Block Practical: Connectionist models and cognitive processes

Part 2: Introduction to artificial neural networks

Olivia Guest

What is a neural network?

A mathematical model

Inspired by the nervous system

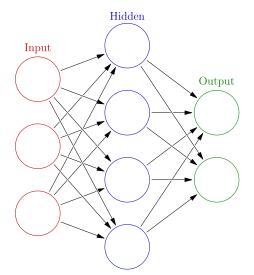


Figure: Glosser.ca / CC-BY-SA-3.0



What is a neural network?

A mathematical model

- ► Inspired by the nervous system
- ► A set of *units*, connected by *weights*

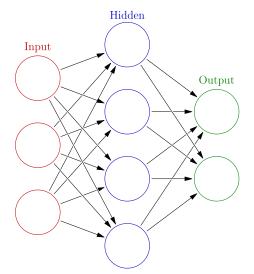


Figure: Glosser.ca / CC-BY-SA-3.0



What is a neural network?

A mathematical model

- Inspired by the nervous system
- A set of units, connected by weights
- ➤ The network *runs* by passing *activations* from the *input* (to the *hidden*) to the *output* units

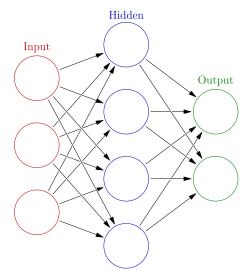


Figure: Glosser.ca / CC-BY-SA-3.0



Some aspects of their behaviour are like their namesake!

► Learn pretty much any input-output data

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- Learn pretty much any input-output data
- Uncover rules on their own about data

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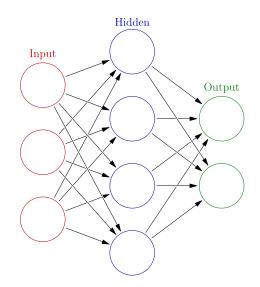
- Learn pretty much any input-output data
- Uncover rules on their own about data
- Generalise from what they have learnt

Some aspects of their behaviour are like their namesake!

- Learn pretty much any input-output data
- Uncover rules on their own about data
- Generalise from what they have learnt
- Cope with noise and damage

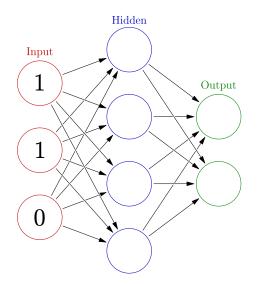
By using maths, predictably!

1. Input units are set to a pattern

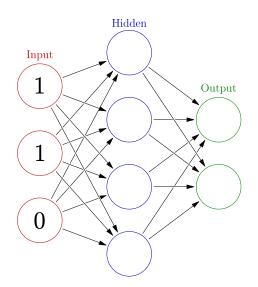


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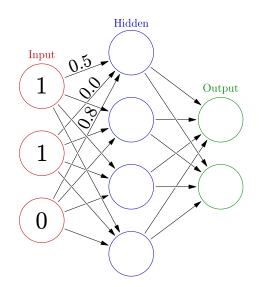
1. Input units are set to a pattern



- 1. Input units are set to a pattern
- 2. Calculate hidden units' states

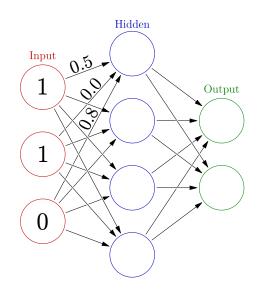


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- 1. Input units are set to a pattern
- 2. Calculate hidden units' states:

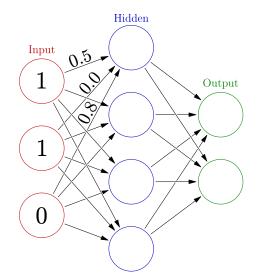
$$1 \times 0.5 = 0.5$$



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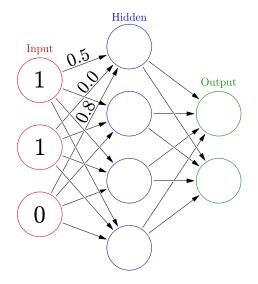
 $1 \times 0.0 = 0.0$



- 1. Input units are set to a pattern
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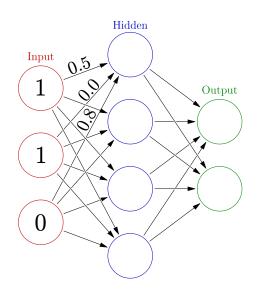
$$1 \times 0.5 = 0.5$$

 $1 \times 0.0 = 0.0$
 $0 \times 0.8 = 0.0$



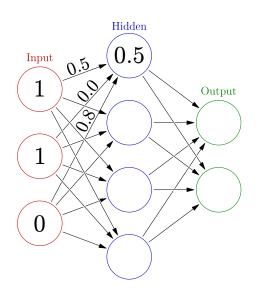
- 1. Input units are set to a pattern
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$$\begin{array}{lll} 1\times 0.5 = & 0.5 \\ 1\times 0.0 = & 0.0 \\ 0\times 0.8 = & 0.0 \end{array} +$$

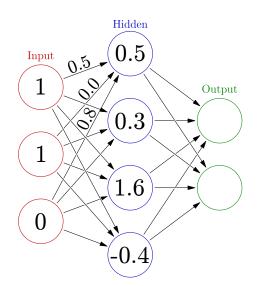


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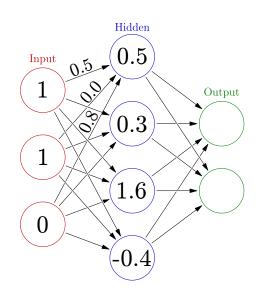
$$\begin{array}{cccc}
1 \times 0.5 &=& 0.5 \\
1 \times 0.0 &=& 0.0 \\
0 \times 0.8 &=& 0.0 & + \\
\hline
& 0.5 & & & & \\
\end{array}$$



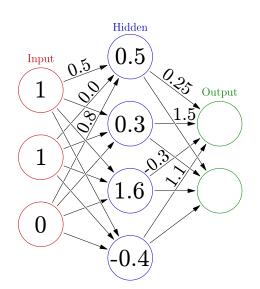
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- 1. Input units are set to a pattern
- 2. Calculate hidden units' states
- 3. Same for output units

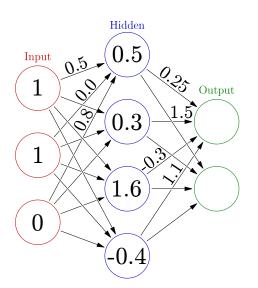


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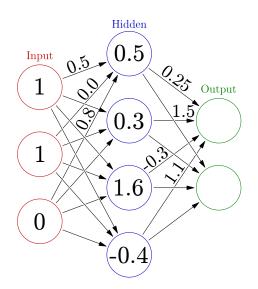
$$0.5 \times 0.25 = 0.125$$



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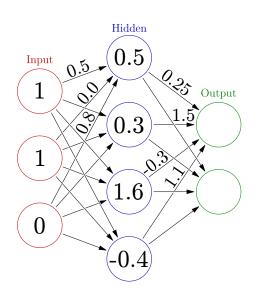
 $0.3 \times 1.5 = 0.45$



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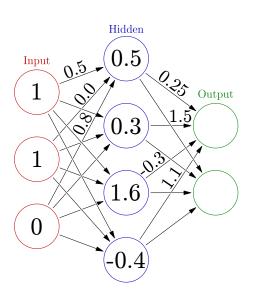
 $0.3 \times 1.5 = 0.45$
 $1.6 \times -0.3 = -0.48$



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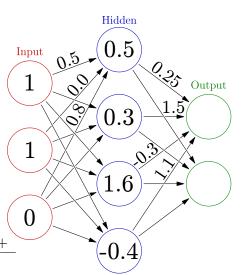
$$0.5 \times 0.25 = 0.125$$

 $0.3 \times 1.5 = 0.45$
 $1.6 \times -0.3 = -0.48$
 $-0.4 \times 1.1 = -0.44$



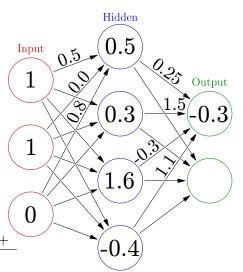
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\hline
& & & & & & & \\
-0.345 & & & & & & \\
\end{array}$$

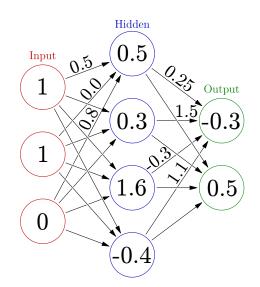


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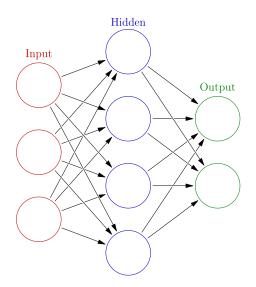


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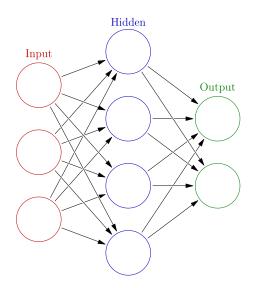
By using maths, predictably!

► But programmers are *lazy*!



By using maths, predictably!

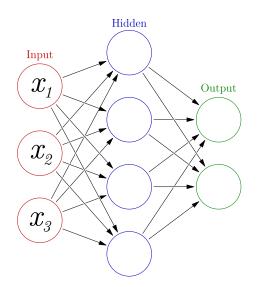
► General names save time



By using maths, predictably!

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ightharpoonup input units: x_i

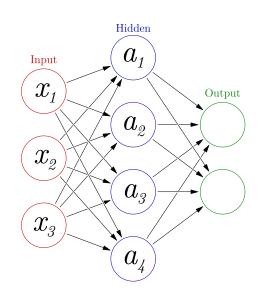


By using maths, predictably!

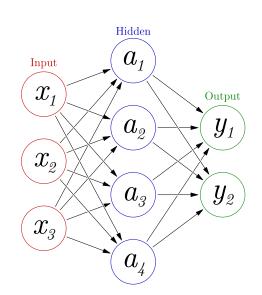
General names save time

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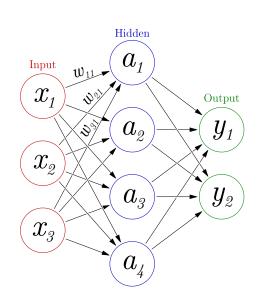
► hidden units: a_j



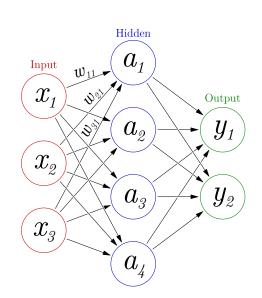
- ► General names save time
- ightharpoonup input units: x_i
- ▶ hidden units: a_i
- ▶ output units: *y_k*



- General names save time
- ightharpoonup input units: x_i
- ▶ hidden units: a_i
- ightharpoonup output units: y_k
- ightharpoonup connection weights: w_{ij}

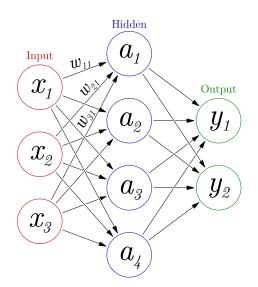


- ► General names save time
- ▶ input units: x_i
- ▶ hidden units: a_i
- ightharpoonup output units: y_k
- ightharpoonup connection weights: w_{ij}
- ► subscripts general: *ijklm*... specific: 12345...



By using maths, predictably!

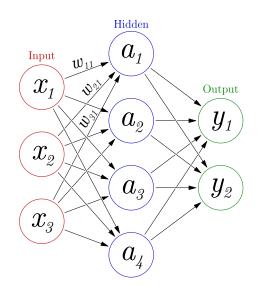
We use general names to write a general equation:



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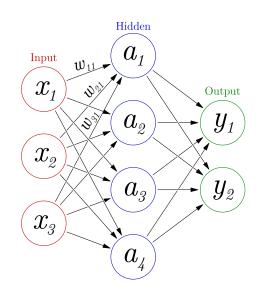
 $a_i =$



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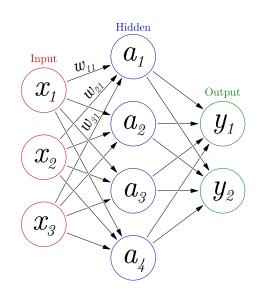
$$a_i = x_j \times w_{ji}$$



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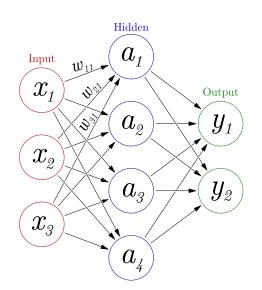
$$a_i = \sum_{j=1}^{N} x_j \times w_{ji}$$



By using maths, predictably!

We use general names to write a general equation:

$$a_i = f\left(\sum_{j=1}^N x_j \times w_{ji}\right)$$

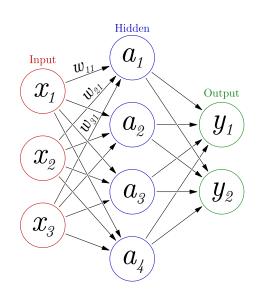


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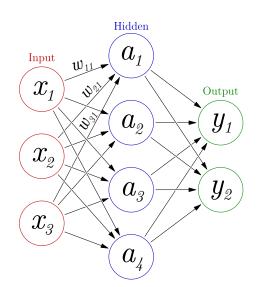
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where a_i is the unit whose state we want to calculate, N is the number of units on the previous layer, w_{ji} is the weight on the connection between i and j, and f is a function that the unit applies.



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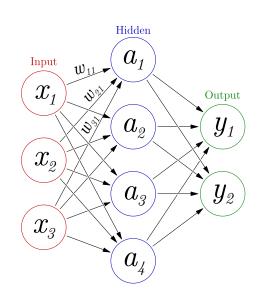
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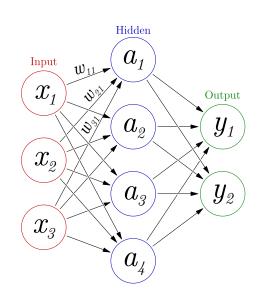
$$a_1 = f\left(\sum_{j=1}^{N} x_j \times w_{j1}\right)$$



By using maths, predictably!

$$a_i = f\left(\sum_{j=1}^N x_j \times w_{ji}\right)$$

$$a_1 = f\left(\sum_{j=1}^3 x_j \times w_{j1}\right)$$

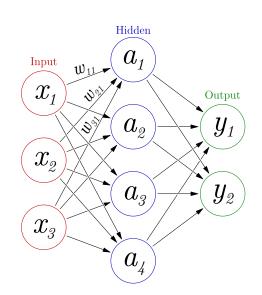


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$$x_1 \times w_{11}$$

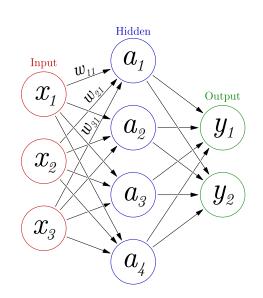


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$$a_i = f\left(\sum_{j=1}^N x_j \times w_{ji}\right)$$

$$a_1 = f\left(\sum_{j=1}^3 x_j \times w_{j1}\right)$$

$$x_1 \times w_{11} + x_2 \times w_{21}$$

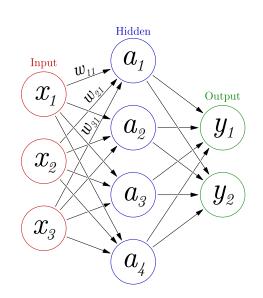


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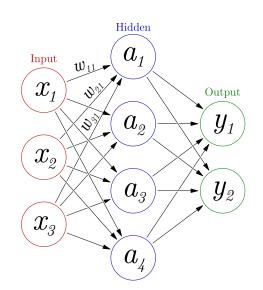
$$x_1 \times w_{11} + x_2 \times w_{21} + x_3 \times w_{31}$$



By using maths, predictably!

$$a_i = f\left(\sum_{j=1}^N x_j \times w_{ji}\right)$$

$$a_2 = f\left(\sum_{j=1}^3 x_j \times w_{j2}\right)$$

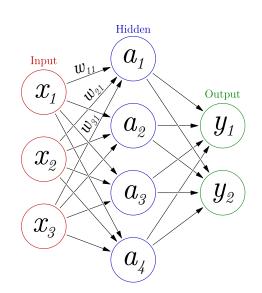


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$$a_2 = f\left(\sum_{j=1}^3 x_j \times w_{j2}\right)$$

$$x_1 \times w_{12}$$

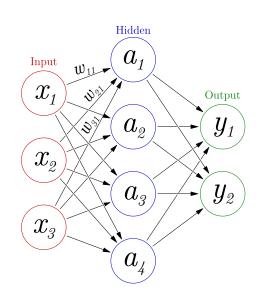


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$$a_i = f\left(\sum_{j=1}^N x_j \times w_{ji}\right)$$

$$a_2 = f\left(\sum_{j=1}^3 x_j \times w_{j2}\right)$$

$$x_1 \times w_{12} + x_2 \times w_{22}$$

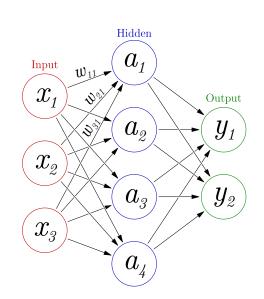


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$$a_i = f\left(\sum_{j=1}^N x_j \times w_{ji}\right)$$

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$$x_1 \times w_{12} + x_2 \times w_{22} + x_3 \times w_{32}$$



How do networks learn?

Cunning!

Many options: Hebbian learning, back-propagation of error, Boltzmann machine learning, self-organising map algorithm, etc.

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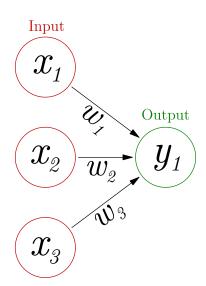
Many options: Hebbian learning, back-propagation of error, Boltzmann machine learning, self-organising map algorithm, etc.

- All learning algorithms work by changing the connection weights
- ► Learning can be divided into *supervised*, *unsupervised*, and *reinforcement*

A very simple learning rule

"Cells that fire together, wire together"

— Carla Shatz

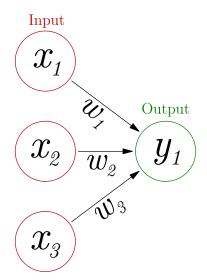


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 $w_i =$

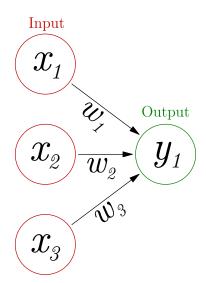


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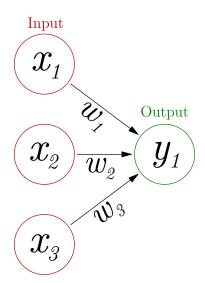
$$w_i = x_i \times y_i$$



A very simple learning rule

"Cells that fire together, wire together" — Carla Shatz

$$w_i = \eta \times x_i \times y_i$$

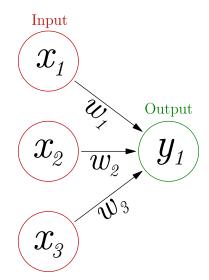


A very simple learning rule

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 $\Delta w_i = \eta \times x_i \times y_i$



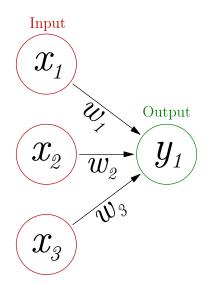
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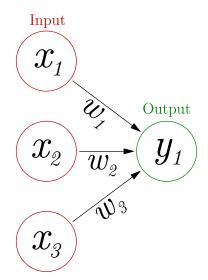
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$$\Delta w_i = \eta \times \underline{x_i} \times y_j$$

which means each weight is changed by a small in/decrement for every pattern

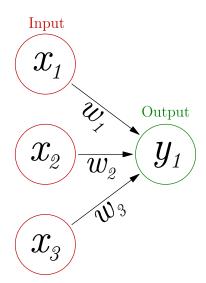


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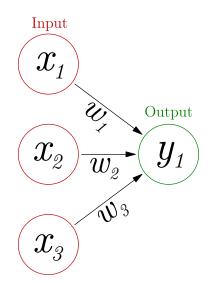
$$\Delta w_i = \eta \times x_i \times y_j$$



"Cells that fire together, wire together" — Carla Shatz

$$\Delta w_i = \eta \times x_i \times y_j$$

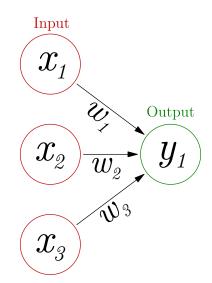
$$\Delta w_1 = \boldsymbol{\eta} \times x_i \times y_j$$



"Cells that fire together, wire together" — Carla Shatz

$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.5 \times x_i \times y_j$$



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$$\Delta w_i = \eta \times x_i \times y_j$$

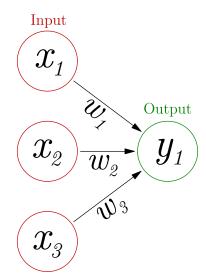
$$\Delta w_1 = 0.5 \times x_1 \times y_j$$



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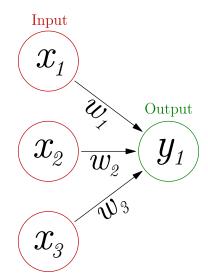
$$\Delta w_1 = 0.5 \times x_1 \times y_1$$



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$$\Delta w_i = \eta \times x_i \times y_j$$

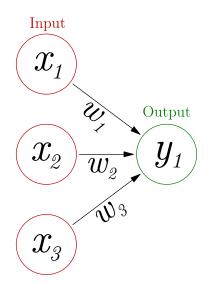
$$\Delta w_1 = 0.5 \times 1.0 \times y_1$$



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$$\Delta w_i = \eta \times x_i \times y_j$$

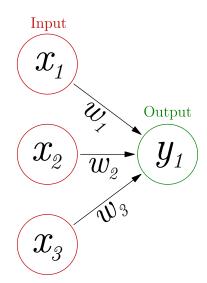
$$\Delta w_1 = 0.5 \times 1.0 \times 0.3$$



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$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

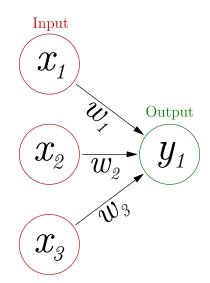


"Cells that fire together, wire together" — Carla Shatz

$$\Delta w_i = \boldsymbol{\eta} \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

$$\mathbf{new} \ w_1 = \mathbf{old} \ w_1 + \Delta w_1$$

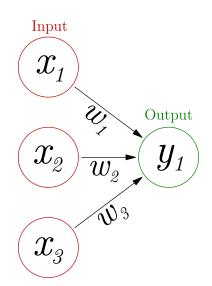


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new
$$w_1 = 0.0 + \Delta w_1$$

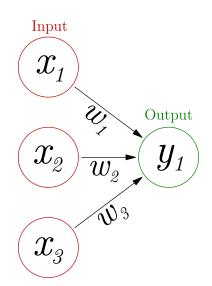


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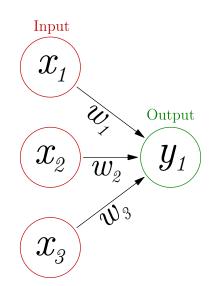


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$$w_1 = 0.15$$



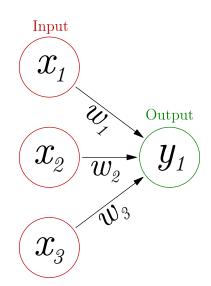
"Cells that fire together, wire together" — Carla Shatz

Hebb's rule is simple, but *very unstable*!

$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

$$w_1 = 0.15$$



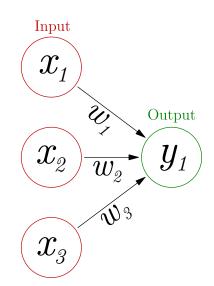
"Cells that fire together, wire together" — Carla Shatz

Hebb's rule is simple, but *very unstable*!

$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

 $w_1 = 0.15 + \text{something}$ positive



"Cells that fire together, wire together" — Carla Shatz

Hebb's rule is simple, but *very unstable*!

$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

 $w_1 = 0.15 + \text{something}$ positive + something else positive +



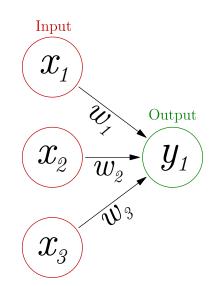
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Hebb's rule is simple, but *very unstable*!

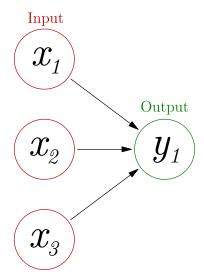
$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

 $w_1 = 0.15 + \text{something}$ positive + something else positive + another positive value + ...

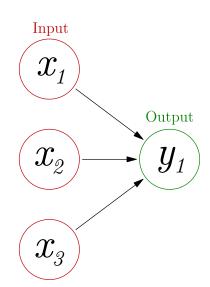


A simple classifier



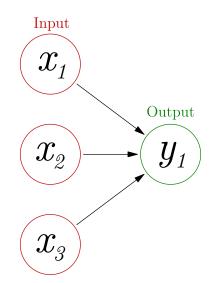
A simple classifier

 Created in 1957 at the Cornell Aeronautical Laboratory by Frank Rosenblatt



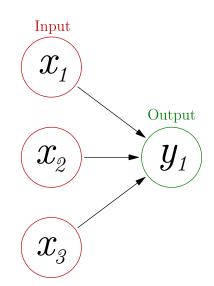
A simple classifier

- Created in 1957 at the Cornell Aeronautical Laboratory by Frank Rosenblatt
- Linear classifier



A simple classifier

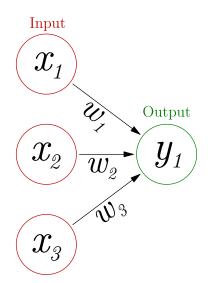
- Created in 1957 at the Cornell Aeronautical Laboratory by Frank Rosenblatt
- ▶ Linear classifier
- Simplest form of feedforward network



How does the perceptron learn?

Maths again!

1. Initialise weights

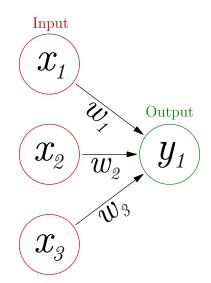


Maths again!

- 1. Initialise weights
- 2. Run network using:

$$y_j = f\left(\sum_{1}^{N} w_i \times x_i\right)$$

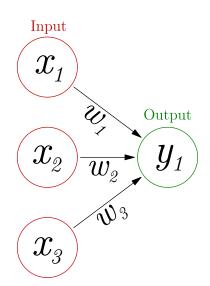
same as always!



Maths again!

- 1. Initialise weights
- 2. Run network
- 3. Update weights using:

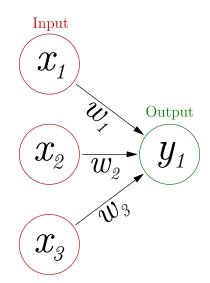
$$\Delta w_i = \eta \quad y_j \times x_i$$



Maths again!

- 1. Initialise weights
- 2. Run network
- 3. Update weights using:

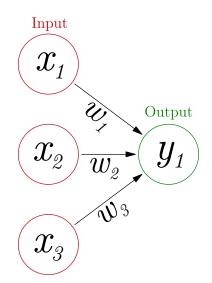
$$\Delta w_i = \eta \ (d_j - y_j) \ \times x_i$$



- 1. Initialise weights
- 2. Run network
- 3. Update weights using:

$$\Delta w_i = \eta \ (d_j - y_j) \ \times x_i$$

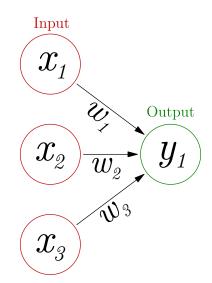
where d is what we want y to be given x, and η is the learning rate.



How does the perceptron learn?

Maths again!

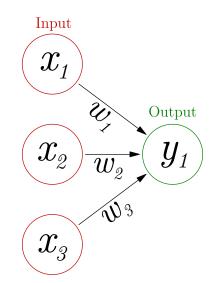
- 1. Initialise weights
- 2. Run network
- 3. Update weights
- 4. Repeat 2 and 3



How does the perceptron learn?

Maths again!

- 1. Initialise weights
- 2. Run network
- 3. Update weights
- 4. Repeat 2 and 3
- 5. When do we stop?



Perceptron coding time

Time to program a perceptron!

```
def Train(self):
```

```
def Train(self):
                x[i] = self.patterns[p][i]
```

```
def Train(self):
            for i in range(N):
                x[i] = self.patterns[p][i]
```

```
def Train(self):
            for i in range(N):
                x[i] = self.patterns[p][i]
                y += x[i] * w[i]
            y = f(y)
            error = d[p][0] - y
           for i in range(N+1):
                w[i] += h * error * x[i]
```

```
def Train(self):
            for i in range(N):
                x[i] = self.patterns[p][i]
            for i in range(N+1):
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            y = f(y)
            error = d[p][0] - y
           for i in range(N+1):
                w[i] += h * error * x[i]
```

```
def Train(self):
            for i in range(N):
                x[i] = self.patterns[p][i]
            y = 0
            for i in range(N+1):
                y += x[i] * w[i]
            y = f(y)
            error = d[p][0] - y
           for i in range(N+1):
                w[i] += h * error * x[i]
```

```
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            y = f(y)
            error = d[p][0] - y
           for i in range(N+1):
                w[i] += h * error * x[i]
```

```
def Train(self):
       for p in range(P):
            for i in range(N):
                x[i] = self.patterns[p][i]
            y = 0
            for i in range(N+1):
                y += x[i] * w[i]
            y = f(y)
            error = d[p][0] - y
           for i in range(N+1):
                w[i] += h * error * x[i]
```

```
def Train(self):
   for t in range (100):
       for p in range(P):
            for i in range(N):
                x[i] = self.patterns[p][i]
            y = 0
            for i in range(N+1):
                y += x[i] * w[i]
            y = f(y)
            error = d[p][0] - y
           for i in range(N+1):
                w[i] += h * error * x[i]
```

Defining some patterns!

Defining some patterns!

```
Patterns =
        [#colour, taste
         #yellow-red, sour-sweet
         [0.1, 0.5], #loquat
         [0.0, 0.0], #lemon
         [1.0, 0.8], #red apple
         [1.0, 0.9], #strawberry
Targets = [ #1.0 = yellow fruit
           [1.0], #target for first pattern
           [1.0], #targets indicate
           [0.0], #which class
           [0.0], #a pattern
          ] #belongs to
```