Welcome to the introduction to computational cognitive modelling workshop!

Part 2: Introduction to artificial neural networks

Part 2: **Introduction to artificial neural networks**

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Chris Brand

Nick Sexton

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What is a neural network?

A mathematical model

- Inspired by the nervous system

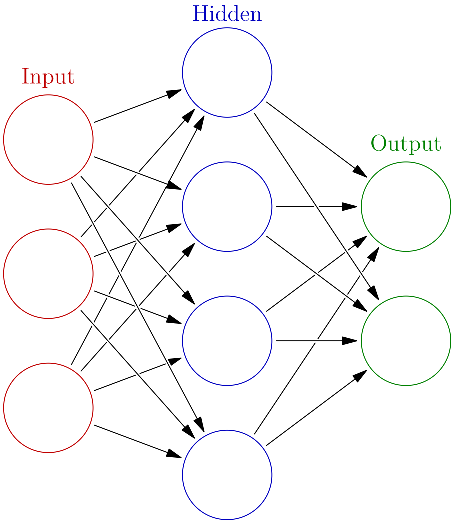


Figure : Glosser.ca / CC-BY-SA-3.0

What is a neural network?

A mathematical model

- Inspired by the nervous system
- A set of *units*, connected by *weights*

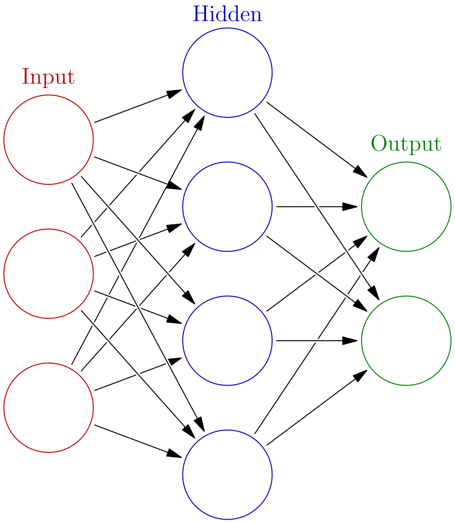
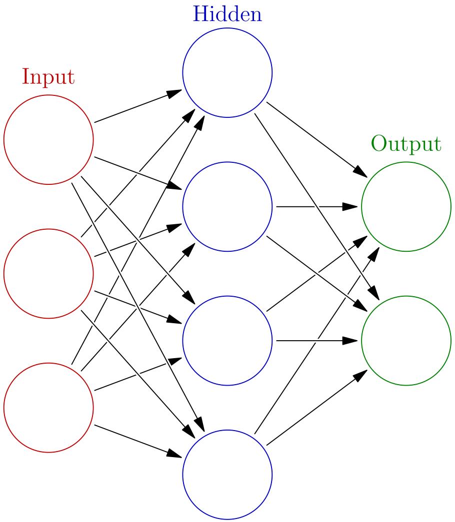


Figure : Glosser.ca / CC-BY-SA-3.0

What is a neural network?

A mathematical model

- Inspired by the nervous system
- A set of *units*, connected by *weights*
- The network *runs* by passing *activations* from the *input* (to the *hidden*) to the *output* units



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Why use artificial neural networks for modelling?

Some aspects of their behaviour are like their namesake!

- Learn pretty much any input-output data

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Why use artificial neural networks for modelling?

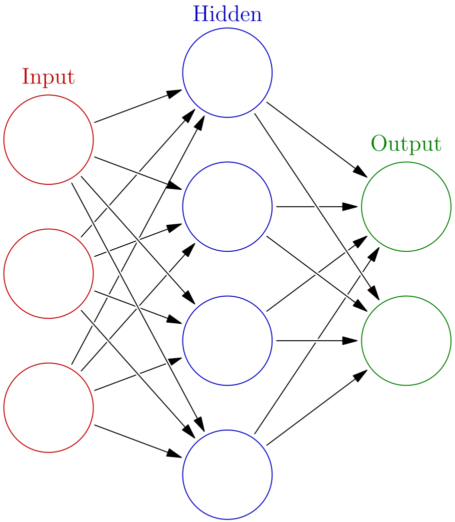
Some aspects of their behaviour are like their namesake!

- Learn pretty much any input-output data
- Uncover rules on their own about data
- Generalise from what they have learnt
- Cope with noise and damage

How does an artificial neural network run?

By using maths, predictably!

1. Input units are set to a *pattern*



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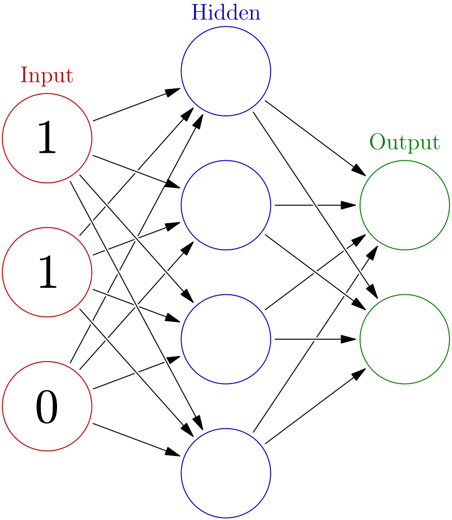

How does an artificial neural network run?

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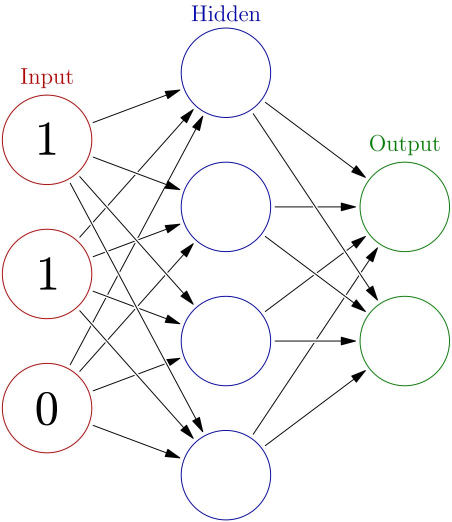
1. Input units are set to a *pattern*
2. Calculate hidden units' states



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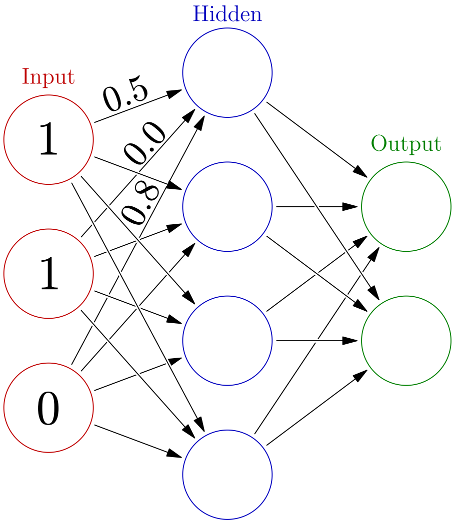

How does an artificial neural network run?

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1. Input units are set to a *pattern*

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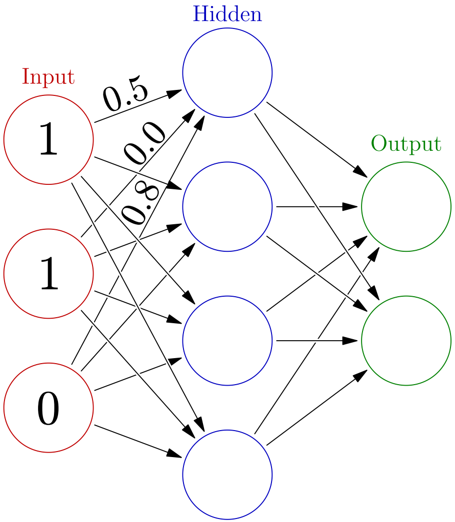
   $$1 \times 0.5 = \quad 0.5$$



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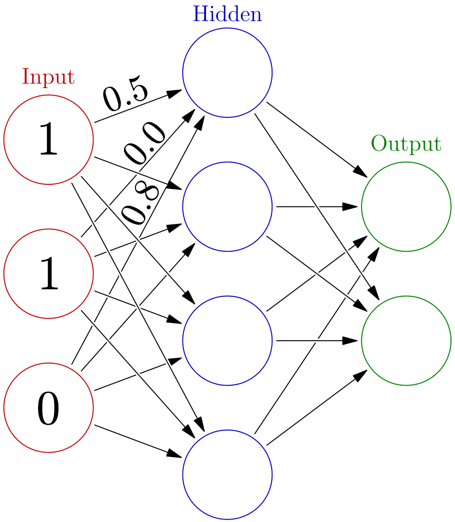

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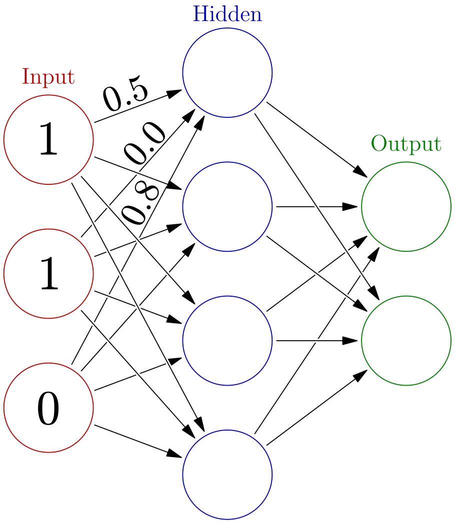
$$\begin{aligned} 1 \times 0.5 &= \quad 0.5 \\ 1 \times 0.0 &= \quad 0.0 \\ 0 \times 0.8 &= \quad 0.0 \end{aligned}$$



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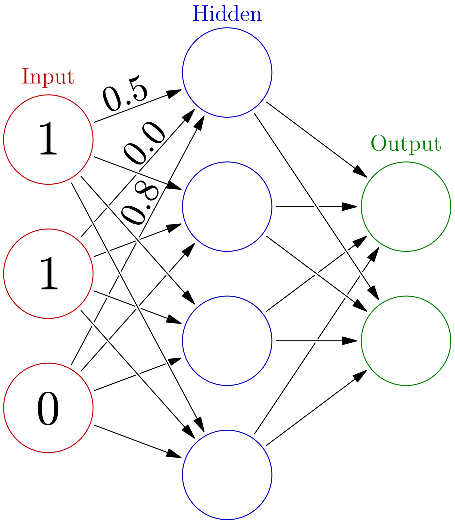

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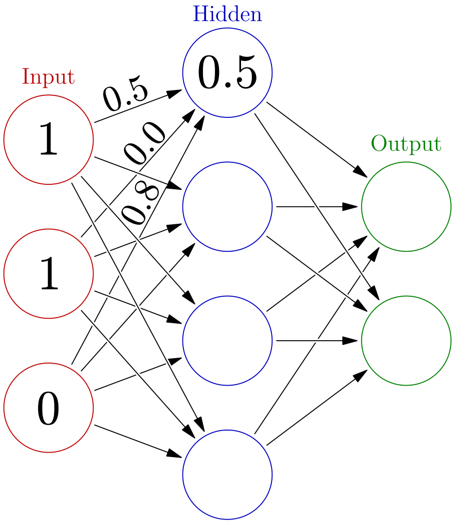

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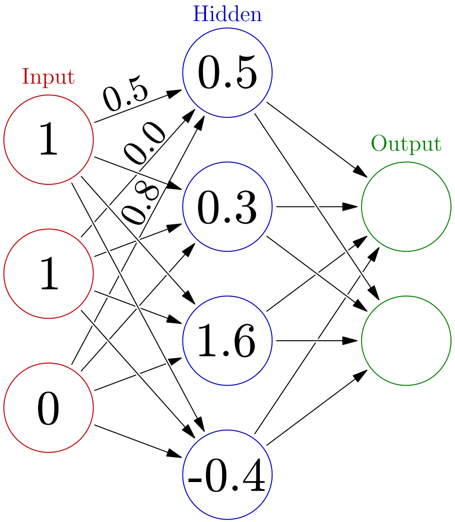

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1. Input units are set to a *pattern*
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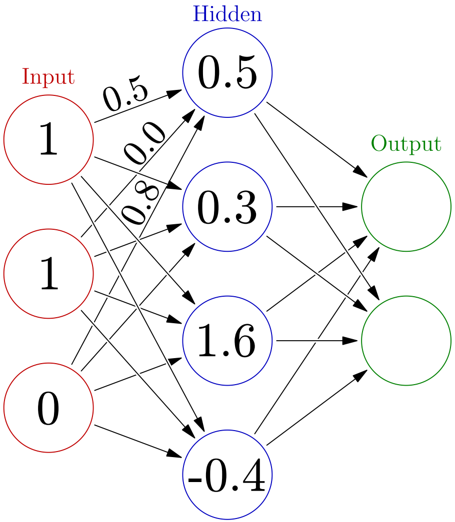
3. Same for output units



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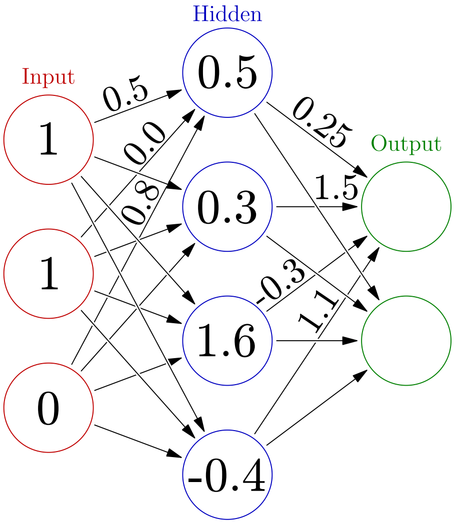

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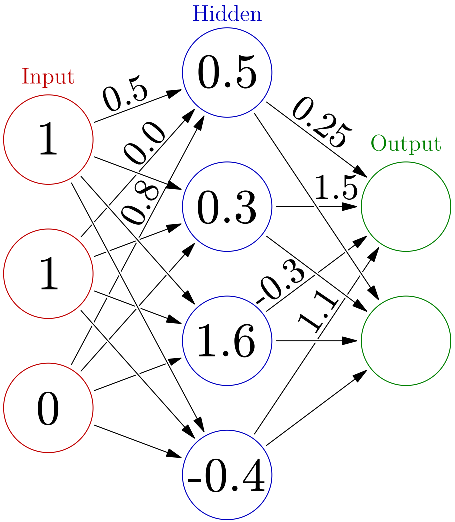
   $0.5 \times 0.25 \quad = \qquad 0.125$



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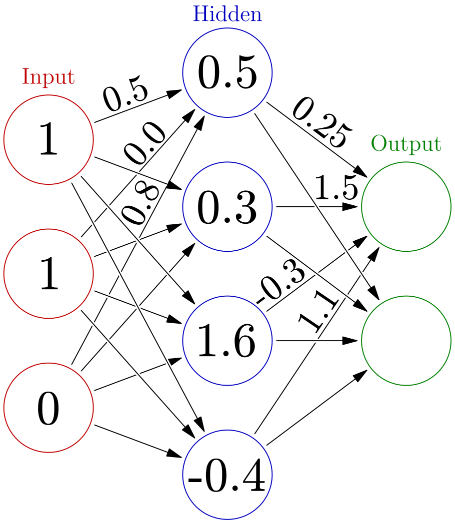
$$\begin{aligned} 0.5 \times 0.25 &= 0.125 \\ 0.3 \times 1.5 &= 0.45 \end{aligned}$$



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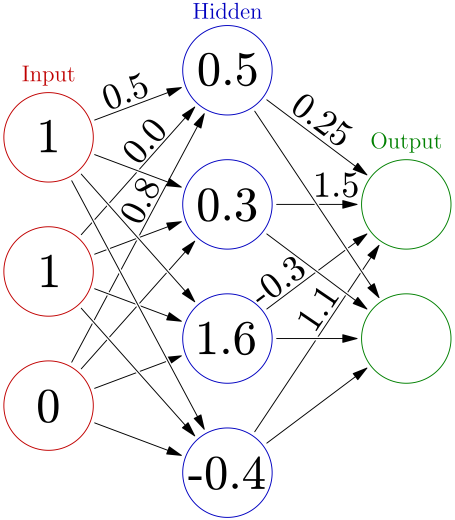
$$\begin{aligned} 0.5 \times 0.25 &= 0.125 \\ 0.3 \times 1.5 &= 0.45 \\ 1.6 \times \text{-}0.3 &= -0.48 \end{aligned}$$



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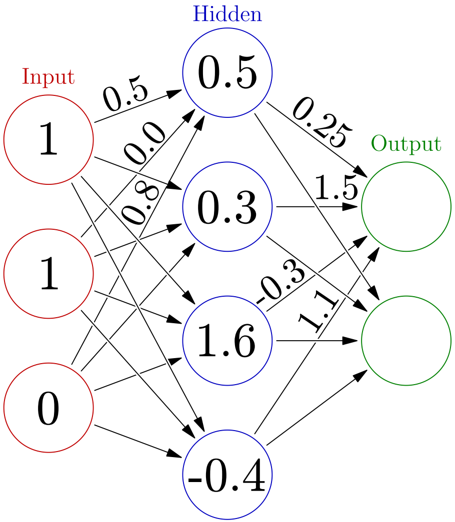
1.6 \times \text{-}0.3 & = & -0.48 \\
\text{-}0.4 \times 1.1 & = & -0.44
\end{array}
$$



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\hline
 & & -0.345 &
\end{array}
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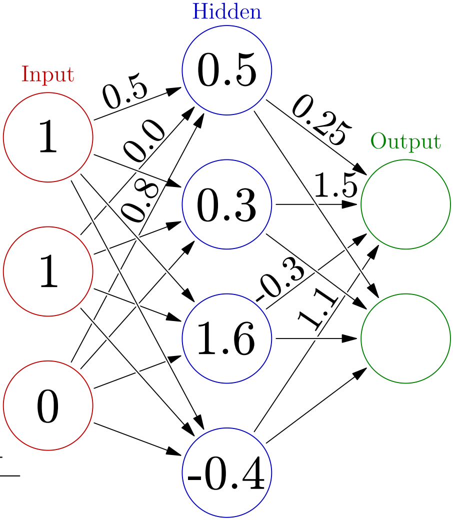

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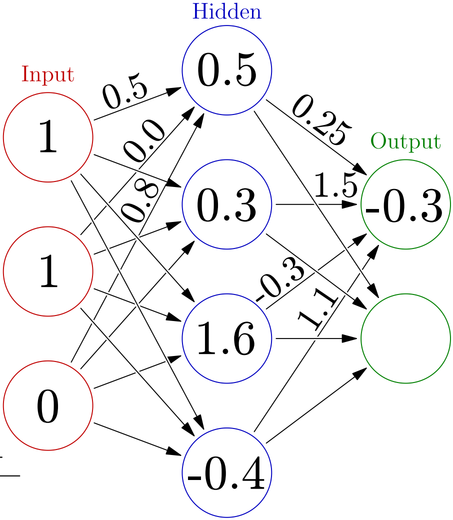

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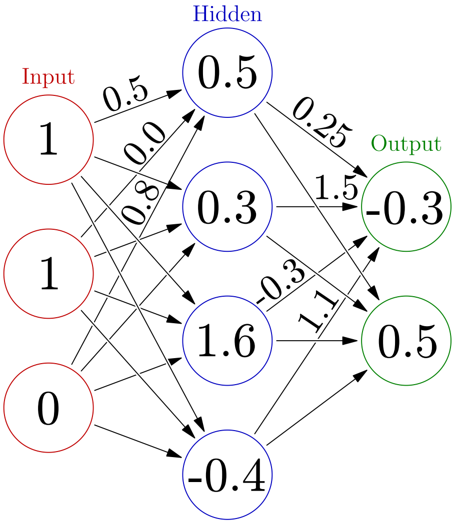
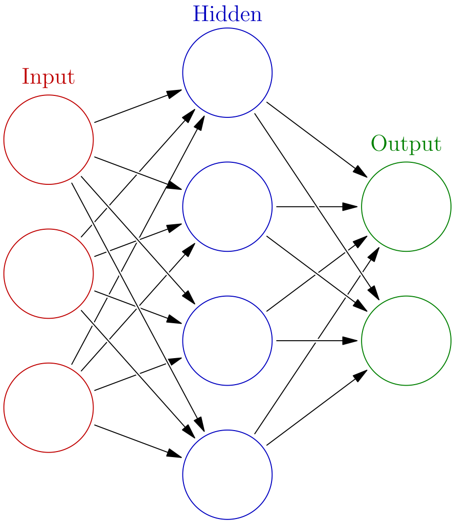


How does an artificial neural network run?

By using maths, predictably!

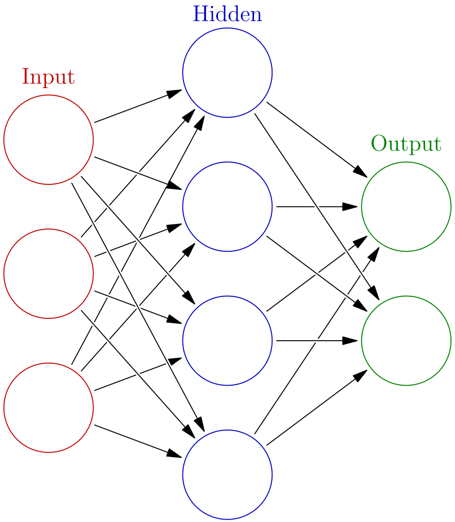
- But programmers are *lazy*!



How does an artificial neural network run?

By using maths, predictably!

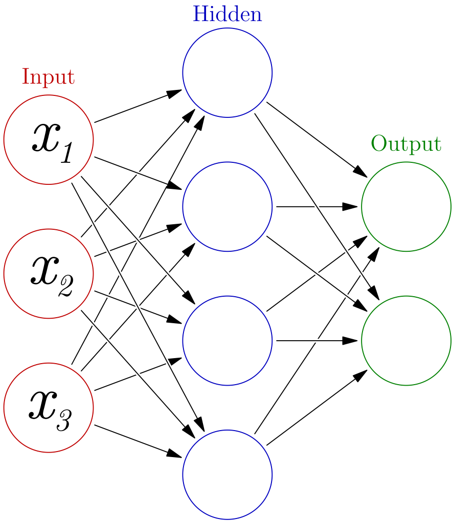
- General names save time



How does an artificial neural network run?

By using maths, predictably!

- General names save time
- input units: x_i



How does an artificial neural network run?

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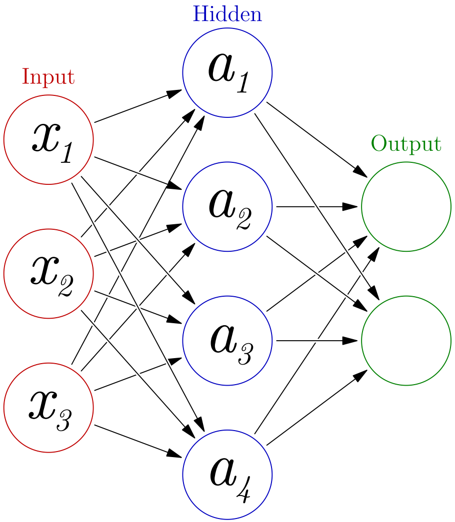
- General names save time
- input units: x_i
- hidden units: a_j



How does an artificial neural network run?

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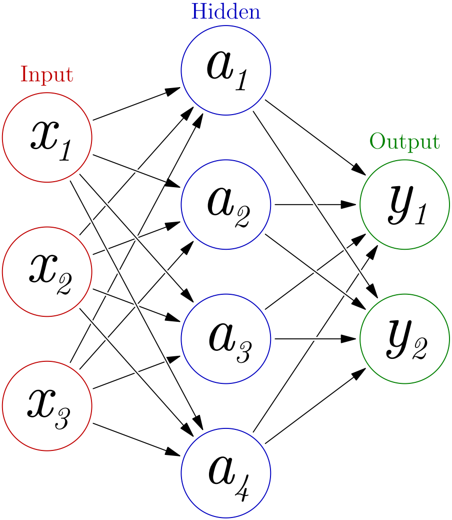
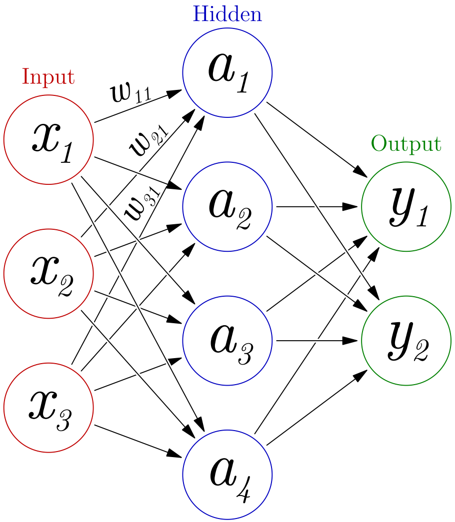
- General names save time
- input units: x_i
- hidden units: a_j
- output units: y_k



How does an artificial neural network run?

By using maths, predictably!

- General names save time
- input units: x_i
- hidden units: a_j
- output units: y_k
- connection weights: w_{ij}



How does an artificial neural network run?

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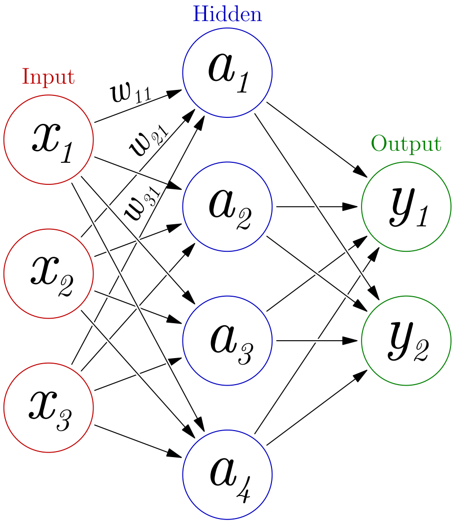
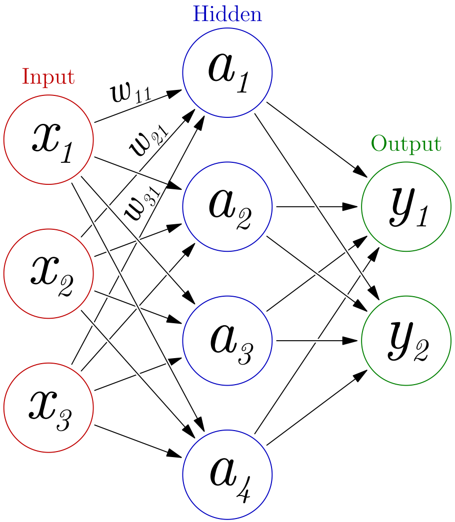
- General names save time
- input units: x_i
- hidden units: a_j
- output units: y_k
- connection weights: w_{ij}
- subscripts
  general: $ijklm\ldots$
  specific: $12345\ldots$



How does an artificial neural network run?

By using maths, predictably!

We use general names to write a general equation:



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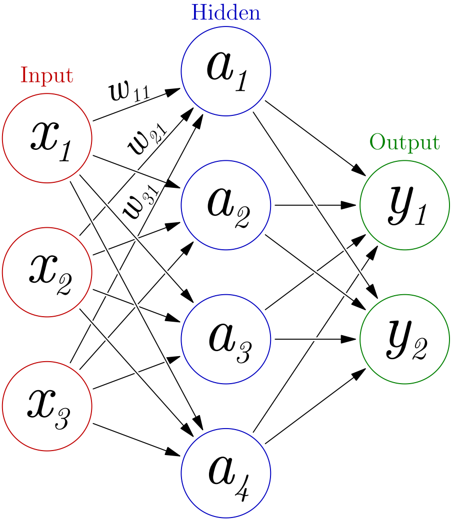
$$a_i =$$

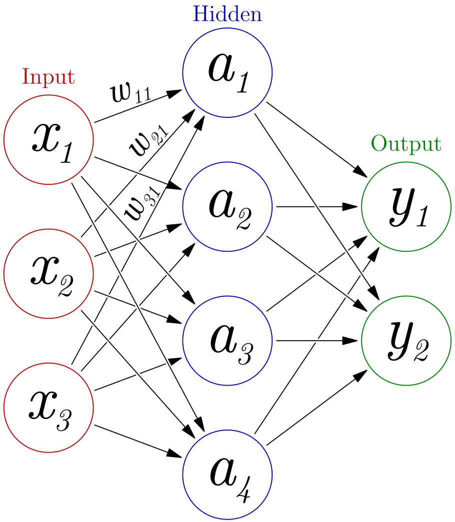


How does an artificial neural network run?

By using maths, predictably!

We use general names to write a general equation:

$$a_i = \qquad x_j \times w_{ji}$$



How does an artificial neural network run?

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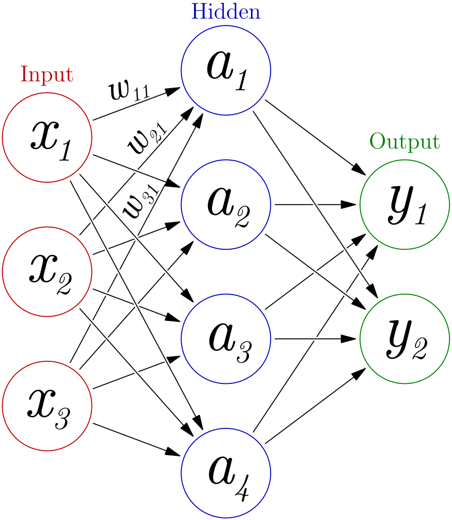
We use general names to write a general equation:

$$a_i = \sum_{j=1}^{N} x_j \times w_{ji}$$

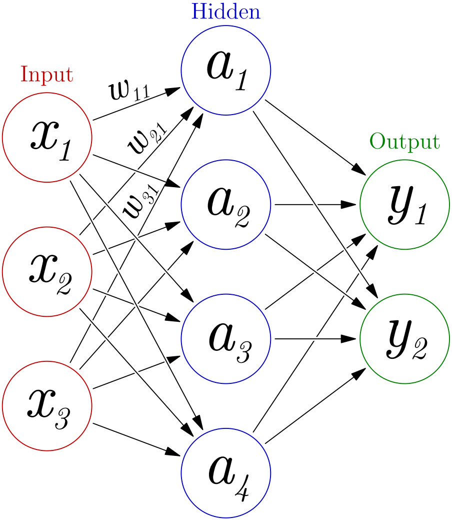


How does an artificial neural network run?

By using maths, predictably!

We use general names to write a general equation:

$$a_i = f\left(\sum_{j=1}^{N} x_j \times w_{ji}\right)$$



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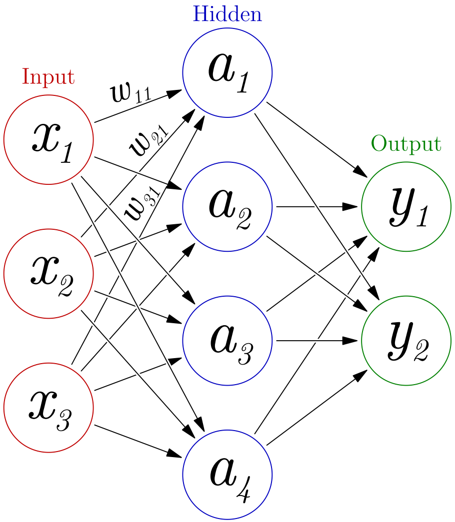
where a_i is the unit whose state we want to calculate, N is the number of units on the previous layer, w_{ji} is the weight on the connection between i and j, and f is a function that the unit applies.



How does an artificial neural network run?

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We use this equation by replacing *iterators* i and j:

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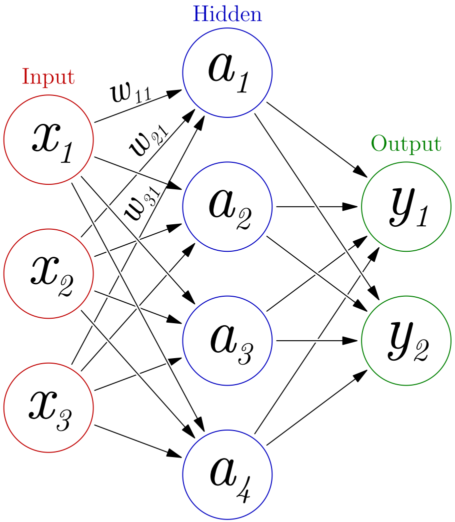

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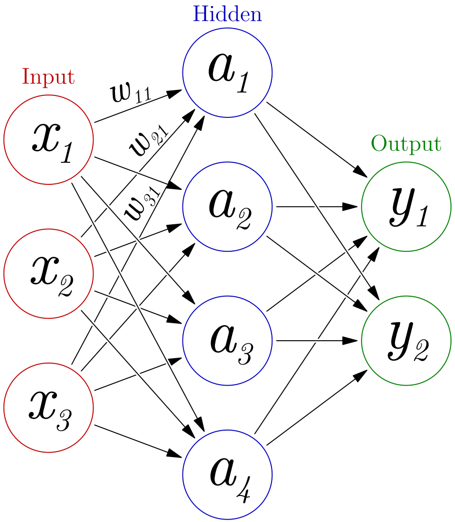
$$a_i = f\left(\sum_{j=1}^{N} x_j \times w_{ji}\right)$$

$$a_{\color{blue}1} = f\left(\sum_{j=1}^{N} x_j \times w_{j{\color{blue}1}}\right)$$



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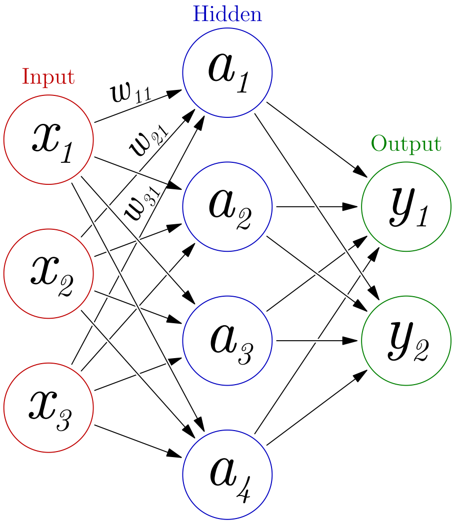
We use this equation by replacing *iterators* i and j:

$$a_i = f\left(\sum_{j=1}^{N} x_j \times w_{ji}\right)$$

$$a_1 = f\left(\sum_{j=1}^{3} x_j \times w_{j1}\right)$$



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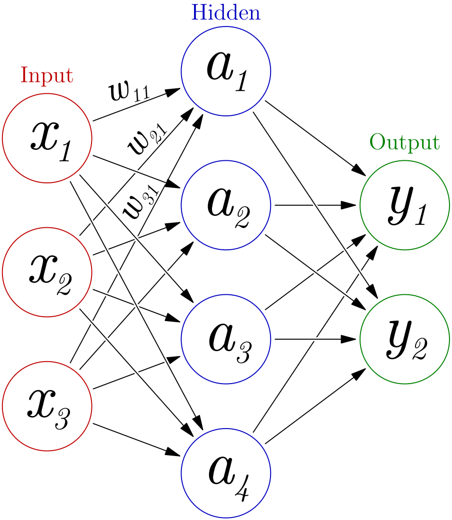
$x_1 \times w_{11}$



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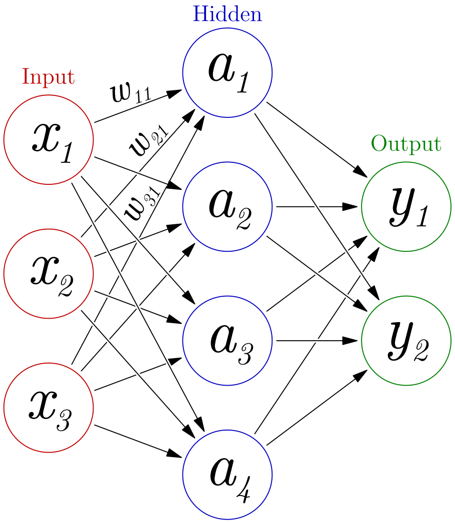
$x_1 \times w_{11} + x_2 \times w_{21}$



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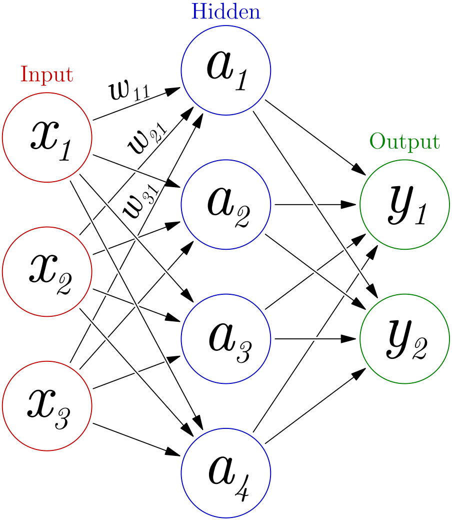
$$x_1 \times w_{11} + x_2 \times w_{21} + x_3 \times w_{31}$$



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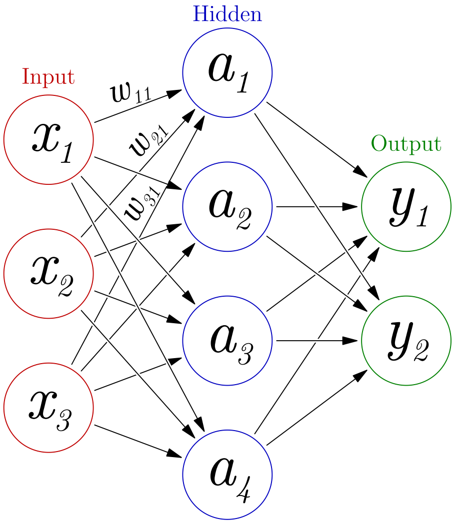
We use this equation by replacing *iterators* i and j:

$$a_i = f\left(\sum_{j=1}^{N} x_j \times w_{ji}\right)$$

$$a_{2} = f\left(\sum_{j=1}^{3} x_j \times w_{j2}\right)$$



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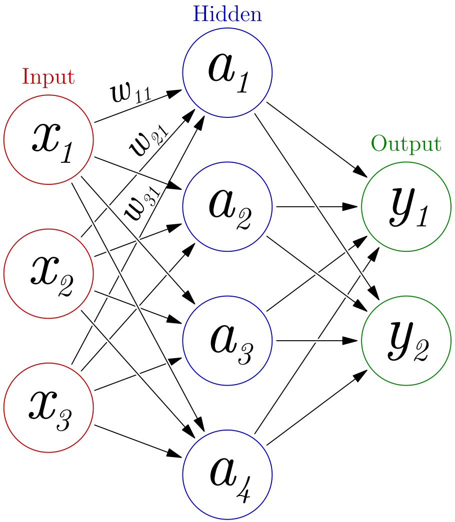
$$a_2 = f\left(\sum_{j=1}^{3} x_j \times w_{j2}\right)$$

$x_1 \times w_{12}$



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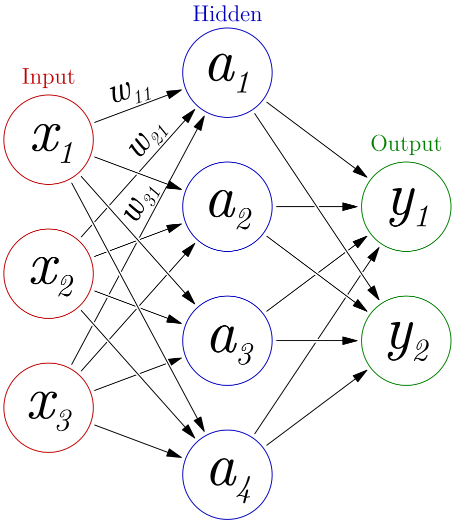
$$a_2 = f\left(\sum_{j=1}^{3} x_j \times w_{j2}\right)$$

$x_1 \times w_{12} + x_2 \times w_{22}$



How does an artificial neural network run?

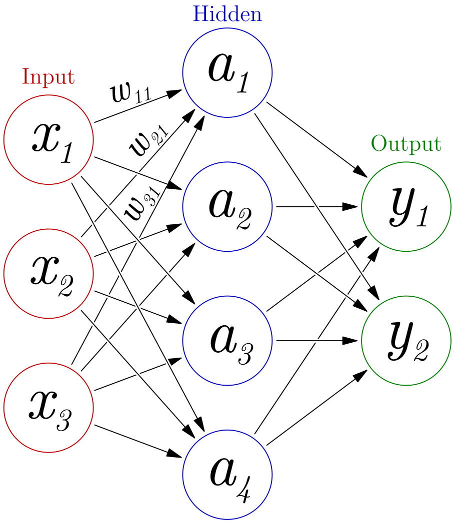
By using maths, predictably!

We use this equation by replacing *iterators* i and j:

$$a_i = f\left(\sum_{j=1}^{N} x_j \times w_{ji}\right)$$

$$a_2 = f\left(\sum_{j=1}^{3} x_j \times w_{j2}\right)$$

$x_1 \times w_{12} + x_2 \times w_{22} + x_3 \times w_{32}$



How do networks learn?

Cunning!

- Many options: Hebbian learning, back-propagation of error, Boltzmann machine learning, self-organising map algorithm, etc.

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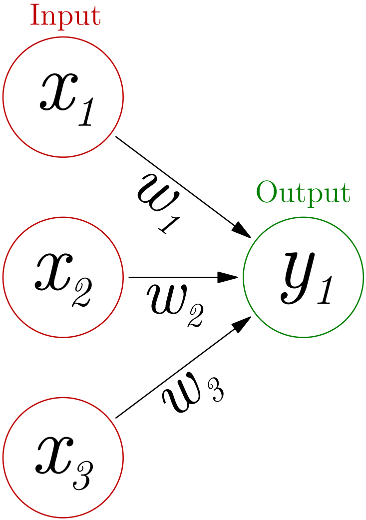
- Many options: Hebbian learning, back-propagation of error, Boltzmann machine learning, self-organising map algorithm, etc.
- All learning algorithms work by changing the connection weights
- Learning can be divided into *supervised*, *unsupervised*, and *reinforcement*

Hebbian learning

A very simple learning rule

"Cells that fire together, wire together"
— Carla Shatz



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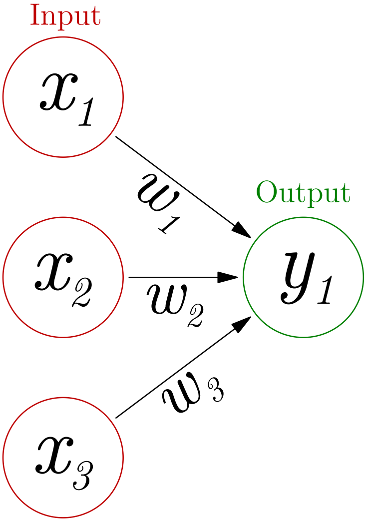
$$w_i =$$



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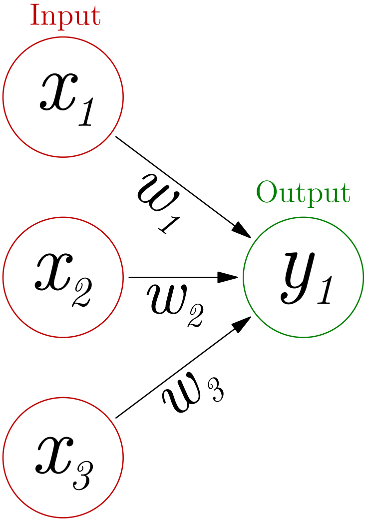
$$w_i = \quad x_i \times y_j$$



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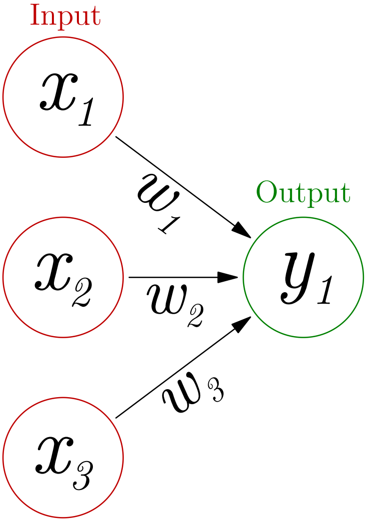

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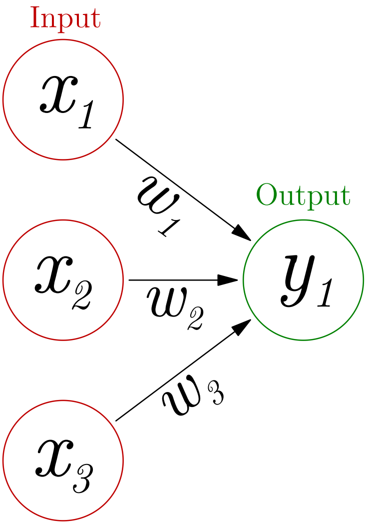
$$\Delta\, w_i = \eta \times {\color{red} x_i} \times {\color{green} y_j}$$



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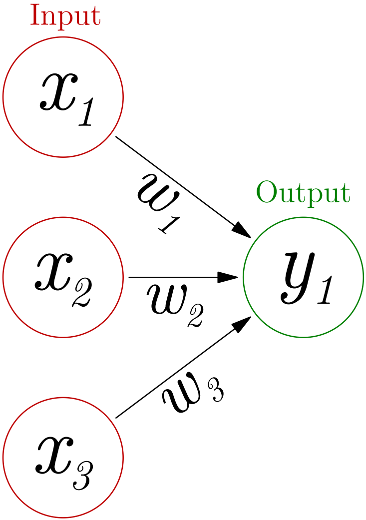
$$\Delta\, w_i = \eta \times x_i \times y_j$$

which means each weight is changed by a small in/decrement for every pattern



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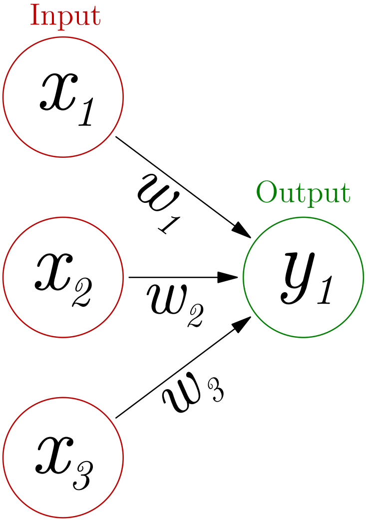

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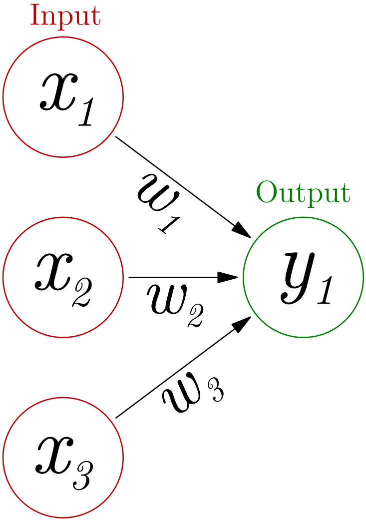

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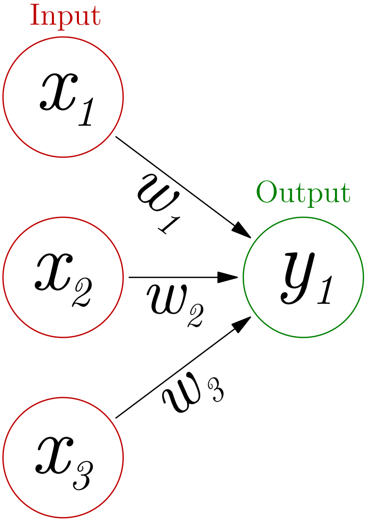

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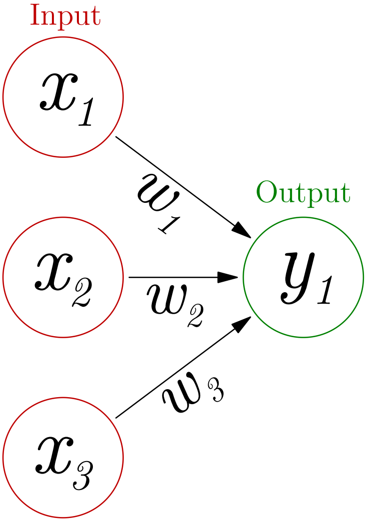

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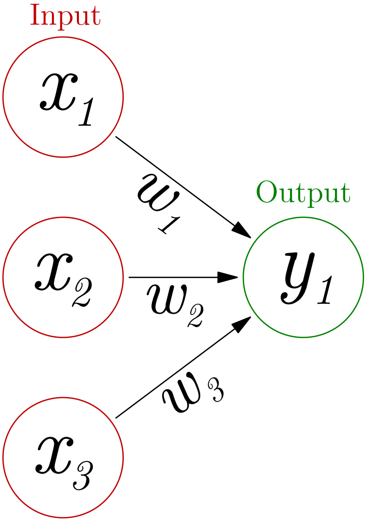

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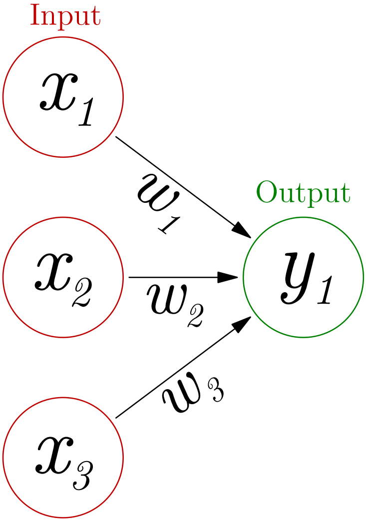

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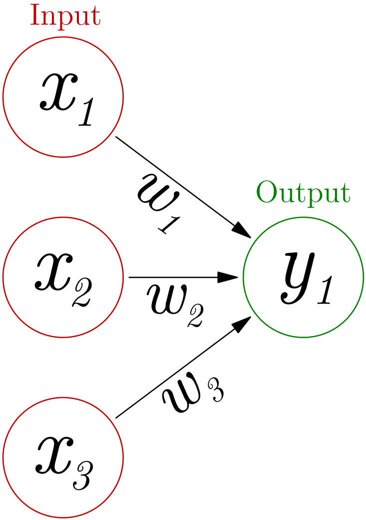

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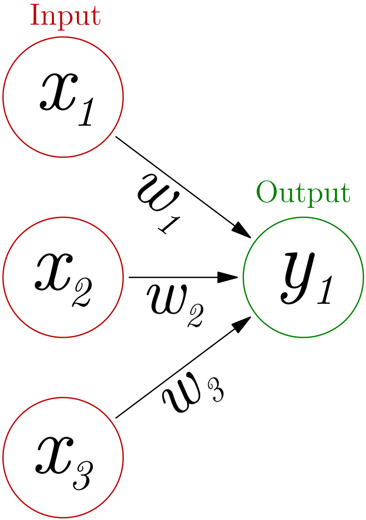

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$$\Delta w_i = \eta \times x_i \times y_j$$

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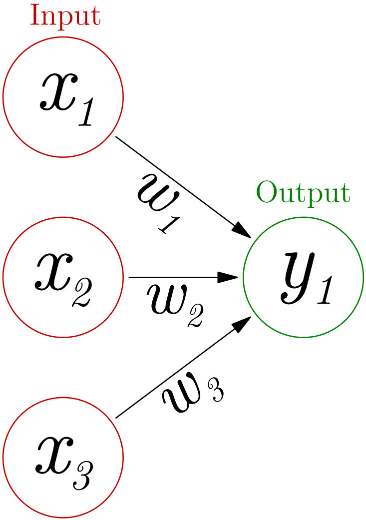

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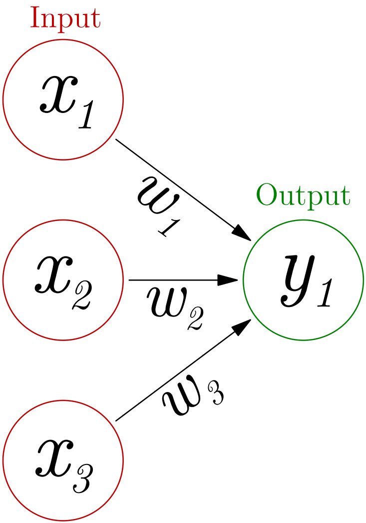
$$\textbf{new } w_1 = \textbf{old } w_1 + \Delta w_1$$



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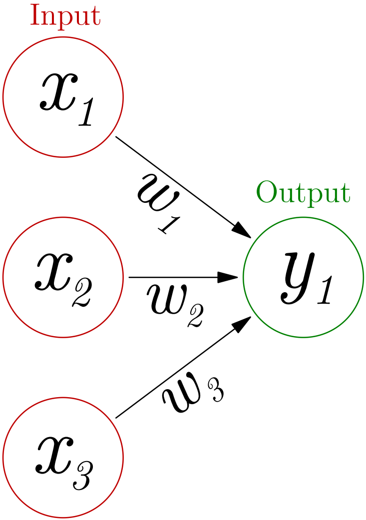

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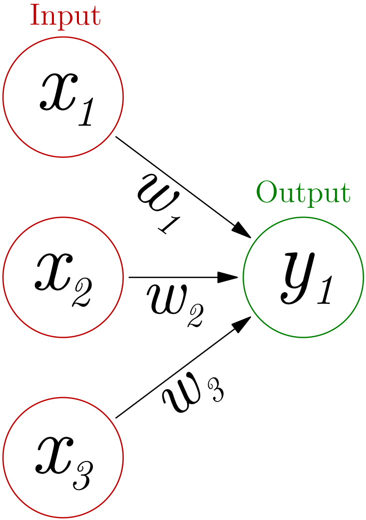

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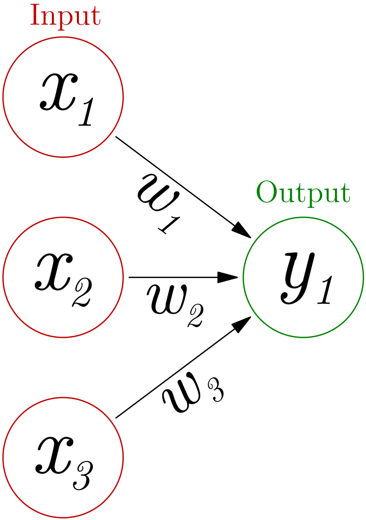

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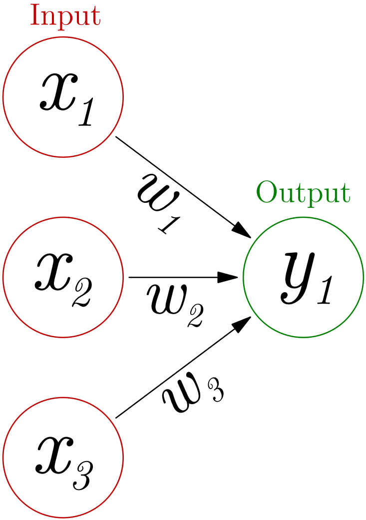

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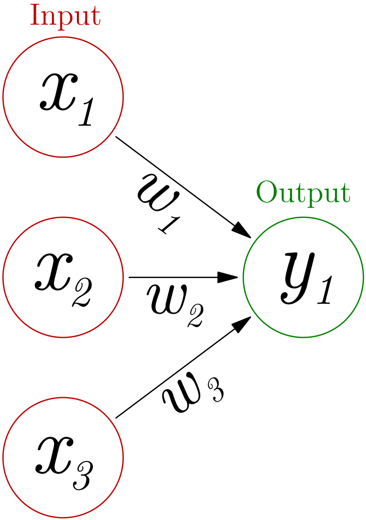
$w_1 = 0.15 +$ something positive



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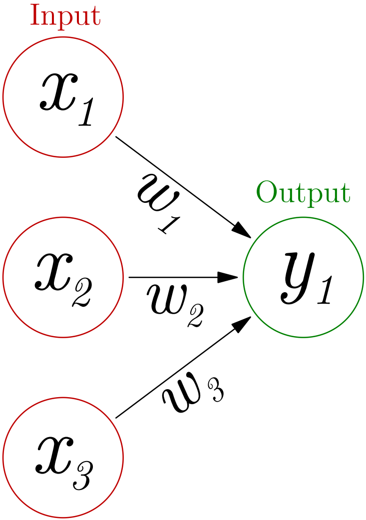

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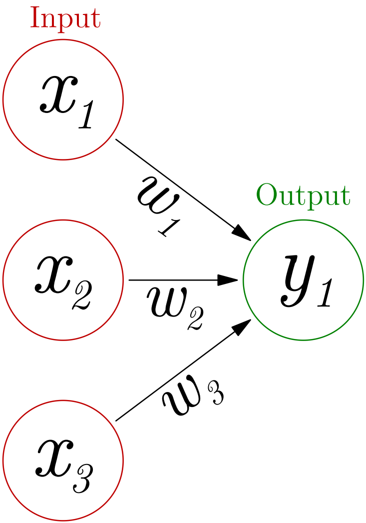
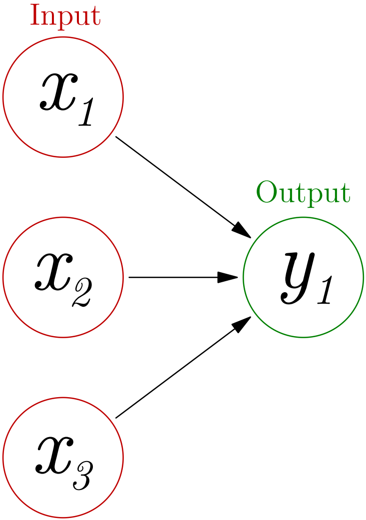
$w_1 = 0.15$ + something positive + something else positive + another positive value + ...



The perceptron

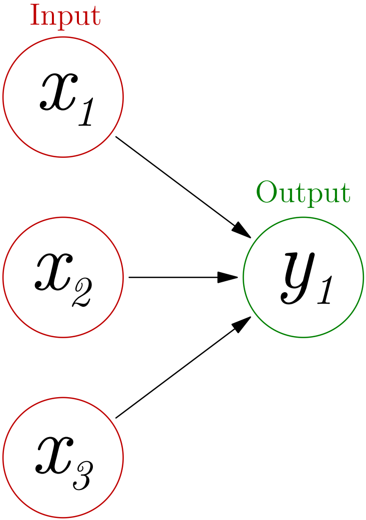
A simple classifier



The perceptron

A simple classifier

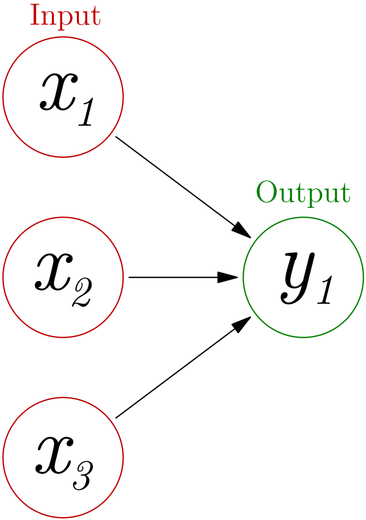
- Created in 1957 at the Cornell Aeronautical Laboratory by Frank Rosenblatt



The perceptron

A simple classifier

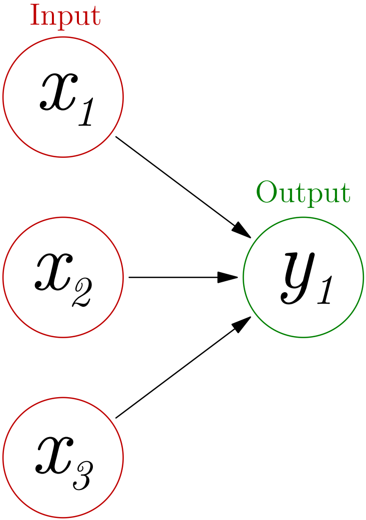
- Created in 1957 at the Cornell Aeronautical Laboratory by Frank Rosenblatt
- Linear classifier



The perceptron

A simple classifier

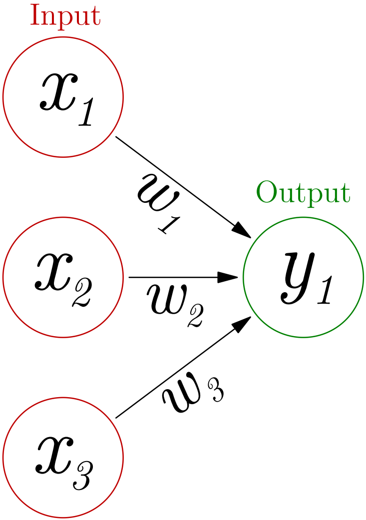
- Created in 1957 at the Cornell Aeronautical Laboratory by Frank Rosenblatt
- Linear classifier
- Simplest form of feedforward network



How does the perceptron learn?

Maths again!

1. Initialise weights



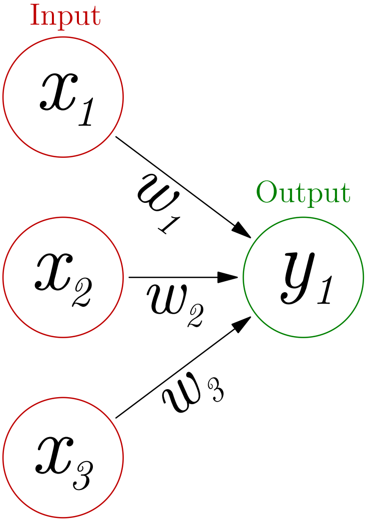
How does the perceptron learn?

Maths again!

1. Initialise weights
2. Run network using:

$$y_j = f\left(\sum_{1}^{N} w_i \times x_i\right)$$

same as always!

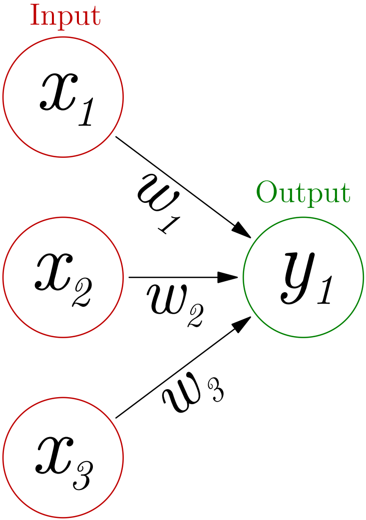


How does the perceptron learn?

Maths again!

1. Initialise weights
2. Run network
3. Update weights using:

$$\Delta w_i = \eta \quad y_j \times x_i$$

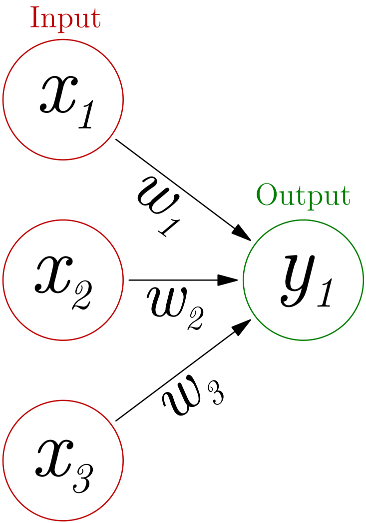


How does the perceptron learn?

Maths again!

1. Initialise weights
2. Run network
3. Update weights using:

$$\Delta w_i = \eta \ (d_j - y_j) \ \ \times x_i$$



How does the perceptron learn?

Maths again!

1. Initialise weights
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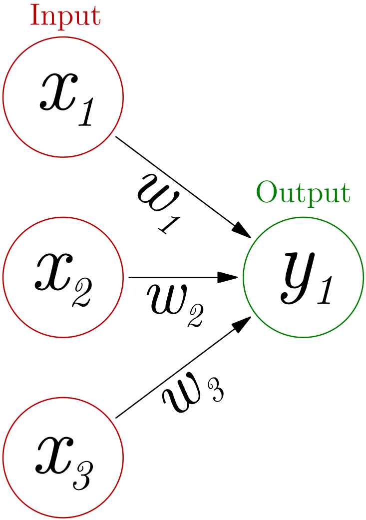
where d is what we want y to be given x, and η is the learning rate.



How does the perceptron learn?

Maths again!

1. Initialise weights
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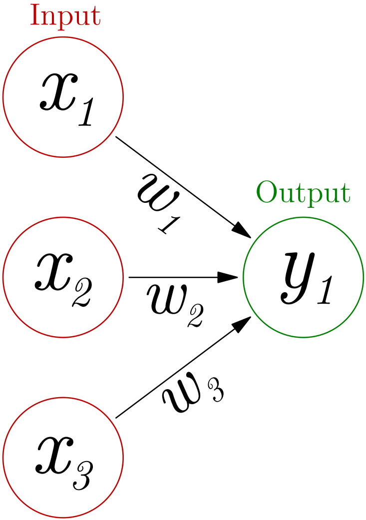
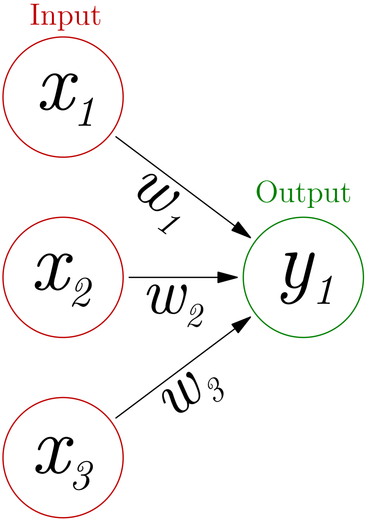
3. Update weights
4. Repeat 2 and 3



How does the perceptron learn?

Maths again!

1. Initialise weights
2. Run network
3. Update weights
4. Repeat 2 and 3
5. When do we stop?



Perceptron coding time

Time to program a perceptron!

Oh, and join our mailing list so we can send you stuff:
https://groups.google.com/d/forum/introcompcog

Training the network

The basic algorithm!

```
def Train(self):
```

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                x[i] = self.patterns[p][i]
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def Train(self):



            for i in       (N):
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The basic algorithm!

```
def Train(self):



            for i in        (N):
                x[i] = self.patterns[p][i]



                y += x[i] * w[i]
            y = f(y)
            error = d[p][0] - y
           for i in        (N+1):
                w[i] += h * error * x[i]
```

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Training the network

The basic algorithm!

```
def Train(self):



            for i in        (N):
                x[i] = self.patterns[p][i]
            y = 0
            for i in        (N+1):
                y += x[i] * w[i]
            y = f(y)
            error = d[p][0] - y
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            error = d[p][0] - y
           for i in       (N+1):
                w[i] += h * error * x[i]
```

Training the network

The basic algorithm!

```
def Train(self):

       for p in      (P):
            for i in      (N):
                x[i] = self.patterns[p][i]
            y = 0
            for i in      (N+1):
                y += x[i] * w[i]
            y = f(y)
            error = d[p][0] - y
           for i in      (N+1):
                w[i] += h * error * x[i]
```

Training the network

The basic algorithm!

```
def Train(self):
    for t in          (100):
        for p in          (P):
             for i in          (N):
                 x[i] = self.patterns[p][i]
             y = 0
             for i in          (N+1):
                 y += x[i] * w[i]
             y = f(y)
             error = d[p][0] - y
            for i in          (N+1):
                 w[i] += h * error * x[i]
```

Training the network

Defining some patterns!

```
Patterns =
        [#colour, shape, taste
         #red-yellow, big-small, sweet-sour
         [0.1, 0.0, 0.2], #loquat
         [0.0, 0.2, 0.0], #lemon
         [1.0, 0.5, 0.8], #red apple
         [1.0, 0.0, 0.9], #strawberry
        ]
```

Training the network

Defining some patterns!

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         #red-yellow, big-small, sweet-sour
         [0.1, 0.0, 0.2], #loquat
         [0.0, 0.2, 0.0], #lemon
         [1.0, 0.5, 0.8], #red apple
         [1.0, 0.0, 0.9], #strawberry
        ]

Targets = [
           [1.0], #first target
           [1.0], #targets indicate
           [0.0], #which class
           [0.0], #a pattern
          ] #belongs to
```