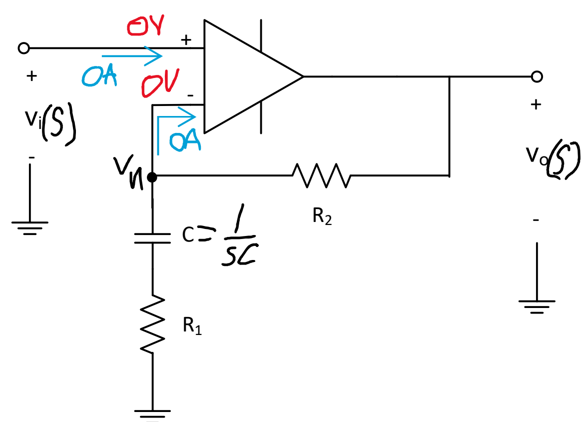
BEE 506 **Lab: Electronic Filter Design** **Winter 2022**

Active Filter Design and Verification

1. In the active circuit shown below the op amp can be considered ideal. Analyze the circuit to derive the expression for the transfer function H(s)=$V_o(s)/V_i(s)$.

$v_n = v_i(s)$



Verification that the transfer function is equal to the following (equation 1):

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_1 + R_2}{R_1} \frac{s + \frac{1}{(R_1 + R_2)C}}{s + \frac{1}{R_1 C}}$$

$$\frac{v_i(s) - v_o(s)}{R_2} + \frac{v_i(s) - 0}{\frac{1}{sC}+R_1} + 0 = 0$$

$$\frac{1}{R_2} v_i(s) - \frac{1}{R_2} v_o(s) = -\frac{v_i(s)}{\frac{1}{sC}+R_1}$$

$$\frac{1}{R_2} v_i(s) + \frac{1}{\frac{1}{sC}+R_1} v_i(s) = \frac{1}{R_2} v_o(s)$$

$$v_i(s)\left[\frac{1}{R_2} + \frac{1}{\frac{1}{sC}+R_1}\right] = \frac{1}{R_2} v_o(s)$$

$$R_2\left[\frac{1}{R_2} + \frac{1}{\frac{1}{sC}+R_1}\right] = \frac{v_o(s)}{v_i(s)}$$

$$1 + \frac{R_2}{\frac{1}{sC}+R_1} = \frac{v_o(s)}{v_i(s)}$$

$$\frac{\frac{1}{sC}+R_1+R_2}{\frac{1}{sC}+R_1} = \frac{v_o(s)}{v_i(s)}$$

$$\frac{\frac{1+R_1 sC+R_2 sC}{sC}}{\frac{1+R_1 sC}{sC}} = \frac{v_o(s)}{v_i(s)}$$

$$\frac{1 + R_1sC + R_2sC}{1 + R_1sC} = \frac{v_o(s)}{v_i(s)}$$
$$\frac{1 + sC(R_1 + R_2)}{\left[s + \frac{1}{CR_1}\right] CR_1} = \frac{v_o(s)}{v_i(s)}$$
$$\frac{C(R_1 + R_2)}{CR_1} * \frac{s + \frac{1}{(R_1+R_2)C}}{s + \frac{1}{R_1C}} = \frac{v_o(s)}{v_i(s)}$$
$$\frac{R_1 + R_2}{R_1} \frac{s + \frac{1}{(R_1+R_2)C}}{s + \frac{1}{R_1C}} = \frac{v_o(s)}{v_i(s)}$$

2. Choose numerical values for the resistors (in kΩ range) and the capacitor (in microfarad range). Choose $R_2 > R_1$. Substitute the values of the resistors and the capacitor in the equation above. Input the transfer function into MATLAB and generate its Bode plot.

let $R_2 = 2$ kΩ & $R_1 = 1$ kΩ & C = 2 μF:

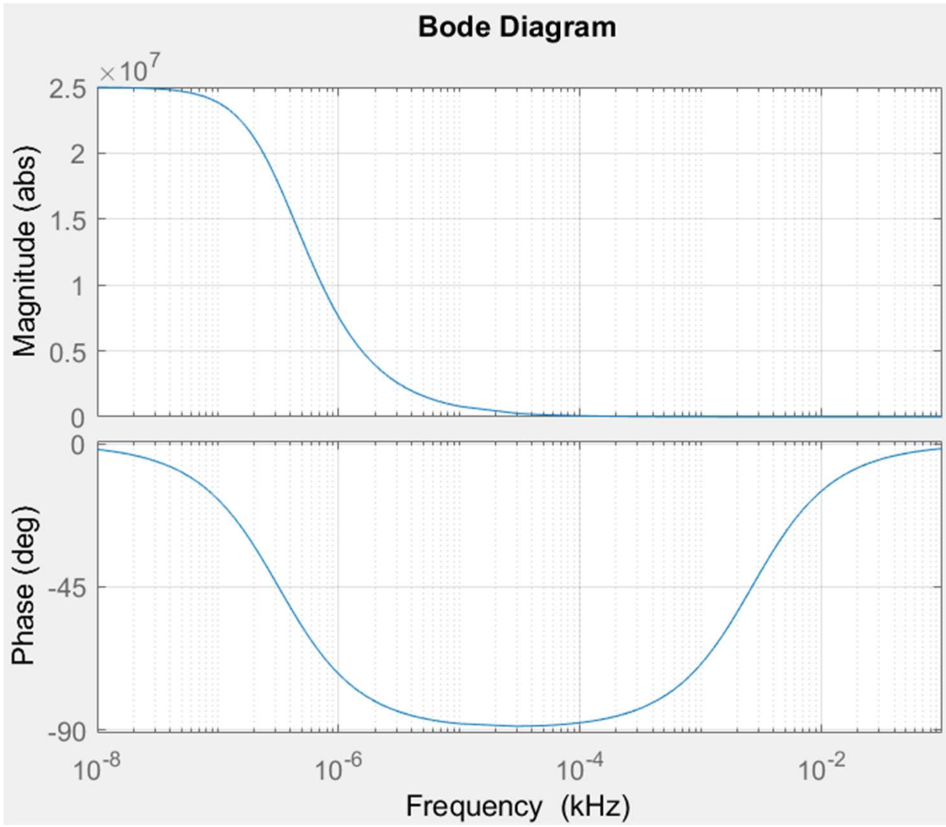
$$H(s) = \frac{1k + 2k}{1k} \frac{s + \frac{1}{(1k+2k)2\mu}}{s + \frac{1}{(1k)(2\mu)}} = 3000 \left[\frac{s + 166.67}{s + 0.002}\right] = \frac{3000s + 500000}{s + 0.002}$$

note:

$$H(s) = \frac{K}{s + 0.002}$$
$$K = (s + 0.002) * H(s) = 3000s + 500000|_{s=-0.002} = 499994$$

$$L^{-1}\{H(s)\} = h(t) = Ke^{-at}u(t)$$
$$h(t) = 499994e^{-0.002t}u(t)$$

Bode plot from MATLAB: plot is of a Low-pass filter



Command Window

```
>> n = [3000 50000];
>> d = [1 0.002];
>> h = tf(n.d)
Dot indexing is not supported for variables of this type.

>> h = tf(n,d)

h =

  3000 s + 50000
  --------------
    s + 0.002

Continuous-time transfer function.

fx >> bode(h)
```

3. Substitute s=jω in equation 1 above to obtain H(jω). Find the magnitude function of H(jω), |H(jω)|, in terms of R_1, R_2, C and ω. Evaluate |H(jω)| at ω=0 and at ω=infinity. Verify that these results agree with the numerical results from MATLAB.

$$\mathrm{H}(j\omega) = \frac{R_1 + R_2}{R_1} \frac{\frac{1}{(R_1+R_2)C} + j\omega}{\frac{1}{R_1 C} + j\omega}$$

$$\mathrm{H}(j\omega) = \frac{R_1 + R_2}{R_1} * \frac{1 + j\omega(R_1 + R_2)C}{(R_1 + R_2)C} * \frac{R_1 C}{1 + j\omega R_1 C} = \frac{1 + j\omega(R_1 + R_2)C}{1 + j\omega R_1 C}$$

$$|\mathrm{H}(j\omega)| = \frac{\sqrt{1^2 + \omega^2 (R_1 + R_2)^2 C^2}}{\sqrt{1^2 + \omega^2 {R_1}^2 C^2}} = \frac{\sqrt{1 + \omega^2 (R_1 + R_2)^2 C^2}}{\sqrt{1 + \omega^2 {R_1}^2 C^2}}$$

|H(jω)| @ ω = 0:

$$|\mathrm{H}(j0)| = \frac{\sqrt{1+0}}{\sqrt{1+0}} = 1$$

|H(jω)| @ ω = ∞:

$$|\mathrm{H}(j\infty)| = \frac{(R_1 + R_2)}{R_1} \frac{\sqrt{\frac{1}{(R_1+R_2)^2 C^2} + \infty^2}}{\sqrt{\frac{1}{{R_1}^2 C^2} + \infty^2}}$$

This term → 0

$$|\mathrm{H}(j\infty)| = \frac{(R_1 + R_2)}{R_1} \lim_{x\to\infty} \left[\frac{\sqrt{\frac{1}{(R_1+R_2)^2 C^2} + x^2}}{\sqrt{\frac{1}{{R_1}^2 C^2} + x^2}} \right] = \frac{(R_1 + R_2)}{R_1} * \left[\frac{\sqrt{\frac{1/(R_1+R_2)^2 C^2}{\infty^2} + 1}}{\sqrt{\frac{1/{R_1}^2 C^2}{\infty^2} + 1}} \right]$$

This term → 0

$$|\mathrm{H}(j\infty)| = \frac{(R_1 + R_2)}{R_1} \frac{\sqrt{0+1}}{\sqrt{0+1}} = \frac{(R_1 + R_2)}{R_1}$$

4. The filter cutoff frequency is the frequency where the magnitude is equal to its maximum value divided by sqrt(2) (or 0.7071 $|H(j\omega)|_{max}$). Show that for $R_2 > R_1$ the expression for the filter cutoff frequency ω_c is equal to the following:

$$w_c = \frac{1}{C}\sqrt{\frac{1}{R_1^2} - \frac{2}{2(R_1+R_2)^2}}$$

$$\frac{|H(j\omega)|_{max}}{\sqrt{2}} = \frac{\frac{(R_1+R_2)}{R_1}}{\sqrt{2}} = \frac{\sqrt{1+\omega^2(R_1+R_2)^2C^2}}{\sqrt{1+\omega^2{R_1}^2C^2}}$$

$$(R_1+R_2)^2\left(1+{\omega_c}^2{R_1}^2C^2\right) = 2{R_1}^2\left(1+{\omega_c}^2(R_1+R_2)^2C^2\right)$$

$$\left(R_1+R_2\right)^2 + (R_1+R_2)^2(R_1+R_2)^2\left(1+{\omega_c}^2{R_1}^2C^2\right) = 2{R_1}^2 + 2{R_1}^2{\omega_c}^2(R_1+R_2)^2C^2\right)$$

$$(R_1+R_2)^2{R_1}^2{\omega_c}^2C^2 = (R_1+R_2)^2 - 2R_1$$

$${R_1}^2{\omega_c}^2C^2 = 1 - \frac{2R_1}{(R_1+R_2)^2}$$

$${\omega_c}^2 = \frac{1}{{R_1}^2C^2}\left[1 - \frac{2R_1}{(R_1+R_2)^2}\right]$$

$$\sqrt{{\omega_c}^2 = \frac{1}{C^2}\left[\frac{1}{{R_1}^2} - \frac{2{R_1}^2}{{R_1}^2(R_1+R_2)^2}\right]}$$

$$\omega_c = \frac{1}{C}\sqrt{\frac{1}{{R_1}^2} - \frac{2{R_1}^2}{{R_1}^2(R_1+R_2)^2}}$$

5. Design the filter circuit for a cutoff frequency of 2000 Hz and a gain of 10 (Vo/Vin) in its passband. This step involves choosing values for R_1, R_2 and C to meet the given specifications.

$$\text{gain} = \frac{V_o}{V_i} = 10 = \frac{R_1+R_2}{R_1} \quad ; \text{ let } R_1 = 1\text{k}\Omega$$

$$\omega_c = 2000(2\pi) = 12566.37\frac{\text{rad}}{\text{s}} = \frac{1}{C}\sqrt{\frac{1}{R_1^2} - \frac{2}{2(R_1+R_2)^2}}$$

$$10 = \frac{1\text{k}+R_2}{1\text{k}}$$

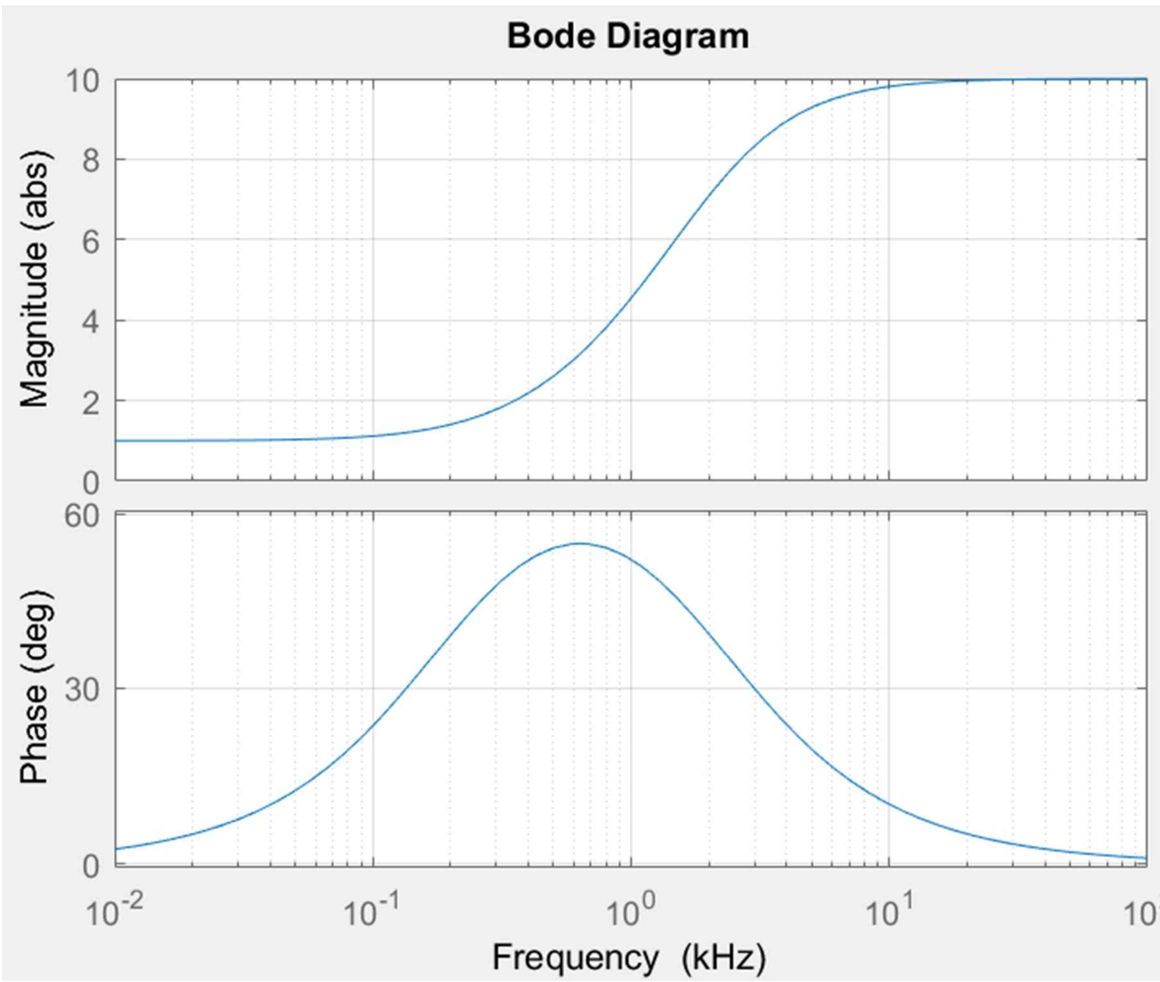
$$10\text{k} = 1\text{k} + R_2 \;\rightarrow\; R_2 = 9\text{ k}\Omega$$

$$\omega_c = 4000\pi = \frac{1}{C}\sqrt{\frac{1}{1\text{k}^2} - \frac{2}{2(1\text{k}+9\text{k})^2}} = \frac{1}{C}\sqrt{9.9 * 10^{-7}}$$

$$C = \frac{9.9499 * 10^{-4}}{4000\pi} = 7.9179 * 10^{-8} \rightarrow \; C = 79.18\ \mathrm{nF}$$

$$H(s) = \frac{10\mathrm{k}}{1\mathrm{k}} \frac{s + \frac{1}{10\ \ (79.18\ \ )}}{s + \frac{1}{1\mathrm{k}(79.18\mathrm{n})}} = 10\frac{s + 1262.97}{s + 12629.7} = \frac{10s + 12629.7}{s + 12629.7}$$

6. Verify your filter design with MATLAB and LTspice. (Is a High-pass op amp filter)



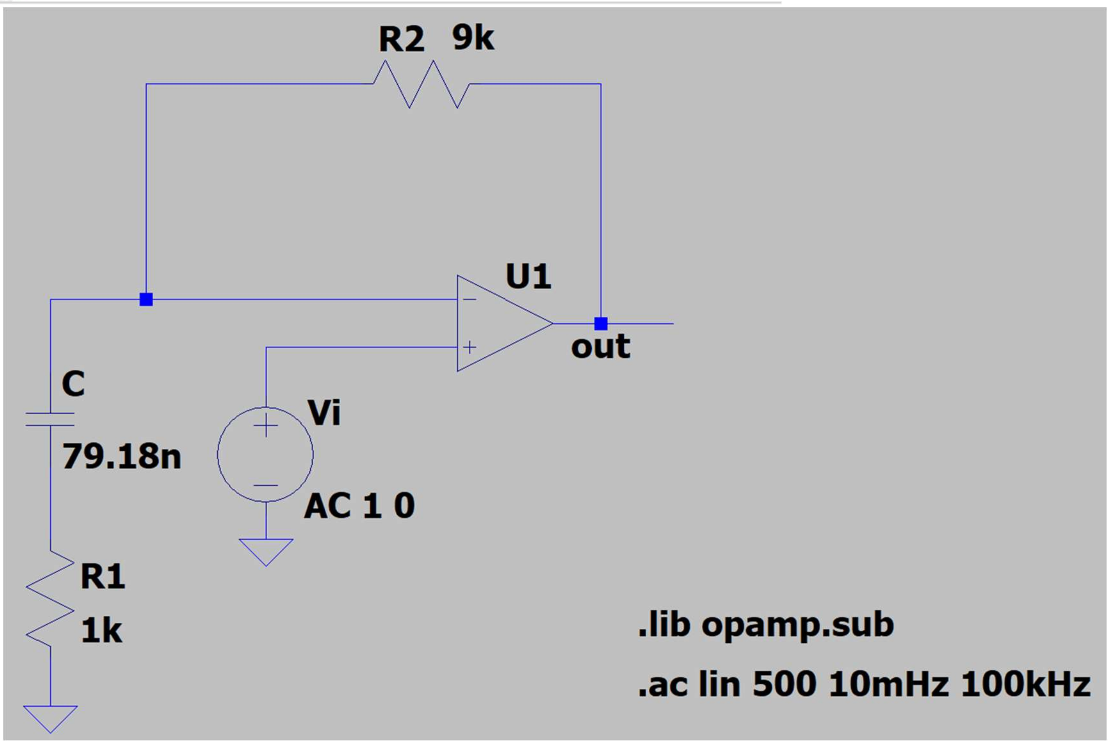
```
>> clear
>> n = [10 12629.7];
>> d = [1 12629.7];
>> h = tf(n,d)

h =

  10 s + 1.263e04
  ---------------
   s + 1.263e04

Continuous-time transfer function.

>> bode(h)
>>
```



vertical axis is in volts and V(n002) = Vi on schematic → so graph is of function: V(out)/ Vi