ECON880: Computational Economics
Fall 2024, University of Wisconsin
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Problem Set #1 - Goal Due Date 9/11/24

This problem introduces you to dynamic programming in an infinite horizon growth model. In particular, you are to modify the deterministic dynamic programming code that appears on my website:[1]

- deterministic_growth_script.jl and deterministic_growth_functions.jl (julia code).
- parallel_script.jl and parallel_functions.jl (julia code to parallelize determinstic problem).

These programs use value function iteration to solve for the decision rule $K_{t+1}(K_t)$ in a non-stochastic setting. You are to modify these programs to add uncertainty over technology shocks in Julia. The modified code will be similar to the inner loop of a nested fixed point problem that you will compute in problem set #2. Since value function iteration is well-suited to parallelization, you also need to modify the provided parallel code. This can speed up finding decision rules in more complicated environments that we will study later in the class. As you will see when you run these, there is some startup loss of time in the parallel code which makes it slower in the deterministic case but then it gets faster in the stochastic case.[2]

Specifically, assume that households have log preferences with $\beta = 0.99$, the production technology satisfies $Y_t = Z_t K_t^{\theta}$ where $\alpha = 0.36$, and capital

[1] Since we have several cohorts which took the summer Julia course, I am presuming everyone will write this semester in Julia. But just in case, there is also matlab code vfigrowth07.m on my website and I can also supply fortran code.

[2] In the non-parallelized version (and the stochastic model) there's an object 'choice_lower' which stores the index of the policy function on the capital grid so that for the next point on the grid you only have to check higher values of capital. It's using the fact that the value function is monotone to limit the size of the loops. However, you can't do this when you parallelize on the capital grid because each of the workers would be writing to the 'choice_lower' object simultaneously. You could have a workaround where you store the indices in a shared vector and have the workers write to that, but that would be more complicated.

depreciates at rate $\delta = 0.025$. We will assume technology shocks follow a 2 state Markov Process. The transition matrix is calibrated to NBER business cycle data where we take an expansion to be an instance of a positive technology shock and recession to be an instance of a negative technology shock. The transition matrix is given by

$$\Pi = \begin{bmatrix} 0.977 & 0.023 \\ 0.074 & 0.926 \end{bmatrix}.$$

where, for instance, $prob(Z_{t+1} = Z_g | Z_t = Z_g) = 0.977$. The support of the markov process is given by $\{Z_g = 1.25, Z_b = 0.2\}$. I chose these values in order to satisfy that $\overline{Z} = 1$.To see this, note that Π implies an invariant distribution over the two states of $\overline{p}^g = 0.763$ and $\overline{p}^b = 0.237$. In that case, I chose $Z_g = 1.25$ and solved for Z_b in $\overline{Z} = \overline{p}_g Z_g + \overline{p}_b Z_b$.

The dynamic programming problem you are to solve is thus:

$$V(K, Z) = \max_{K'} \log\left(ZK^\alpha + (1-\delta)K - K'\right) + \beta \mathbb{E}\left[V(K', Z')\right]$$

where to save on notation any variable $X_t = X$ and $X_{t+1} = X'$.

1. Solve the stochastic dynamic programming problem via value function iteration over a discrete grid on K of 1000 linearly spaced grid points over $[0.01, 90]$. Specifically, let $K_i \in \{0.01, 0.1001, ..., 90\}$. What are the differences in computing time between the un-parallelized versus parallelized versions for both deterministic and stochastic cases?

2. Plot the value function over K for each state Z. Is it increasing (i.e. is $V(K_{i+1}, Z) \geq V(K_i, Z)$ for $K_{i+1} > K_i$)? Is it "concave" (in the sense that $V(K_{i+1}, Z) - V(K_i, Z)$ is decreasing)?

3. Plot the decision rules $K'(K, Z)$ for each state Z including a 45 degree line. Is the decision rule increasing/decreasing in K and Z (i.e. is $K'(K_{i+1}, Z) \geq K'(K_i, Z)$ for $K_{i+1} > K_i$ and is $K'(K, Z_g) \geq K'(K, Z_b)$)? Is saving increasing in K and Z (to see this, plot the change in the decision rule $K'(K, Z) - K$ across K for each possible exogenous state Z)? Provide intuition in terms of the marginal benefit versus marginal cost of increasing capital holdings. What in the environment might lead to saving at low levels of capital and dissaving at high levels of capital?