Welcome to the introduction to computational cognitive modelling workshop!

Part 2: Introduction to artificial neural networks

# Part 2: Introduction to artificial neural networks

Olivia Guest Chris Brand

Nick Sexton Nicole Cruz De Echeverria Loebell

#### What is a neural network?

#### A mathematical model

Inspired by the nervous system

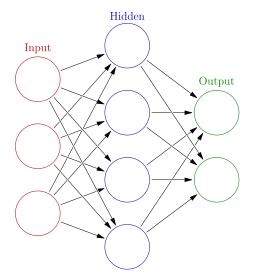


Figure: Glosser.ca / CC-BY-SA-3.0



#### What is a neural network?

#### A mathematical model

- Inspired by the nervous system
- ► A set of *units*, connected by *weights*

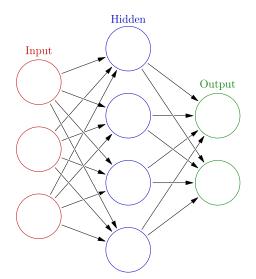


Figure: Glosser.ca / CC-BY-SA-3.0



#### What is a neural network?

#### A mathematical model

- Inspired by the nervous system
- A set of units, connected by weights
- The network runs by passing activations from the input (to the hidden) to the output units

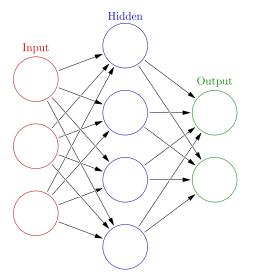


Figure: Glosser.ca / CC-BY-SA-3.0



Some aspects of their behaviour are like their namesake!

► Learn pretty much any input-output data

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- Learn pretty much any input-output data
- Uncover rules on their own about data

Some aspects of their behaviour are like their namesake!

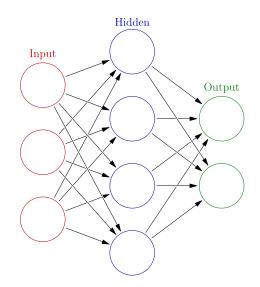
- Learn pretty much any input-output data
- Uncover rules on their own about data
- Generalise from what they have learnt

Some aspects of their behaviour are like their namesake!

- Learn pretty much any input-output data
- Uncover rules on their own about data
- Generalise from what they have learnt
- Cope with noise and damage

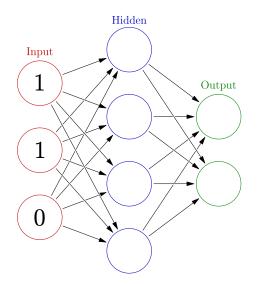
By using maths, predictably!

1. Input units are set to a pattern

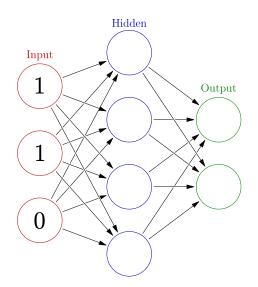


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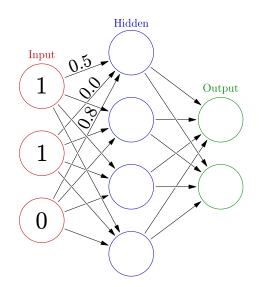
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- 1. Input units are set to a pattern
- 2. Calculate hidden units' states

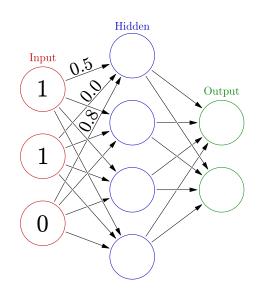


- 1. Input units are set to a pattern
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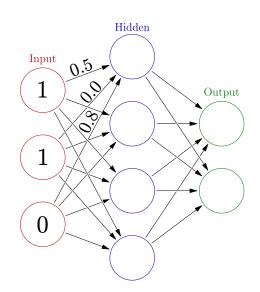
- 1. Input units are set to a pattern
- 2. Calculate hidden units' states:

$$1 \times 0.5 = 0.5$$



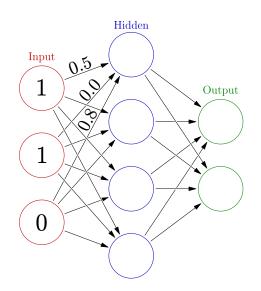
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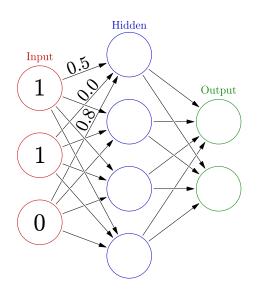
- 1. Input units are set to a pattern
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$$1 \times 0.5 = 0.5$$
  
 $1 \times 0.0 = 0.0$   
 $0 \times 0.8 = 0.0$ 



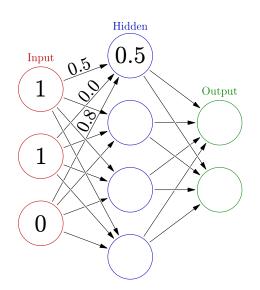
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$$\begin{array}{rcl}
1 \times 0.5 & = & 0.5 \\
1 \times 0.0 & = & 0.0 \\
0 \times 0.8 & = & 0.0 & +
\end{array}$$

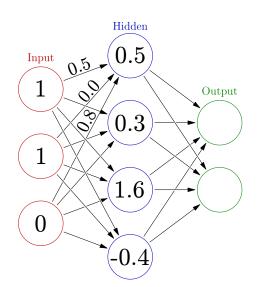


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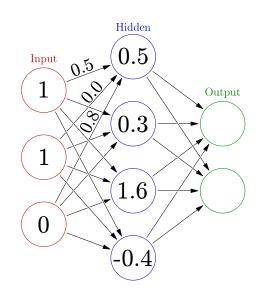
$$\begin{array}{rcl}
1 \times 0.5 & = & 0.5 \\
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\hline
0.5
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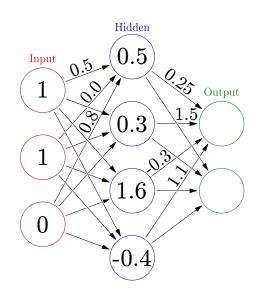
- 1. Input units are set to a pattern
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- 1. Input units are set to a pattern
- 2. Calculate hidden units' states
- 3. Same for output units

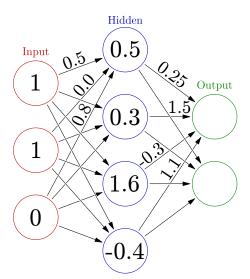


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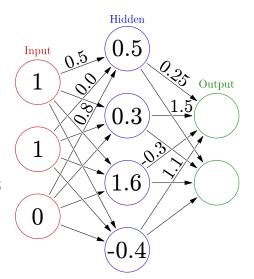
- 1. Input units are set to a pattern
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$$0.5 \times 0.25 = 0.125$$



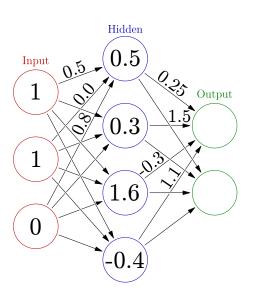
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$$0.5 \times 0.25 = 0.125$$
  
 $0.3 \times 1.5 = 0.45$ 



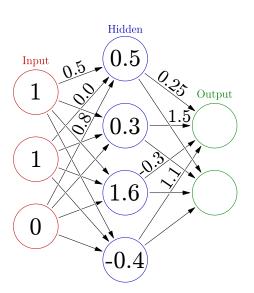
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 $-0.4 \times 1.1 = -0.44$ 



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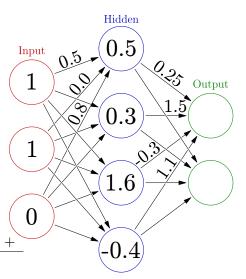
$$0.5 \times 0.25 = 0.125$$

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$$-0.345$$



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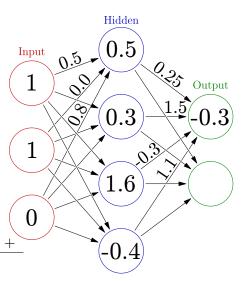
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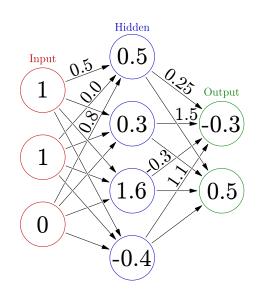
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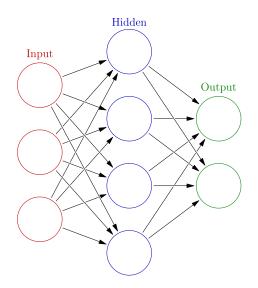


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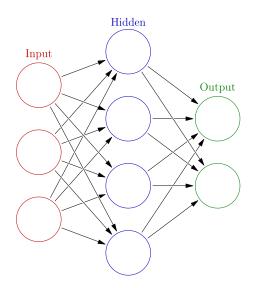
By using maths, predictably!

► But programmers are *lazy*!



By using maths, predictably!

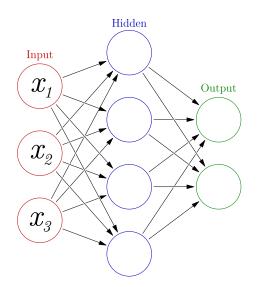
► General names save time



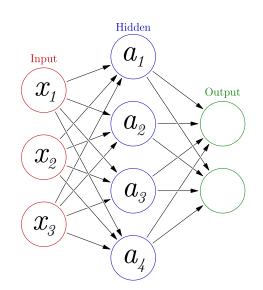
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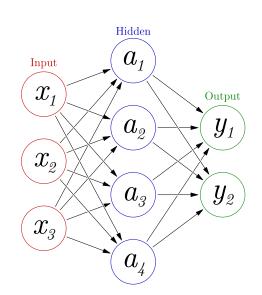
ightharpoonup input units:  $x_i$ 



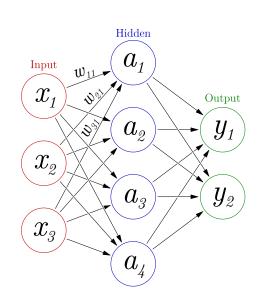
- ► General names save time
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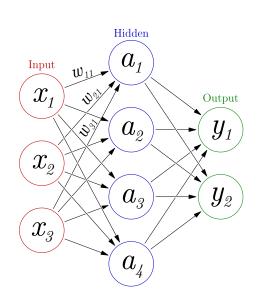
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- General names save time
- ightharpoonup input units:  $x_i$
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- ightharpoonup output units:  $y_k$
- ightharpoonup connection weights:  $w_{ij}$

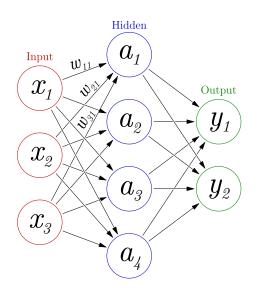


- ► General names save time
- ightharpoonup input units:  $x_i$
- $\blacktriangleright$  hidden units:  $a_j$
- ightharpoonup output units:  $y_k$
- ightharpoonup connection weights:  $w_{ij}$
- ► subscripts general: *ijklm*... specific: 12345...



By using maths, predictably!

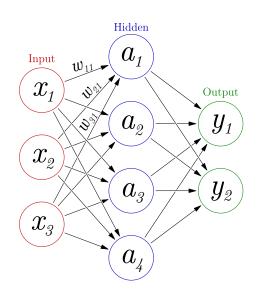
We use general names to write a general equation:



By using maths, predictably!

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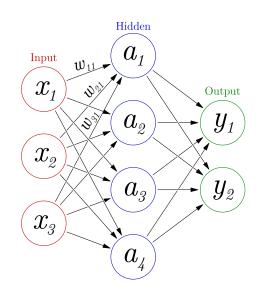
 $a_i =$ 



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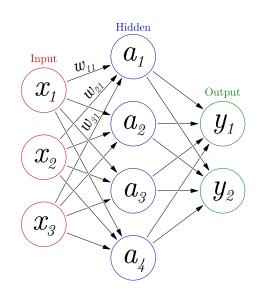
$$a_i = x_j \times w_{ji}$$



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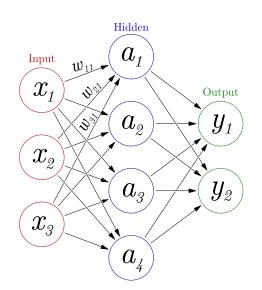
$$a_i = \sum_{j=1}^{N} x_j \times w_{ji}$$



By using maths, predictably!

We use general names to write a general equation:

$$a_i = f\left(\sum_{j=1}^N x_j \times w_{ji}\right)$$

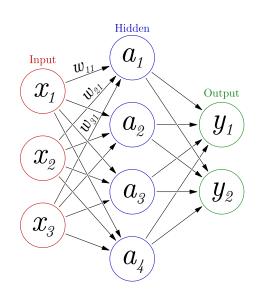


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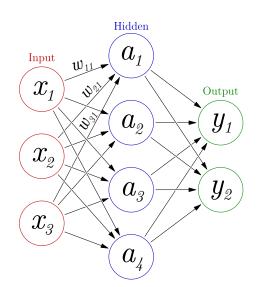
$$a_i = f\left(\sum_{j=1}^N x_j \times w_{ji}\right)$$

where  $a_i$  is the unit whose state we want to calculate, N is the number of units on the previous layer,  $w_{ji}$  is the weight on the connection between i and j, and f is a function that the unit applies.



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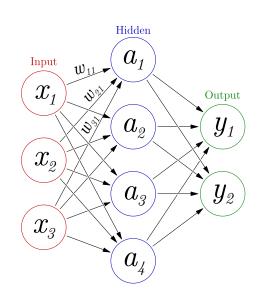
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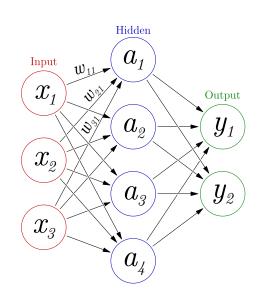
$$a_1 = f\left(\sum_{j=1}^{N} x_j \times w_{j1}\right)$$



By using maths, predictably!

$$a_i = f\left(\sum_{j=1}^N x_j \times w_{ji}\right)$$

$$a_1 = f\left(\sum_{j=1}^3 x_j \times w_{j1}\right)$$

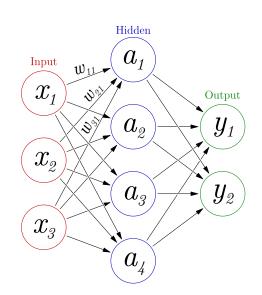


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$$x_1 \times w_{11}$$

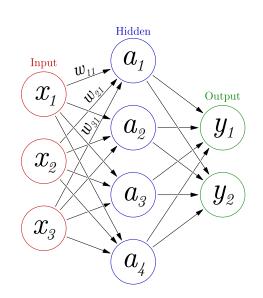


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$$a_i = f\left(\sum_{j=1}^N x_j \times w_{ji}\right)$$

$$a_1 = f\left(\sum_{j=1}^3 x_j \times w_{j1}\right)$$

$$x_1 \times w_{11} + x_2 \times w_{21}$$

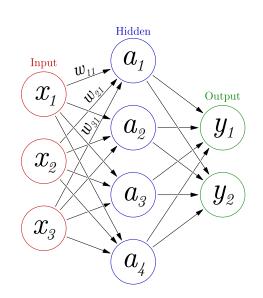


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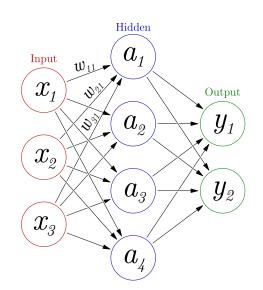
$$x_1 \times w_{11} + x_2 \times w_{21} + x_3 \times w_{31}$$



By using maths, predictably!

$$a_i = f\left(\sum_{j=1}^N x_j \times w_{ji}\right)$$

$$a_2 = f\left(\sum_{j=1}^3 x_j \times w_{j2}\right)$$

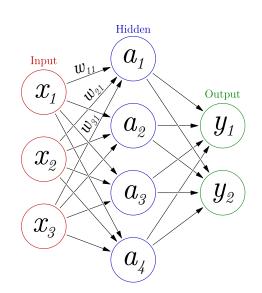


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$$a_2 = f\left(\sum_{j=1}^3 x_j \times w_{j2}\right)$$

$$x_1 \times w_{12}$$

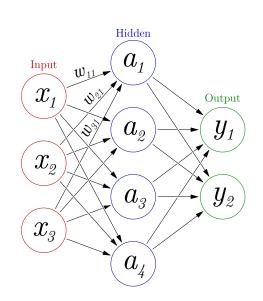


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$$a_i = f\left(\sum_{j=1}^N x_j \times w_{ji}\right)$$

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$$x_1 \times w_{12} + x_2 \times w_{22}$$

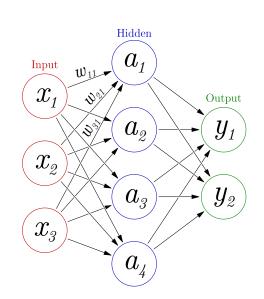


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$$a_i = f\left(\sum_{j=1}^N x_j \times w_{ji}\right)$$

$$a_2 = f\left(\sum_{j=1}^3 x_j \times w_{j2}\right)$$

$$x_1 \times w_{12} + x_2 \times w_{22} + x_3 \times w_{32}$$



#### How do networks learn?

Cunning!

Many options: Hebbian learning, back-propagation of error, Boltzmann machine learning, self-organising map algorithm, etc.

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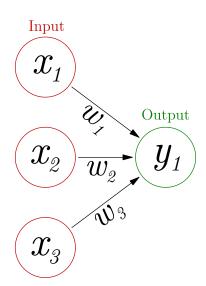
Many options: Hebbian learning, back-propagation of error, Boltzmann machine learning, self-organising map algorithm, etc.

- All learning algorithms work by changing the connection weights
- ► Learning can be divided into *supervised*, *unsupervised*, and *reinforcement*

A very simple learning rule

"Cells that fire together, wire together"

— Carla Shatz

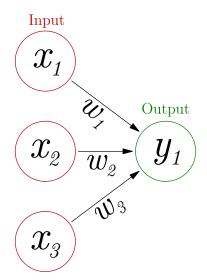


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 $w_i =$ 

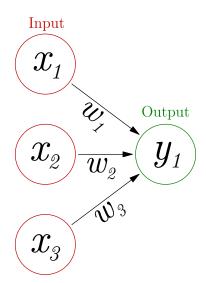


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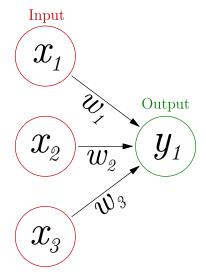
$$w_i = x_i \times y_i$$



A very simple learning rule

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$$w_i = \eta \times x_i \times y_i$$

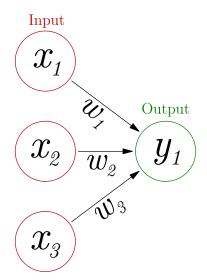


A very simple learning rule

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$$\Delta w_i = \eta \times x_i \times y_i$$



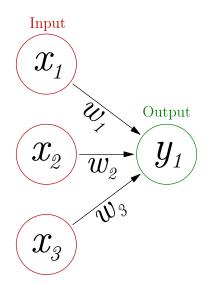
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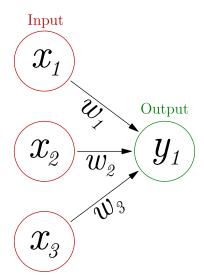
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$$\Delta w_i = \eta \times \underline{x_i} \times y_j$$

which means each weight is changed by a small in/decrement for every pattern

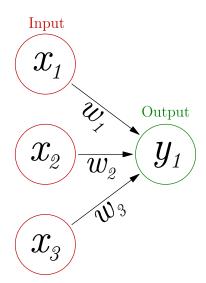


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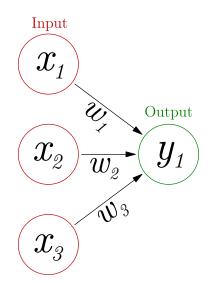
$$\Delta w_i = \eta \times x_i \times y_j$$



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$$\Delta w_i = \eta \times x_i \times y_j$$

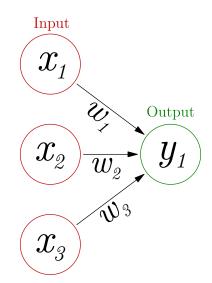
$$\Delta w_1 = \boldsymbol{\eta} \times x_i \times y_j$$



"Cells that fire together, wire together" — Carla Shatz

$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.5 \times x_i \times y_j$$



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$$\Delta w_i = \boldsymbol{\eta} \times x_i \times y_j$$

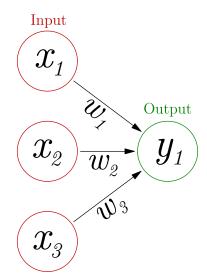
$$\Delta w_1 = 0.5 \times x_1 \times y_j$$



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$$\Delta w_i = \boldsymbol{\eta} \times x_i \times y_j$$

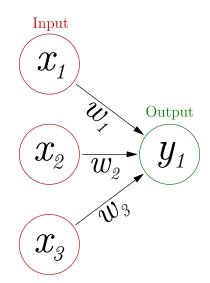
$$\Delta w_1 = 0.5 \times 1.0 \times y_1$$



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$$\Delta w_i = \boldsymbol{\eta} \times x_i \times y_j$$

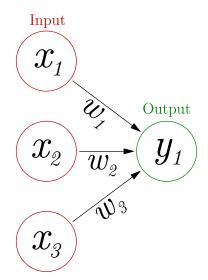
$$\Delta w_1 = 0.5 \times 1.0 \times 0.3$$



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$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

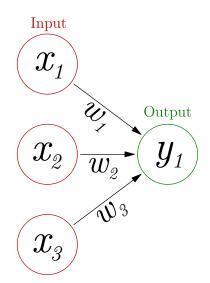


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$$\Delta w_i = \boldsymbol{\eta} \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

$$\mathbf{new} \ w_1 = \mathbf{old} \ w_1 + \Delta w_1$$

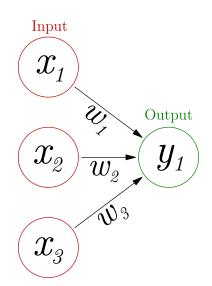


"Cells that fire together, wire together" — Carla Shatz

$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

**new** 
$$w_1 = 0.0 + \Delta w_1$$

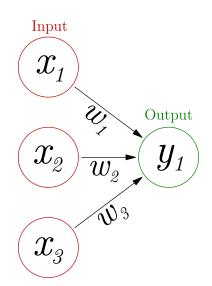


"Cells that fire together, wire together" — Carla Shatz

$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

**new** 
$$w_1 = 0.0 + 0.15$$



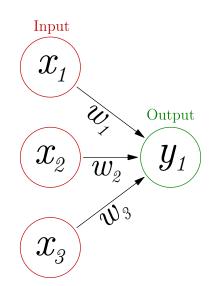
"Cells that fire together, wire together" — Carla Shatz

Hebb's rule is simple, but *very unstable*!

$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

**new** 
$$w_1 = 0.15$$



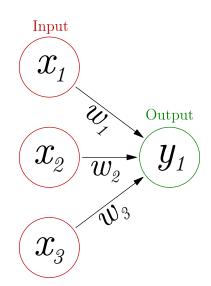
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Hebb's rule is simple, but *very unstable*!

$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

$$w_1 = 0.15$$



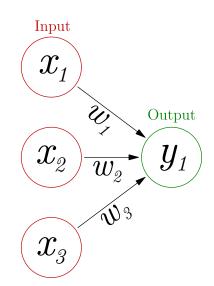
"Cells that fire together, wire together" — Carla Shatz

Hebb's rule is simple, but *very unstable*!

$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

 $w_1 = 0.15 + \text{something}$  positive



"Cells that fire together, wire together" — Carla Shatz

Hebb's rule is simple, but *very unstable*!

$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

 $w_1 = 0.15 + \text{something}$ positive + something else positive +



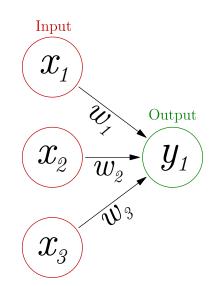
"Cells that fire together, wire together" — Carla Shatz

Hebb's rule is simple, but *very unstable*!

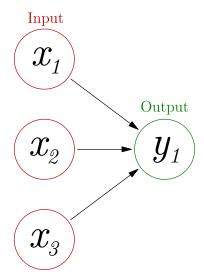
$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

 $w_1 = 0.15 + \text{something}$ positive + something else positive + another positive value + ...

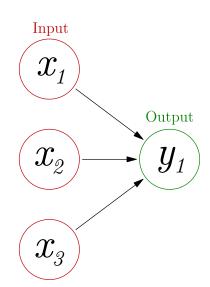


A simple classifier



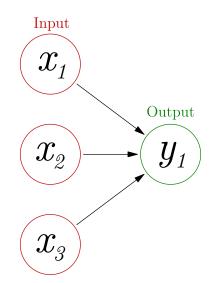
A simple classifier

 Created in 1957 at the Cornell Aeronautical Laboratory by Frank Rosenblatt



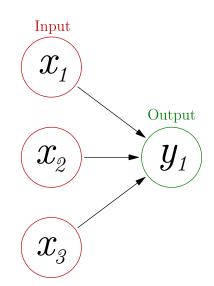
A simple classifier

- Created in 1957 at the Cornell Aeronautical Laboratory by Frank Rosenblatt
- Linear classifier



#### A simple classifier

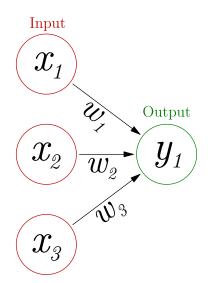
- Created in 1957 at the Cornell Aeronautical Laboratory by Frank Rosenblatt
- ▶ Linear classifier
- Simplest form of feedforward network



# How does the perceptron learn?

Maths again!

1. Initialise weights

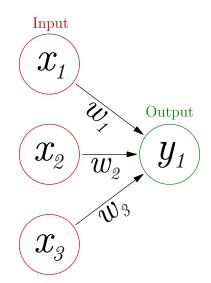


Maths again!

- 1. Initialise weights
- 2. Run network using:

$$y_j = f\left(\sum_{1}^{N} w_i \times x_i\right)$$

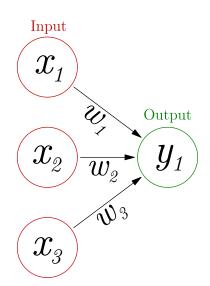
same as always!



Maths again!

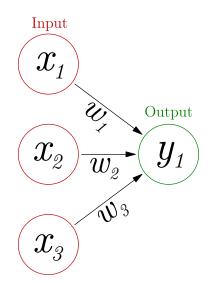
- 1. Initialise weights
- 2. Run network
- 3. Update weights using:

$$\Delta w_i = \eta \quad y_j \times x_i$$



- 1. Initialise weights
- 2. Run network
- 3. Update weights using:

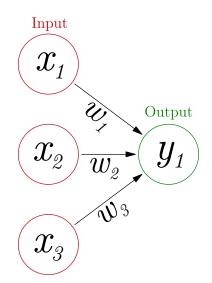
$$\Delta w_i = \eta \ (d_j - y_j) \ \times x_i$$



- 1. Initialise weights
- 2. Run network
- 3. Update weights using:

$$\Delta w_i = \eta \ (d_j - y_j) \ \times x_i$$

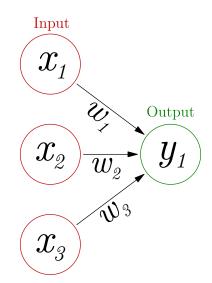
where d is what we want y to be given x, and  $\eta$  is the learning rate.



# How does the perceptron learn?

Maths again!

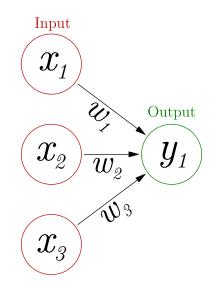
- 1. Initialise weights
- 2. Run network
- 3. Update weights
- 4. Repeat 2 and 3



# How does the perceptron learn?

Maths again!

- 1. Initialise weights
- 2. Run network
- 3. Update weights
- 4. Repeat 2 and 3
- 5. When do we stop?



# Perceptron coding time

# Time to program a perceptron!

Oh, and join our mailing list so we can send you stuff: https://groups.google.com/d/forum/introcompcog

```
def Train(self):
             error = d[p][0] - y
```

```
def Train(self):
                x[i] = self.patterns[p][i]
             error = d[p][0] - y
```

```
def Train(self):
            for i in range(N):
                x[i] = self.patterns[p][i]
             error = d[p][0] - y
```

```
def Train(self):
            for i in range(N):
                x[i] = self.patterns[p][i]
                y += f(x[i] * w[i])
             error = d[p][0] - y
```

```
def Train(self):
            for i in range(N):
                x[i] = self.patterns[p][i]
            for i in range(N+1):
                y += f(x[i] * w[i])
             error = d[p][0] - y
```

```
def Train(self):
            for i in range(N):
                x[i] = self.patterns[p][i]
            y = 0
            for i in range(N+1):
                y += f(x[i] * w[i])
             error = d[p][0] - y
```

```
def Train(self):
            for i in range(N):
                x[i] = self.patterns[p][i]
            y = 0
            for i in range(N+1):
                y += f(x[i] * w[i])
             error = d[p][0] - y
```

```
def Train(self):
            for i in range(N):
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```

```
def Train(self):
            for i in range(N):
                x[i] = self.patterns[p][i]
            y = 0
            for i in range(N+1):
                y += f(x[i] * w[i])
             error = d[p][0] - y
                w[i] += h * error * x[i]
```

```
def Train(self):
            for i in range(N):
                x[i] = self.patterns[p][i]
            y = 0
            for i in range(N+1):
                y += f(x[i] * w[i])
             error = d[p][0] - y
           for i in range(N+1):
                w[i] += h * error * x[i]
```

```
def Train(self):
       for p in range(P):
            for i in range(N):
                x[i] = self.patterns[p][i]
            y = 0
            for i in range(N+1):
                y += f(x[i] * w[i])
             error = d[p][0] - y
           for i in range(N+1):
                w[i] += h * error * x[i]
```

```
def Train(self):
   for t in range(100):
       for p in range(P):
            for i in range(N):
                x[i] = self.patterns[p][i]
            y = 0
            for i in range(N+1):
                y += f(x[i] * w[i])
             error = d[p][0] - y
           for i in range(N+1):
                w[i] += h * error * x[i]
```

Defining some patterns!

```
Patterns =
    [#colour, shape, taste
        #red-yellow, big-small, sweet-sour
        [0.1, 0.0, 0.2], #loquat
        [0.0, 0.2, 0.0], #lemon
        [1.0, 0.5, 0.8], #red apple
        [1.0, 0.0, 0.9], #strawberry
]
```

Defining some patterns!

```
Patterns =
        [#colour, shape, taste
         #red-yellow, big-small, sweet-sour
         [0.1, 0.0, 0.2], #loquat
         [0.0, 0.2, 0.0], #lemon
         [1.0, 0.5, 0.8], #red apple
         [1.0, 0.0, 0.9], #strawberry
Targets = [
           [1.0], #first target
           [1.0], #targets indicate
           [0.0], #which class
           [0.0], #a pattern
          ] #belongs to
```