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ECE 516
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Milestone 2

Introduction

The goal of the assignment is to analyze the unmanned vehicle (UV) dynamics to draw conclusions about the controllability, observability, stability and system modes of the 5-state model. Our model is nonlinear, due to the terms $\cos(\theta)$ and $\sin(\theta)$, which means the model needs to be linearized. There are two different approaches that can be used to obtain the linearized model that will both provide the same results – Taylor series approach and trig approximation. Once these concepts are understood, the information will be used to draw real life conclusions about how noise and disturbances affect the dynamics of the UV.

After the system has been linearized, the stability of the model can be analyzed by obtaining the eigenvalues (λ_i). In order to obtain constant terms in each matrix of the linearized model, a nominal point has to be defined. The nominal point will be a description of the initial state inputs. One of the nominal points should be an equilibrium point as well, which is achieved in this model when all initial state variables are zero. Once the matrix is in this form, eigenvalues are calculated and used to determine the stability surrounding that point. The eigenvalues for a linearized model are categorized into 3 types of stability: (1) *unstable*, when λ_i have positive real parts (2) *stable*, when λ_i have negative real parts (3) *hard to say/dependent on higher order terms*, when $\lambda_i = 0$.

Once the system is in the linearized model, we can draw conclusions about the controllability of the matrix. The controllability of a system relates the state variables to the input variables. When the model is in a state space representation, controllability can be determined by looking at the A and B matrices. Complete controllability will be achieved *iff* $P \triangleq [B \mid AB \mid A^2B \mid \cdots \mid A^{n-1}B]$ has a full rank of *n* when the matrix is a size of $n \times rn$. If a system does not achieve full rank and therefore is not completely controllable, the input is disconnected from the state. When a system is not completely controllable some initial states will have no inputs that can drive the system to the zero state which is undesirable.

The observability of a system relates the state variables to the output variables - which is done by looking at the A and C matrices when the system is the linearized state space representation. Complete observability can be achieved *iff* $Q \triangleq \left[\overline{C^T}\middle|\ \overline{A^T}\ \overline{C^T}\ \middle|\overline{A^{2\,T}}\ \overline{C^T}\middle| \cdots \mid \overline{A^{n-1\,T}}\ \overline{C^T}\right]$ achieves full rank of the same value (*n)*. If the system does not achieve full rank and therefore is not completely observable, then the initial state will never be able to be determined from the output.

Linearized systems can be decomposed into each mode, or subsystem, to test the observability and controllability of each individual mode by forming the normal form matrices. The state variables of a system will be directly correlated to each mode of a system. Observations can be made from the normal form C_n and B_n matrices to determine if the mode is one of four types: controllable and observable, controllable but unobservable, observable but uncontrollable, or uncontrollable and unobservable.

An ideal model will be stable, completely controllable and completely observable. Once the model has been analyzed, different forms of noise and disturbances will be used to demonstrate the response of the UV. This will provide insight to how the UV would operate in the real world since it will not always be in ideal conditions.

Problem Formulation

As mentioned before, since the system is nonlinear the first step will be to linearize the model. The linearized model can be formed by taking the partial derivative of each differential equations to form linearized equations as shown below. The new linearized matrices will be evaluated at the nominal points since the equations include the following brackets, $[\ ]_n$.

$$\delta\dot{x} = \left[\frac{\partial f}{\partial x}\right]_n \delta x + \left[\frac{\partial f}{\partial u}\right]_n \delta u \ \rightarrow \delta\dot{x} = \frac{\partial f}{\partial x}|_{x_n,u_n}\delta x + \frac{\partial f}{\partial x}|_{x_n,u_n}\delta u$$

$$\delta y = \left[\frac{\partial h}{\partial x}\right]_n \delta x + \left[\frac{\partial h}{\partial u}\right]_n \delta u \ \rightarrow \delta y = \frac{\partial h}{\partial x}|_{x_n,u_n}\delta x + \frac{\partial h}{\partial x}|_{x_n,u_n}\delta u$$

The nominal point will be defined as a series of values that are plugged in to their respective space within the linearized matrix. The nominal point can be expressed as $x_n = [x\ y\ \theta\ \omega_r\ \omega_l]$. By choosing different nominal points, we can demonstrate different cases. There are 4 cases that will be explored to demonstrate the UV dynamics: parked therefore $x_n = [0\ 0\ 0\ 0\ 0]$, moving forward therefore $x_n = [0\ 0\ 0\ \omega_r\ \omega_l]$ where $\omega_r = \omega_l$, turning right therefore $x_n = [0\ 0\ 0\ \omega_r\ \omega_l]$ where $\omega_r < \omega_l$ and turning left therefore $x_n = [0\ 0\ 0\ \omega_r\ \omega_l]$ where $\omega_r > \omega_l$. These values must stay below the max angular velocity which can be determined by setting the following equation equal to zero and using the max input value (10V):

$$\dot{\omega}_a = \frac{K_{eq}}{J_{eq}R_a}v_a - \left(\frac{B_{eq}R_a + K_{eq}K_b}{J_{eq}R_a}\right)\omega_a \rightarrow \left(\frac{B_{eq}R_a + K_{eq}K_b}{J_{eq}R_a}\right)\omega_a = \frac{K_{eq}}{J_{eq}R_a}v_a \ \rightarrow \omega_a\left(B_{eq}R_a + K_{eq}K_b\right) = K_{eq}v_a$$

$$\boldsymbol{\omega_{max}} = \frac{v_a K_{eq}}{B_{eq}R_a + K_{eq}K_b} = \frac{10 * 0.74}{0.2963 * 7.4 + 0.74 * 0.0022179} = \mathbf{3.349}\ \boldsymbol{rad/sec}$$

Our nominal points will provide us with a matrix of constants about that point that can be plugged into the linearized model and evaluated. In order to check that the model is correct, the nonlinearized model and linearized model should display the same information when all initial conditions are zero. A Simulink simulation can be used to check that the output, described as $v = \frac{r}{2}(\omega_r + \omega_l)$, is the same for the nonlinear and linearized model. The output of the 5-state model can be achieved by using the outputs of the DC motor, ω_r and ω_l. The linearized model can be simulated by defining the new matrices in the MatLAB code as A, B, C and D. These variables will change based on which nominal point is chosen. The same inputs will be used for this model and will be sent into the state-space block in Simulink. The output for this block will be a single output, *v*, that should match the 5-state model output.

Within the same MatLAB code, the new A matrix will be used to determine the eigenvalues of the system at each nominal point. Since the system is described as a 5 by 5 matrix it will be time consuming to calculate the eigenvalues at each nominal point by hand. In order to efficiently identify the eigenvectors and eigenvalues, I will use the following MatLAB code: [eigen_vector, eigen_value] =eig(A). The system will have *five* eigenvalues that can be used to determine the stability of the system. Since the linearized system will have zero eigenvalues, we cannot tell the stability. However, the state variable outputs of the 5-state nonlinear model can be looked at in order to determine if the system is BIBS and/or BIBO stable.

The next step is to use our new A and B matrices to determine the controllability. As mentioned in order for the system to be completely controllable it needs full rank which in this case is 5. This means that the equations mentioned earlier changes to $P \triangleq [B \mid AB \mid A^2B \mid A^3B \mid A^4B]$ where complete controllability is achieved when rank(P)=5. Only when a system is completely controllable; it has a controllability index which is the number of perturbations needed to achieve full rank. In case our system achieves full rank, I will set each perturbation as its own constant in MatLAB so that the controllability index can be easily calculated. After formulating a matrix P with all the perturbations, the controllability can be determined in MatLAB with the following code: rank(P). If it is full rank start testing each perturbation to determine the controllability index. The controllability index is defined by the perturbation that achieves full rank.

Observability will be analyzed using the new A and C matrices. As mentioned in order for the system to be completely observable it needs full rank which in this case is 5. The equation mentioned earlier changes to $Q \triangleq [\overline{C^T} | \ \overline{A^T}\,\overline{C^T}\ | \overline{A^{2\,T}}\ \overline{C^T} | \overline{A^{3\,T}}\ \overline{C^T} | \ \overline{A^{4\,T}}\ \overline{C^T}]$ where complete observability is achieved when rank(Q)=5. Only when a system is completely observable; it will have an observability index which is the number of perturbations needed to achieve full rank. For the same reason as the controllability, I will set each perturbation as its own constant in MatLAB so the observability index can be easily calculated if needed. After formulating a matrix Q with all the perturbations, the observability can be determined in MatLAB with the following code: rank(Q). If it is full rank start testing each perturbation to determine the observability index.

The state variables each correspond to their own mode so in our case we have five modes to analyze. When a model has distinct eigenvalues the normal form of the model can be formed as shown below:

$$\dot{q} = \Lambda q + B_n u(t)$$
$$y(t) = C_n q + Du(t)$$

The variables can be determined with the following equations $\Lambda = M^{-1}AM$, $B_n = M^{-1}B$ and $C_n = CM$. The eigenvector that was determined earlier is the modal matrix, *M*, that is necessary to compute these matrices. When the system has repeated eigenvalues, look at the eigenvectors to determine if they are linearly independent or linearly dependent. When the eigenvectors are linearly independent, the modes are independent and therefore can be decoupled. When we have repeated eigenvalues that have dependent eigenvectors, the modes cannot be decoupled unless a generalized eigenvector is created. Generalized eigenvectors are not a topic that was covered in class so I will not move forward with finding generalized eigenvectors. Other observations will be used to draw conclusions about each mode.

If we are able to move forward with finding the normal form of the model, observability and controllability can be analyzed by only looking at the B_n and C_n matrices. In order to determine controllability, if the B_n matrix has an all zero *row*, that mode is not completely controllable. For example, the first mode in our model is $\dot{x}$ so if the top row of the B_n matrix is zero for both inputs that particular mode is uncontrollable. However, if even one of the terms is nonzero then the matrix is controllable. A similar approach can be used to determine observability. If the C_n matrix has an all zero *column,* then that mode is not completely observable. Using the same example, if the first column of the C_n matrix is all zeros, the mode is not completely observable. By using this approach, each mode can be labeled as any combination as mentioned in the introduction.

The final step of the project is to incorporate disturbances into the model which can be seen below where u_D represents the disturbances on the system and B_D represents the corresponding disturbance matrix:

$$\dot{x} = A(t)x(t) + B(t)u(t) + B_D(t)u_D(t)$$

The disturbance equation used in the model is included below where: α is the slope of the road which needs to be defined in rad, μ is the friction coefficient between the tires and the road and F_l is an extra load force that is added to the system in kg. All other values are given parameters for the model.

$$u_d = \frac{-1}{J_{eq}}(Mgsin(\alpha) + Mg\mu\cos(\alpha) + F_l)r$$

Mathematical Modeling

The five differential equations are defined as:

(1) $f1 = \dot{x}_I = \frac{r}{2}(\omega_l + \omega_r) * cos\theta$

(2) $f2 = \dot{y}_I = \frac{r}{2}(\omega_l + \omega_r) * sin\theta$

(3) $f3 = \dot{\theta} = \frac{r}{W_r}(-\omega_l + \omega_r)$

(4) $f4 = \dot{\omega}_r = \frac{K_{eq}}{J_{eq}R_a} v_{ar} - (\frac{B_{eq} + \frac{K_{eq}K_b}{R_a}}{J_{eq}})\omega_r$

(5) $f5 = \dot{\omega}_l = \frac{K_{eq}}{J_{eq}R_a} v_{al} - (\frac{B_{eq} + \frac{K_{eq}K_b}{R_a}}{J_{eq}})\omega_l$

Use the generalized equation below to come up with the 5 corresponding linearized equations below:

$$\delta\dot{x}_n = \frac{\partial f_n}{\partial x}|_{x_n,u_n}\delta x + \frac{\partial f_n}{\partial y}|_{x_n,u_n}\delta y + \frac{\partial f_n}{\partial \theta}|_{x_n,u_n}\delta\theta + \frac{\partial f_n}{\partial \omega_r}|_{x_n,u_n}\delta\omega_r + \frac{\partial f_n}{\partial \omega_l}|_{x_n,u_n}\delta\omega_l + \frac{\partial f_n}{\partial v_{ar}}|_{x_n,u_n}\delta v_{ar} + \frac{\partial f_n}{\partial v_{al}}|_{x_n,u_n}\delta v_{al}$$

(1) $\boldsymbol{\delta\dot{x}} = 0\delta x + 0\delta y + \frac{-r*\sin(\theta)*(\omega_R+\omega_L)}{2}|_{x_n,u_n}\delta\theta + \frac{r*\cos(\theta)}{2}|_{x_n,u_n}\delta\omega_r + \frac{r*\cos(\theta)}{2}|_{x_n,u_n}\delta\omega_l + 0\delta v_{ar} + 0\delta v_{al}$

(2) $\boldsymbol{\delta\dot{y}} = 0\delta x + 0\delta y + \frac{r*\cos(\theta)*(\omega_R+\omega_L)}{2}|_{x_n,u_n}\delta\theta + \frac{r*\sin(\theta)}{2}|_{x_n,u_n}\delta\omega_r + \frac{r*\sin(\theta)}{2}|_{x_n,u_n}\delta\omega_l + 0\delta v_{ar} + 0\delta v_{al}$

(3) $\boldsymbol{\delta\dot{\theta}} = 0\delta x + 0\delta y + 0\delta\theta + \frac{r}{W_r}|_{x_n,u_n}\delta\omega_r - \frac{r}{W_r}|_{x_n,u_n}\delta\omega_l + 0\delta v_{ar} + 0\delta v_{al}$

(4) $\boldsymbol{\delta\dot{\omega}_r} = 0\delta x + 0\delta y + 0\delta\theta + \frac{-(B_{eq}+\frac{K_e*K_b}{R_a})}{J_{eq}}|_{x_n,u_n}\delta\omega_r + 0\delta\omega_l + \frac{K_{eq}}{R_a*J_{eq}}|_{x_n,u_n}\delta v_{ar} + 0\delta v_{al}$

(5) $\boldsymbol{\delta\dot{\omega}_l} = 0\delta x + 0\delta y + 0\delta\theta + 0\delta\omega_r + \frac{-(B_{eq}+\frac{K_e*K_b}{R_a})}{J_{eq}}|_{x_n,u_n}\delta\omega_l + 0\delta v_{ar} + \frac{K_{eq}}{R_a*J_{eq}}|_{x_n,u_n}\delta v_{al}$

Which can be placed into the following linearized 5-state space model:

$$\begin{bmatrix}\delta\dot{x}\\ \delta\dot{y}\\ \delta\dot{\theta}\\ \delta\dot{\omega}_r\\ \delta\dot{\omega}_l\end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{-r*\sin(\theta)*(\omega_r+\omega_l)}{2} & \frac{r}{2}\cos(\theta) & \frac{r}{2}\cos(\theta) \\ 0 & 0 & \frac{r*\cos(\theta)*(\omega_r+\omega_l)}{2} & \frac{r}{2}\sin(\theta) & \frac{r}{2}\sin(\theta) \\ 0 & 0 & 0 & \frac{r}{W_r} & -\frac{r}{W_r} \\ 0 & 0 & 0 & \frac{-(B_{eq}+\frac{K_e*K_b}{R_a})}{J_{eq}} & 0 \\ 0 & 0 & 0 & 0 & \frac{-(B_{eq}+\frac{K_e*K_b}{R_a})}{J_{eq}} \end{bmatrix} \begin{bmatrix}\delta x\\ \delta y\\ \delta\theta\\ \delta\omega_r\\ \delta\omega_l\end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_{eq}}{R_a*J_{eq}} & 0 \\ 0 & \frac{K_{eq}}{R_a*J_{eq}} \end{bmatrix} \begin{bmatrix}\delta v_{ar}\\ \delta v_{al}\end{bmatrix}$$

$$[\delta y] = \begin{bmatrix} 0 & 0 & 0 & \frac{r}{2} & \frac{r}{2} \end{bmatrix} \begin{bmatrix}\delta x\\ \delta y\\ \delta\theta\\ \delta\omega_r\\ \delta\omega_l\end{bmatrix}$$

Example of Calculations at Nominal Point - [0 0 0 0 0]

$$A = \begin{bmatrix} 0 & 0 & 0 & 0.0185 & 0.0185 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2056 & -0.2056 \\ 0 & 0 & 0 & -16.4429 & 0 \\ 0 & 0 & 0 & 0 & -16.4429 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 5.5494 & 0 \\ 0 & 5.5494 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 0 & 0.0185 & 0.0185 \end{bmatrix}$$

Evaluate the nominal point to get eigenvectors:

$$M = eigenvector = \begin{bmatrix} 1 & 0 & 0 & -0.0011 & -0.0011 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.125 & 0.0125 \\ 0 & 0 & 0 & 0.999 & 0 \\ 0 & 0 & 0 & 0 & 0.9999 \end{bmatrix}$$

$$eigenvalues = 0,0,0, -16.44299 - 16.4429$$

Use A and B to get P matrix:

$$P = \begin{bmatrix} 0 & 0 & 0.1 & 0.1 & -1.7 & -1.7 & 28 & 28 & -460 & -460 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.1 & -1.1 & -18.8 & 18.8 & 308 & -308 & -5070 & 5070 \\ 5.5 & 0 & -91.2 & -16.4429 & 1500.4 & 0 & -24671 & 0 & 405650 & 0 \\ 0 & 5.5 & 0 & -91.2 & 0 & 1500.4 & 0 & -24671 & 0 & 405650 \end{bmatrix} = rank(P) = 4$$

The top two columns can never be unique, they will always be multiples of each other, so rank is 4.

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.3 & -0.3 \\ 0 & 0 & 0 & 5 & 5 \\ 0 & 0 & 0 & -82.2 & -82.2 \\ 0 & 0 & 0 & 1352.3 & 1352.3 \end{bmatrix} = rank(Q) = 1$$

Since none of the neither of columns are unique the rank is 1.

Use the M matrix to get the inverse M:

$$M^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0.0011 & 0.0011 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.125 & -0.0125 \\ 0 & 0 & 0 & 1.0001 & 0 \\ 0 & 0 & 0 & 0 & 1.0001 \end{bmatrix},$$

Use to find the normal for matrices:

$$B_n = M^{-1}B = \begin{bmatrix} 0.00624 & 0.00624 \\ 0 & 0 \\ 0.06837 & -0.06837 \\ 5.54982 & 0 \\ 0 & 5.54982 \end{bmatrix}$$

$$C_n = CM = \begin{bmatrix} 0 & 0 & 0 & 0.01849 & 0.01849 \end{bmatrix}$$

Simulink Models

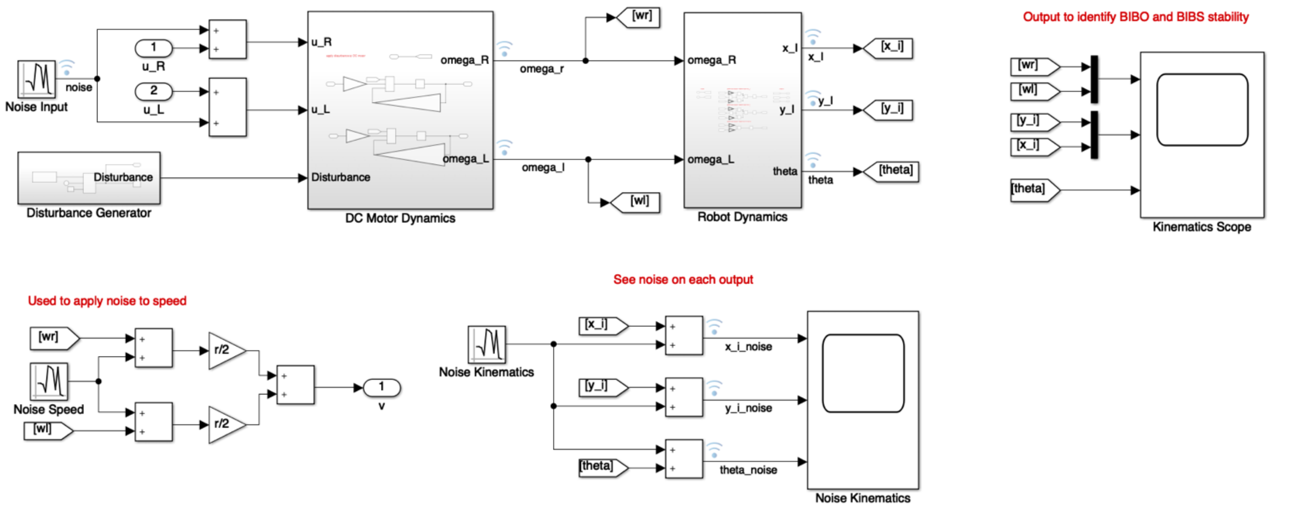


Figure 1: Implementation of noise and disturbance into the 5-state model

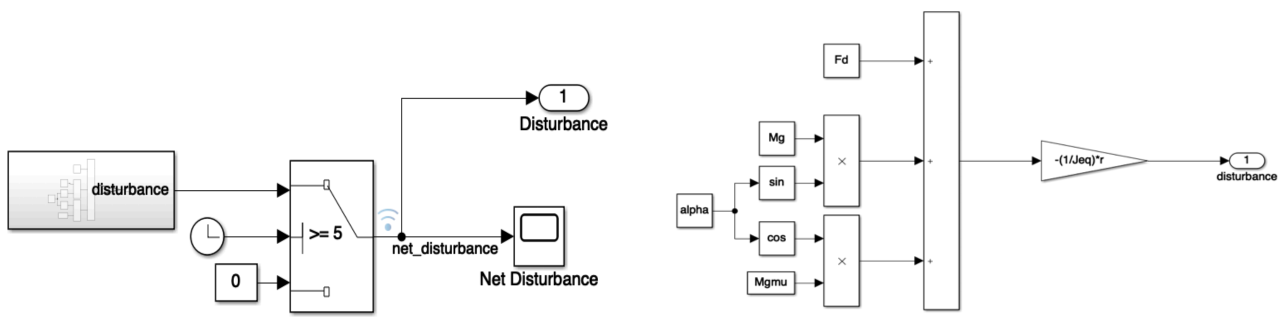


Figure 2: (Left) Disturbance subsystem which shows the switch for when to implement the disturbance (Right) Implementation of the disturbance matrix

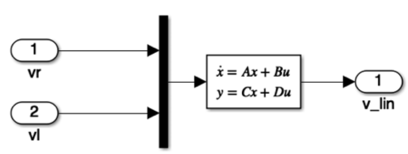
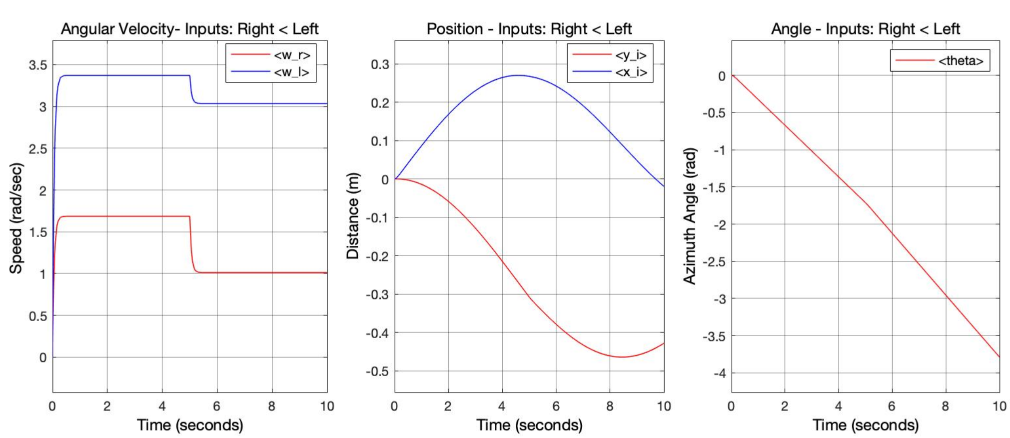


Figure 3: Linearized model that will experience no noise or disturbances and be used for error analysis

Results



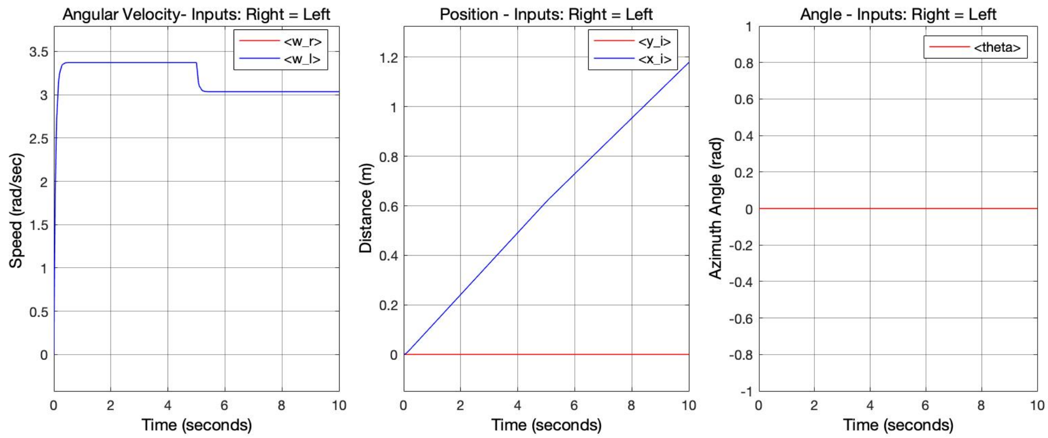


Figure 4: The figures above are used to determine if the system is BIBS and/or BIBO (Top) Right input velocity is greater which will cause the theta to continuously increase (Bottom) Inputs are equal so the UV is continuously moving in the x direction

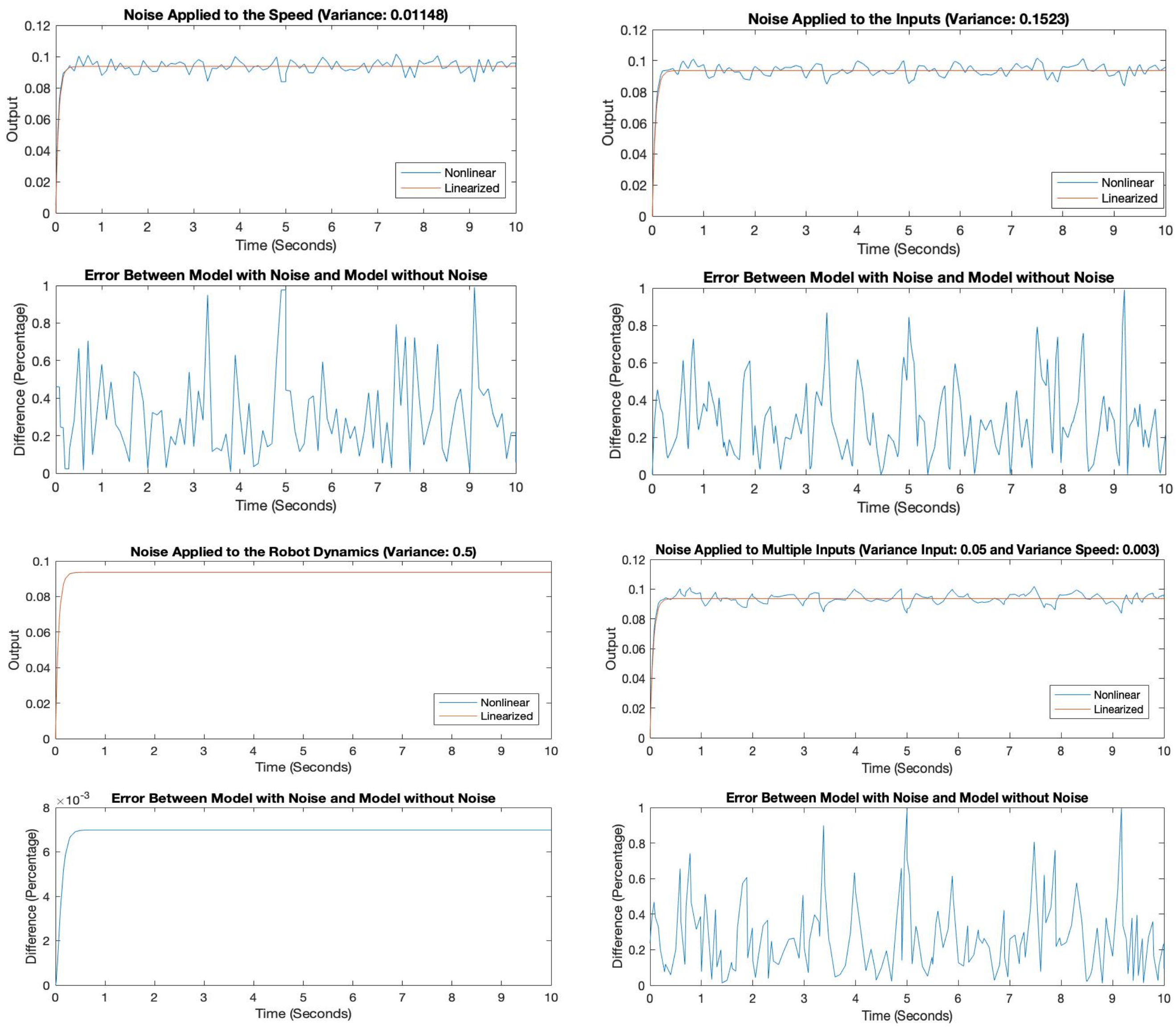


Figure 5: The graphs show the effects for the maximum tolerance being placed at each input to stay below error of one

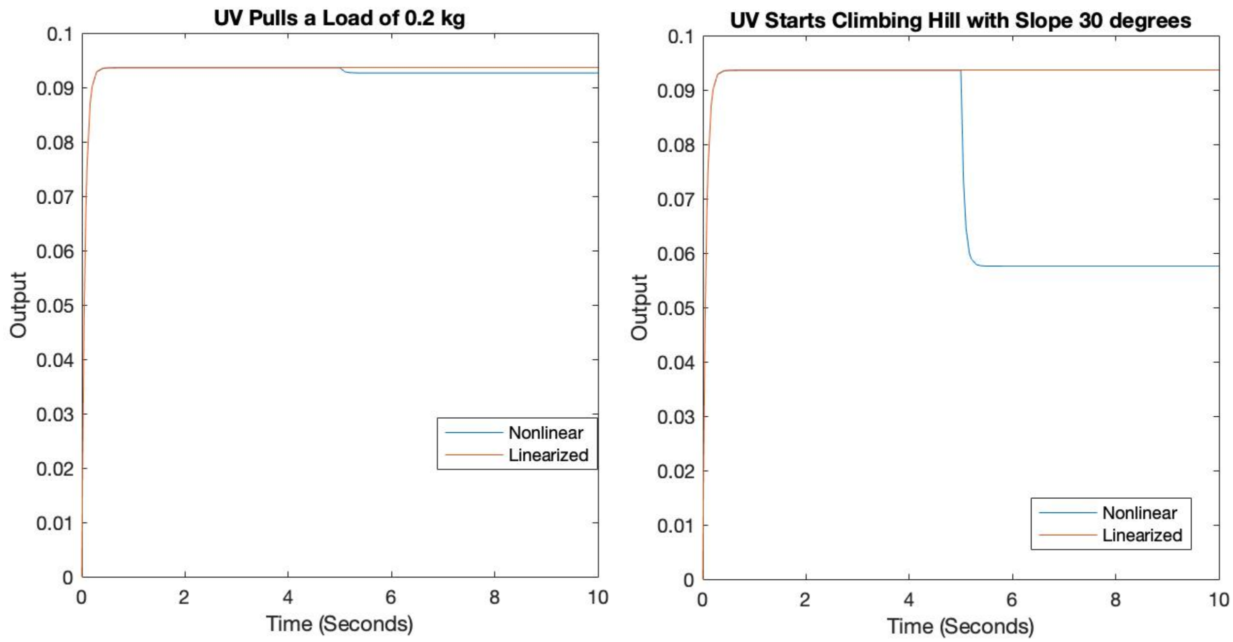


Figure 6: Disturbance applied to the model (Left) An additional load of 0.2 kg gets added to the model (Right) The UV starts to move up a hill that has a slope of 30 degrees

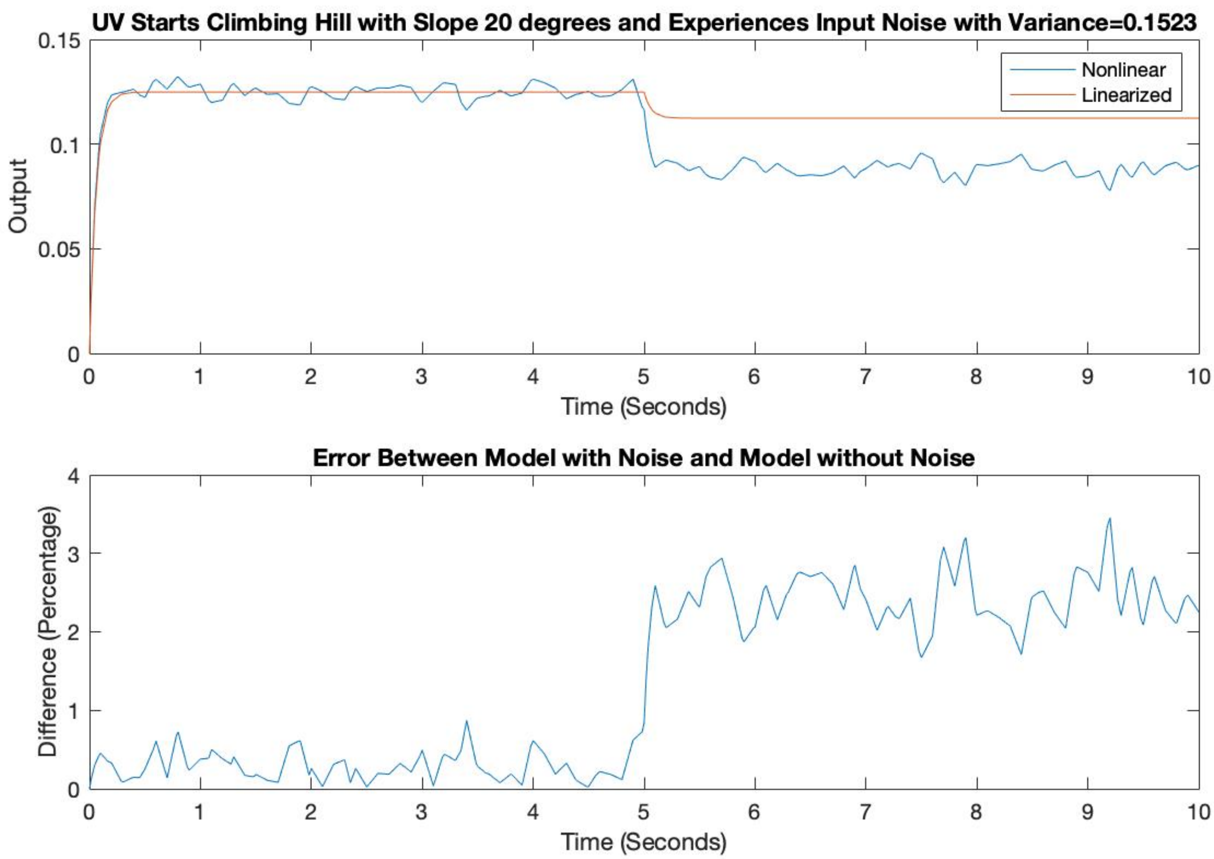


Figure 7: Implementing e maximum input tolerance to keep output error below 1 along with a slope change at 5 seconds

Discussion

The controllability, observability and stability of each mode will need to be calculated for each of the nominal points mentioned before:

The first case that was analyzed was when the car is parked so the nominal point is set as x_n=[0 0 0 0 0]. The new A and B matrices were then used to find the controllability of the matrix. The P matrix achieves a rank of 4 which is not full rank. This means that at this nominal point the system is not completely controllable and has no controllability index. The new A and C matrices were then used to determine the observability of the matrix. The Q matrix only achieves a rank of 1 which also does not achieve full rank. This means that the system is also not completely observable at this nominal point and does not have an observability index. The normal form is necessary to look at the mode decomposition of the system to determine the observability and/or controllability of each mode. The model provides these five eigenvalues: [0 0 0 -16.442878 -16.442878]. The model therefore gives us two negative real repeated roots and three zero repeated roots. By looking at the eigenvectors it was determined that this nominal point has independent eigenvectors, which meant the modes could be decoupled, and each mode can be analyzed without finding generalized eigenvectors. By using the eigenvector created in MatLAB (shown in the mathematical modeling section), we got B_n and C_n matrices.

The first mode corresponds to the x position state variable which can be analyzed by looking at the first row of B_n and the first column of C_n. The first mode is completely controllable but not completely observable. The second mode corresponds to the y position state variable. Since the second row of B_n is all zeros and the second C_n column is zero, the system is neither completely controllable or completely observable. The third mode corresponds to the theta state variable. The information in the matrices tell us that this mode is completely controllable but is not completely observable. The fourth and fifth modes correspond to the right and left angular velocities respectively. These modes are both completely controllable and completely observable which was determined from the nonzero values in their corresponding B_n rows and C_n columns.

Since the system is linearized and has three zero eigenvalues, we are not able to determine the stability of the system. Whether or not the system is BIBO or BIBS stable can be observed by looking at the state variables provided in the nonlinear model which is demonstrated above in figure 4. The system is BIBO stable since the input is bounded to an input between 0-10V and the output is also bounded. However, the system is not BIBS stable since one of the states will always be growing in an unbounded fashion over the 10 second time period. The state that will grow exponentially will depend on the inputs to each wheel. When the initial conditions for the system are zero and the inputs to each wheel are equal, the x position state will grow exponentially. When the initial conditions for the system are zero but the inputs of each wheel are different, the UV continuously turns which leads to an unbounded angle state.

The second case that was analyzed represents when the car is moving forward. This is represented by selecting any values that are less than the max angular velocity of 3.349 *rad/sec* for both of the angular velocities. In my model, I choose to stay well below the max and set both the left and right wheels to 2 *rad/sec* which gives the nominal point of x_n=[0 0 0 2 2]. The MatLAB code shows that the new linearized A matrix, now achieves a full rank of 5. The model was then tested to see when it achieved full rank and it was determined that when $P \triangleq [B \mid AB \mid A^2B]$ the P matrix achieves a rank of 5. This means that the system has a controllability index of 3 at this nominal point since it achieves full rank at the third perturbation. Although the system is now completely controllable, it still is not completely observable. This is because the Q matrix still had a rank of 1. The system has the same eigenvalues at this nominal point which means the eigenvectors need to be analyzed in order to move forward.

The second case has eigenvectors that are not distinct so the normal form cannot be made with the eigenvalues that we currently have. In order to look at each system mode more closely, you need to find the generalized eigenvectors which requires a process we have yet to cover. The generalized eigenvector will provide a new M value that will have an inverse matrix so the B_n and C_n matrices can be created. When the model is run without the generalized eigenvectors as it currently is, the B_n matrix is filled with very large numbers (raised to the power of 298) that are inaccurate since our current M does not have a real inverse. Since the system is completely controllable, we know that each of the modes in this system is completely controllable. With the given output, only the fourth and fifth states will be observable. By looking at each mode in this way, it led to the conclusion that states one, two and three are completely controllable but not completely observable. Additionally, states four and five are completely controllable and completely observable.

The eigenvalues are the exact same for this nominal point, which means the system stability still cannot be determined. Therefore, we are led to the same conclusions about the system: it is not BIBS stable since at least one state variable will be always be unbounded and it is BIBO stable due to the fact that the output will always be bounded by the step input for all time t, $||u(t)||\leq 10$. The same conclusions about stability will be true for the two other cases as well (right turn and left turn) so the system is always BIBO stable and never BIBS stable.

The third and fourth cases will be discussed together. The third case displays the UV making a right turn, which can be described when the right angular velocity is less than the left angular velocity. To achieve this, the right wheel angular velocity was dropped down to 1 *rad/sec* which provides the nominal point x_n=[0 0 0 1 2]. The fourth case represents the UV making a left turn, which is described by the left angular velocity being lower than the right. For repeatability, the left wheel angular velocity is dropped down to 1 *rad/sec* to provide another nominal point at x_n=[0 0 0 2 1]. When using each of these nominal points to determine the new linearized matrices, the system achieves a full rank for the P matrix which means it is completely controllable. Full rank is achieved at the third perturbation, so the system still has a controllability index of 3. When analyzing the Q matrix, it still only achieves a rank of 1 so the system is not completely observable.

After looking at a variety of values that fell within the max angular velocity and meet the case requirements, it can be seen that when at least one wheel has a non-zero value for angular velocity the system becomes completely controllable. However, no nominal points were found that also made the system completely observable. In fact, the system never achieved a rank higher than 1 for the Q matrix. This is understandable due to the output matrix and the fact that it is linked directly to the angular velocity values for each wheel.

The final step was to add noise and disturbances to the system. When the nominal point is set to zero for all states, the linearized model and nonlinear model provide the same output. In order to control where and how the noise and disturbances were applied to the model, the non-linear model had to be used. By usin this correlation, the error could be calculated by taking the difference between the linearized model output and the nonlinear model output.

Random noise for the system was implemented by using the random number generator block to create a Gaussian distributed random signal. The noise blocks were controlled within the MatLAB code since the variance of the random number block determines how much noise is added; when the variance within this block is zero, no noise is added to the system. This block was added to three different inputs that are described below. By varying the dynamics of the nonlinear system, the error it created on the output could be calculated by comparing it to the linearized model. I determined the max variance that each input could have while still keeping the output error below one percent. I chose a max error of one percent since the output of the model is overall speed and our model has fairly small output values. The amount error that is tolerated could be adjusted which would lead to increased or decreased variance values. These variance values were obtained by using the Monte Carlo analysis in MatLAB to run various Simulink simulations at a time to find the variance value that made the error less than one percent.

The first noise that I observed was noise is applied to the inputs only, v_{ar} and v_{al}. Noise that is applied here could be a sign that the electrical wires in the UV are starting to come loose or the connections to the battery are poor. Every variation in the road is causing a either an increase or decreased input value to the UV instead of the normal steady input. I found that the system could be subject to a variance of 0.1532 while maintaining an output error of less than one percent for all values.

The second noise that I observed was noise input to the right and left angular velocity state variables, ω_r and ω_l. Noise that is applied to this input can be understood by imagining that the system is being run on cruise control (the linearized model). If you are applying the gas or the break (in the nonlinear model) or dealing with road variation that require different inputs, it will greatly change the overall speed of the system and provide a large error from the linearized model. Since these values are directly correlated to the output, noise in this input has a much lower max tolerance, only 0.01128, to stay below one percent.

The last place that I added noise was to the robot dynamics, x,y and θ. These state variables represent the orientation of the robot on the road that it is on. When there is noise applied to this lane it could be something like switching lanes to get around a car and then getting back in the same lane or a distracted driver swerving within their lane. Since the output is not directly correlated to any of these values, when noise is applied to these inputs there was no change with output which is shown in figure 5.

If the system has noise applied to both the angular velocity the inputs, these max variance values no longer hold true. The max variance of both inputs is greatly reduced as any variance added at the first input will be amplified by the second input. Figure 5 also shows a case where noise is added to both the input and the angular velocity.

In my Simulink model, these disturbances were only applied it to the DC motor subsystem since the dynamics are formed by looking at the wheel dynamics. The disturbances were added five seconds into the simulation to show how it affected the dynamics of a model. The two disturbances are below:

The first disturbance modeled a system that started to climb a hill with a slope of 30 degrees that was applied to the model after five seconds. A slope of 30 degrees is steep, however some cities like San Francisco consist of multiple roads with this amount of slope. The increase of the slope causes a decrease of 0.036 in the output (without the disturbance the value was 0.094 and after the disturbance was added the output was 0.058). The decrease in speed makes sense due to the increase of work that needs to be applied to go the same distance. These results are shown in figure 6.

The other disturbance modeled a system that was put under more load after five seconds. Since the system was only 1.59 kg, I only applied a load of 0.2 kg. This could be representative of the UV picking up some goods that need to be transported to a new location. An increase in the load has very little impact on the output value. By increasing the load by 0.2kg we only decrease the output by 0.001 (the output drops from 0.094 to 0.093) which is shown in figure 6. A wide range of loaded were tested and another case that I found interesting was if the UV was towing another UV behind it that was the same weight. When the load force is increased to 1.59kg the output only drops by 0.008 which shows that a change in slope is more of a disturbance on the output than a change in the load.

Finally, the system was modeled with both disturbances and noise added as shown in figure 7. This shows that these variations to the systems add on top of each other to create more error. The system is not limited to just having noise or disturbances.

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