t}}_{\text{marginal utility of consumption}} = \underbrace{\beta \mathbf{E}_t \left[\frac{\alpha z_{t+1} k_{t+1}^{\alpha-1} + 1 - \delta}{c_{t+1}} \right]}_{\text{marginal cost of saving}}.$$

The LHS captures the marginal utility of consuming more while the RHS is the marginal cost of foregoing consumption today for consumption in the future. When K is low, households might save more in response to a positive productivity shock since they can capture higher future returns. When K is high, a positive productivity shock leads to increased consumption since they have a lot of wealth and the marginal product of capital is low at high K . This explains the non-monotonic shape of the savings curve above for high TFP. On the other hand, a negative productivity shock decreases the marginal product of capital and disincentivizes savings. Another relevant aspect of the role of productivity in the savings decision of households is the persistence of the productivity shocks, which in our case is very high as the states are pretty absorbent. As a consequence, when consumers start with a low productivity their expected value of future marginal product of capital will be very low, whereas it will be higher when they start at a high productivity level. In the latter case, more capital at very low initial capital holdings will decrease marginal product of capital but this effect will be mitigated by the higher returns that are expected to persist due to productivity.