Welcome to the introduction to computational cognitive modelling workshop!

Part 2: Introduction to artificial neural networks

Part 2: **Introduction to artificial neural networks**

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Nick Sexton *Nicole Cruz De Echeverria Loebell*

What is a neural network?

A mathematical model

- Inspired by the nervous system
- A set of *units*, connected by *weights*
- The network *runs* by passing *activations* from the *input* (to the *hidden*) to the *output* units

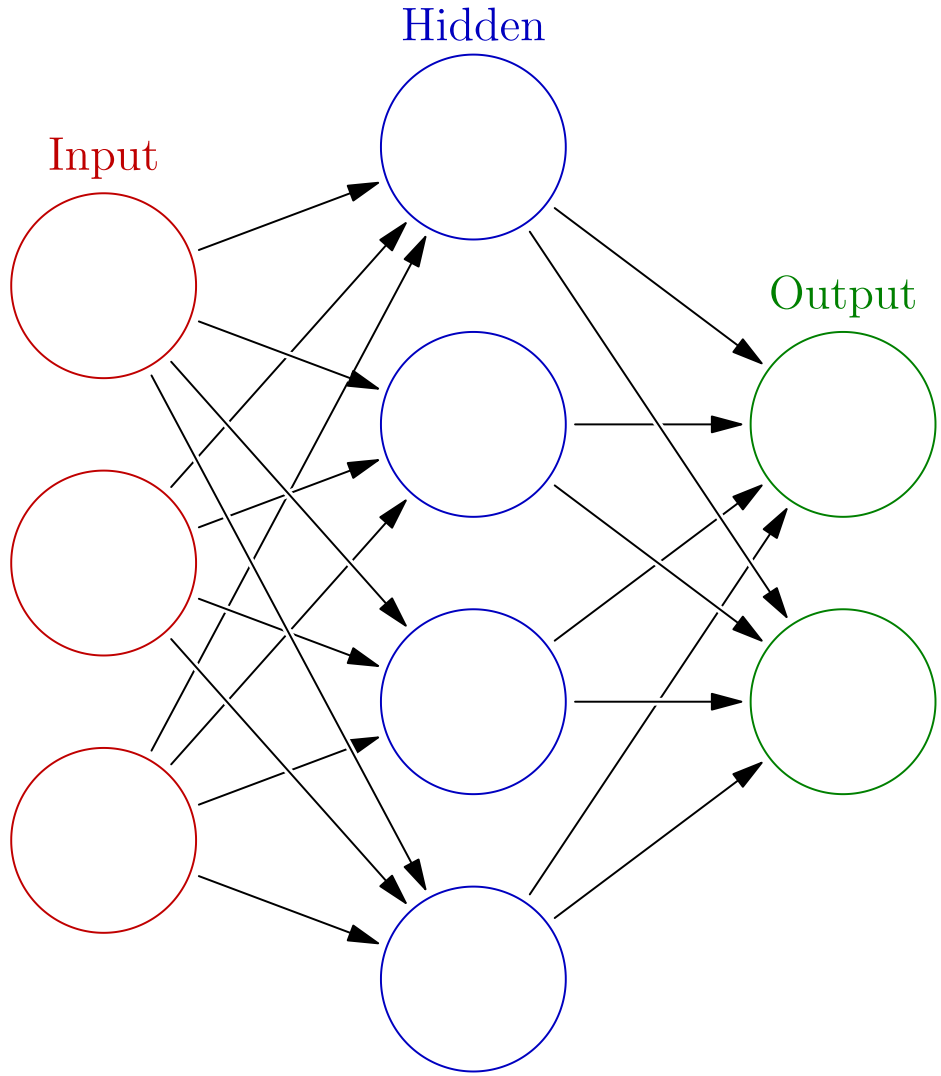


Figure: Glosser.ca / CC-BY-SA-3.0

Why use artificial neural networks for modelling?

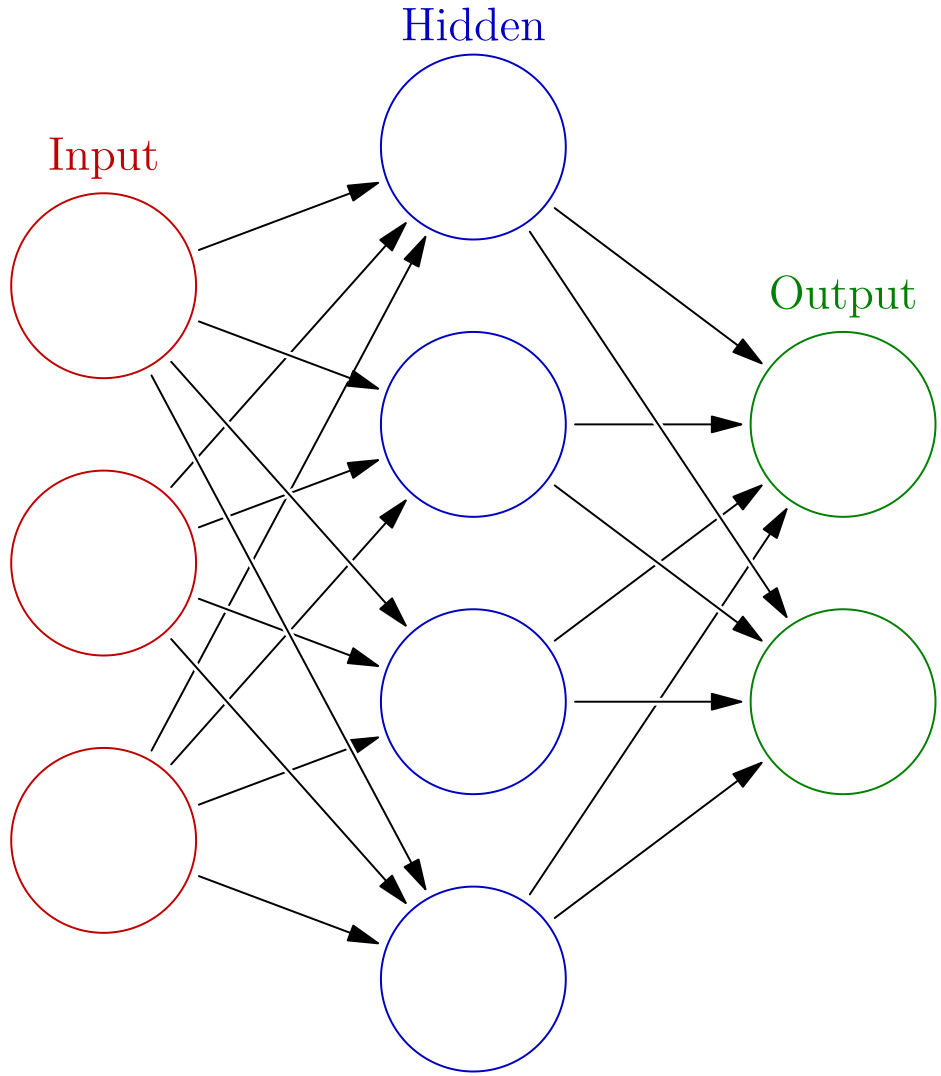
Some aspects of their behaviour are like their namesake!

- Learn pretty much any input-output data
- Uncover rules on their own about data
- Generalise from what they have learnt
- Cope with noise and damage

How does an artificial neural network run?

By using maths, predictably!

1. Input units are set to a *pattern*



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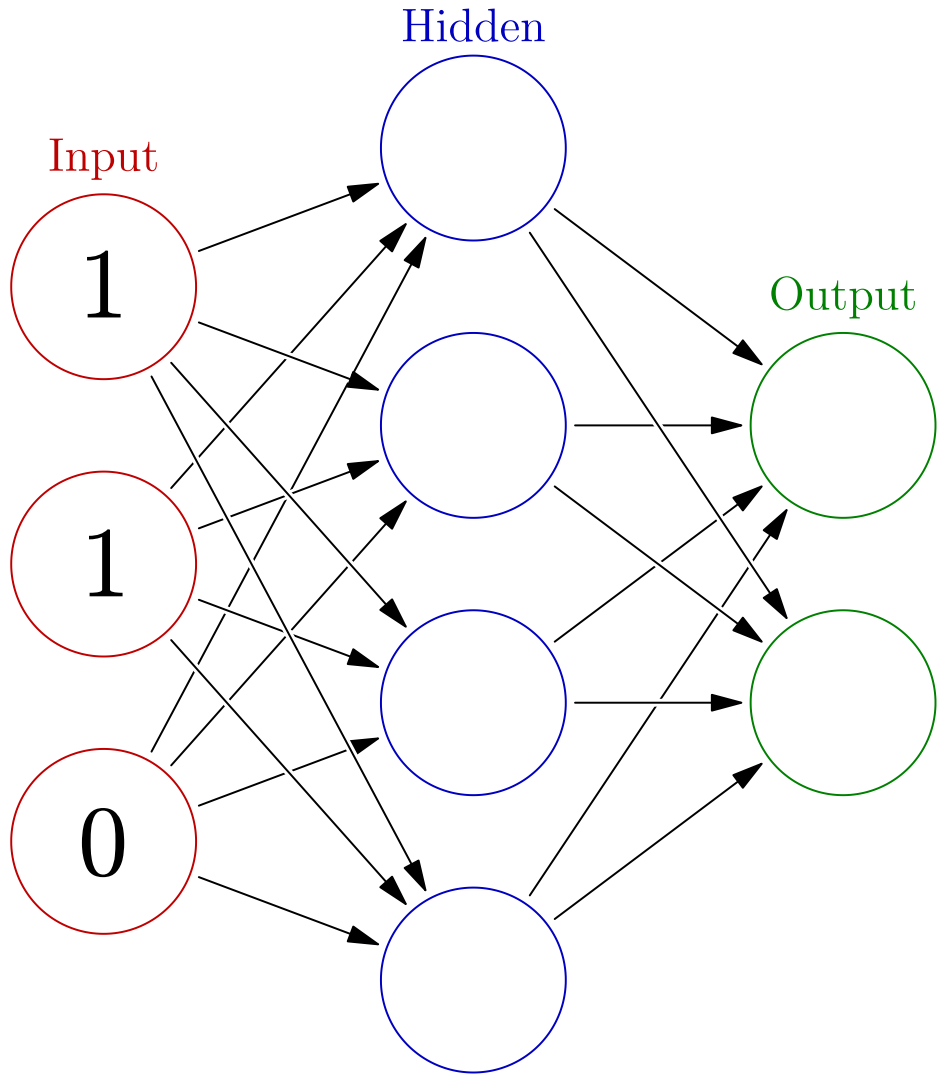

How does an artificial neural network run?

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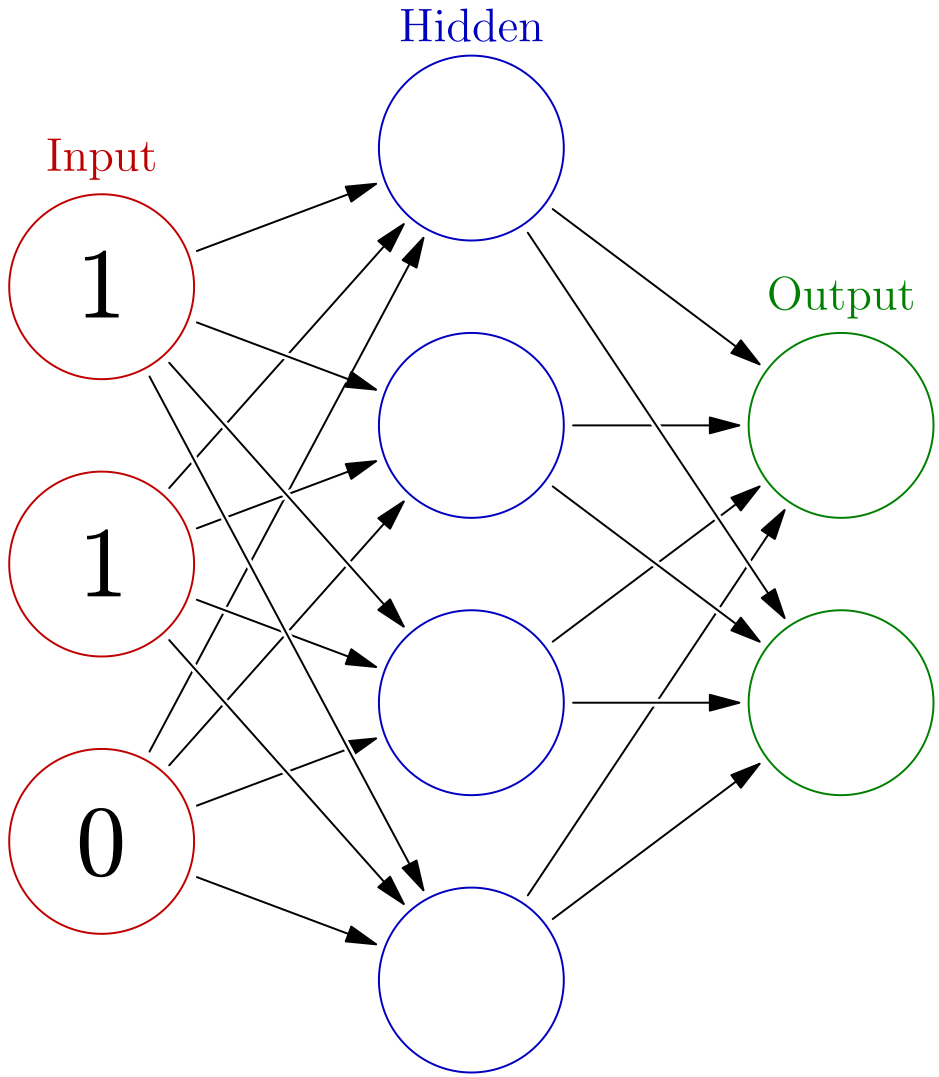
1. Input units are set to a *pattern*
2. Calculate hidden units' states



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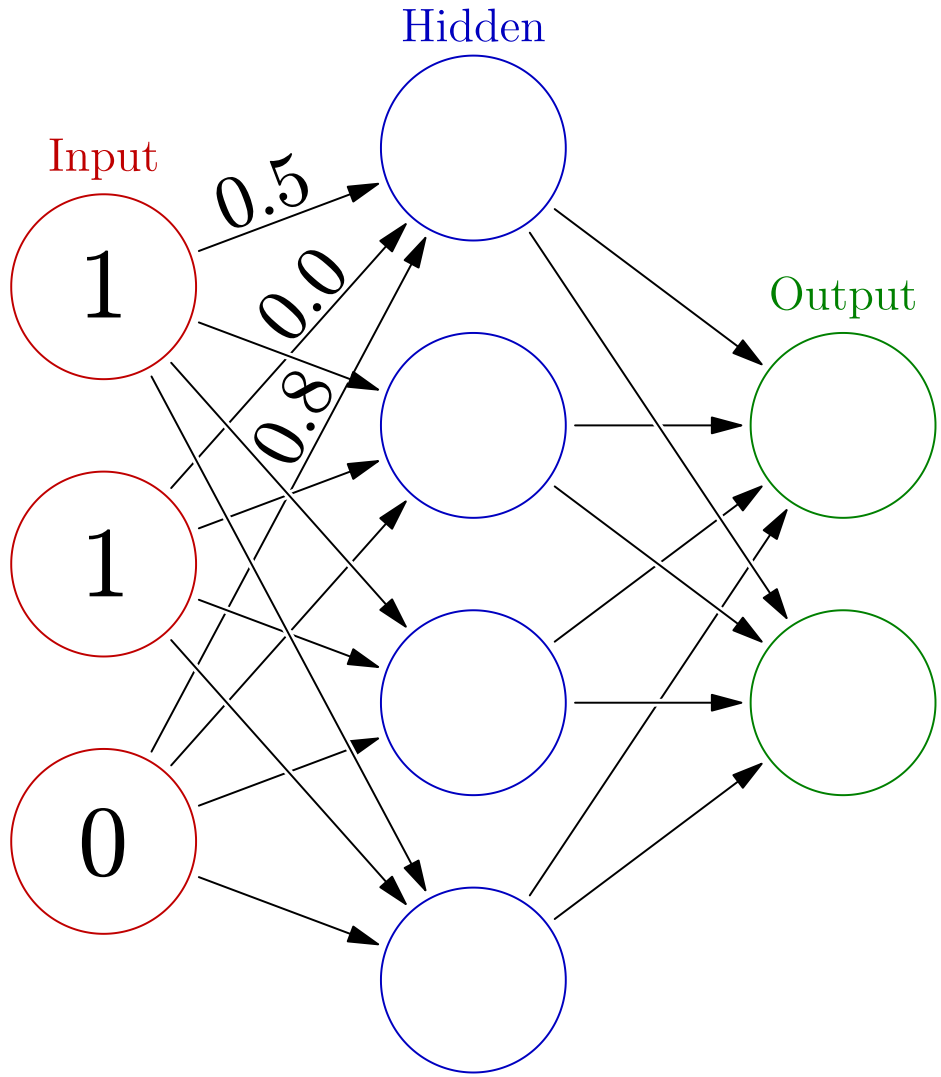

How does an artificial neural network run?

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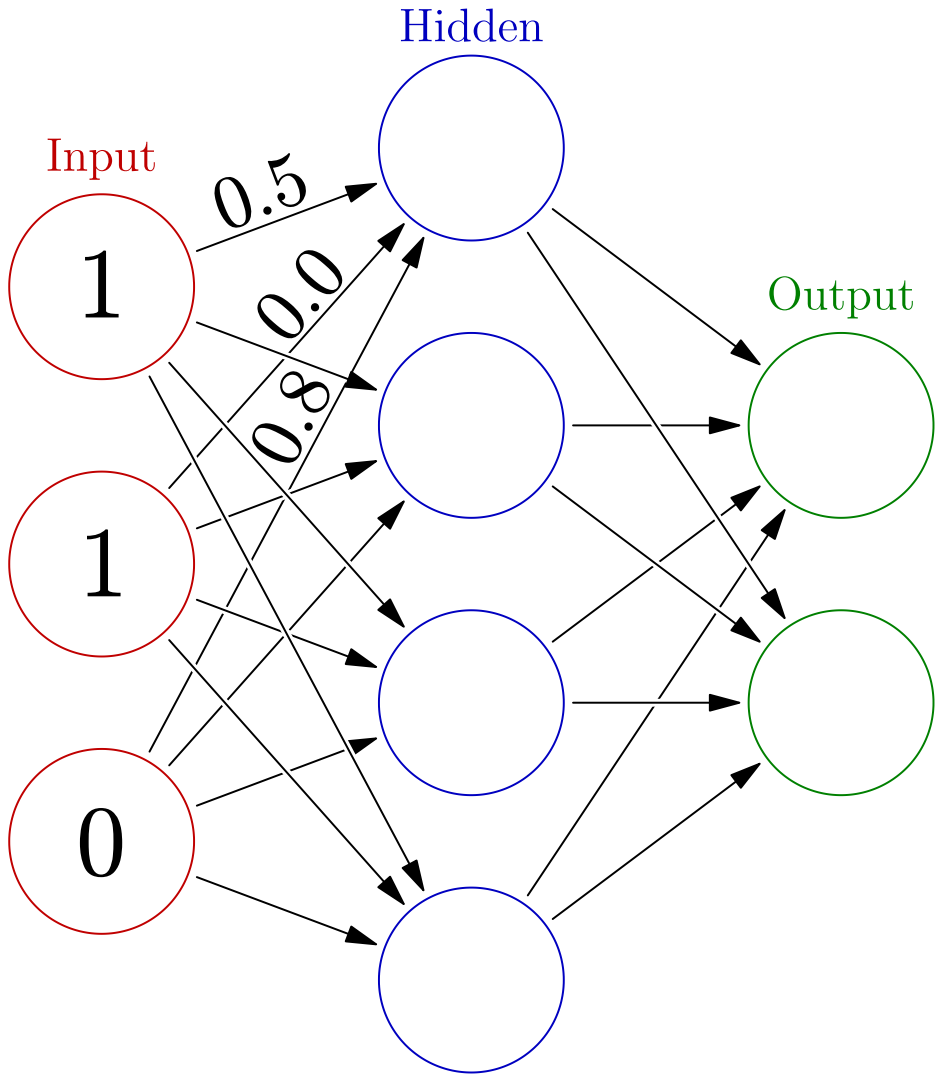
1. Input units are set to a *pattern*

2. Calculate hidden units' states:

$$1 \times 0.5 = 0.5$$



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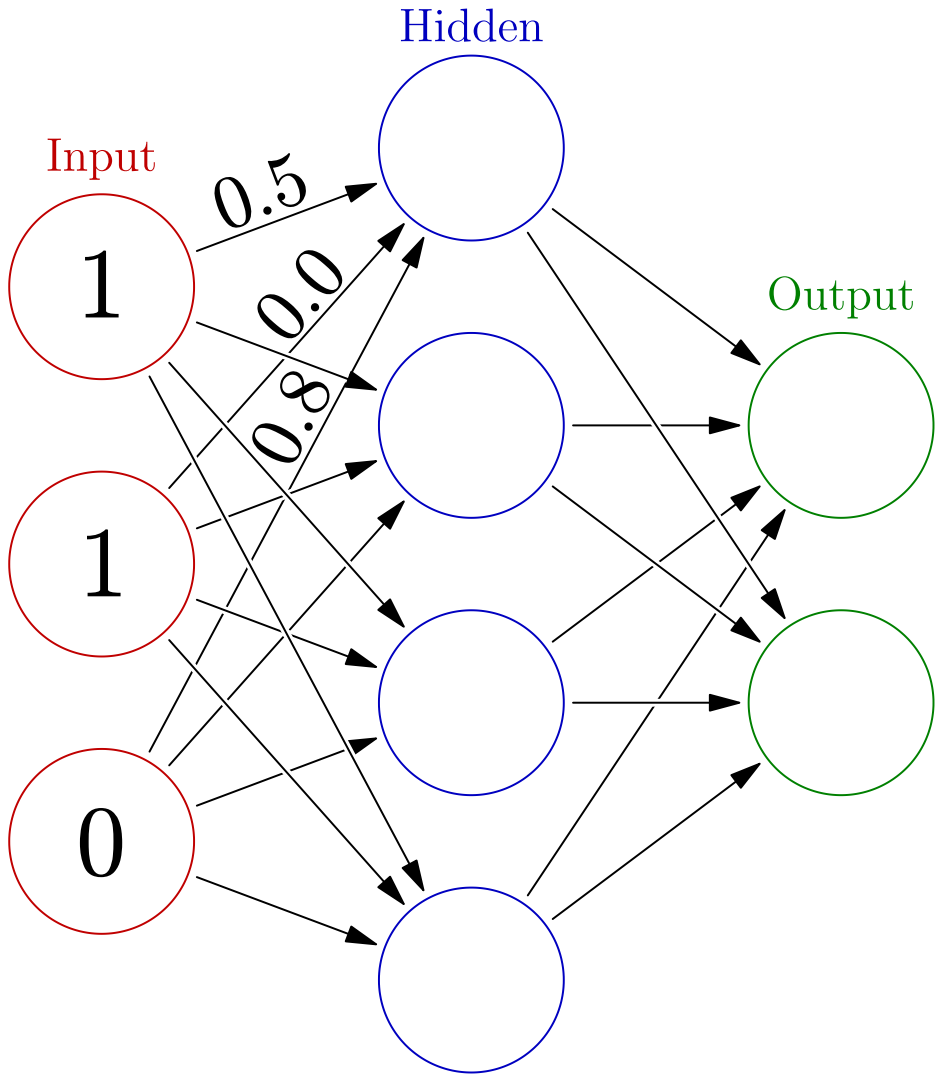

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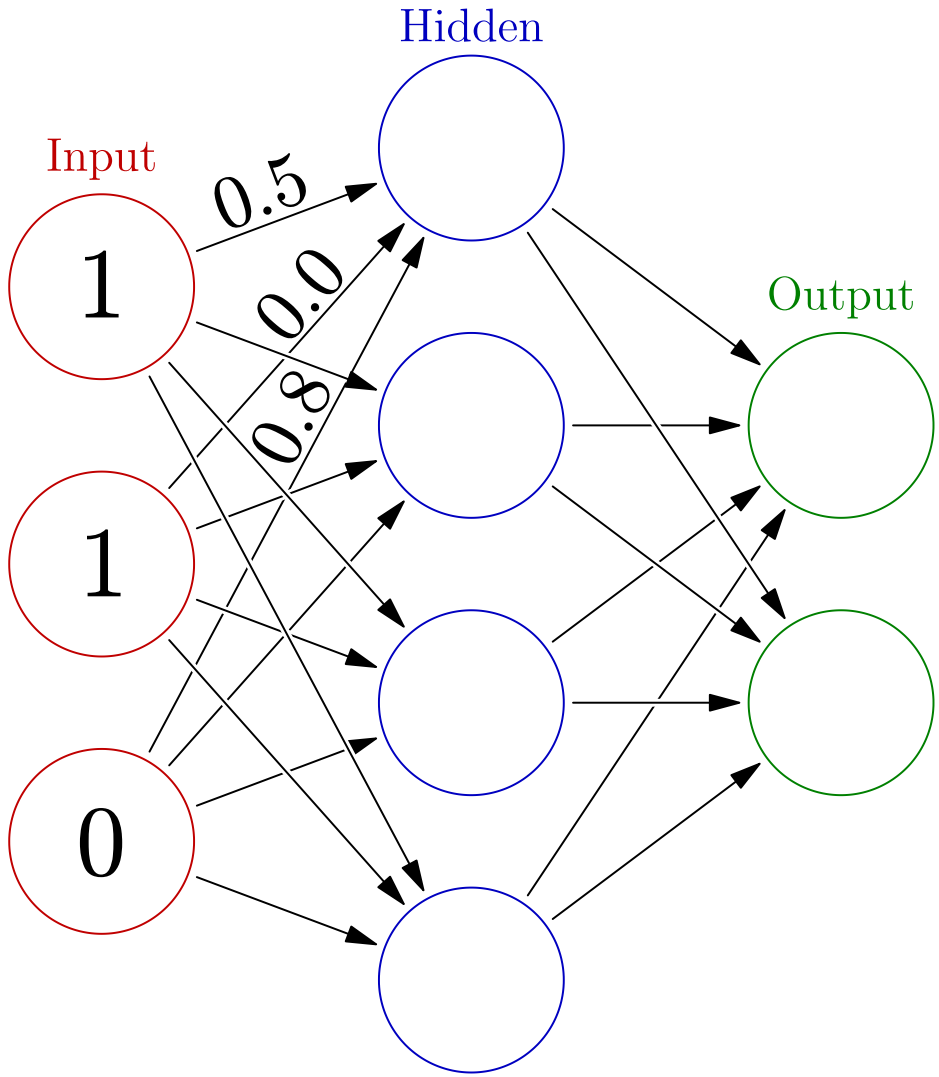
$$\begin{aligned} 1 \times 0.5 &= 0.5 \\ 1 \times 0.0 &= 0.0 \\ 0 \times 0.8 &= 0.0 \end{aligned}$$



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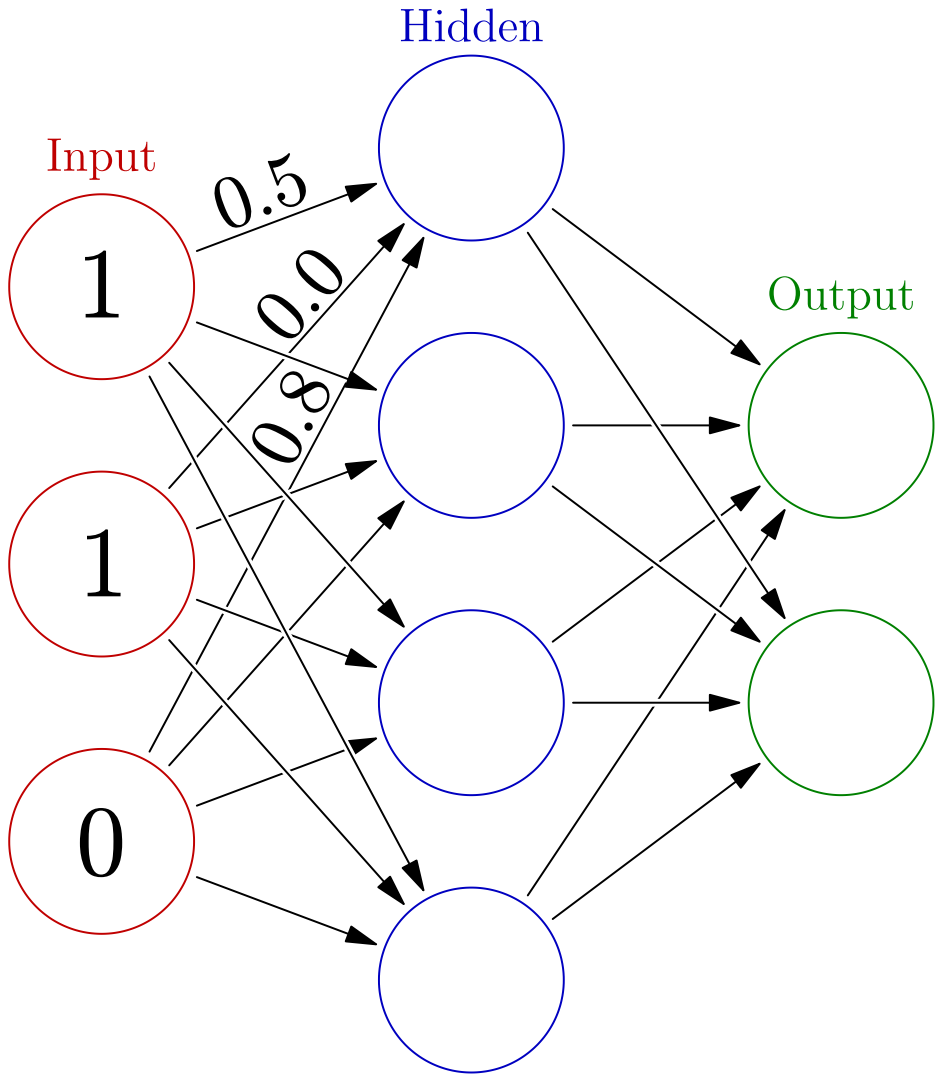

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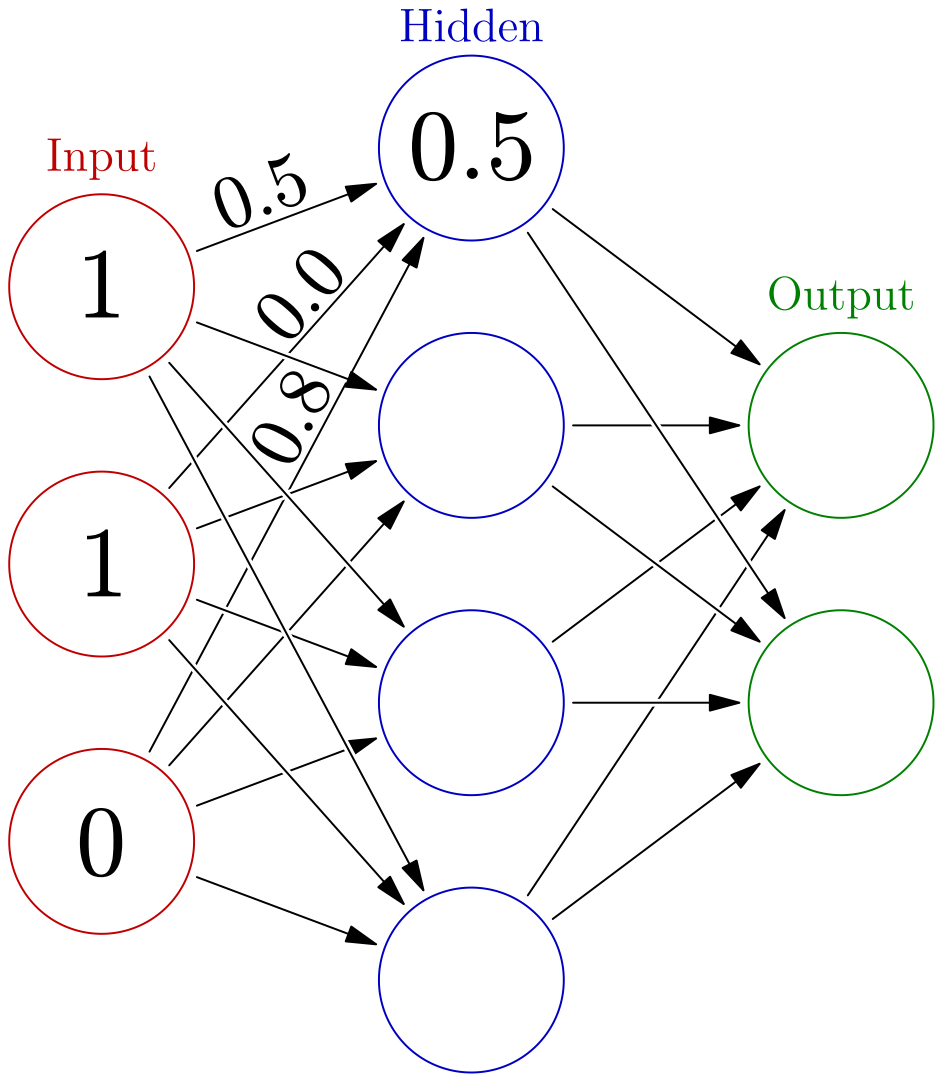

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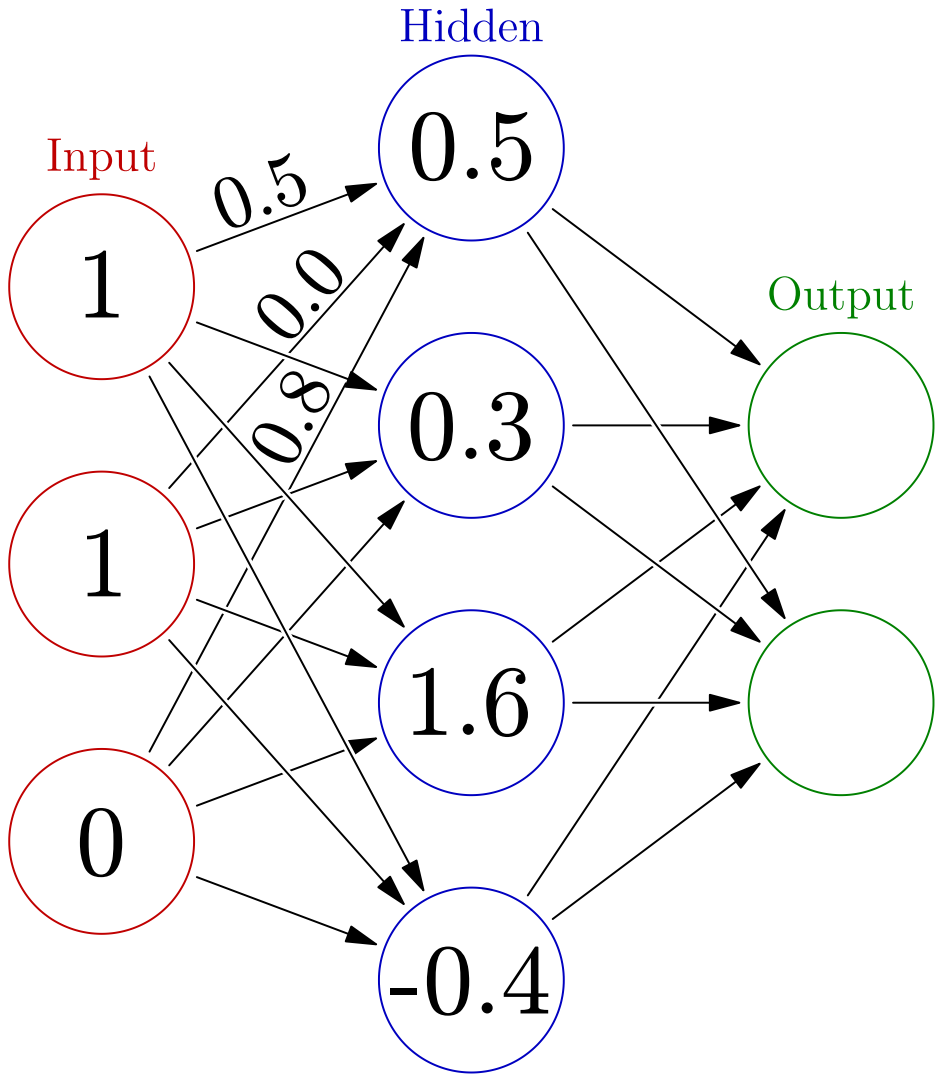

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1. Input units are set to a *pattern*
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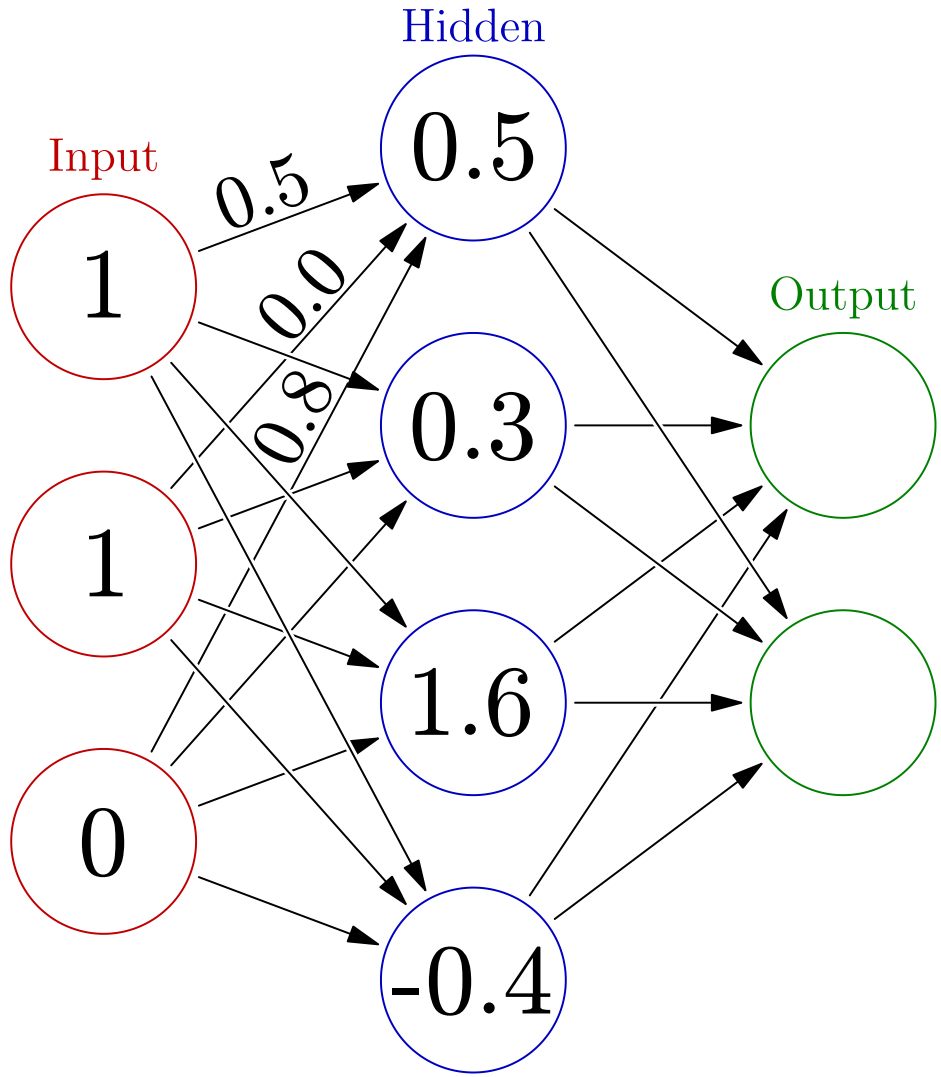
3. Same for output units



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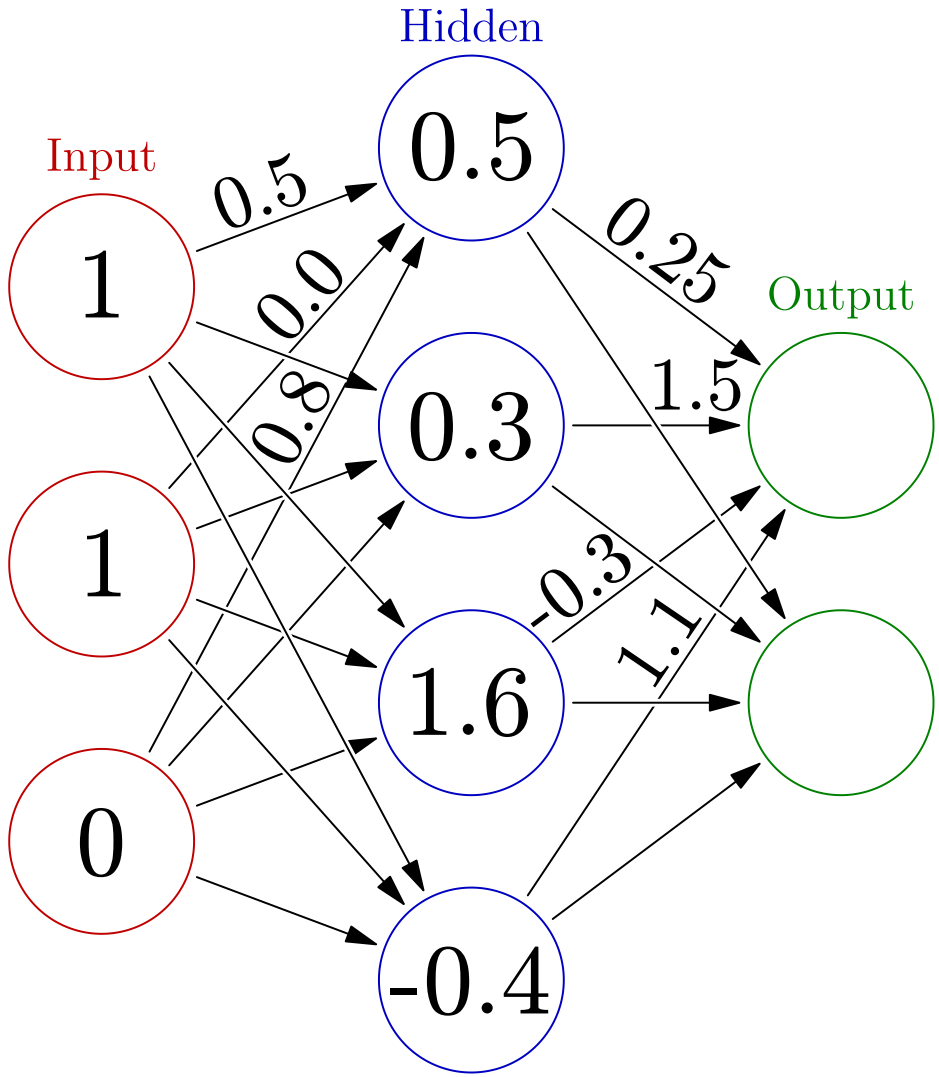

How does an artificial neural network run?

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1. Input units are set to a *pattern*
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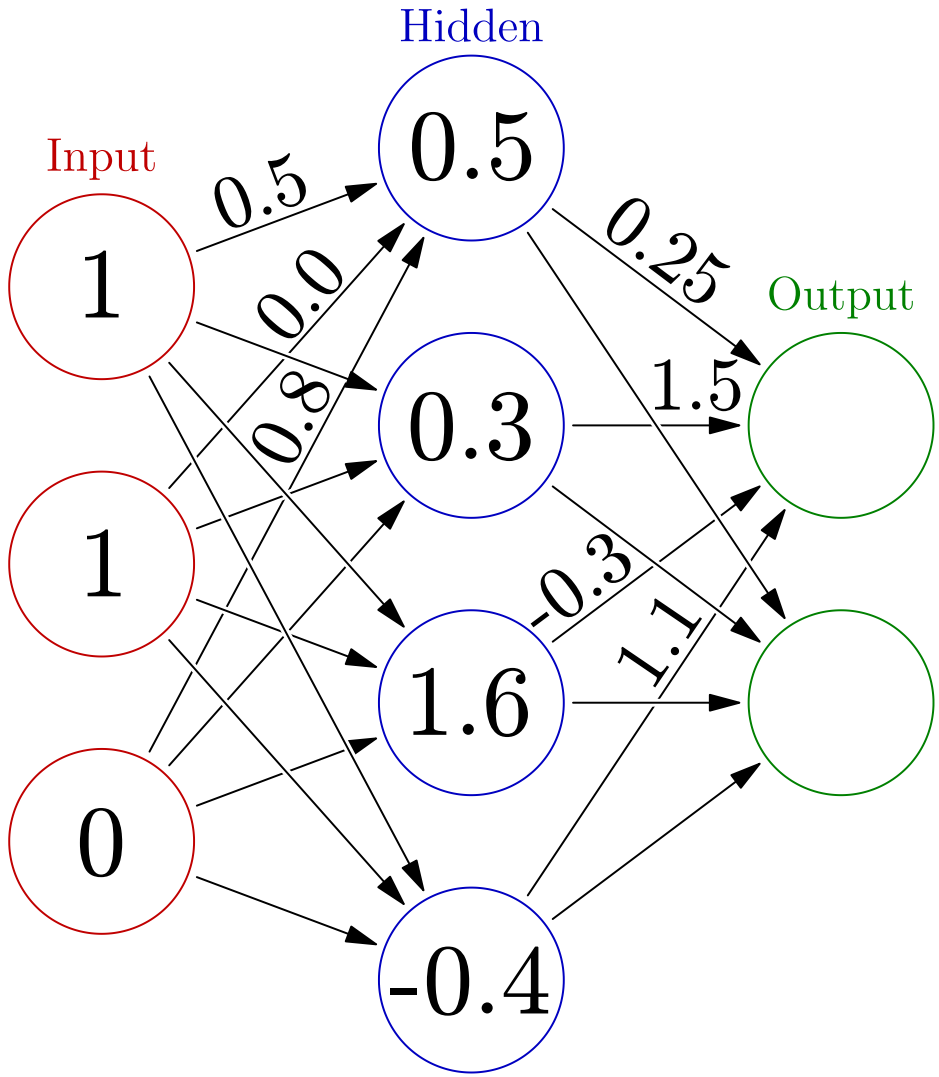
3. Same for output units:

$$0.5 \times 0.25 = 0.125$$



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$$0.5 \times 0.25 = 0.125$$
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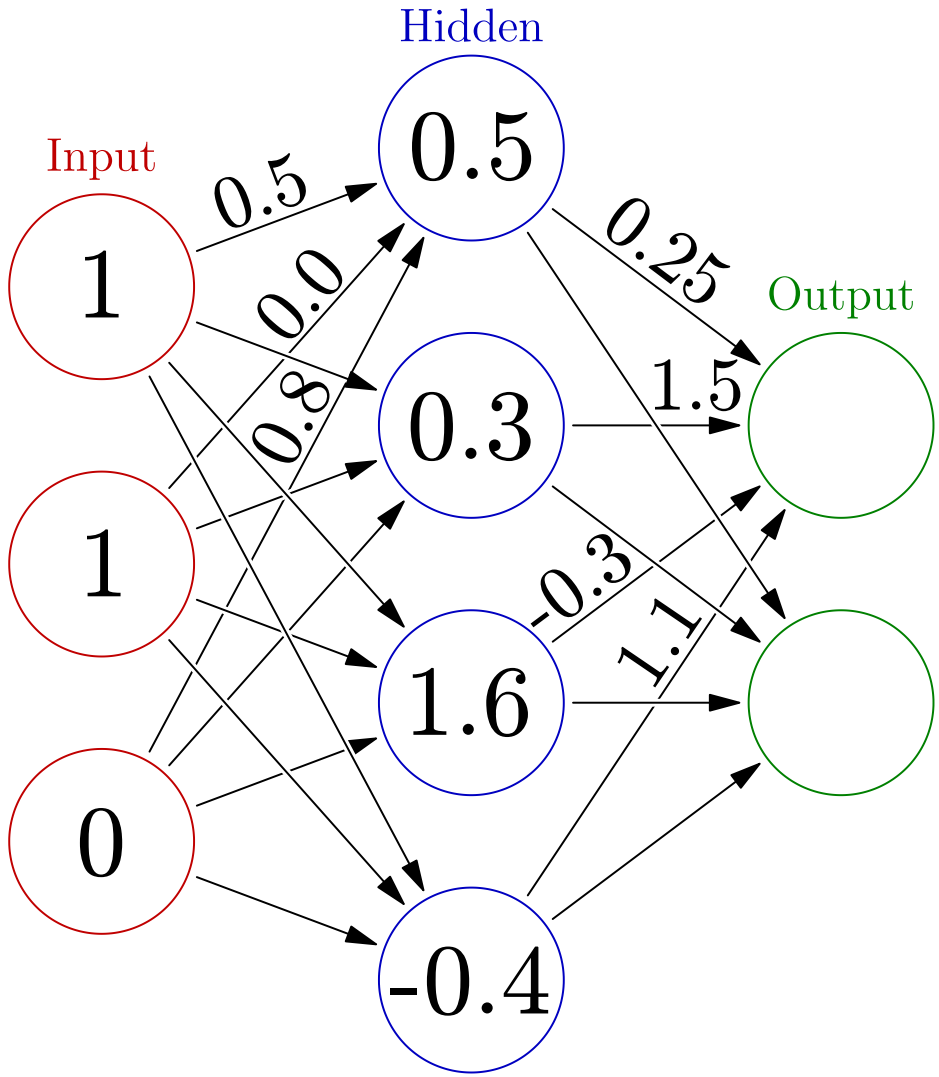

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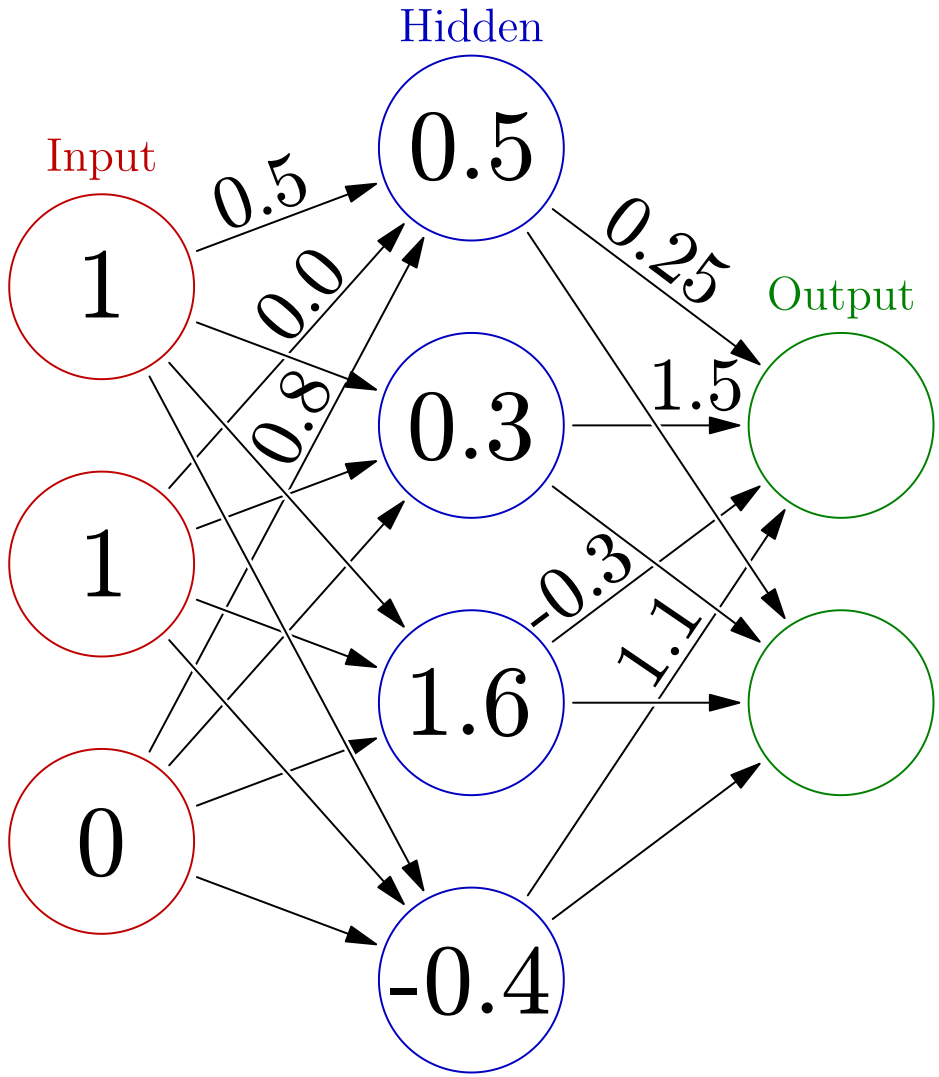
$$\begin{aligned} 0.5 \times 0.25 &= 0.125 \\ 0.3 \times 1.5 &= 0.45 \\ 1.6 \times -0.3 &= -0.48 \end{aligned}$$



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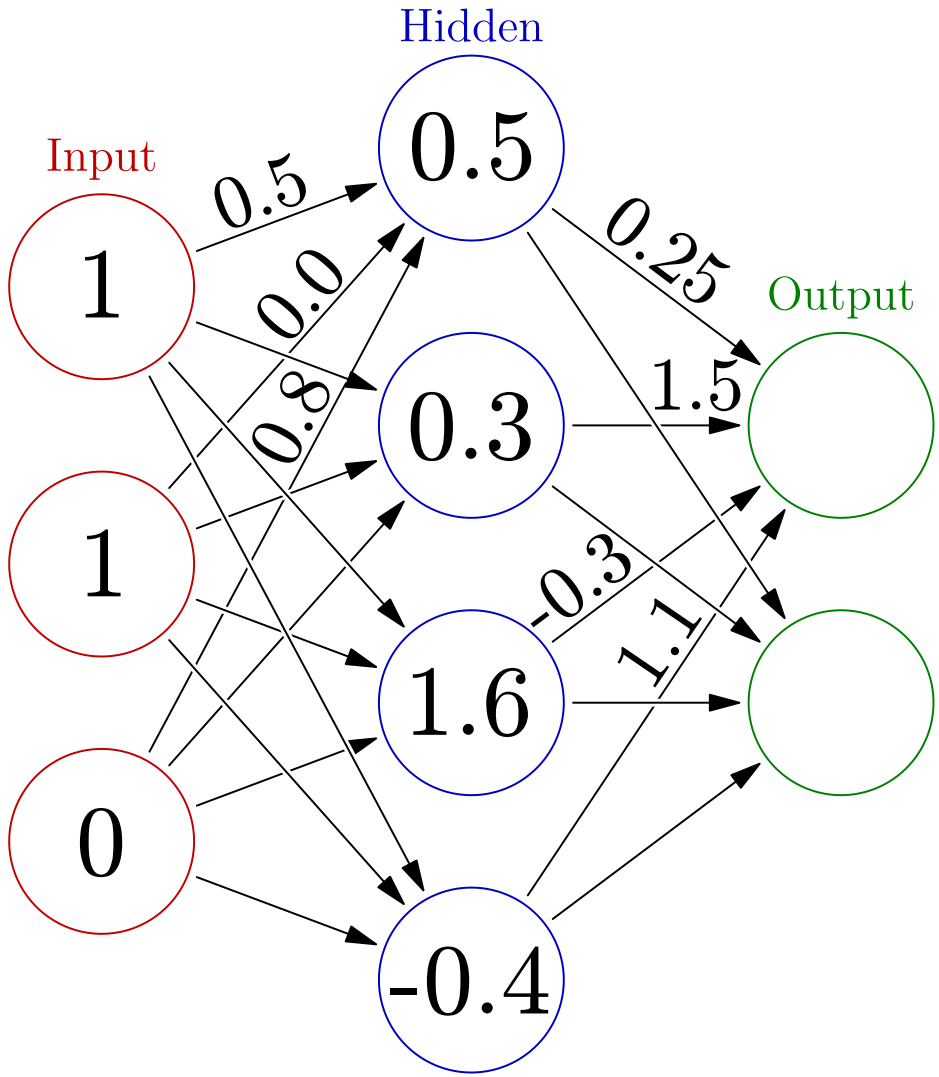
1.6 \times -0.3 &= -0.48 \\
-0.4 \times 1.1 &= -0.44
\end{aligned}
$$



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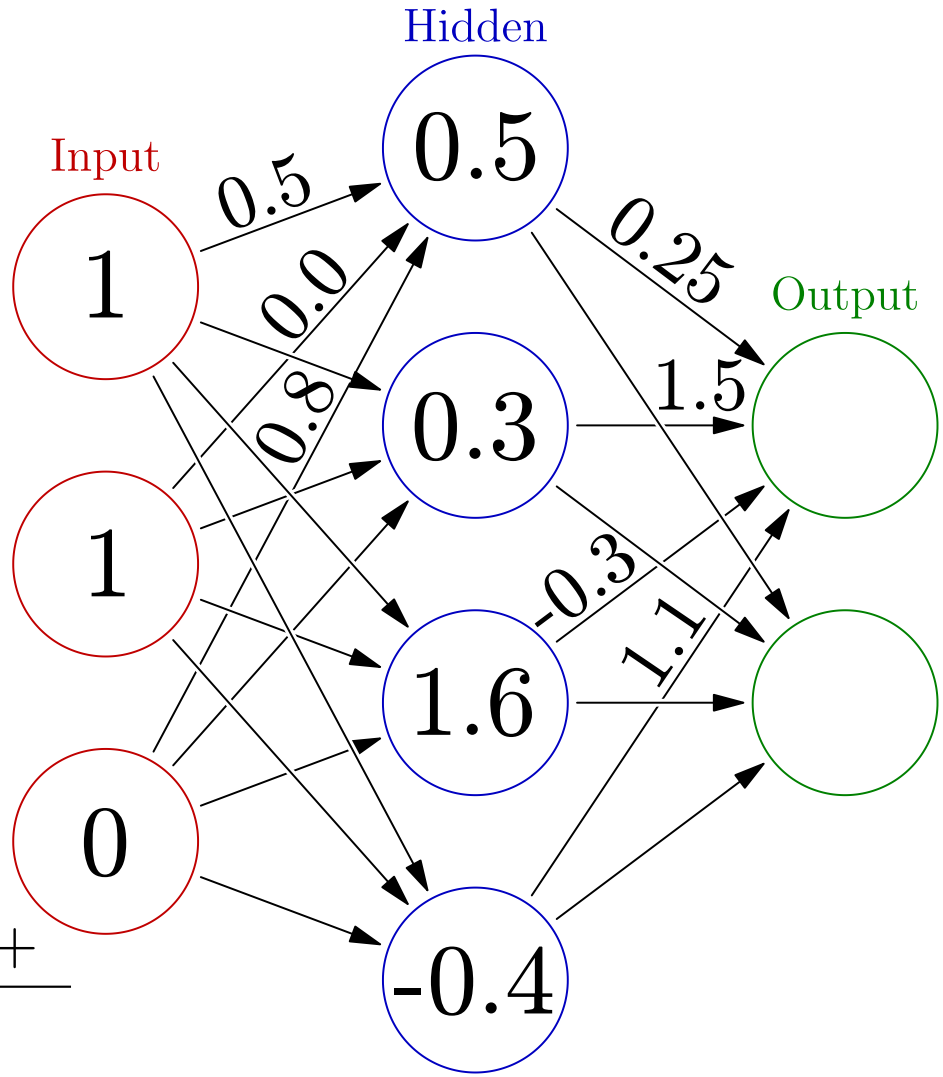
\hline
 & & -0.345 &
\end{array}
$$



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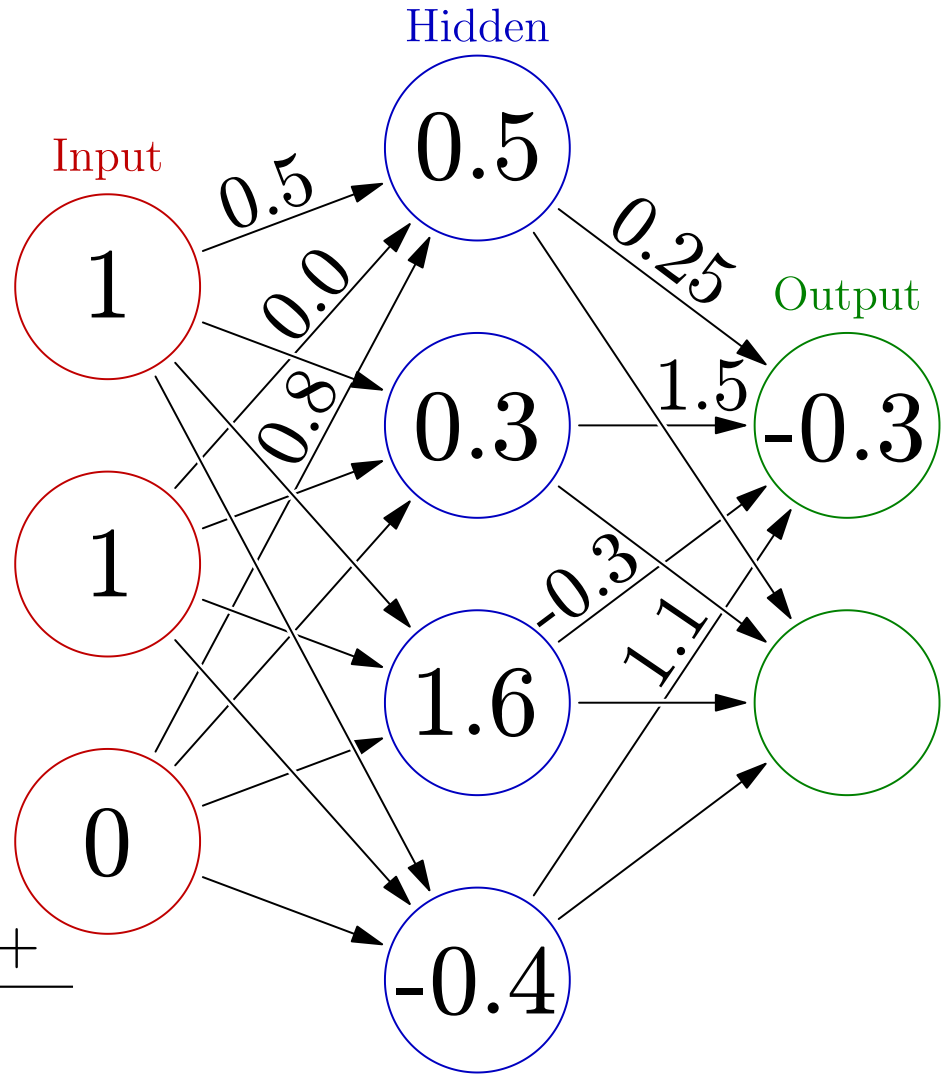

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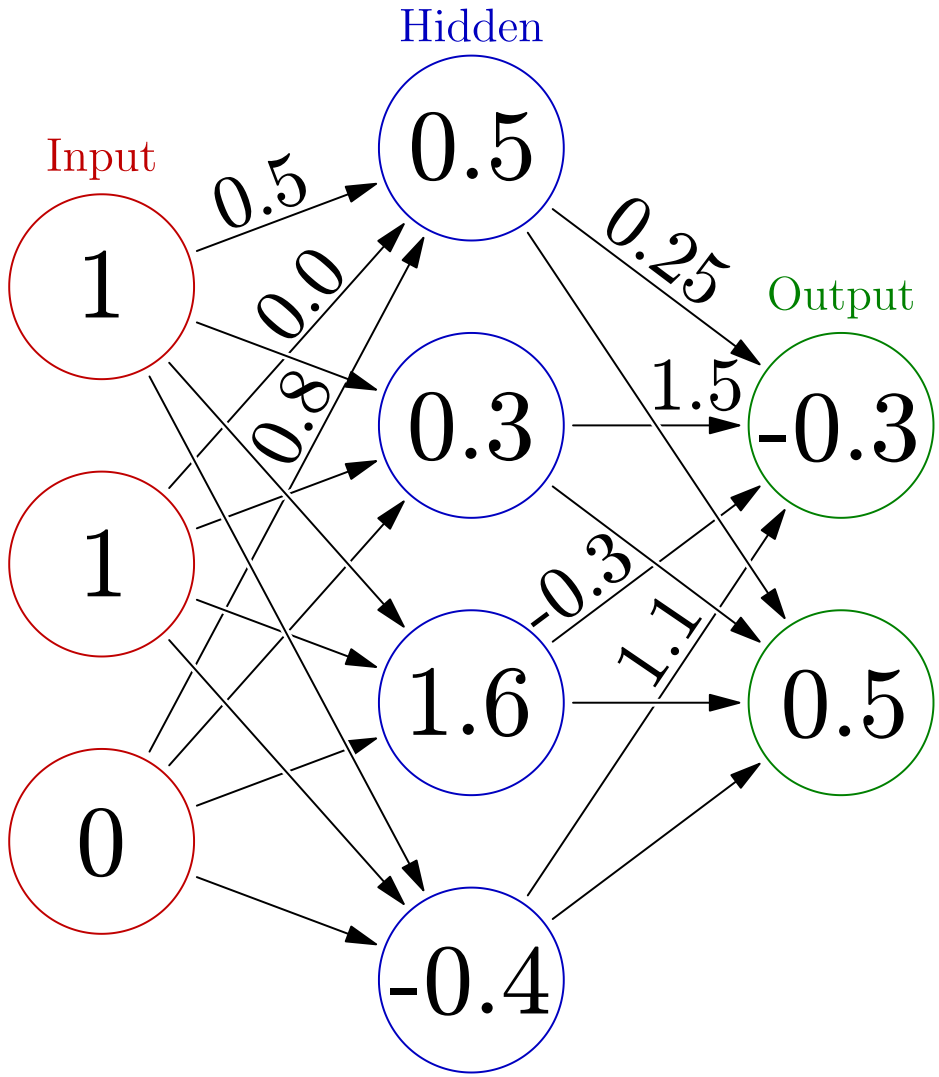
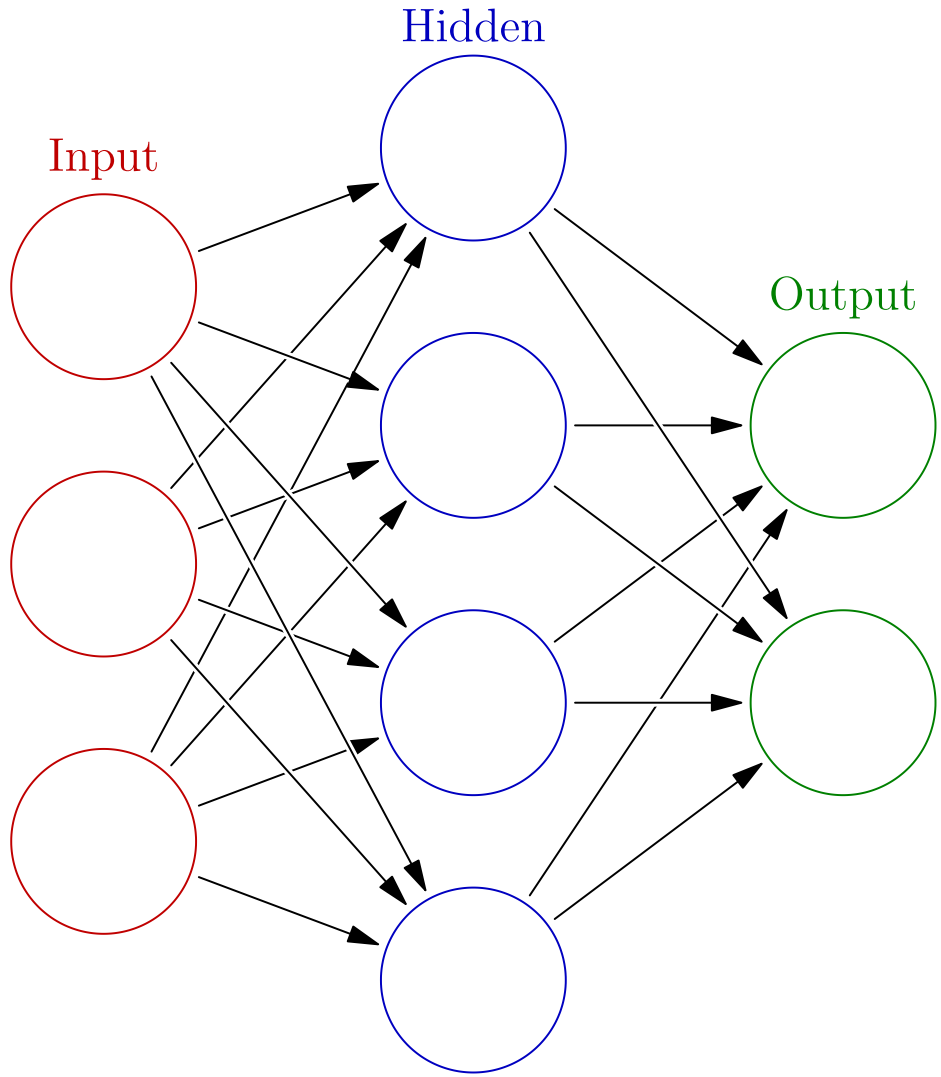


How does an artificial neural network run?

By using maths, predictably!

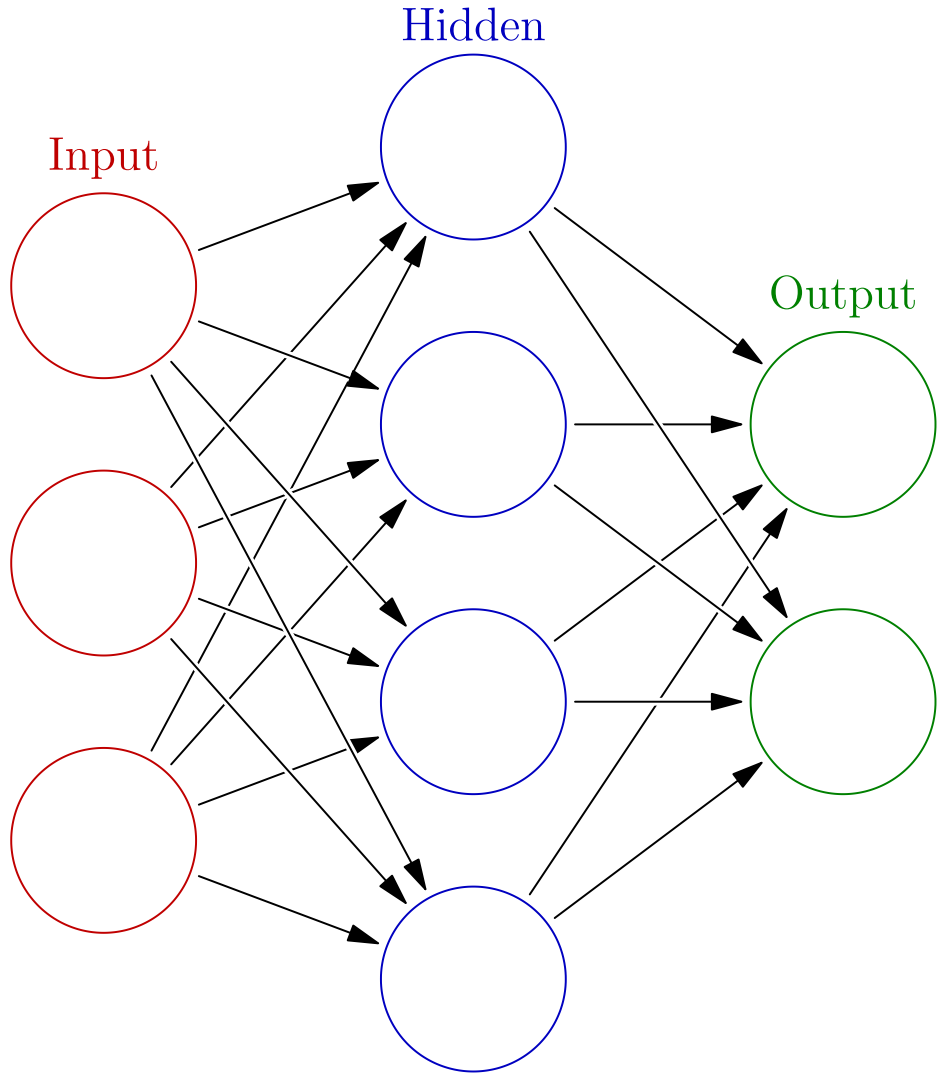
- But programmers are *lazy*!



How does an artificial neural network run?

By using maths, predictably!

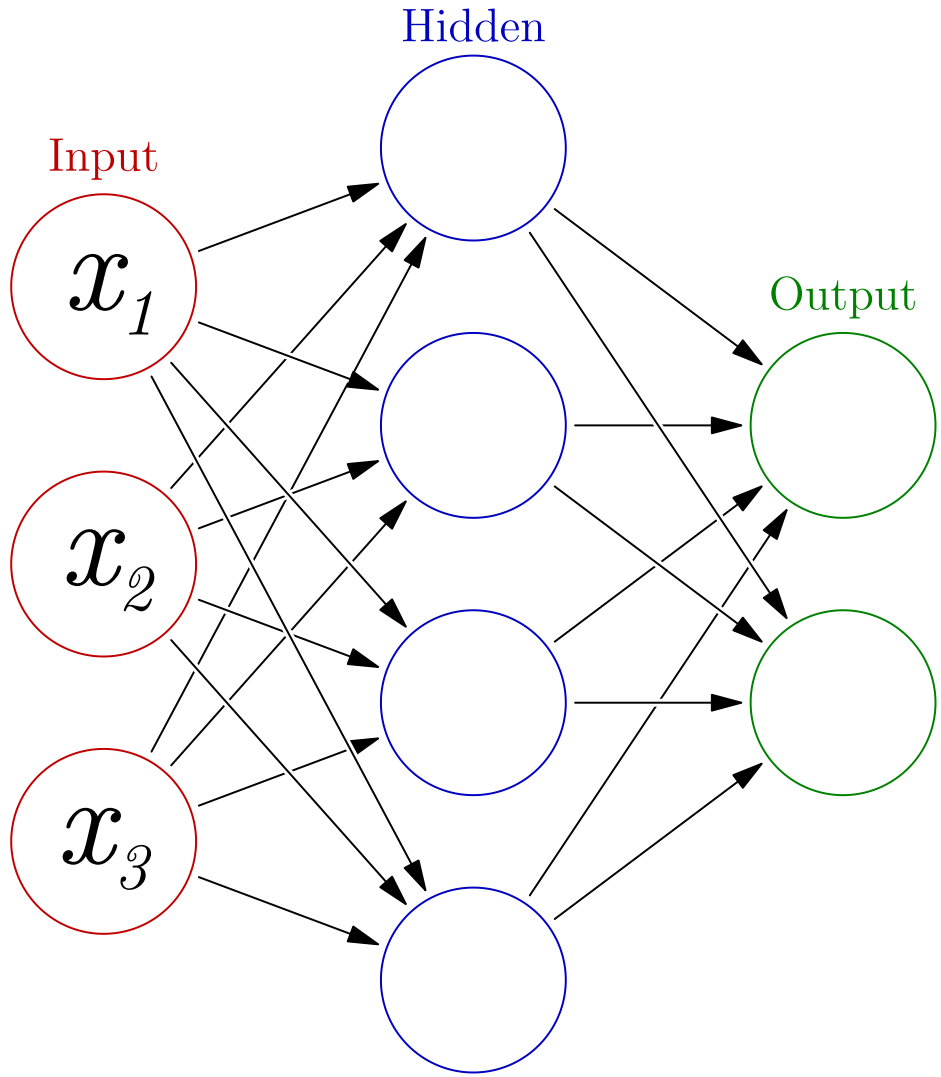
- General names save time



How does an artificial neural network run?

By using maths, predictably!

- General names save time
- input units: x_i



How does an artificial neural network run?

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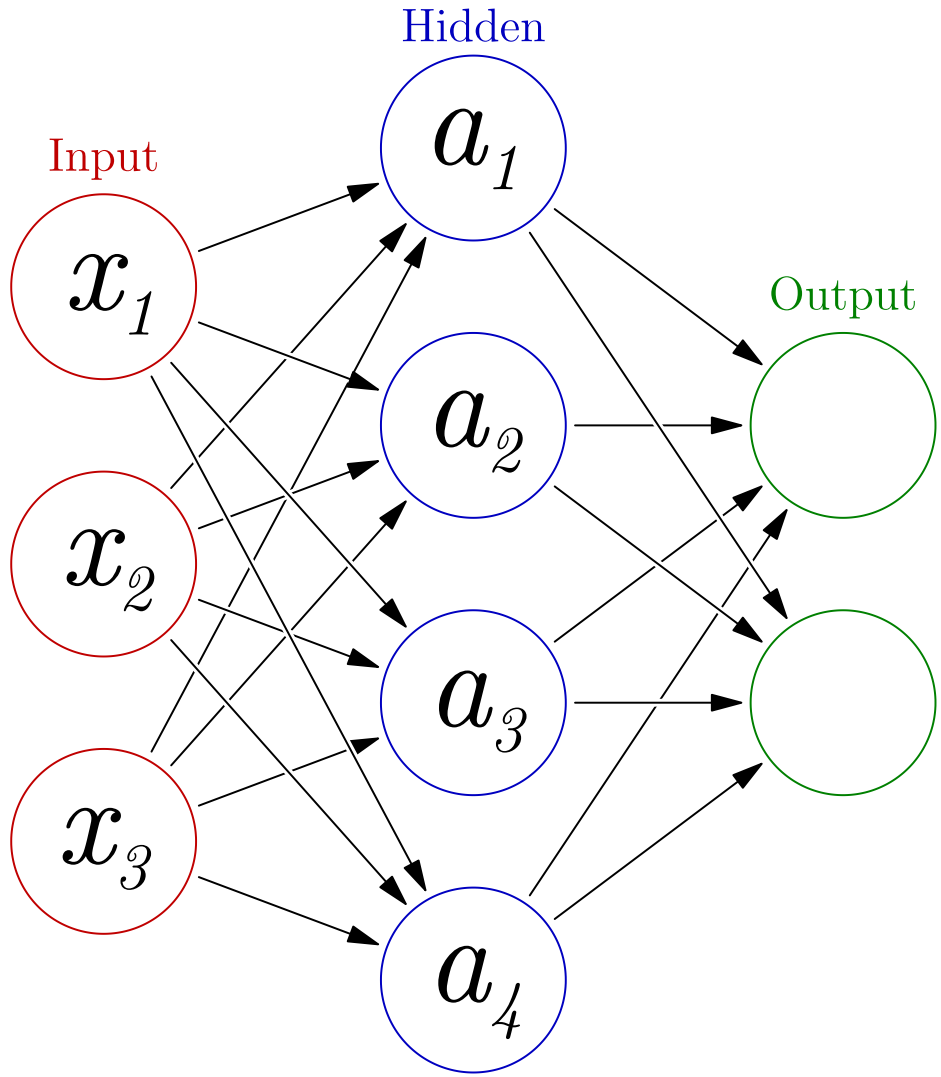
- General names save time
- input units: x_i
- hidden units: a_j



How does an artificial neural network run?

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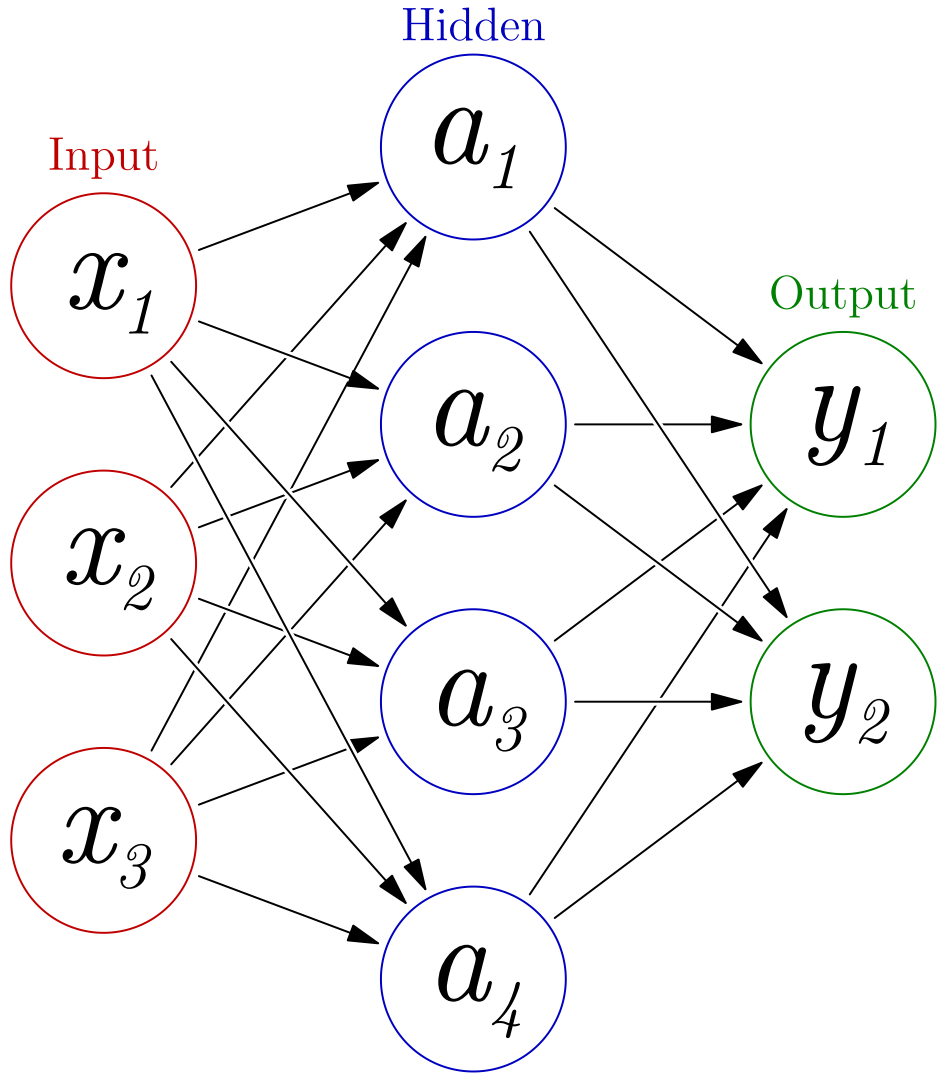
- hidden units: a_j
- output units: y_k



How does an artificial neural network run?

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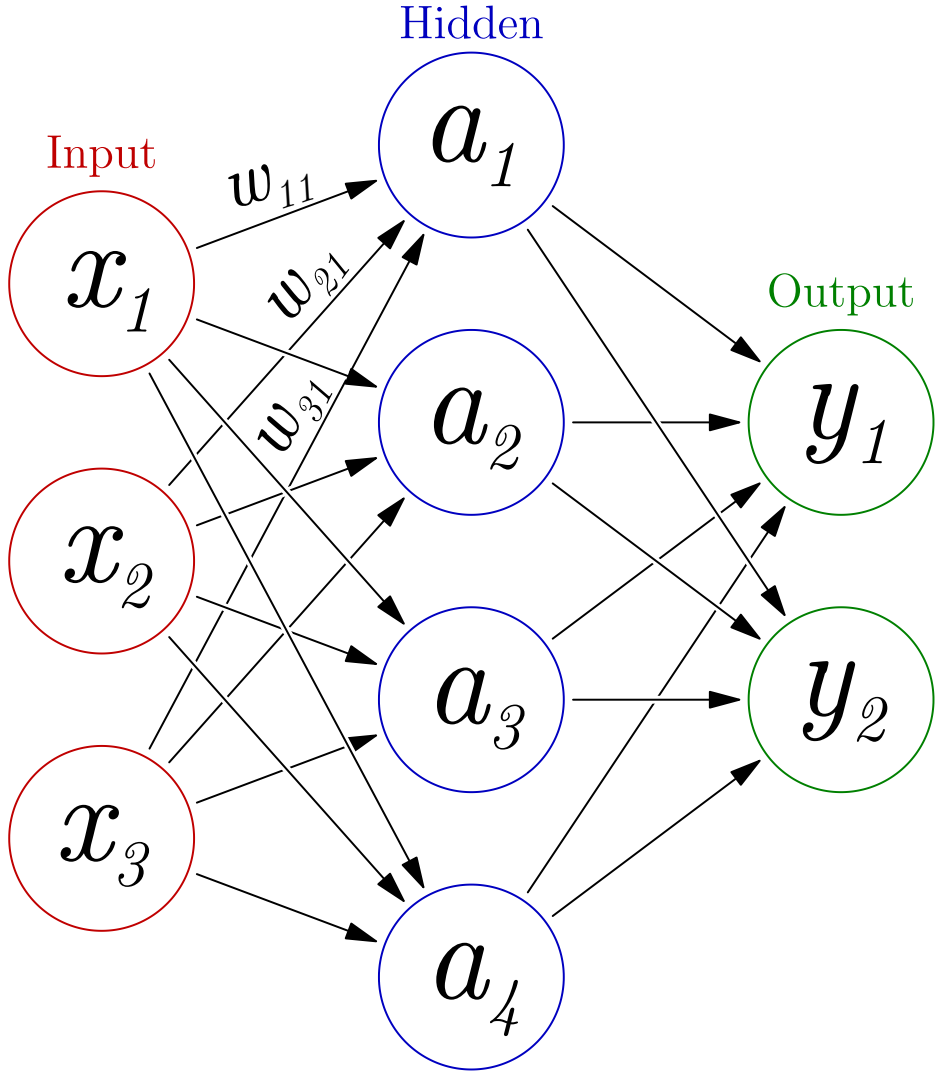
- General names save time
- input units: x_i
- hidden units: a_j
- output units: y_k
- connection weights: w_{ij}



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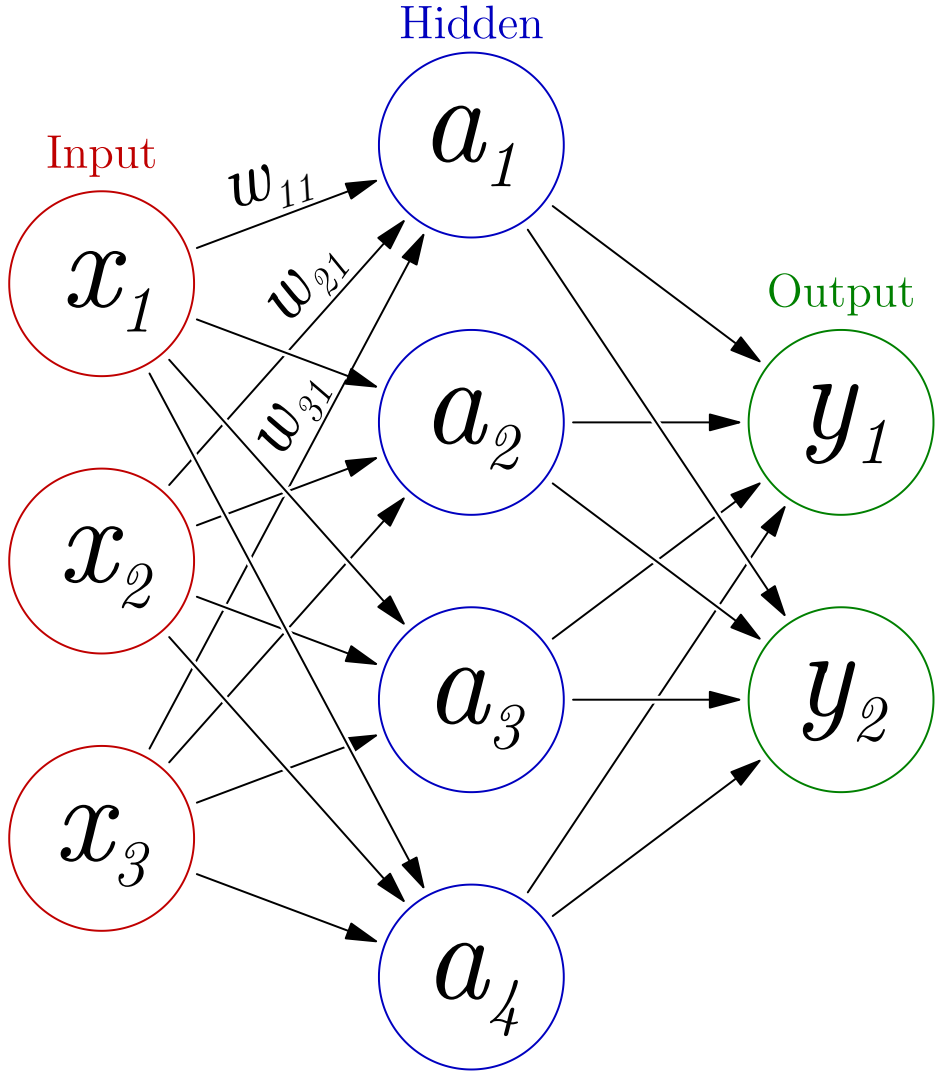
- connection weights: w_{ij}
- subscripts
  general: $ijklm...$
  specific: $12345...$



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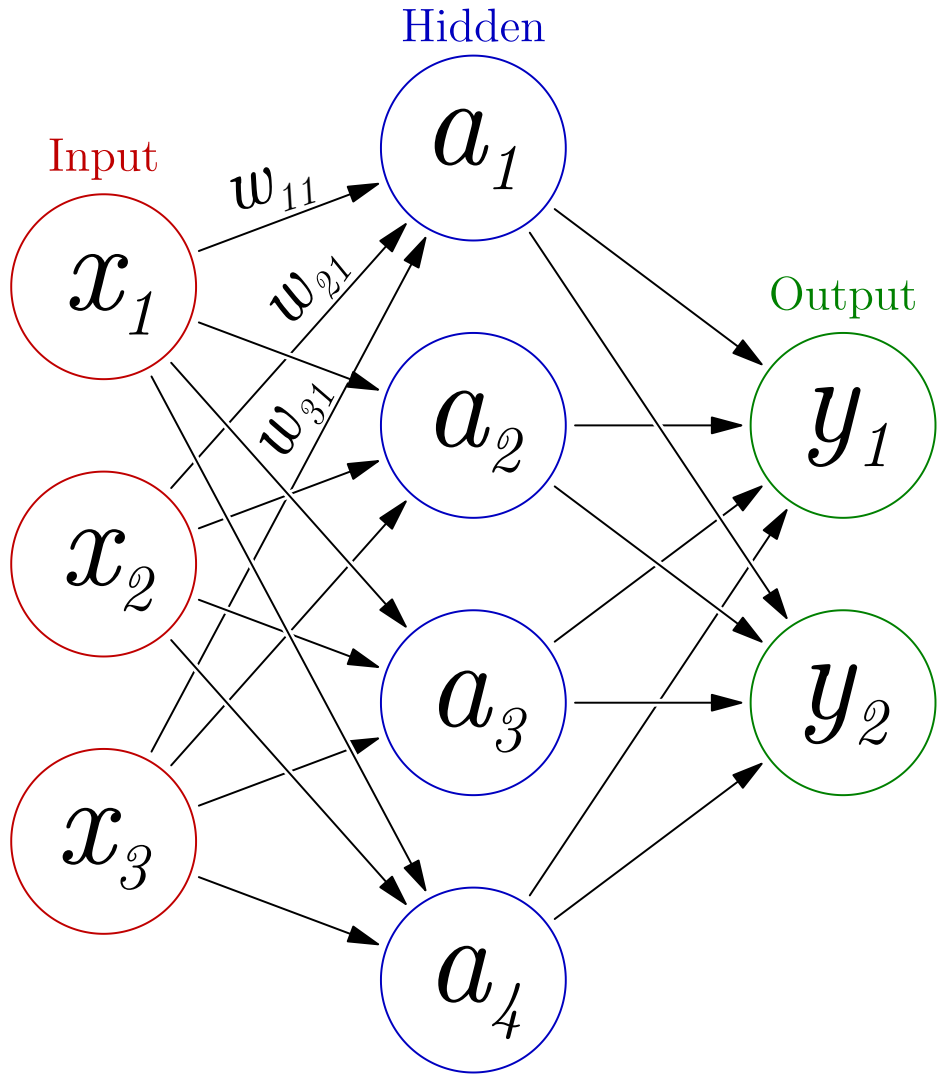
We use general names to write a general equation:



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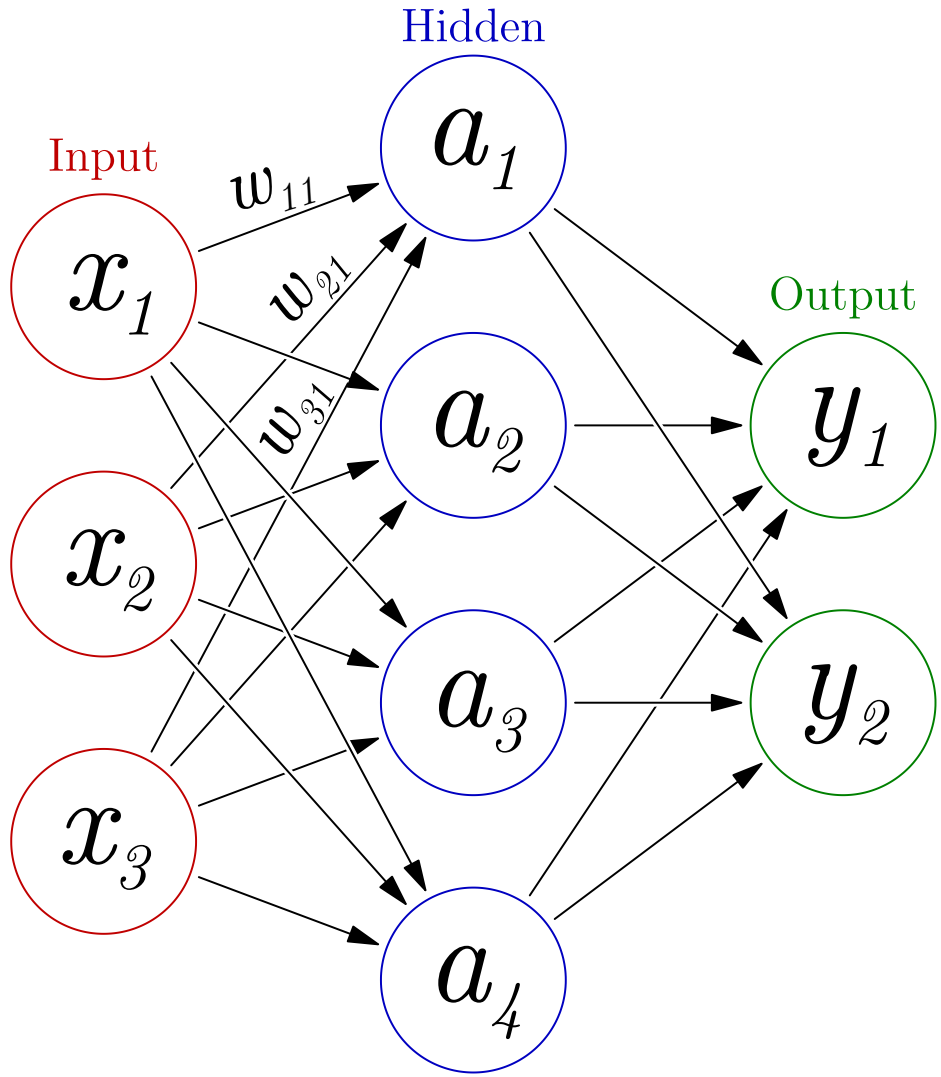
$$a_i =$$



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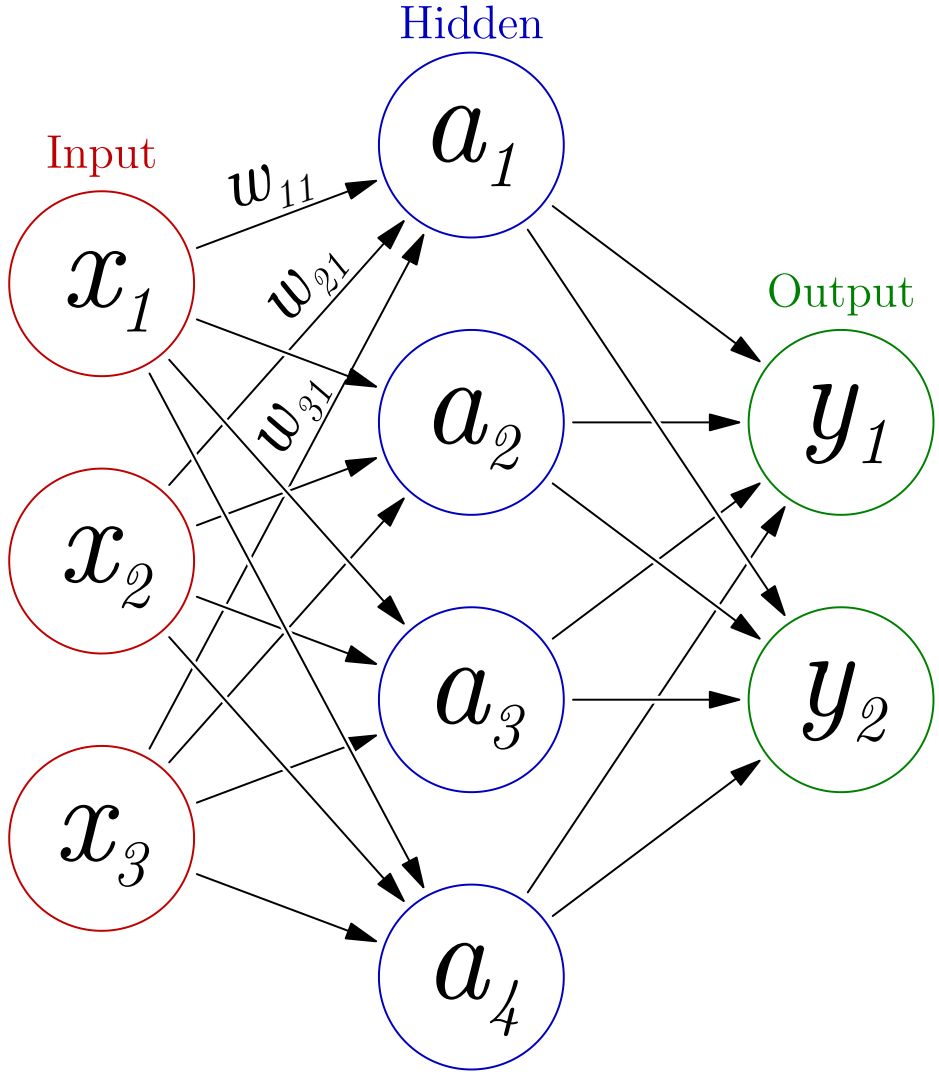
We use general names to write a general equation:

$$a_i = \qquad x_j \times w_{ji}$$



How does an artificial neural network run?

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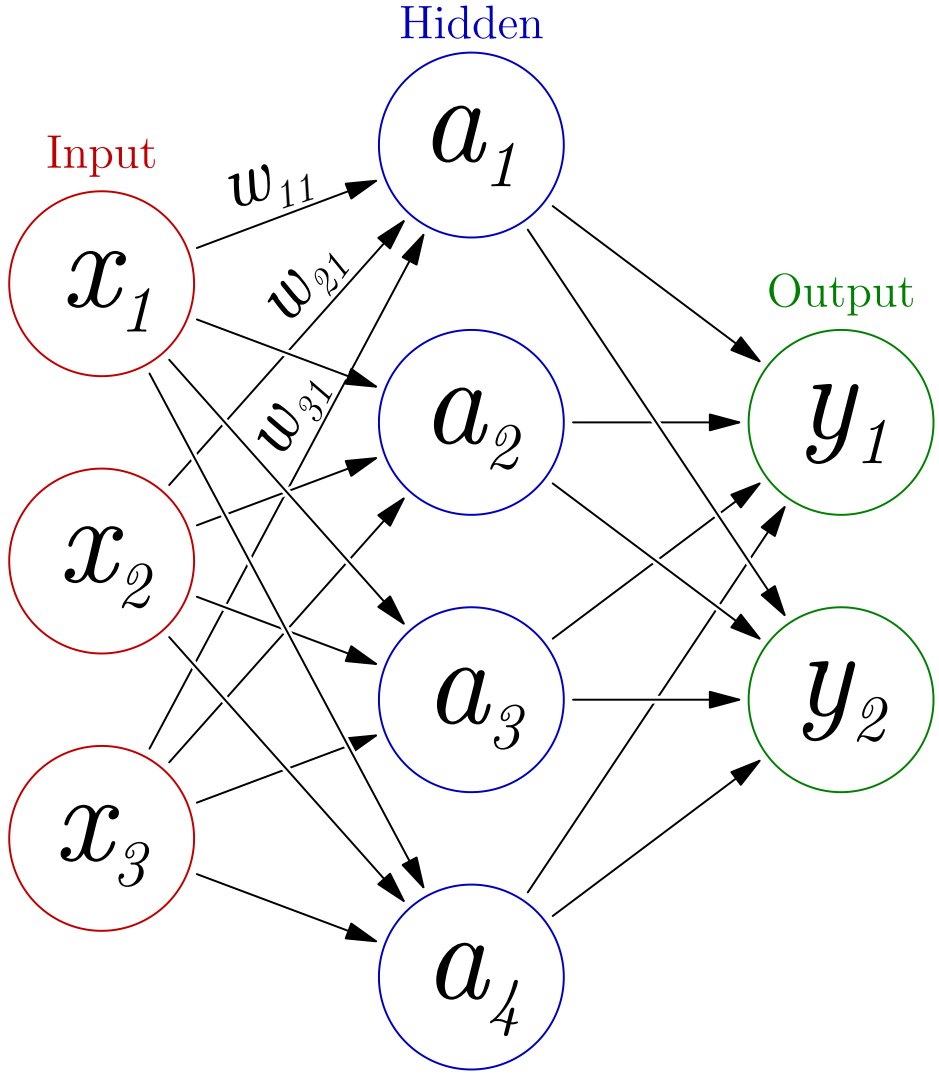
We use general names to write a general equation:

$$a_i = \sum_{j=1}^{N} x_j \times w_{ji}$$



How does an artificial neural network run?

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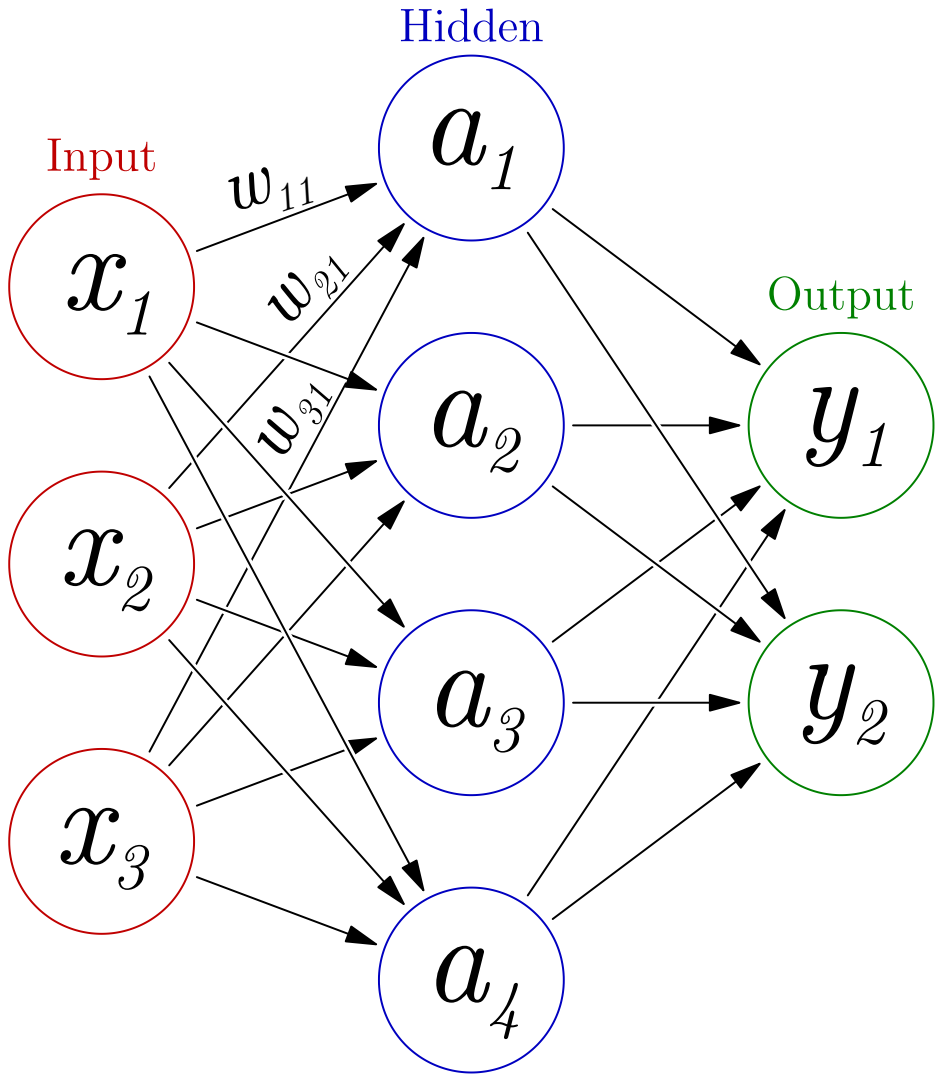
We use general names to write a general equation:

$$a_i = f\left(\sum_{j=1}^{N} x_j \times w_{ji}\right)$$



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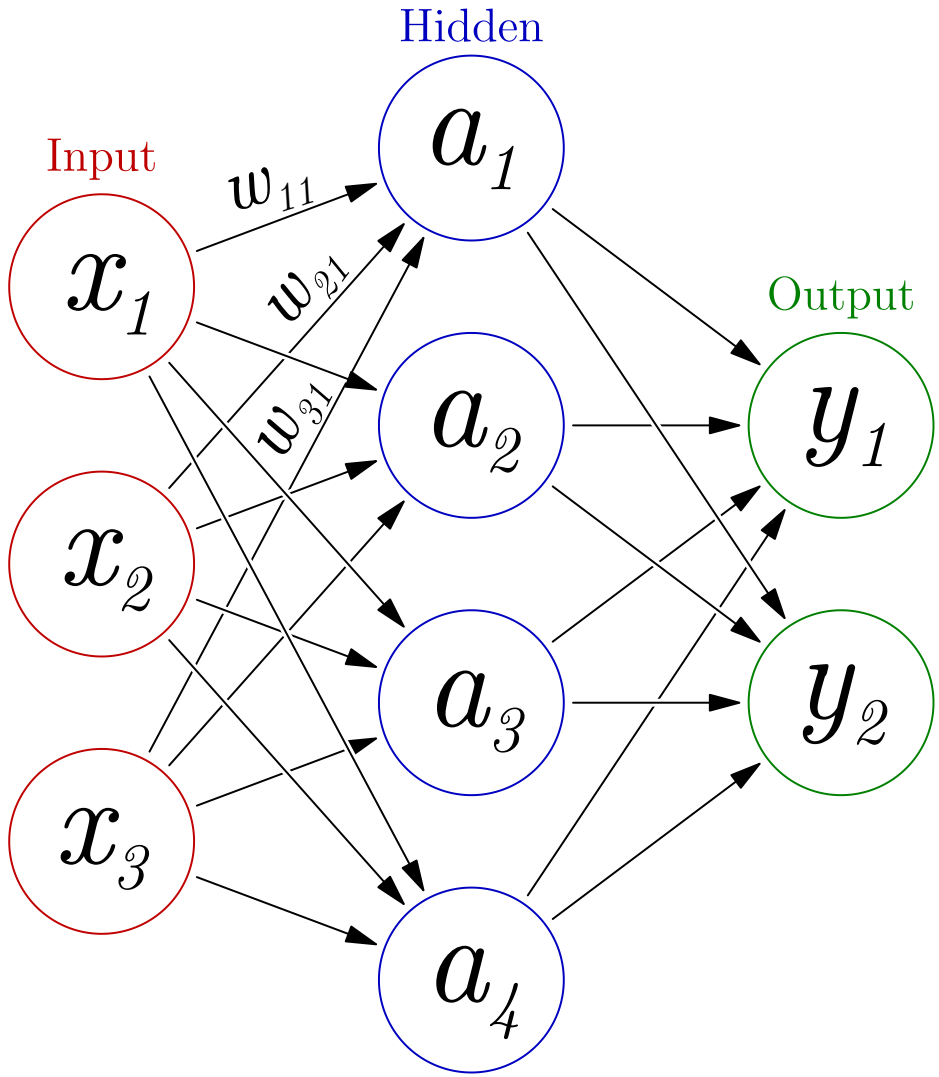

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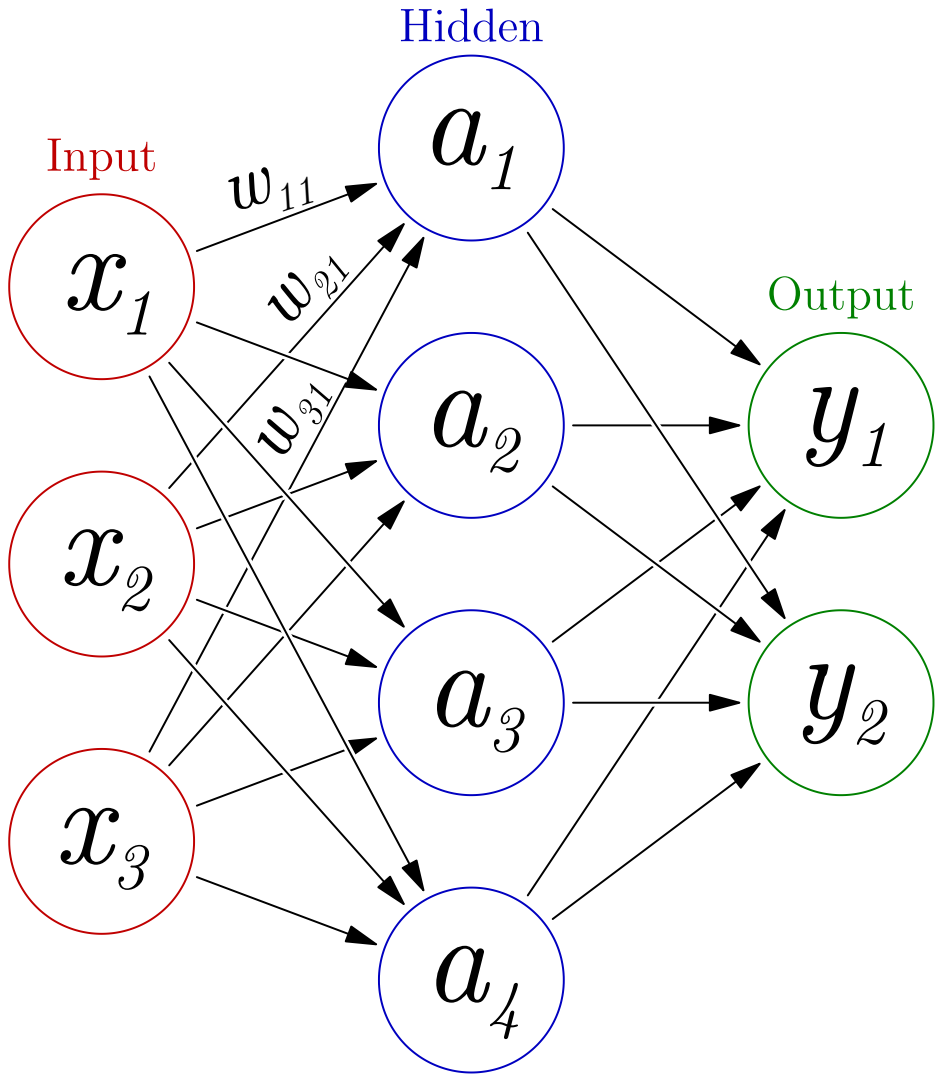
where a_i is the unit whose state we want to calculate, N is the number of units on the previous layer, w_{ji} is the weight on the connection between i and j, and f is a function that the unit applies.

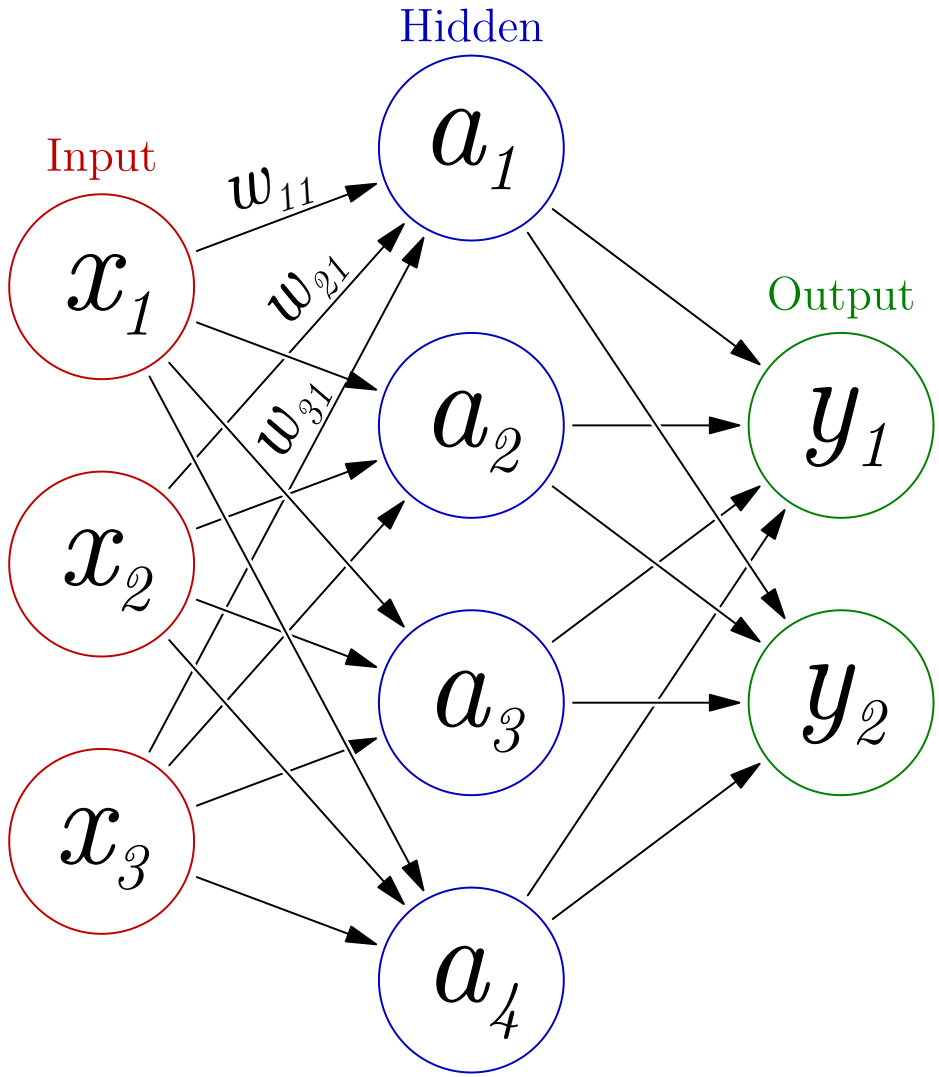


How does an artificial neural network run?

By using maths, predictably!

We use this equation by replacing *iterators* i and j:

$$a_i = f\left(\sum_{j=1}^{N} x_j \times w_{ji}\right)$$



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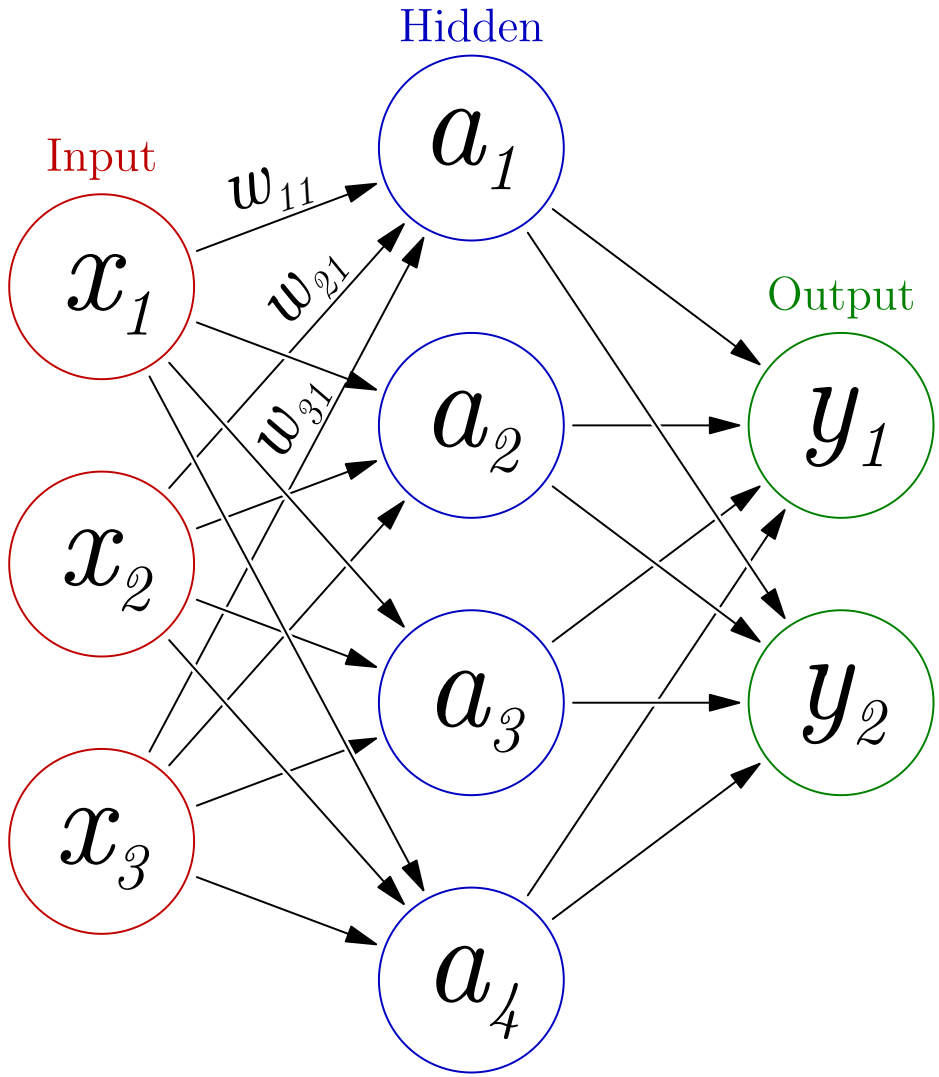
We use this equation by replacing *iterators* i and j:

$$a_i = f\left(\sum_{j=1}^{N} x_j \times w_{ji}\right)$$

$$a_1 = f\left(\sum_{j=1}^{N} x_j \times w_{j1}\right)$$



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We use this equation by replacing *iterators* i and j:

$$a_i = f\left(\sum_{j=1}^{N} x_j \times w_{ji}\right)$$

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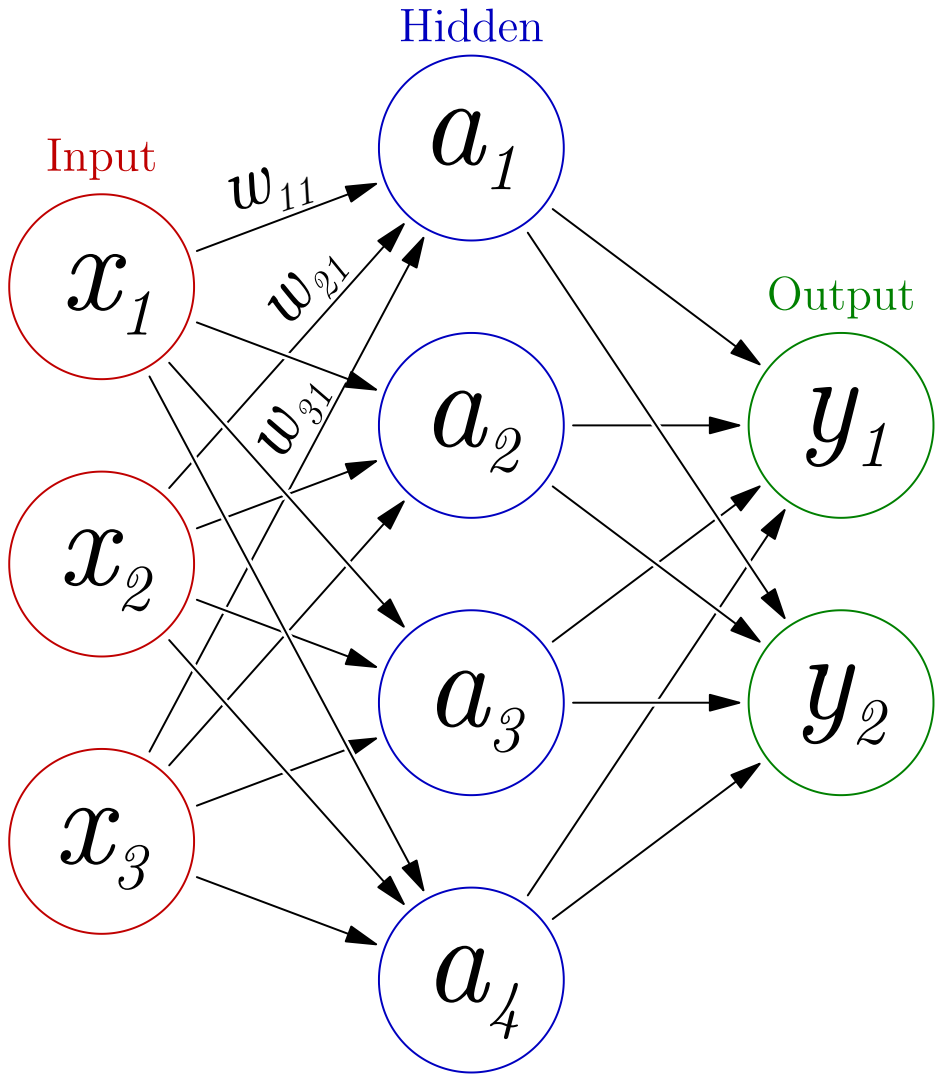

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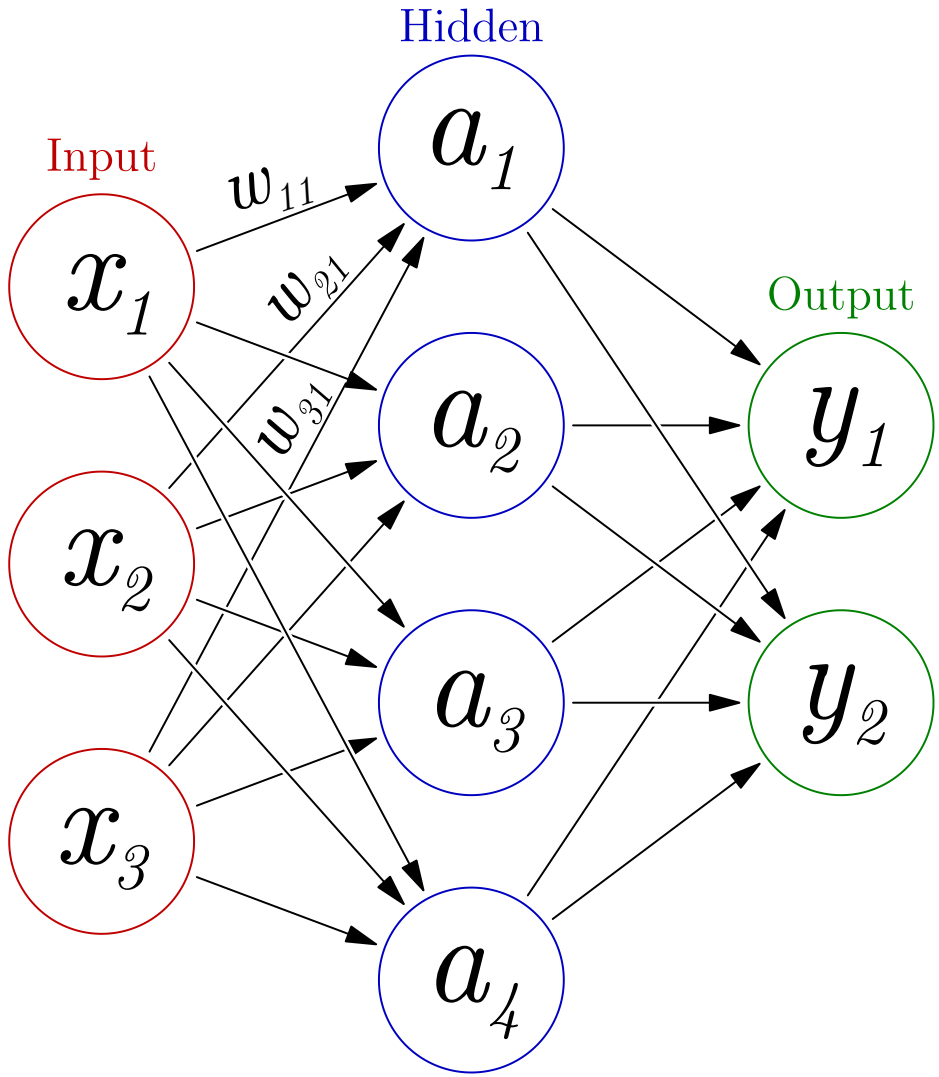
$x_1 \times w_{11}$



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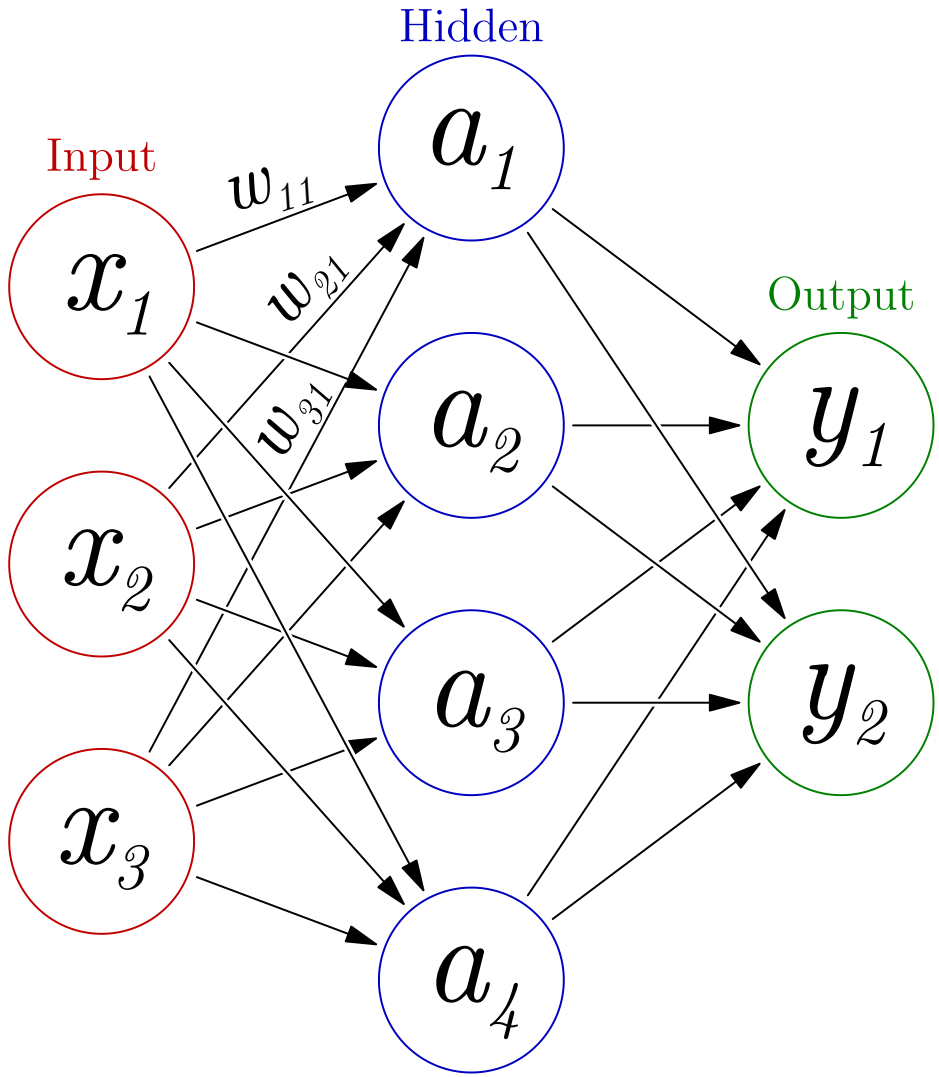
$$x_1 \times w_{11} + x_2 \times w_{21}$$



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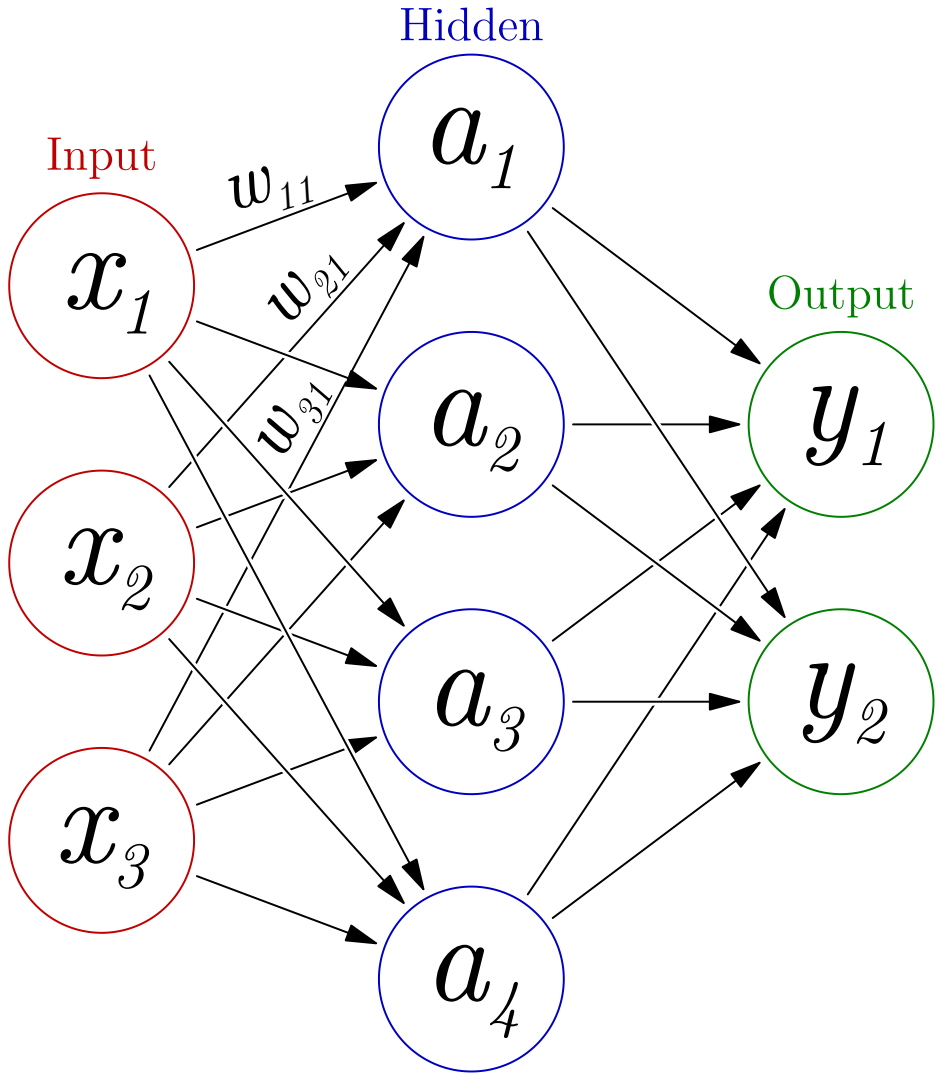
$$x_1 \times w_{11} + x_2 \times w_{21} + x_3 \times w_{31}$$



How does an artificial neural network run?

By using maths, predictably!

We use this equation by replacing *iterators* i and j:

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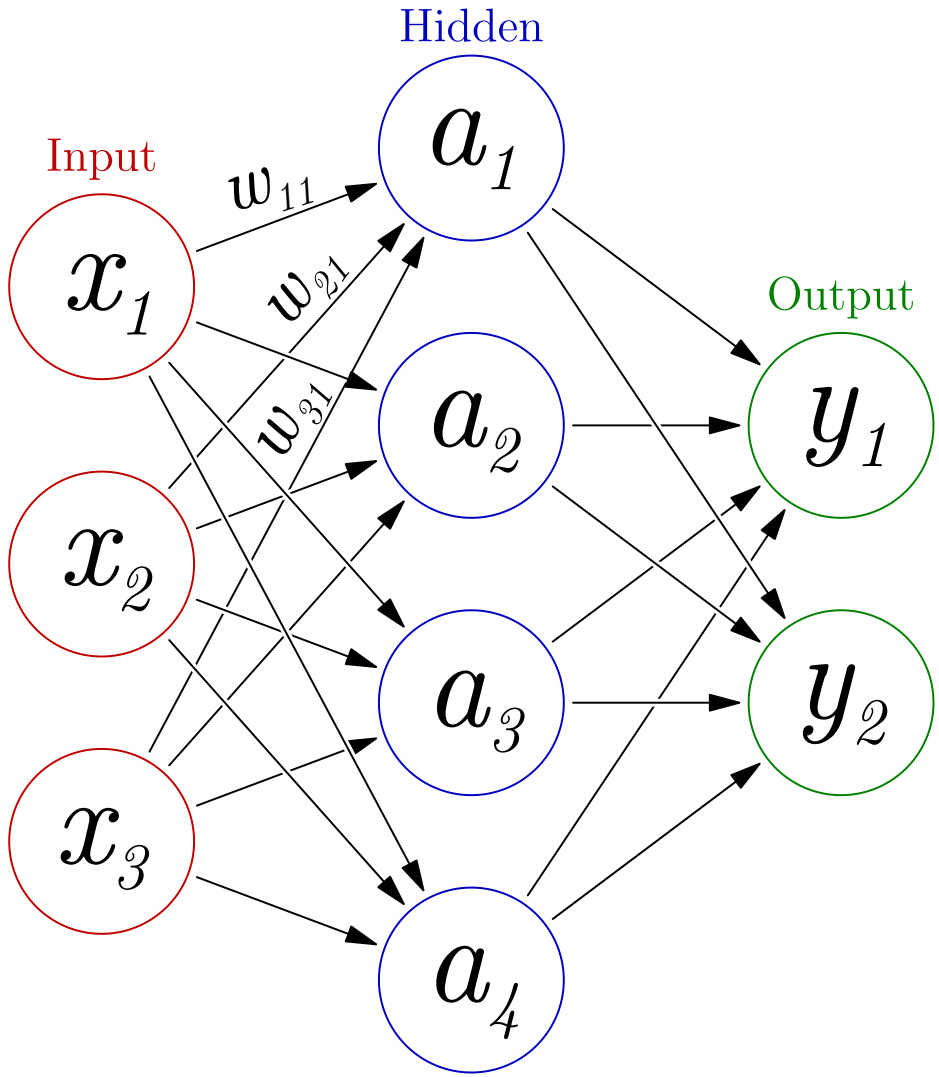

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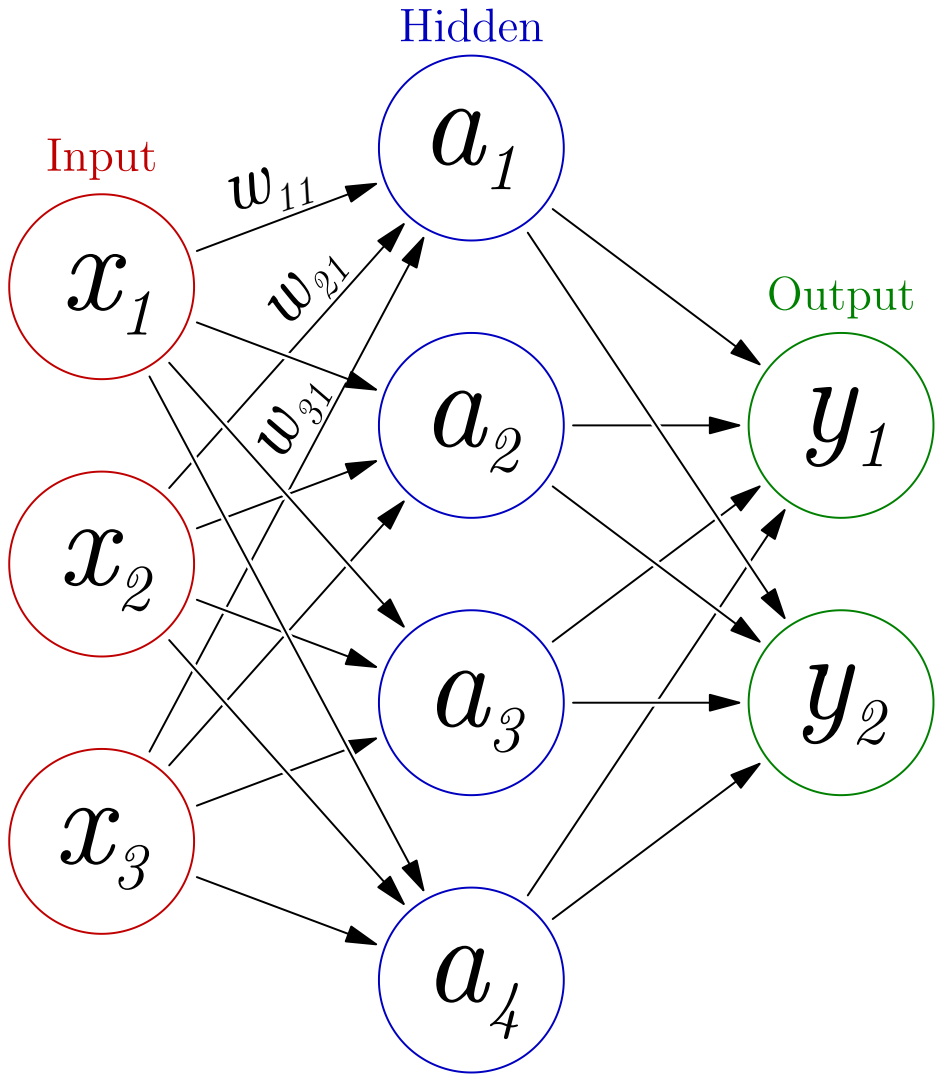
$x_1 \times w_{12}$



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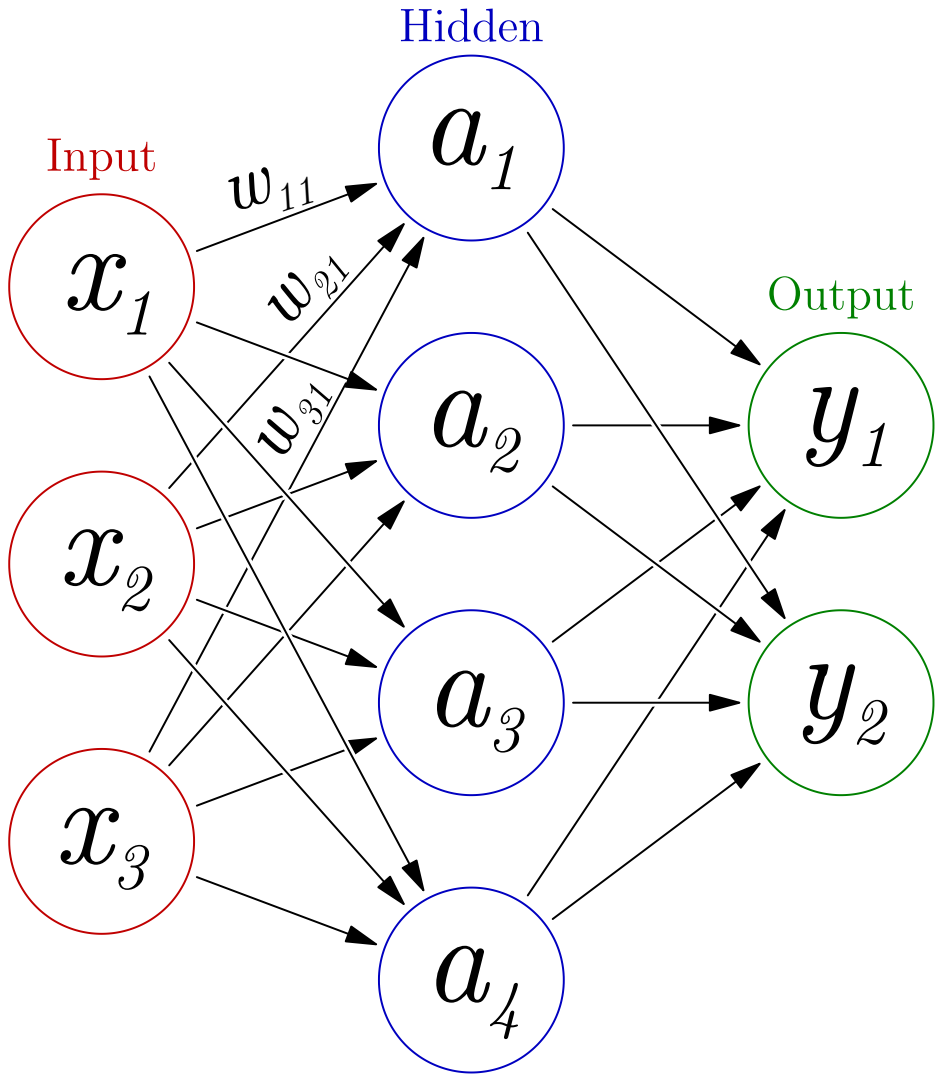

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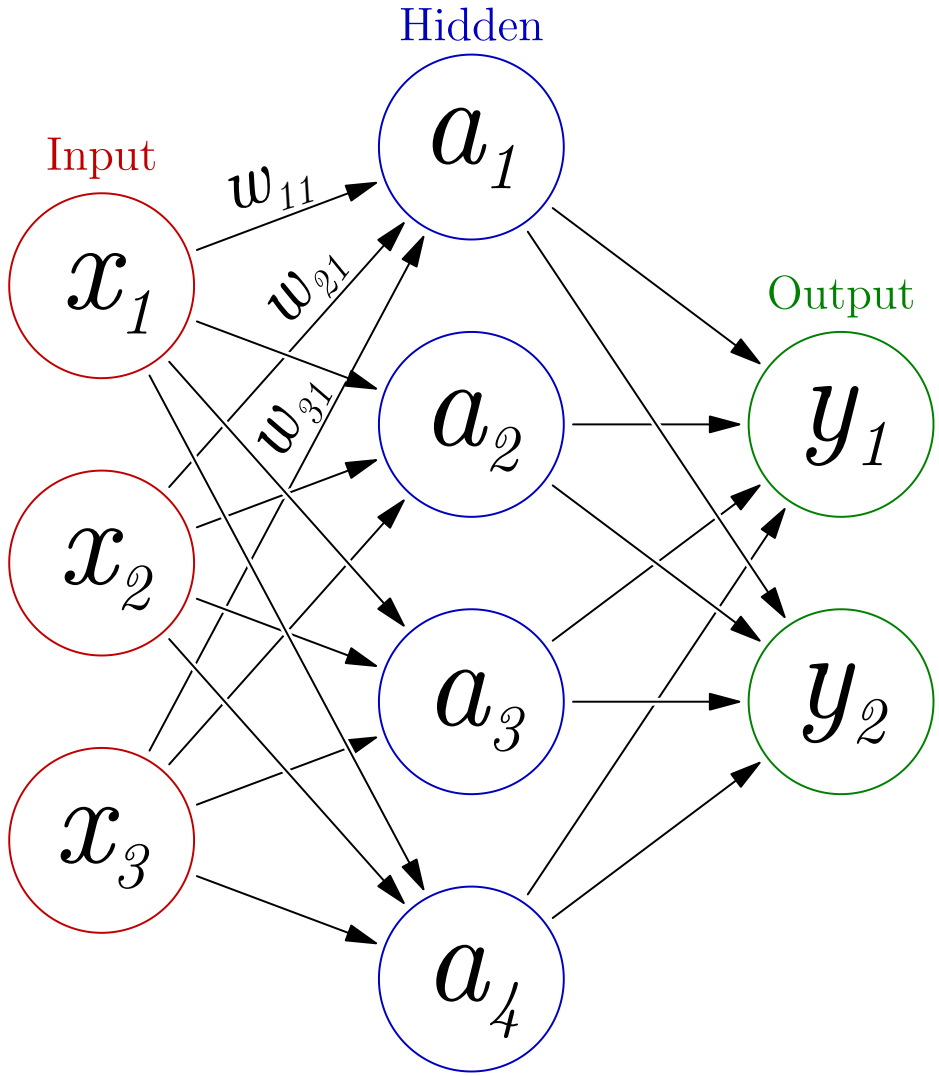
$$x_1 \times w_{12} + x_2 \times w_{22} + x_3 \times w_{32}$$



How do networks learn?

Cunning!

- Many options: Hebbian learning, back-propagation of error, Boltzmann machine learning, self-organising map algorithm, etc.

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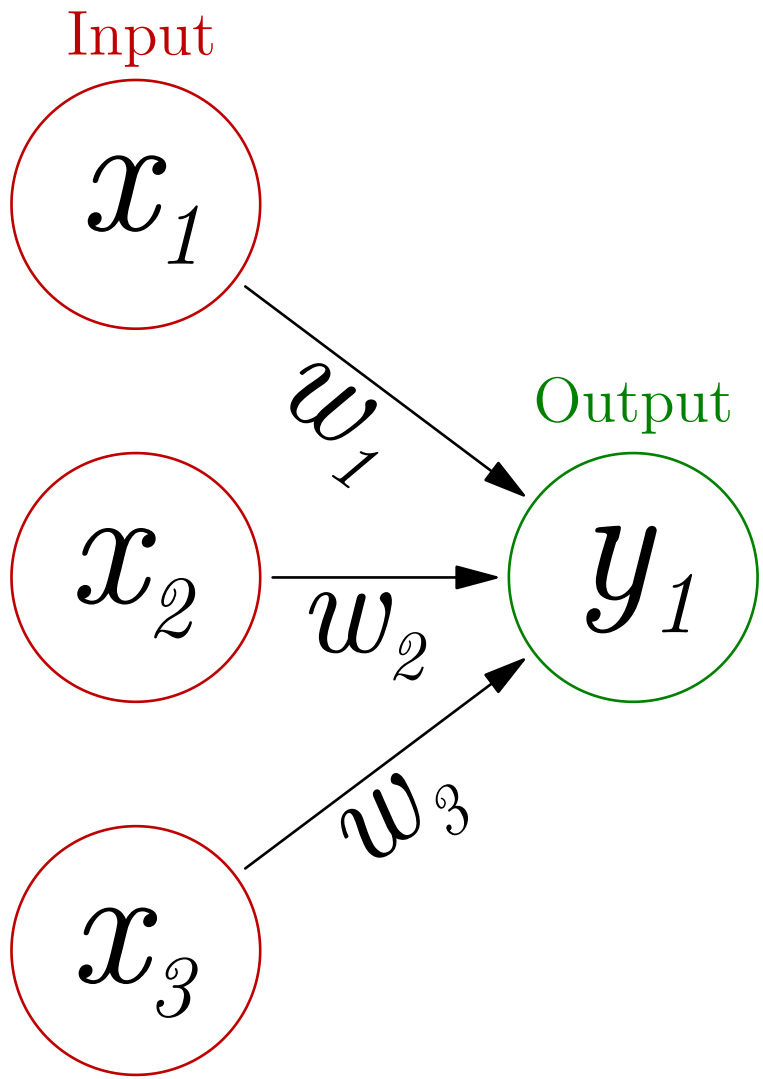
- Many options: Hebbian learning, back-propagation of error, Boltzmann machine learning, self-organising map algorithm, etc.
- All learning algorithms work by changing the connection weights
- Learning can be divided into *supervised*, *unsupervised*, and *reinforcement*

Hebbian learning

A very simple learning rule

"Cells that fire together, wire together"
— Carla Shatz



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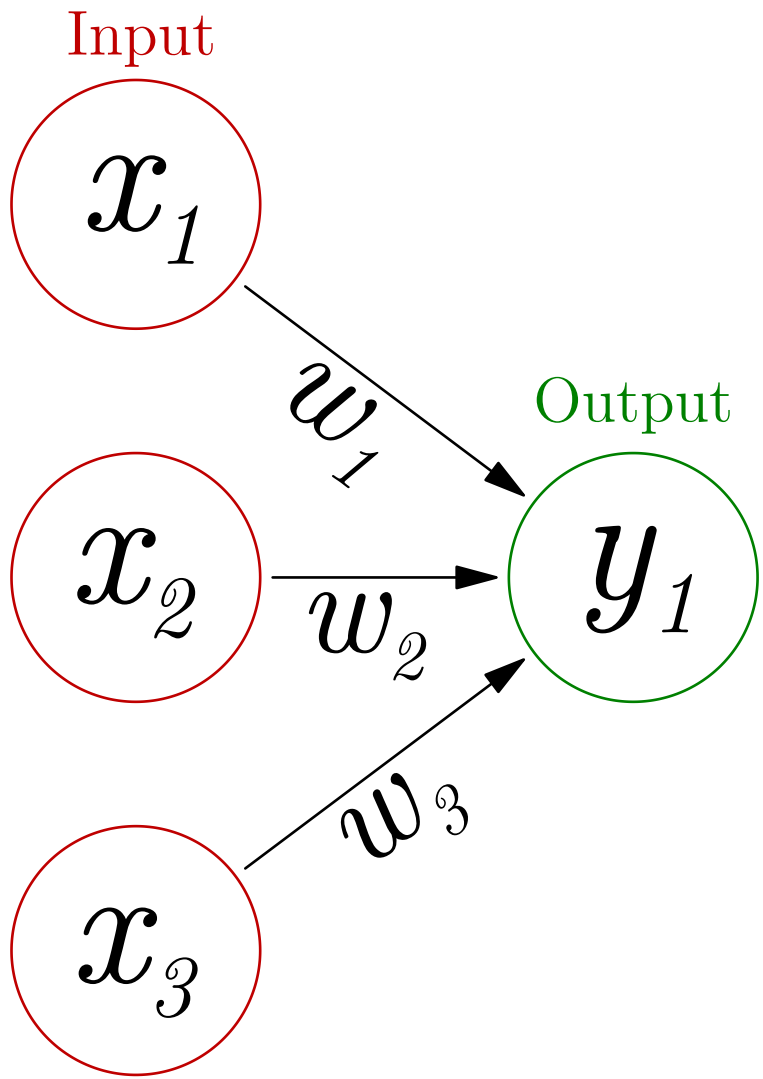
$$w_i =$$



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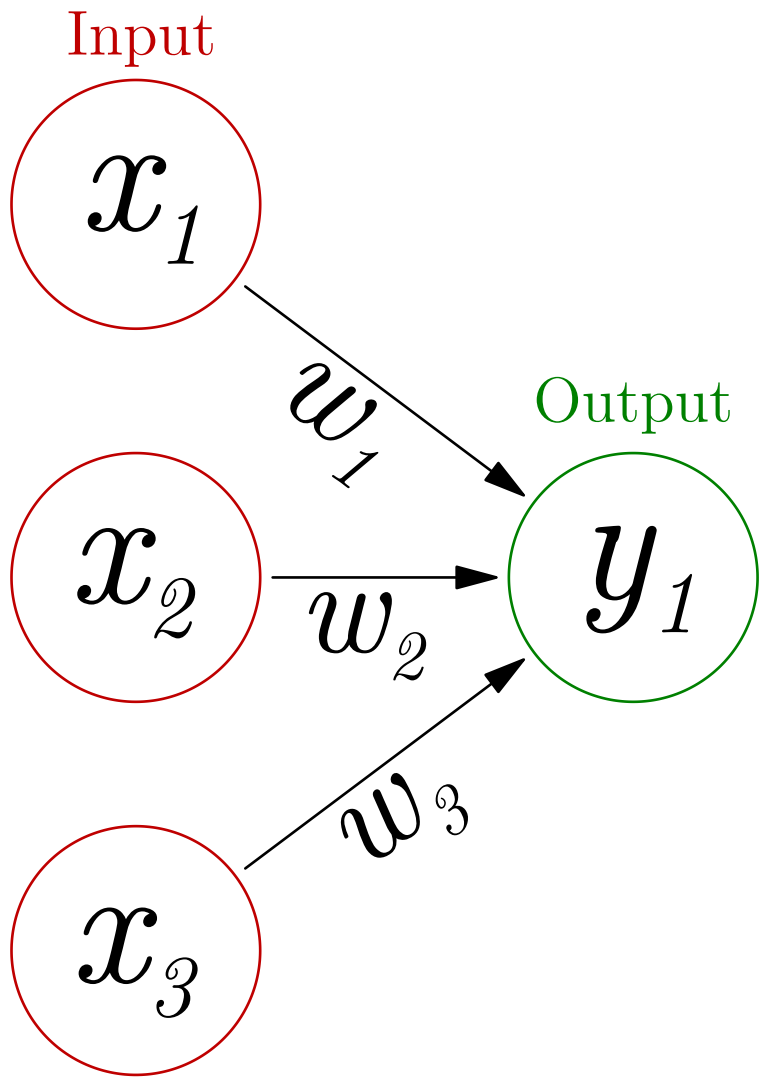

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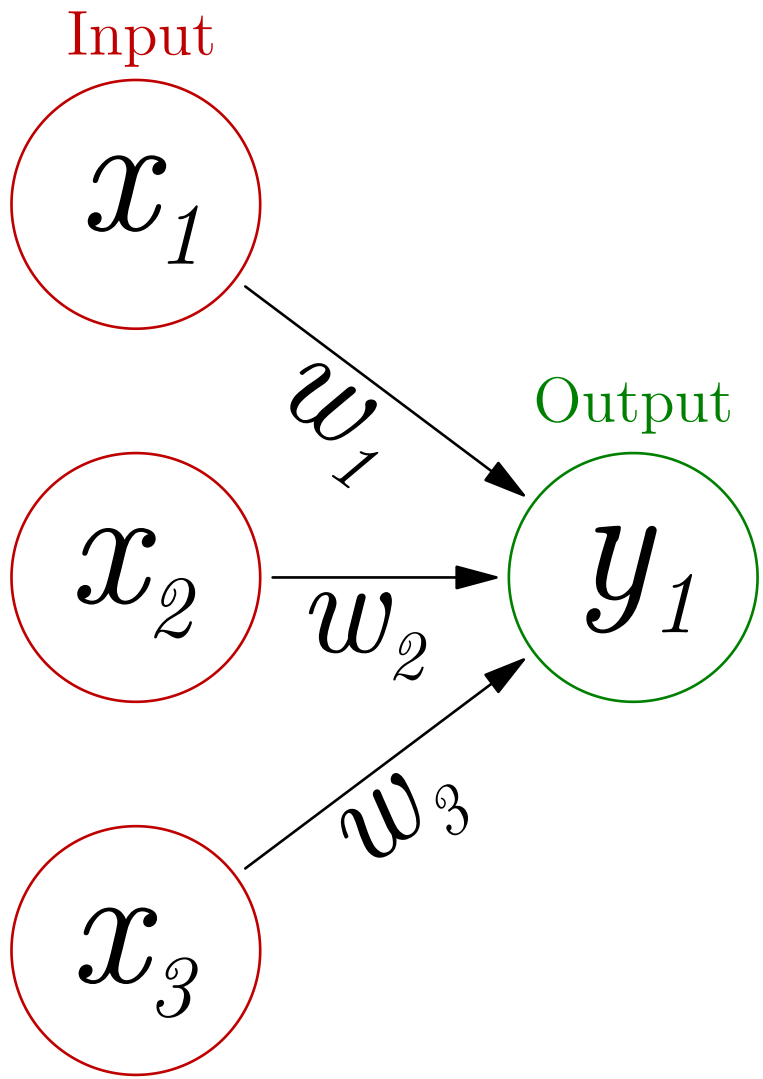

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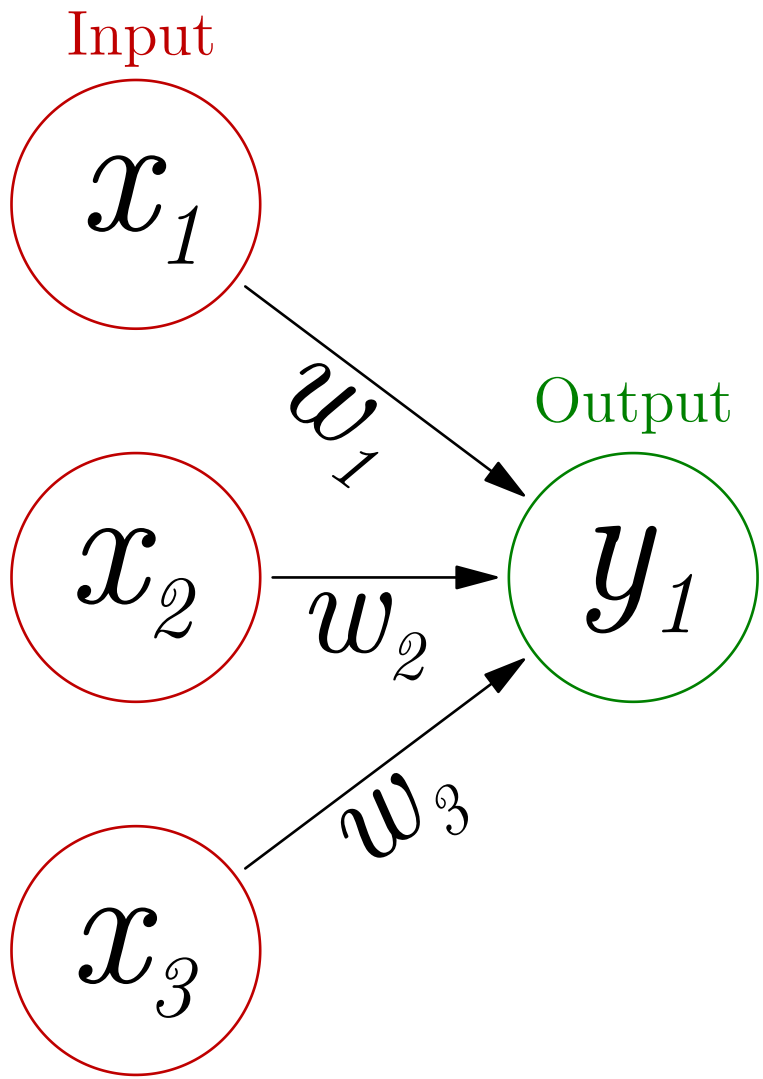

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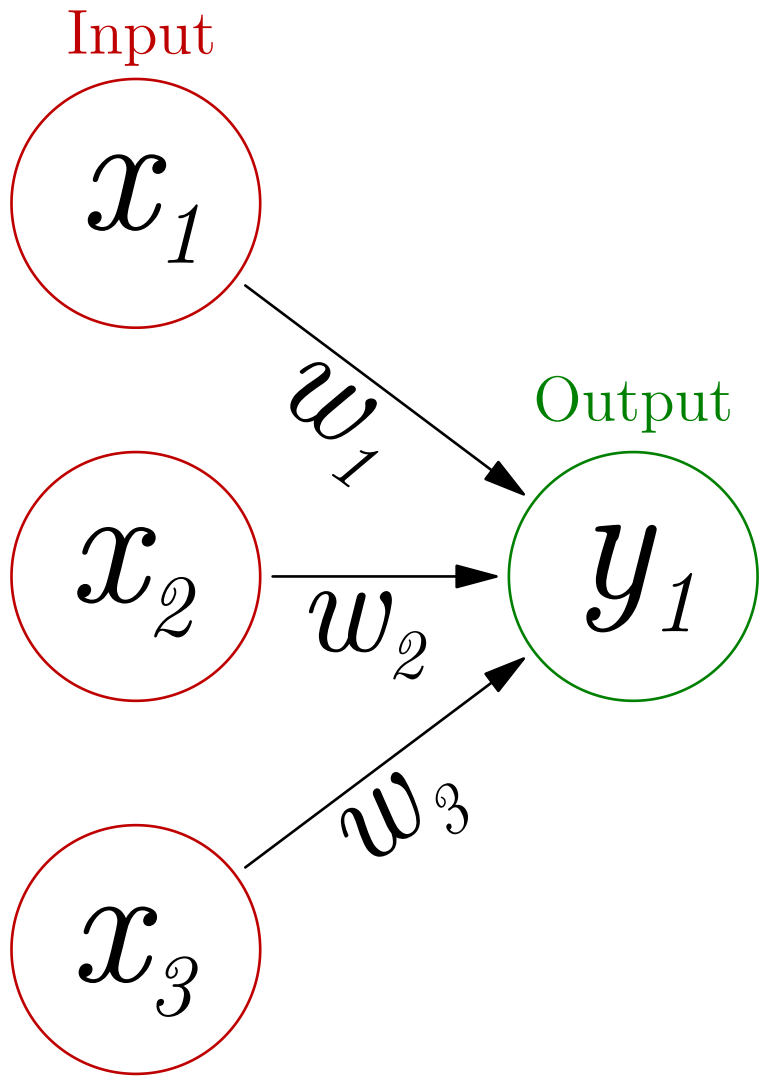
which means each weight is changed by a small in/decrement for every pattern



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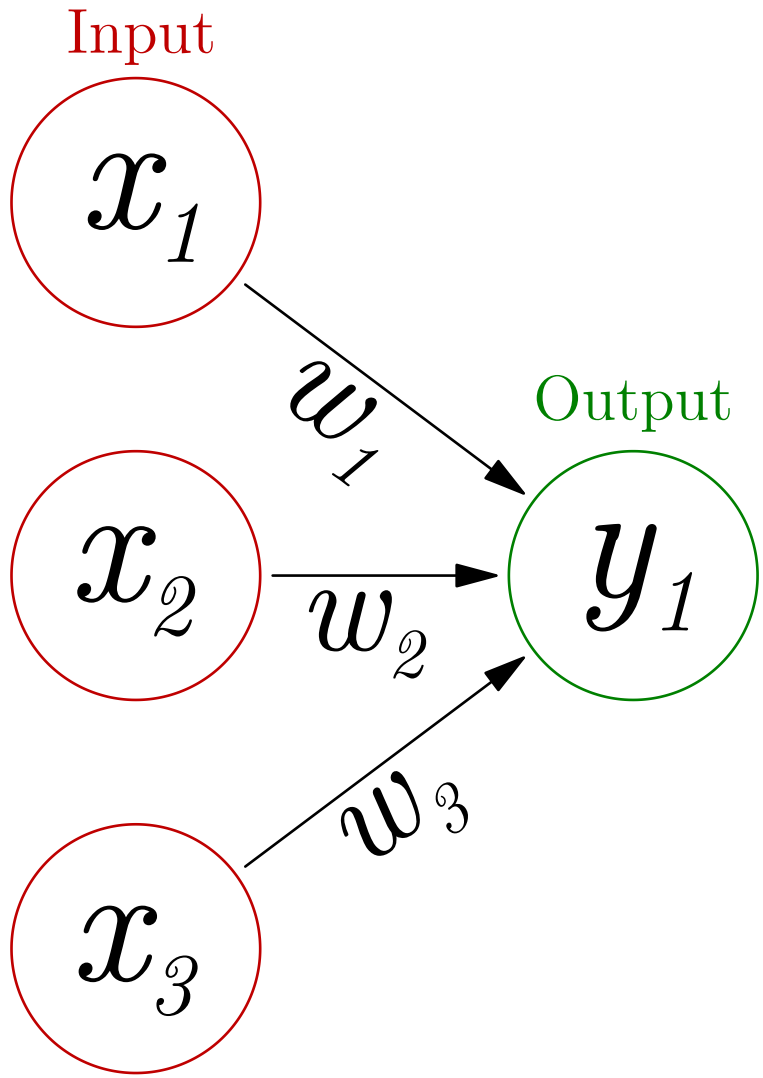
Hebb's rule is simple, but *very unstable*!



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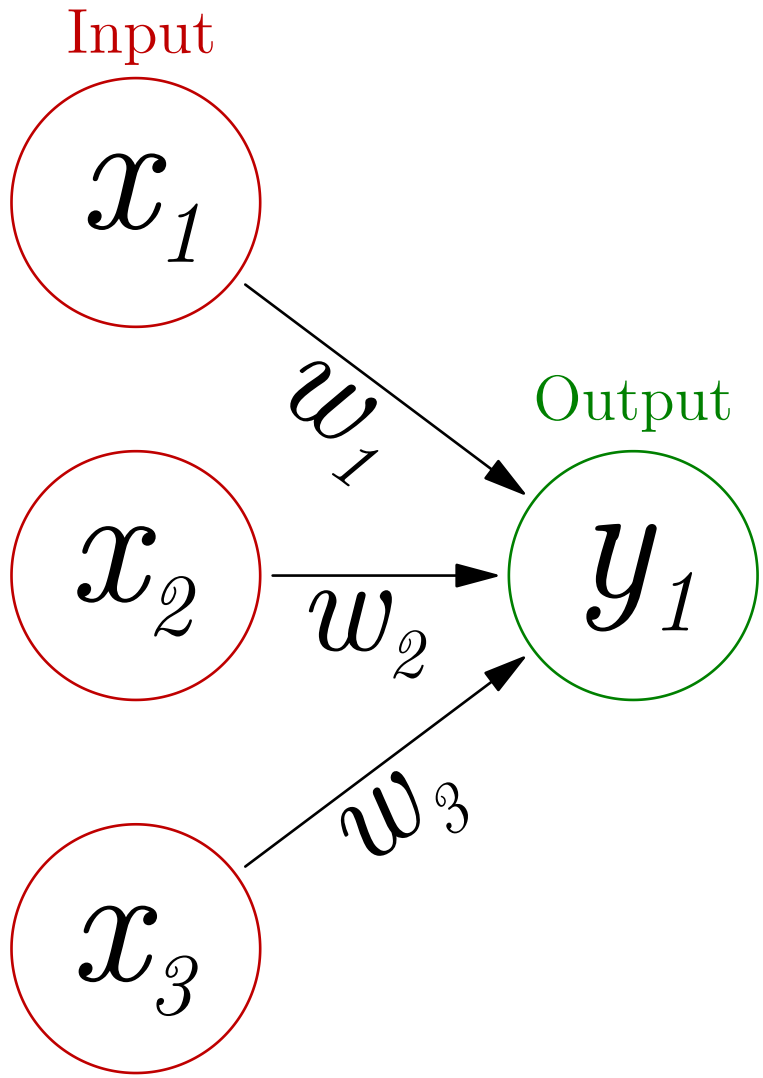

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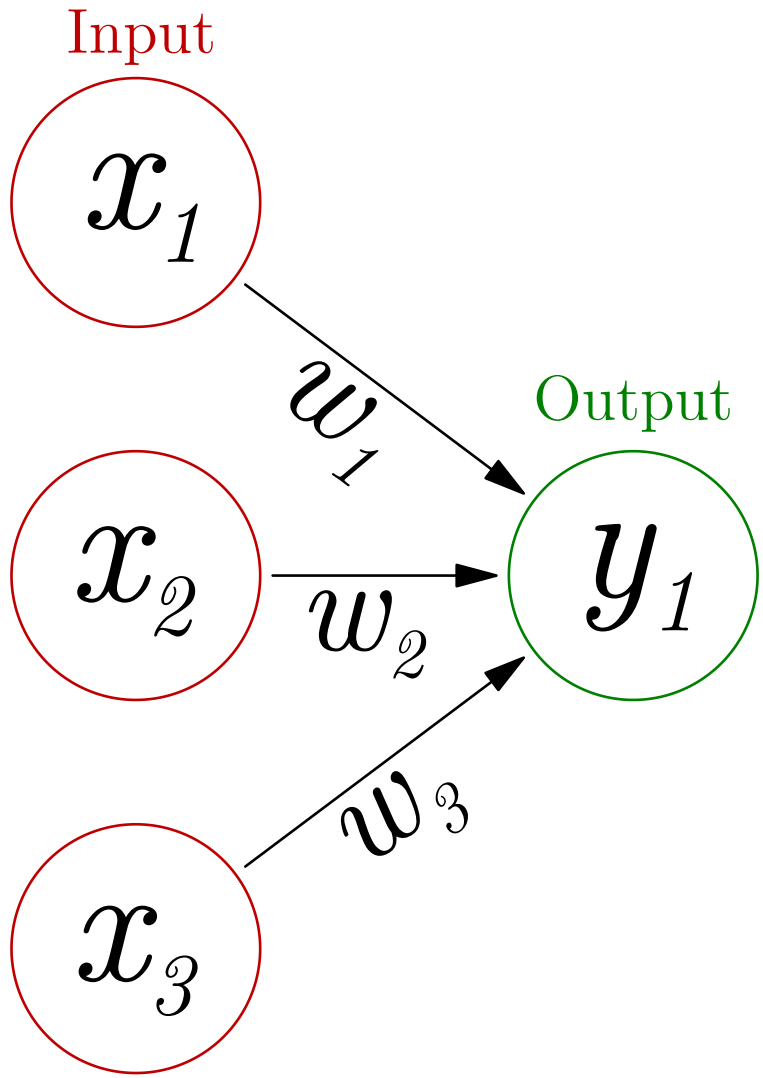

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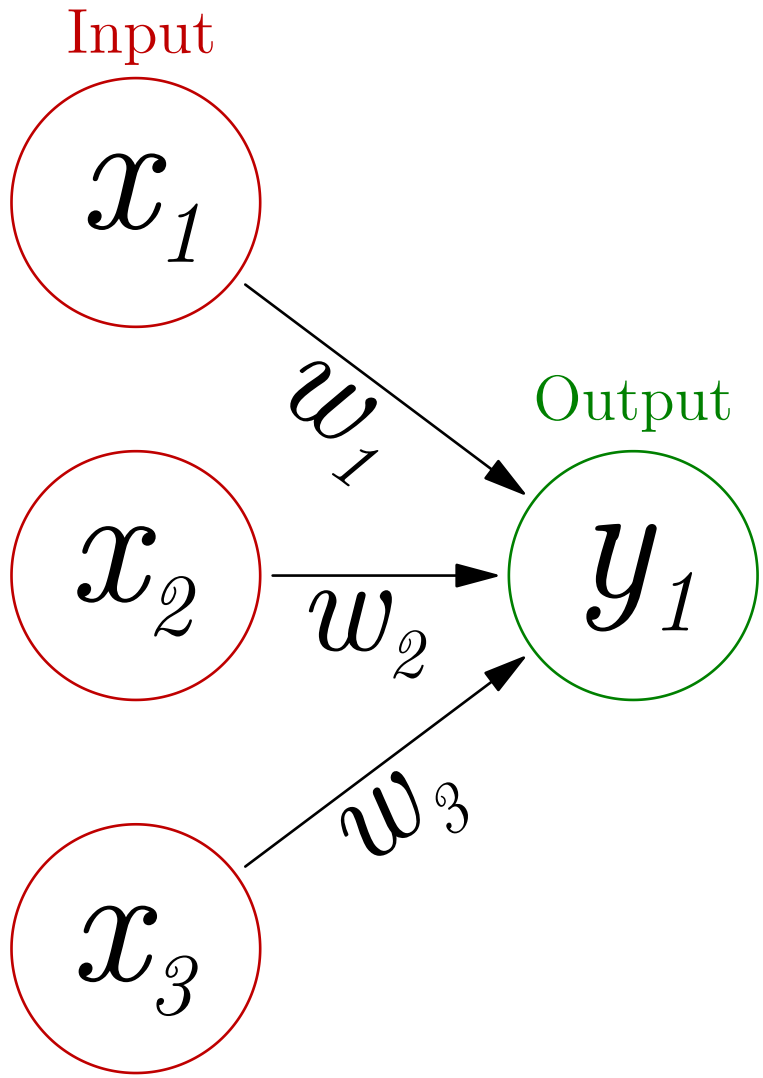

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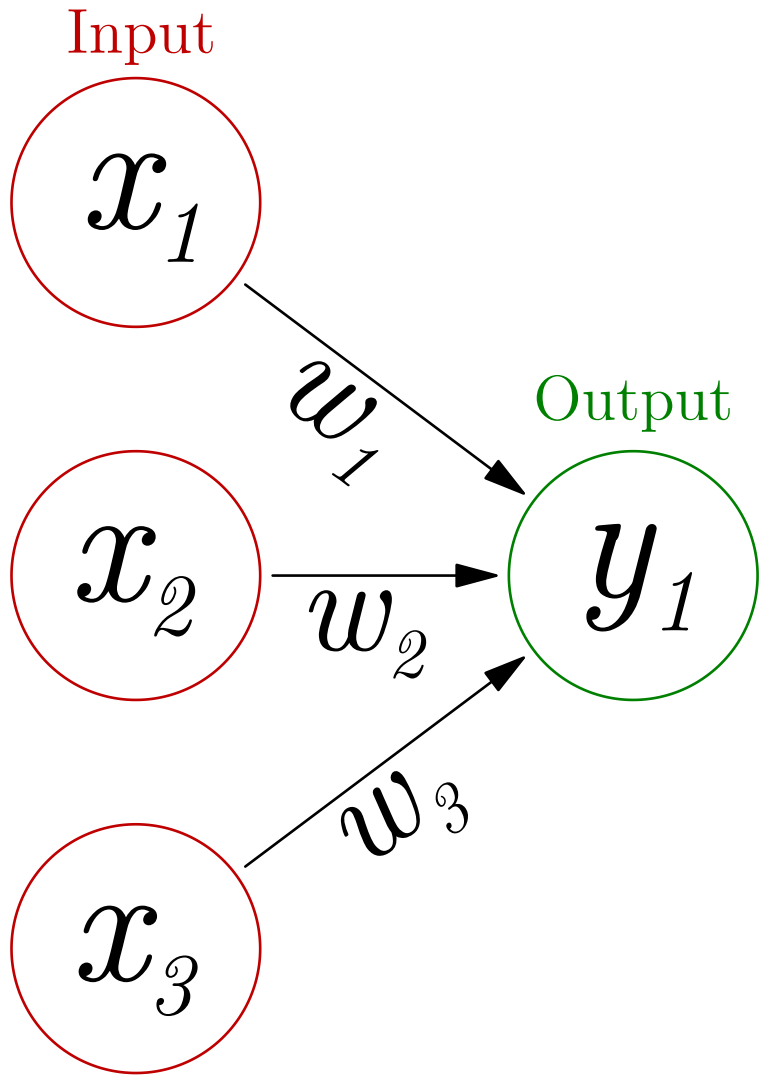

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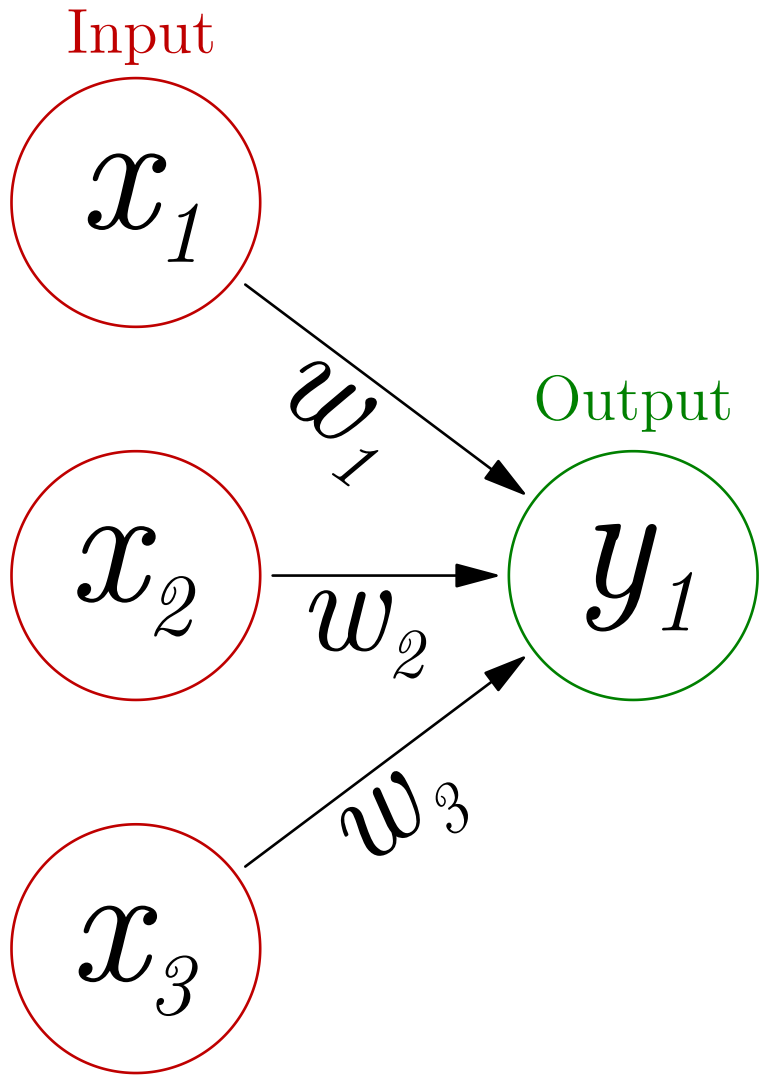

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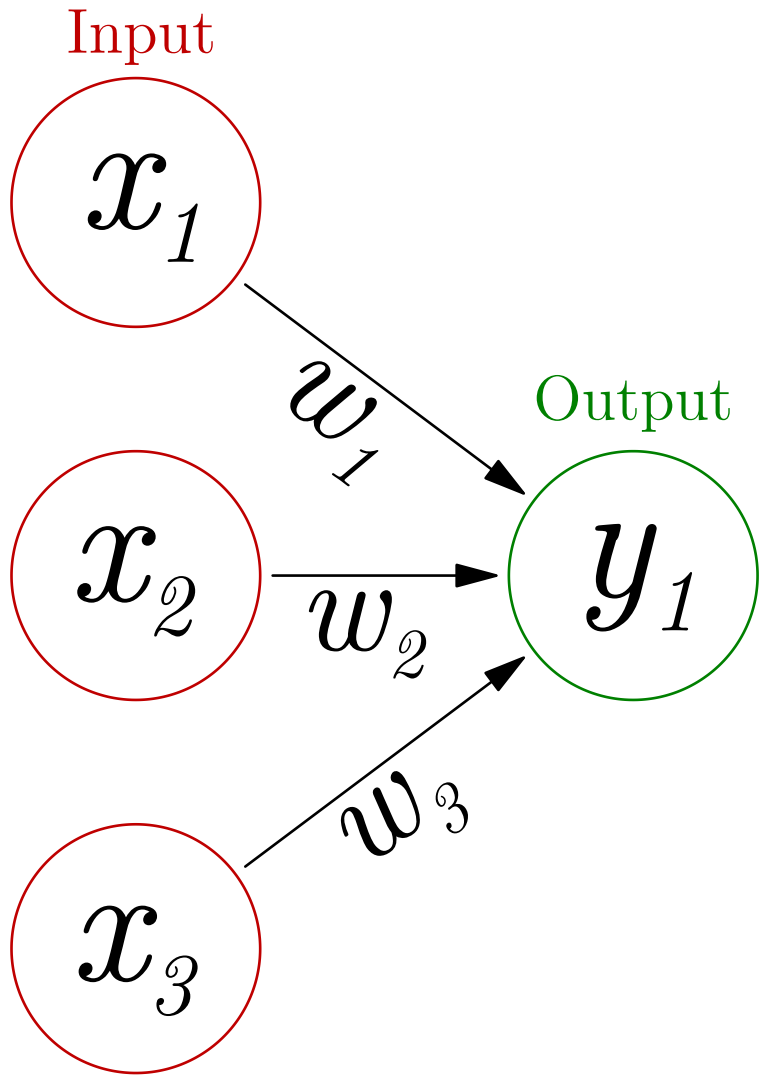

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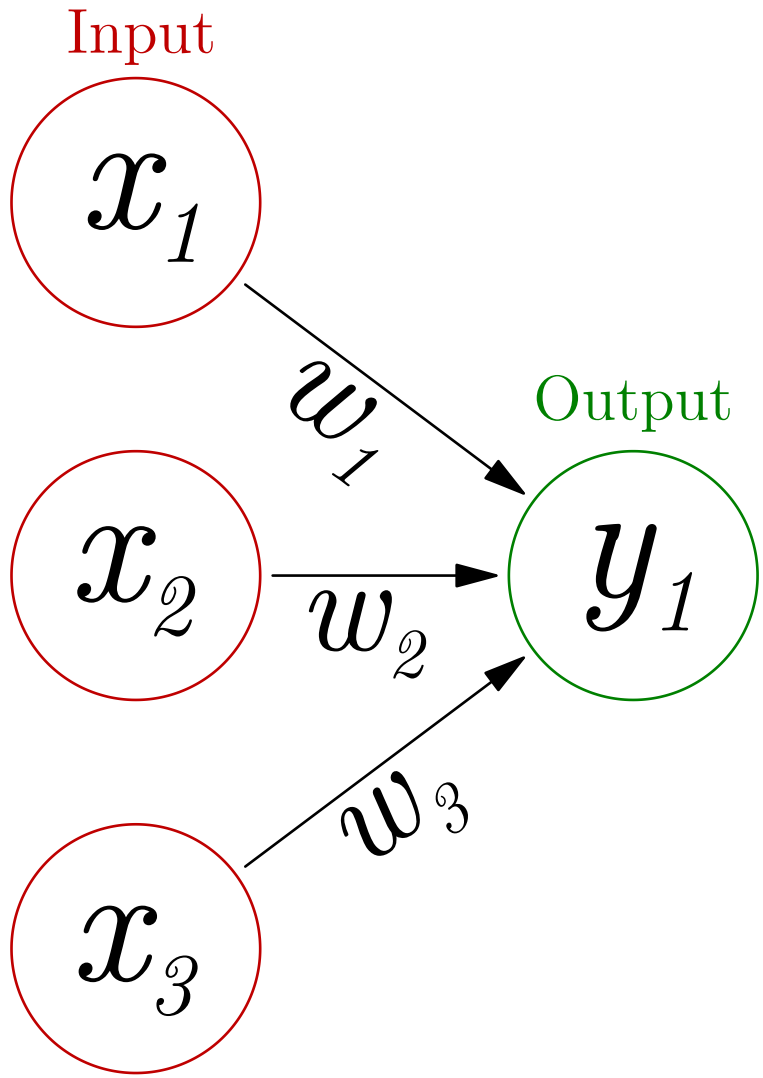

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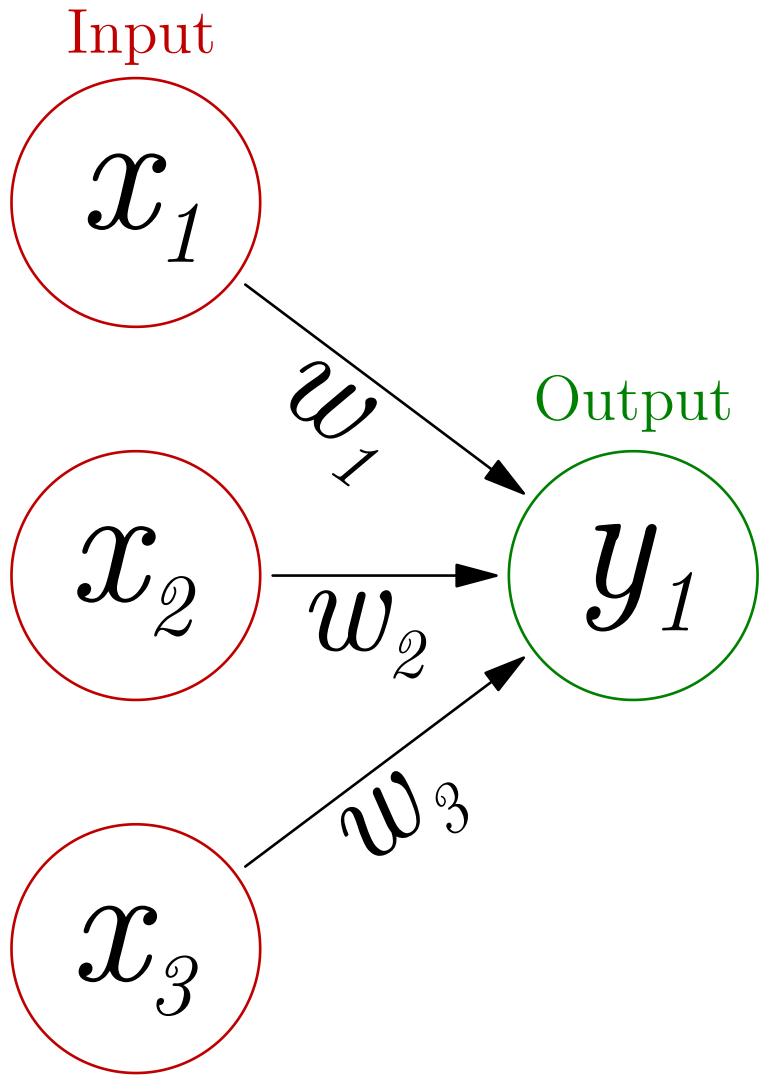

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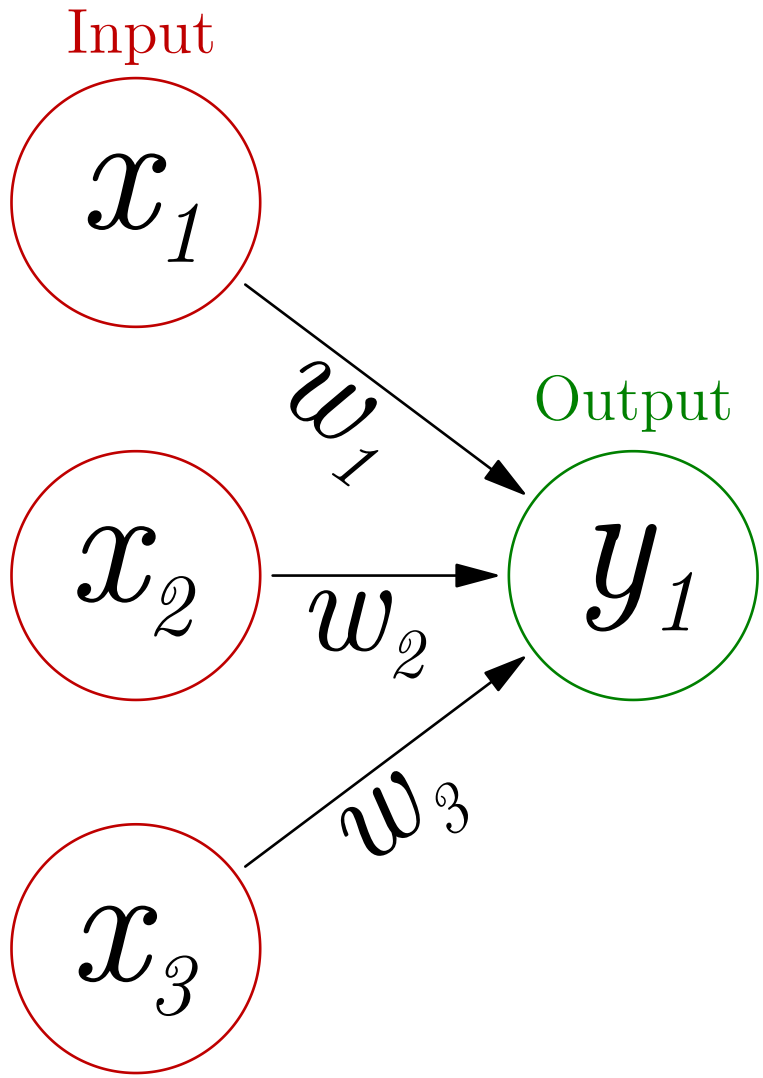

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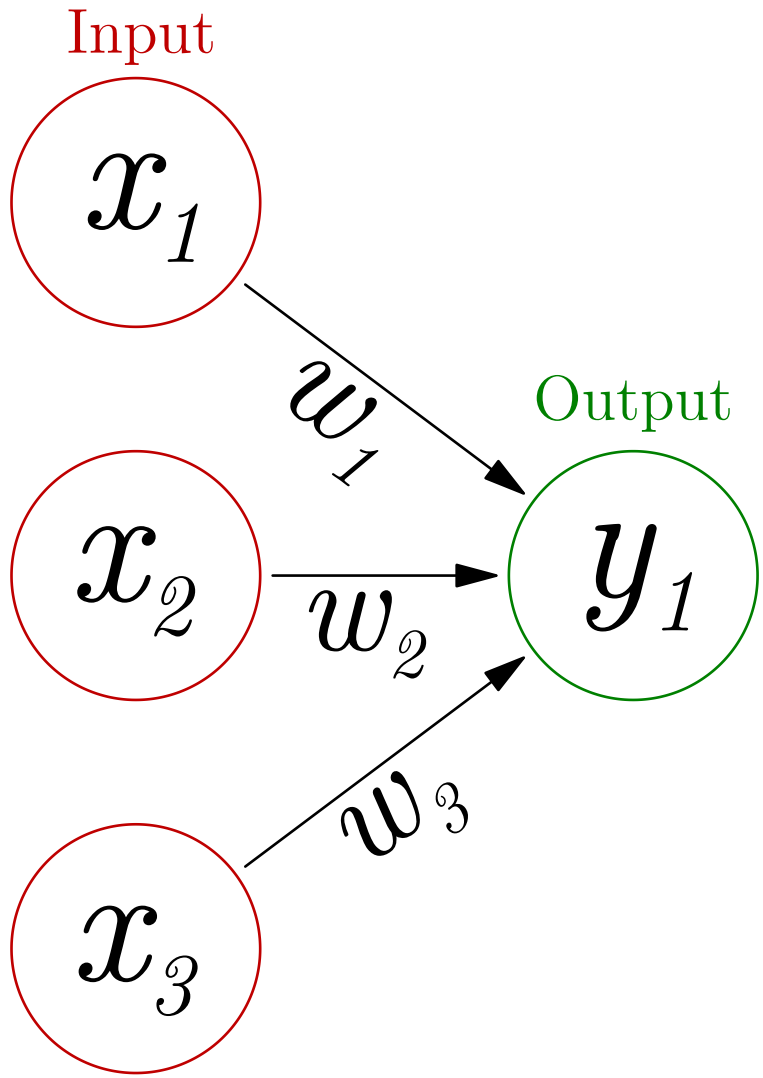

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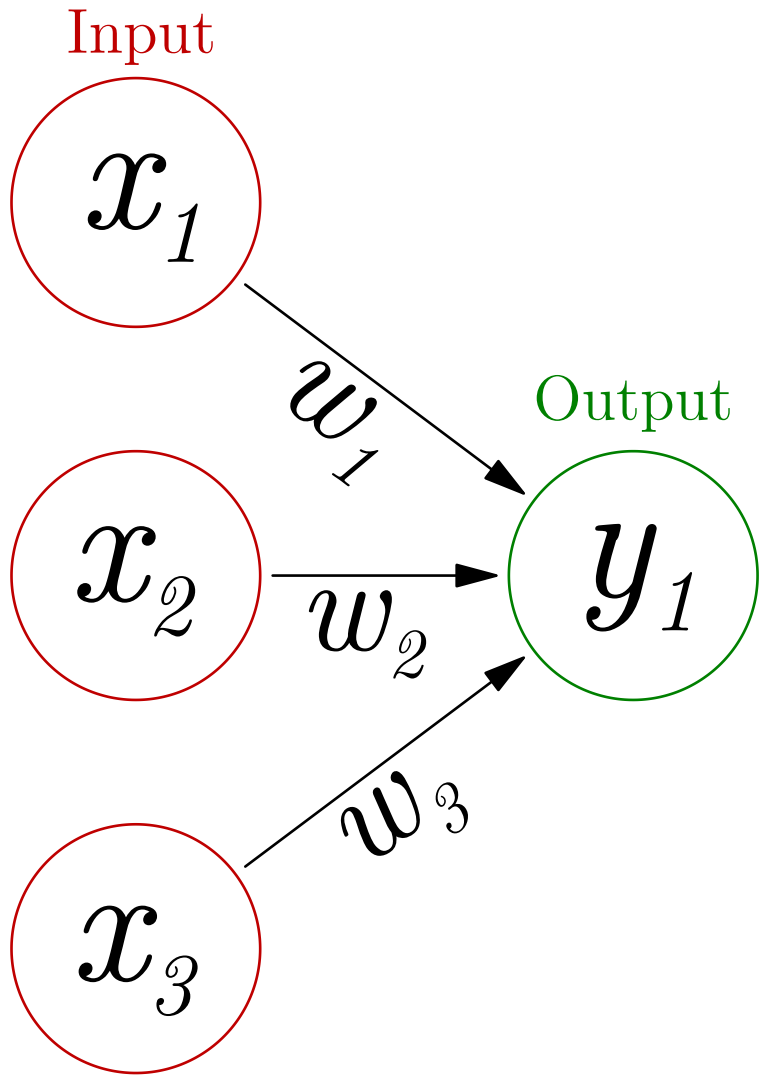

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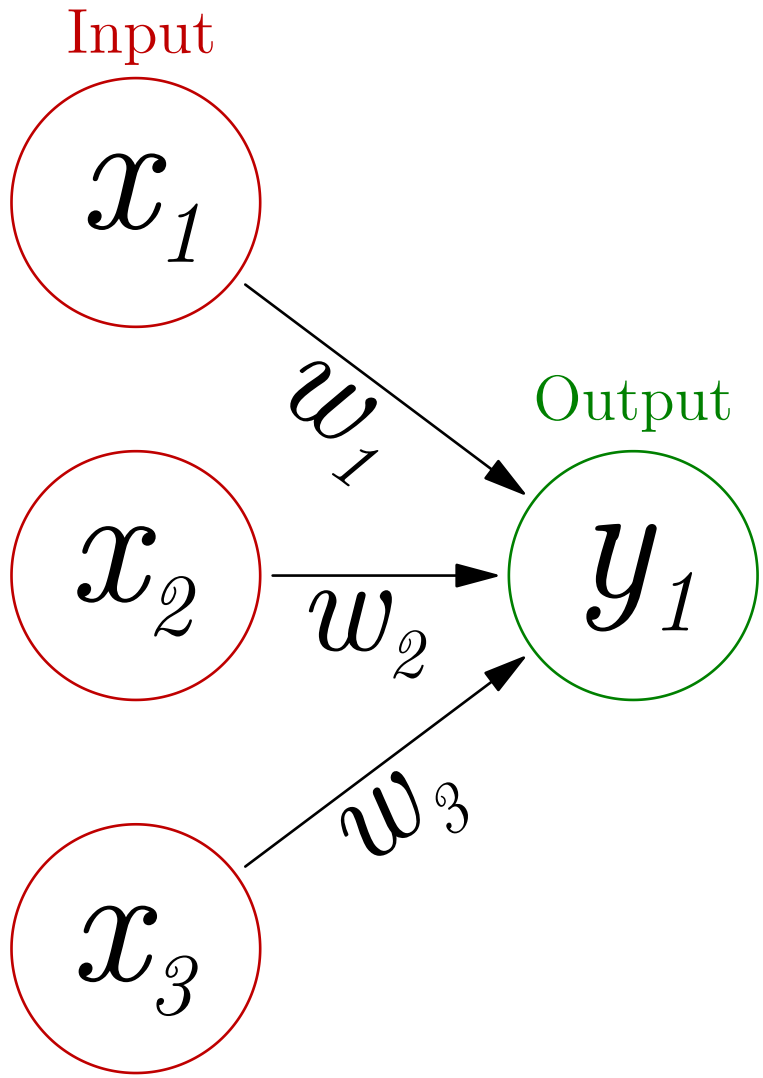

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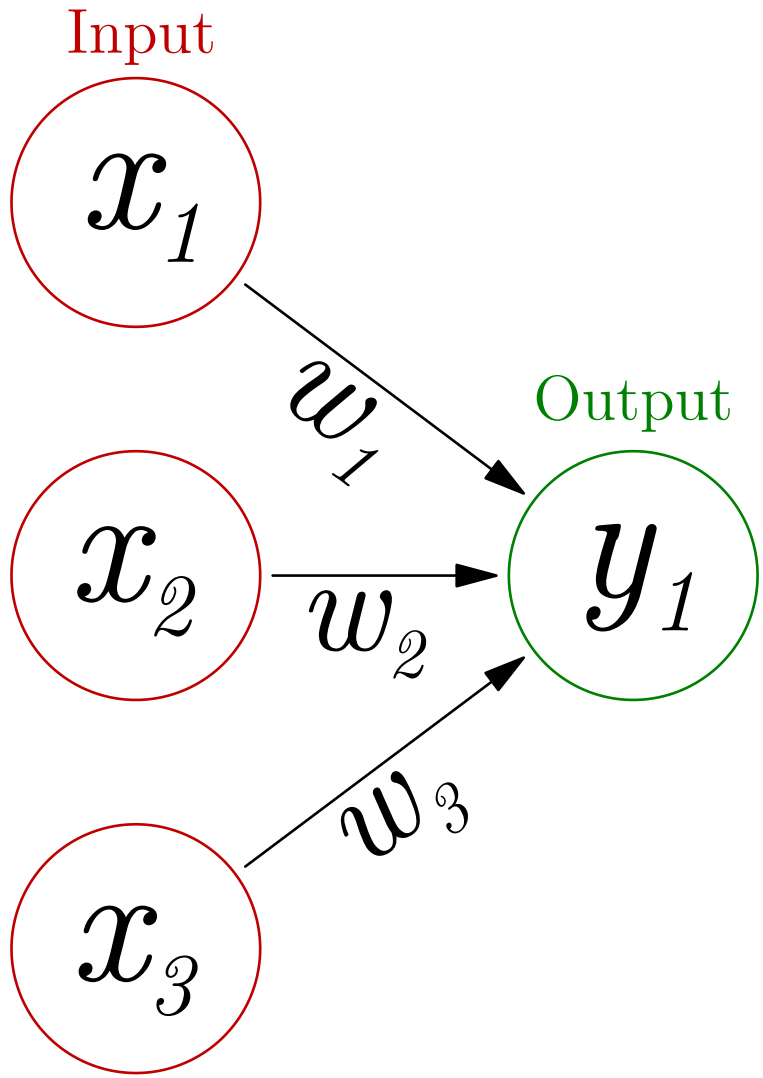

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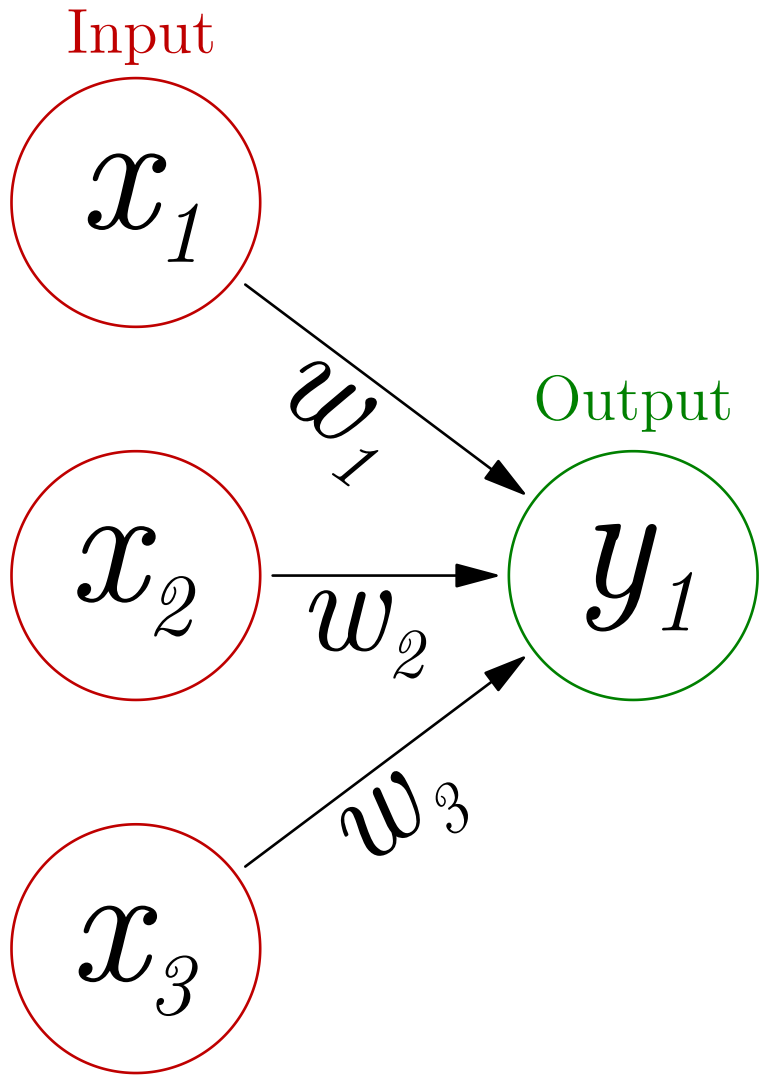
$$w_1 = 0.15 + \text{something positive}$$



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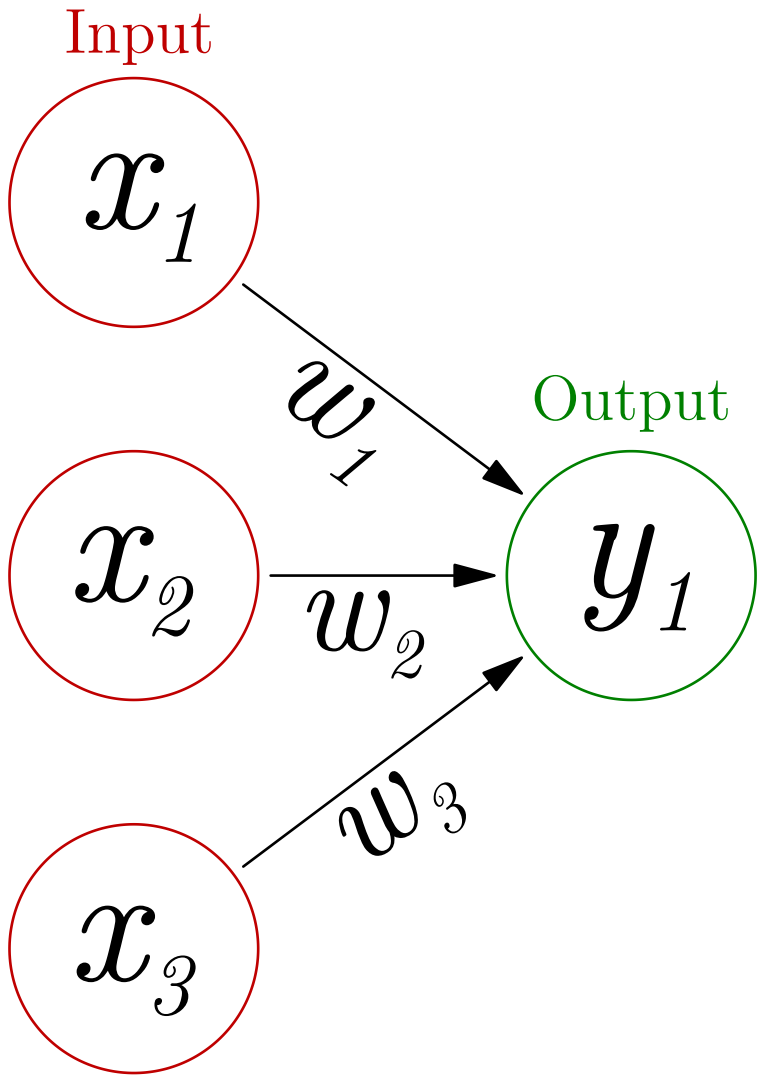

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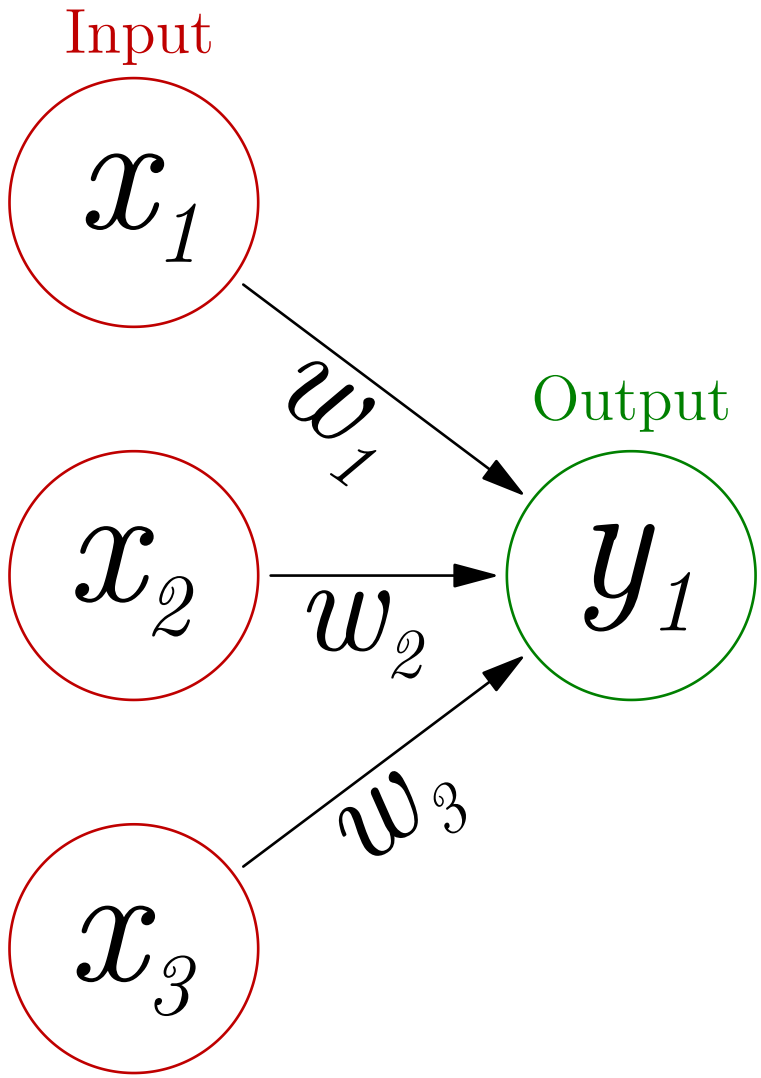
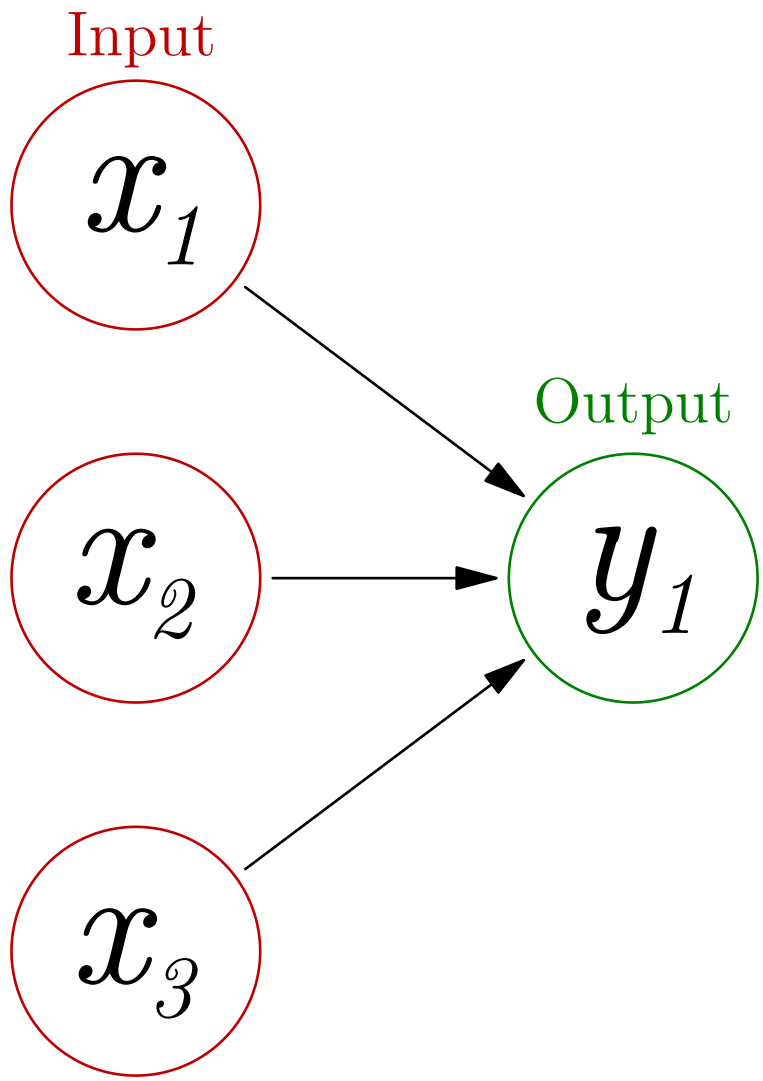


The perceptron

A simple classifier



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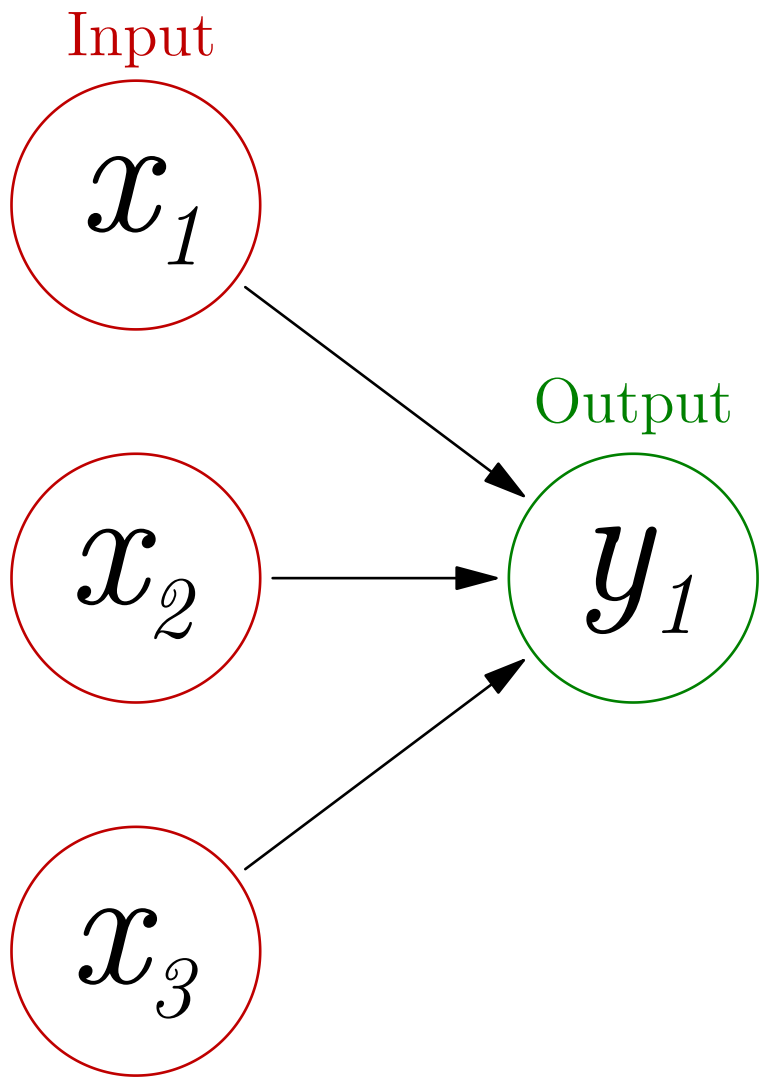
- Created in 1957 at the Cornell Aeronautical Laboratory by Frank Rosenblatt



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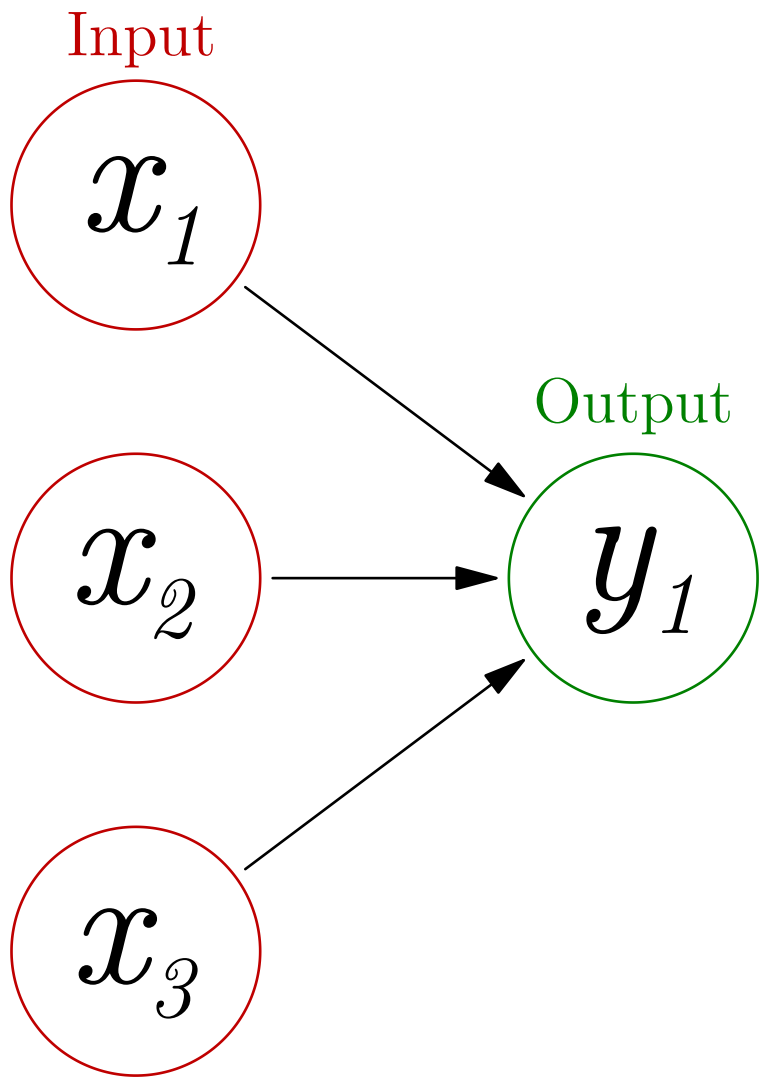

The perceptron

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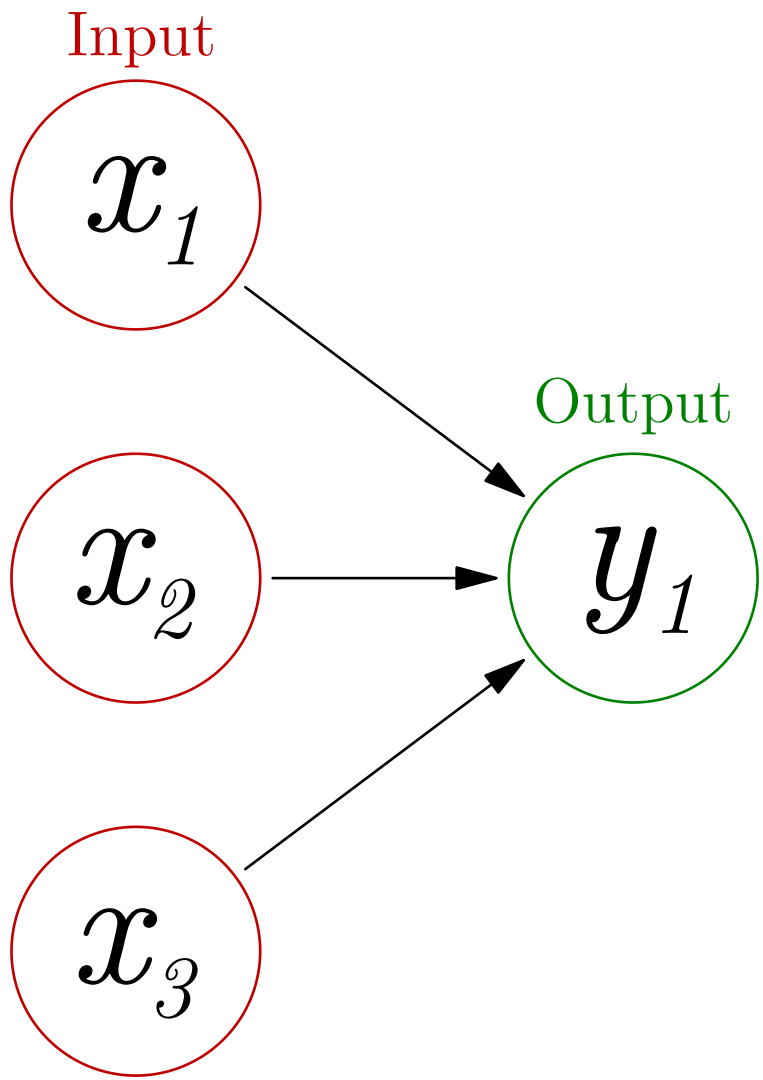
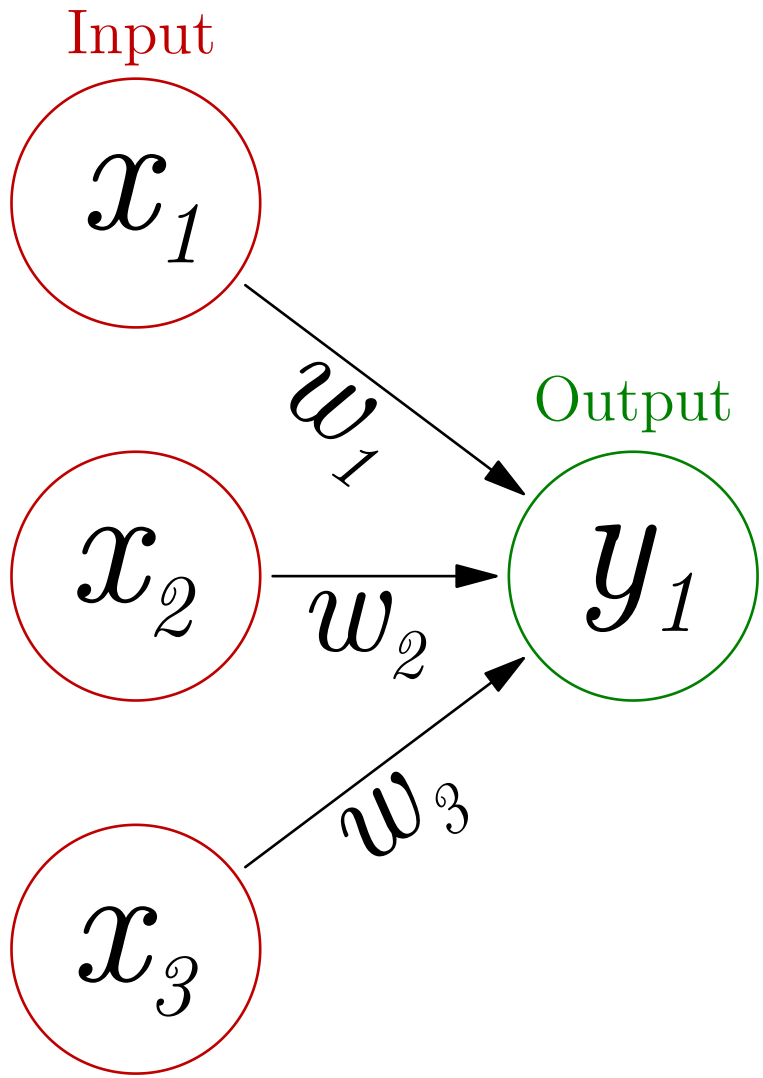
- Created in 1957 at the Cornell Aeronautical Laboratory by Frank Rosenblatt
- Linear classifier
- Simplest form of feedforward network



How does the perceptron learn?

Maths again!

1. Initialise weights



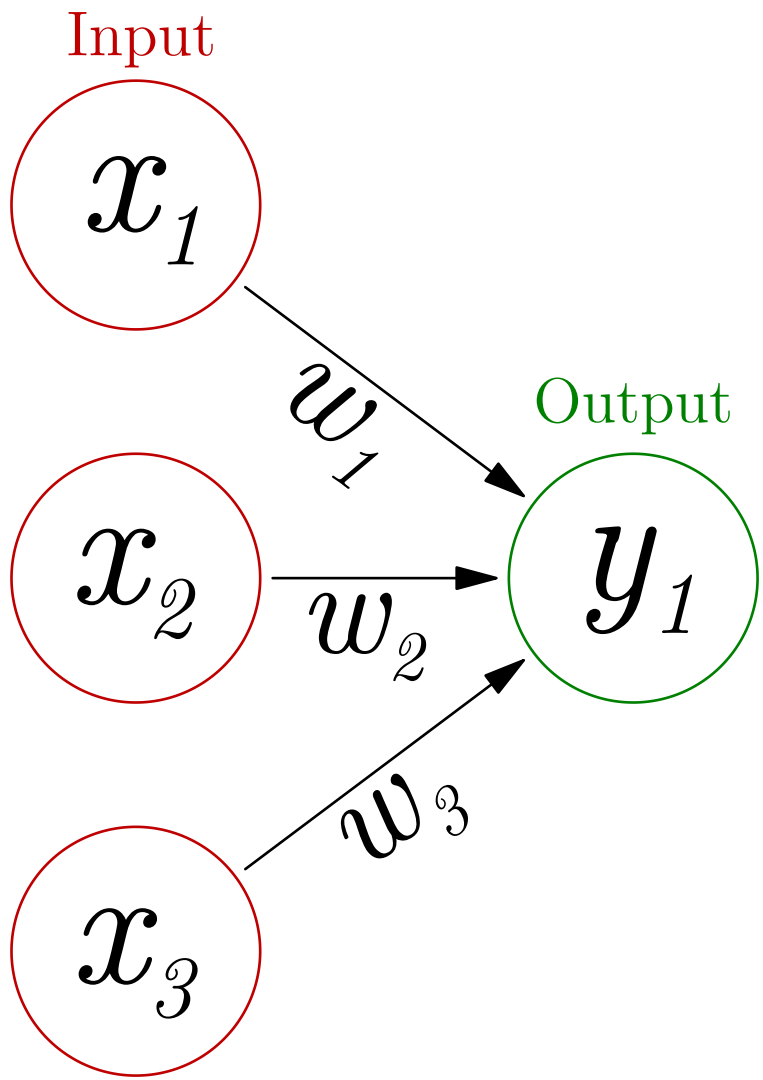
How does the perceptron learn?

Maths again!

1. Initialise weights
2. Run network using:

$$y_j = f\left(\sum_1^N w_i \times x_i\right)$$

same as always!

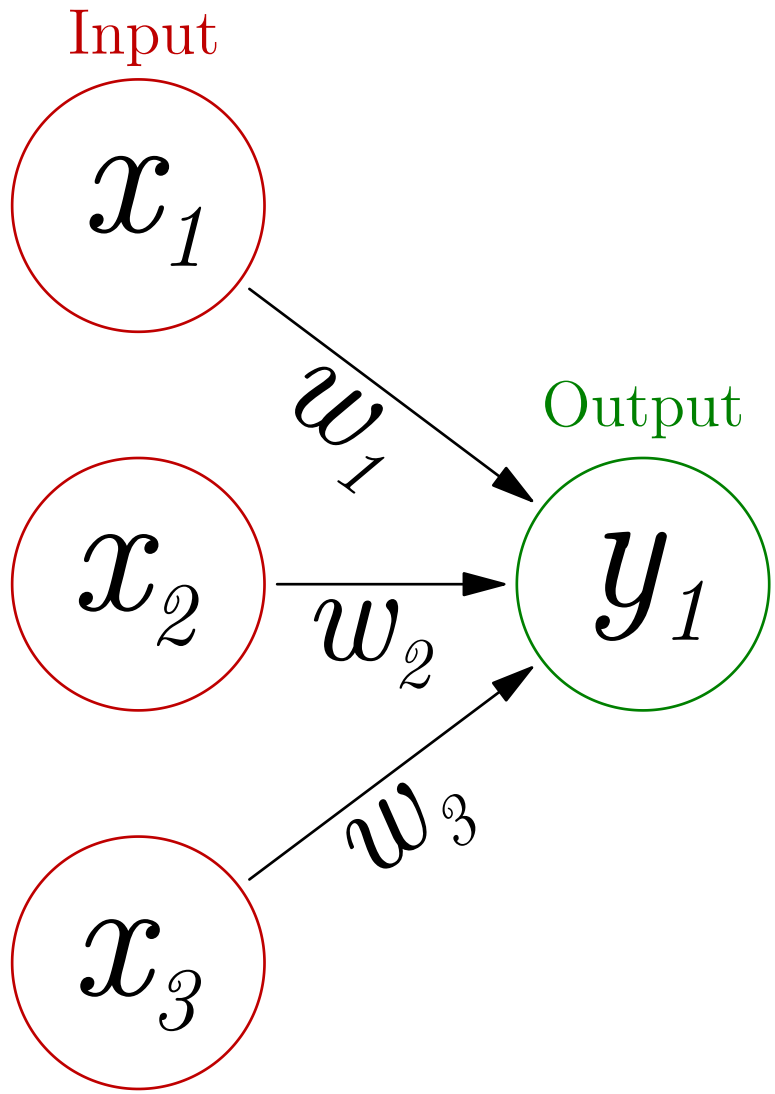


How does the perceptron learn?

Maths again!

1. Initialise weights
2. Run network
3. Update weights using:

$$\Delta w_i = \eta \quad y_j \times x_i$$



How does the perceptron learn?

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1. Initialise weights
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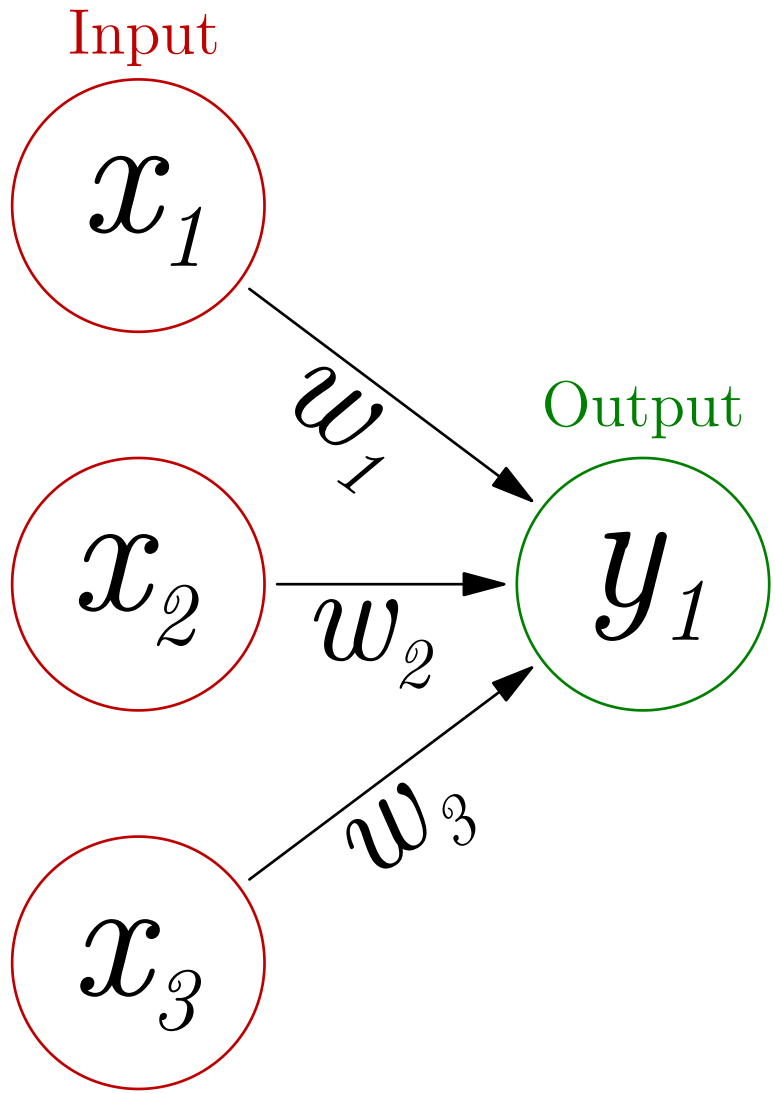
$$\Delta w_i = \eta \ (d_j - y_j) \quad \times x_i$$



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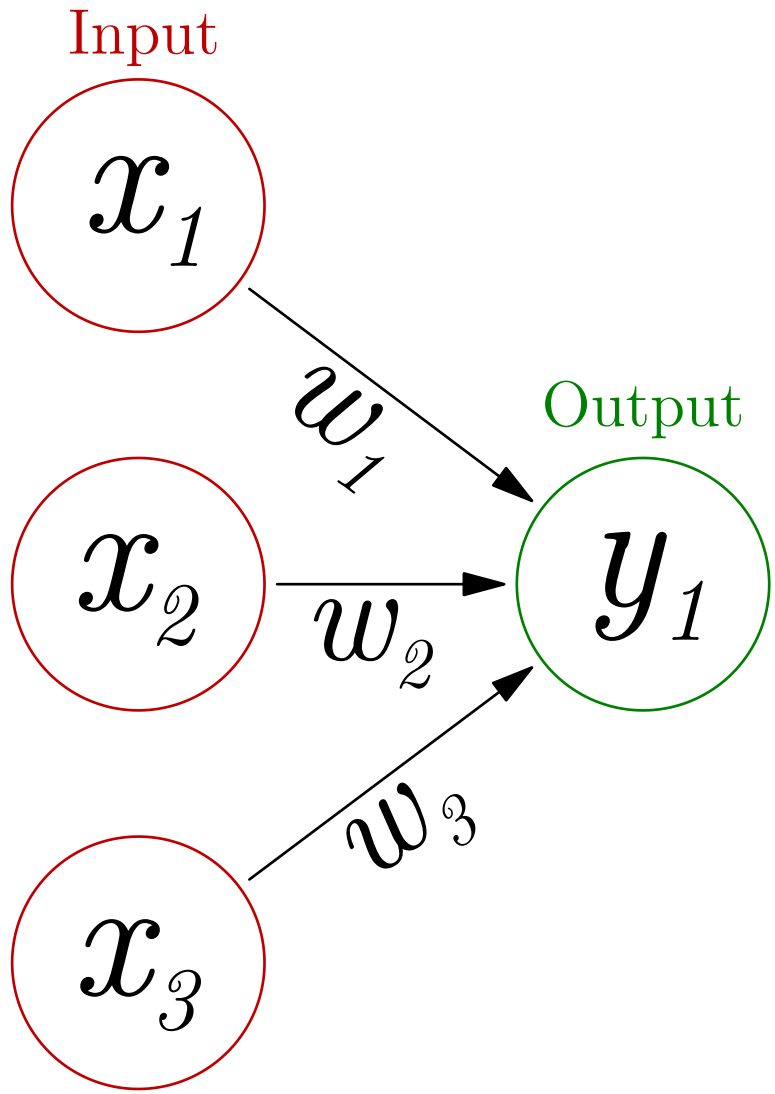
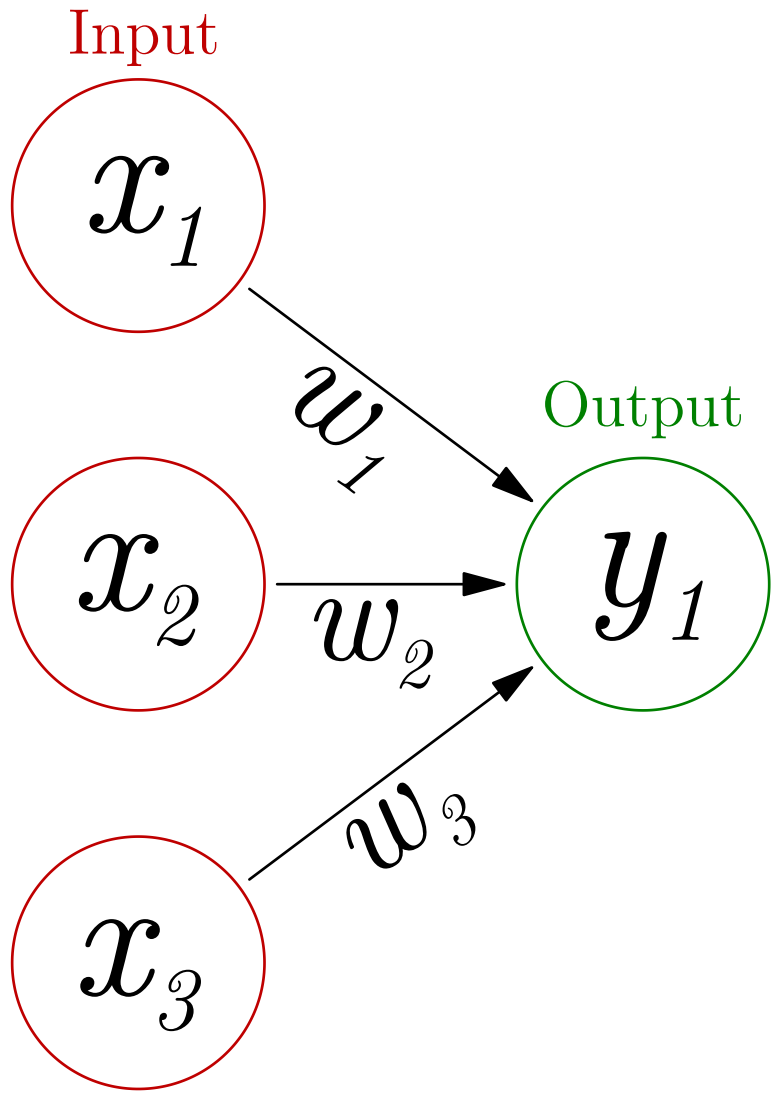
where d is what we want y to be given x, and η is the learning rate.



How does the perceptron learn?

Maths again!

1. Initialise weights
2. Run network
3. Update weights
4. Repeat 2 and 3



How does the perceptron learn?

Maths again!

1. Initialise weights
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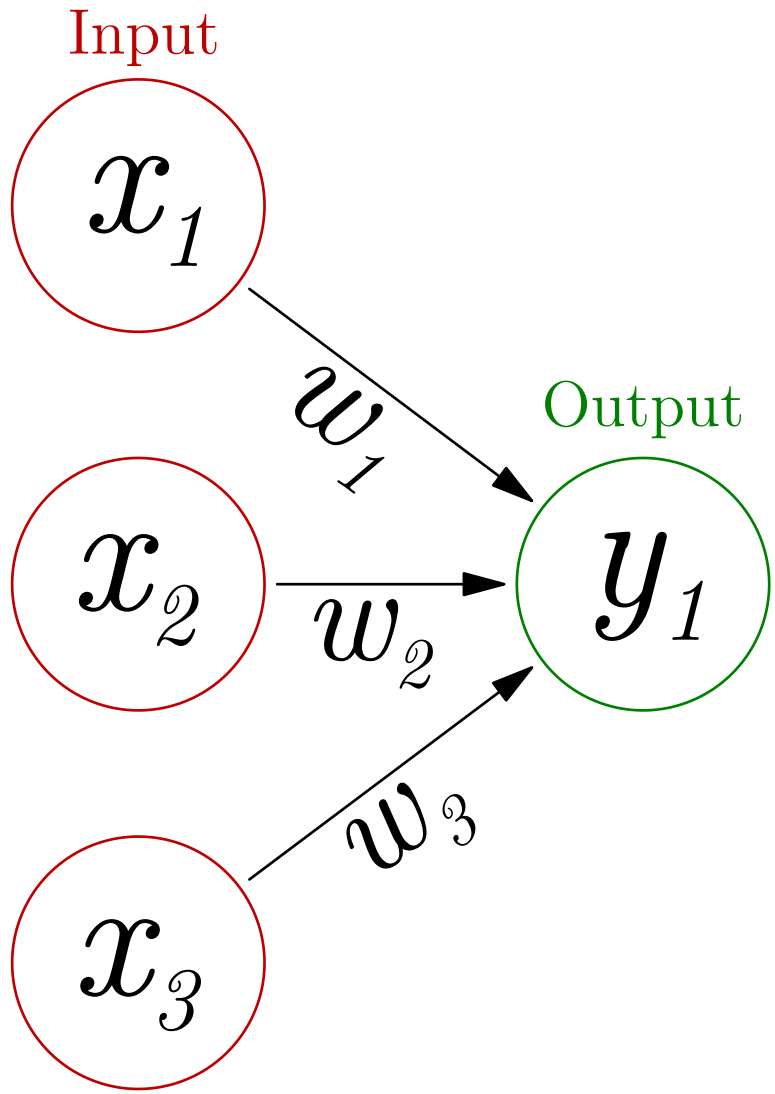
3. Update weights
4. Repeat 2 and 3
5. When do we stop?



The end

Time to program a perceptron!