

Welcome to the introduction
to computational cognitive
modelling workshop!

Part 2: Introduction to artificial neural networks

Part 2: **Introduction to artificial neural networks**

Olivia Guest

Chris Brand

Nick Sexton

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What is a neural network?

A mathematical model

- Inspired by the nervous system

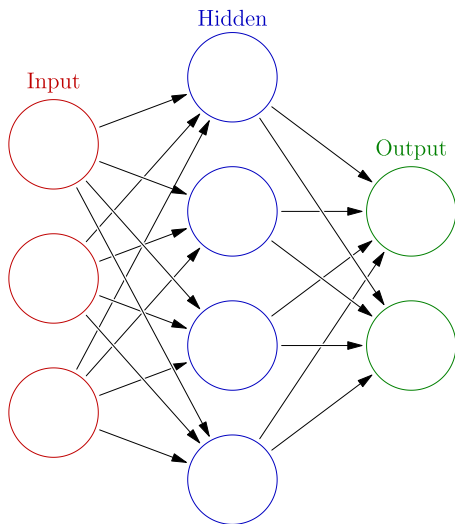


Figure: Glosser.ca / CC-BY-SA-3.0

What is a neural network?

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- ▶ A set of *units*, connected by *weights*

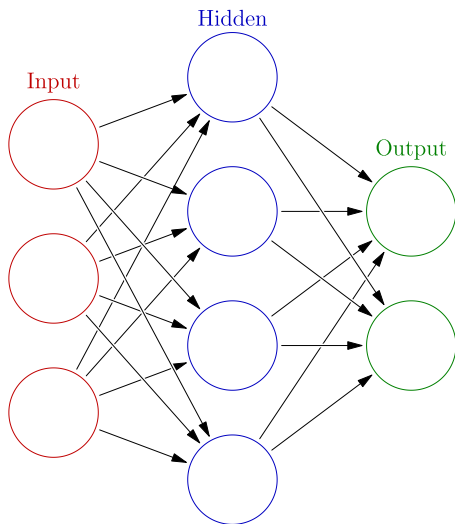


Figure: Glosser.ca / CC-BY-SA-3.0

What is a neural network?

A mathematical model

- ▶ Inspired by the nervous system
- ▶ A set of *units*, connected by *weights*
- ▶ The network *runs* by passing *activations* from the *input* (to the *hidden*) to the *output* units

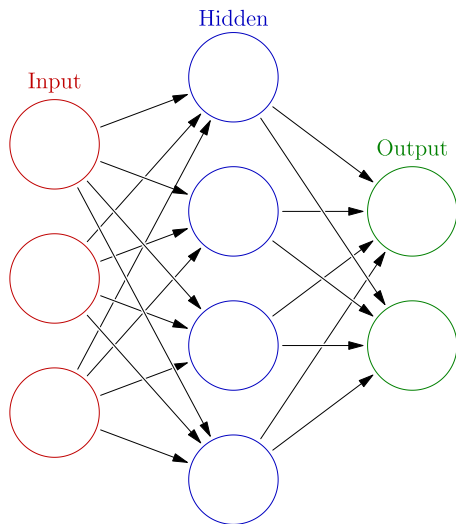


Figure: Glosser.ca / CC-BY-SA-3.0

Why use artificial neural networks for modelling?

Some aspects of their behaviour are like their namesake!

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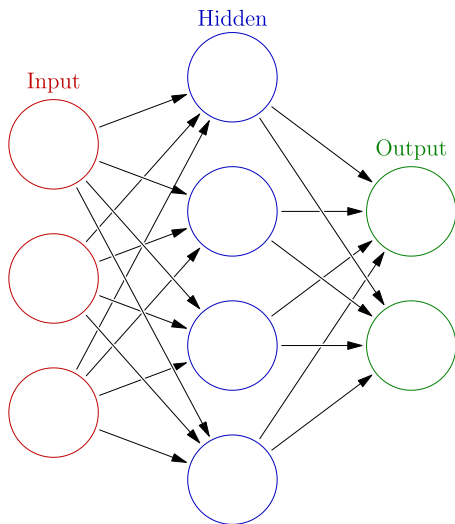
Some aspects of their behaviour are like their namesake!

- ▶ Learn pretty much any input-output data
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- ▶ Generalise from what they have learnt
- ▶ Cope with noise and damage

How does an artificial neural network run?

By using maths, predictably!

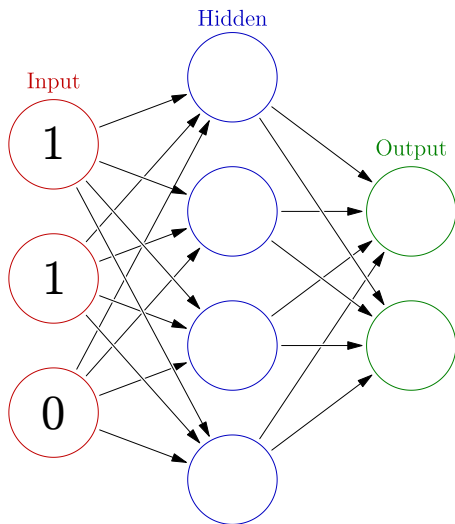
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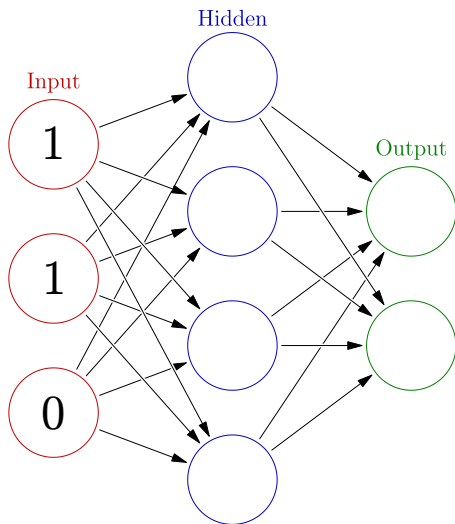
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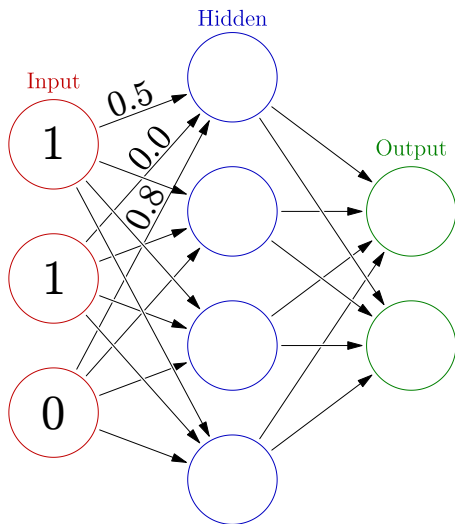
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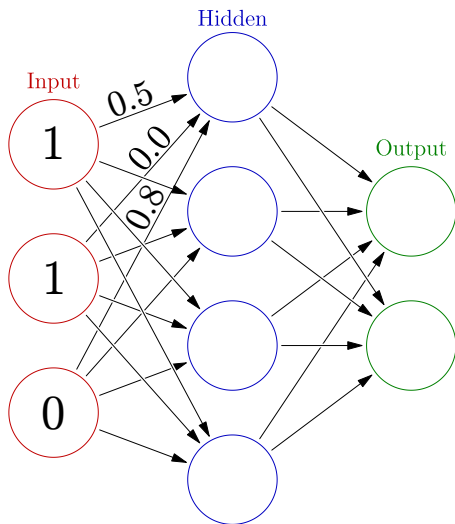


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$$1 \times 0.5 = 0.5$$



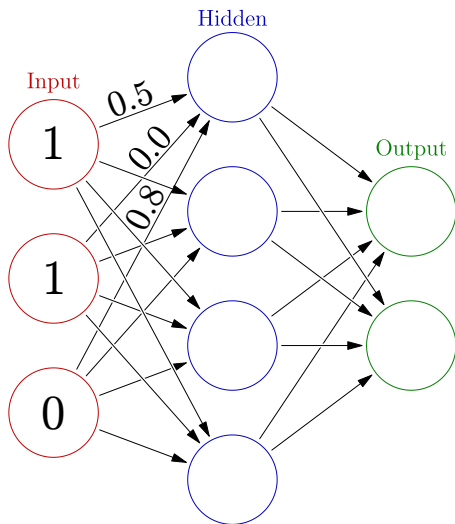
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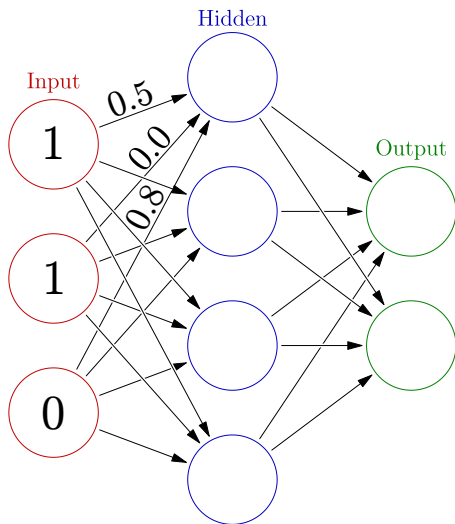
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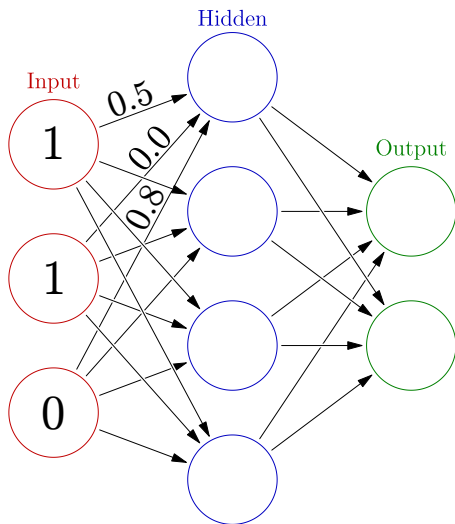


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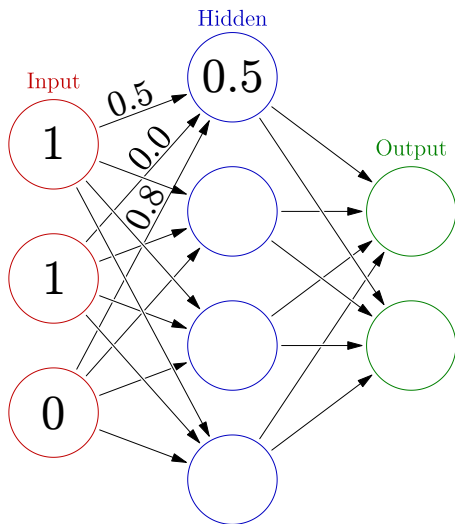


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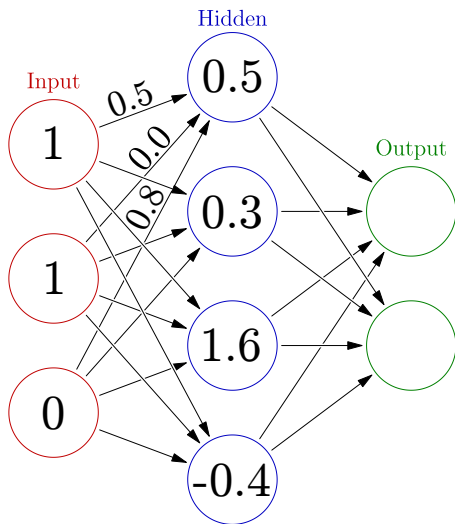
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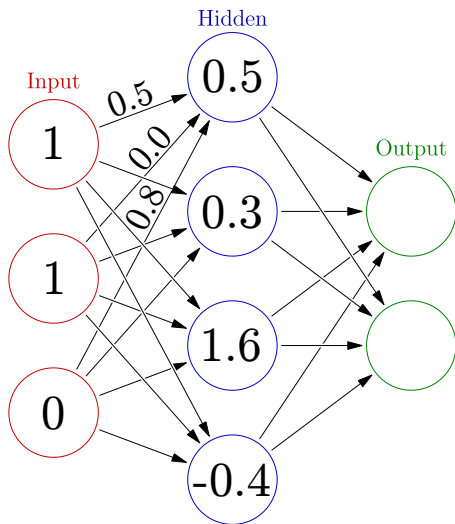
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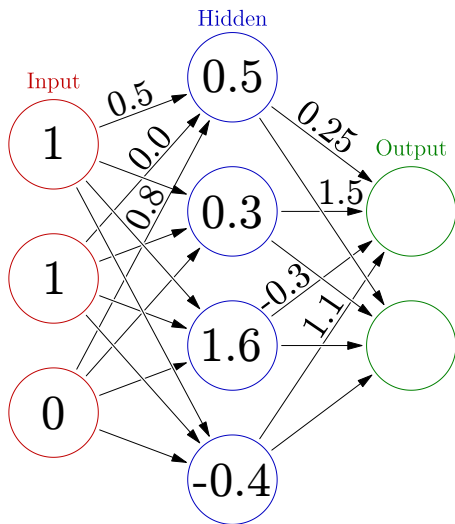
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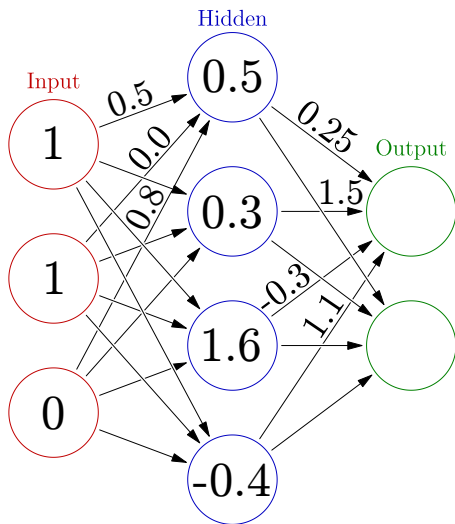


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$$0.5 \times 0.25 = 0.125$$



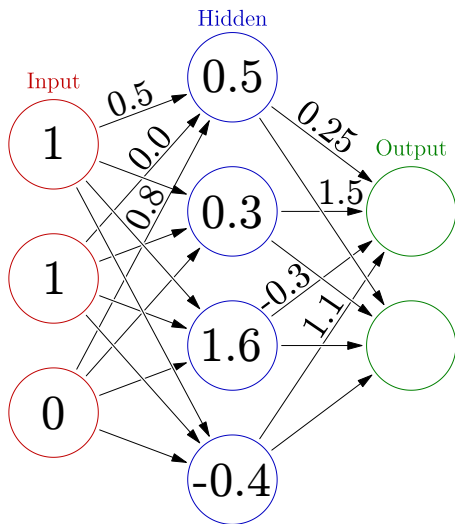
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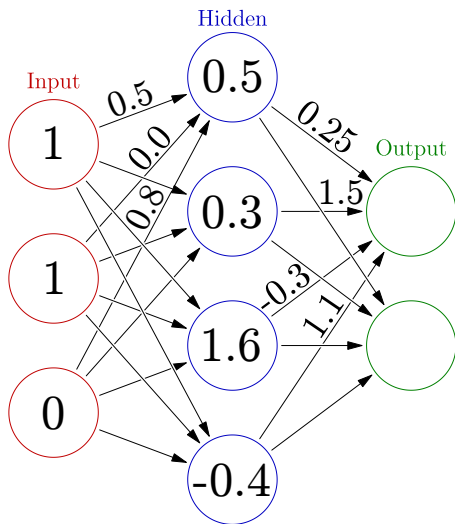


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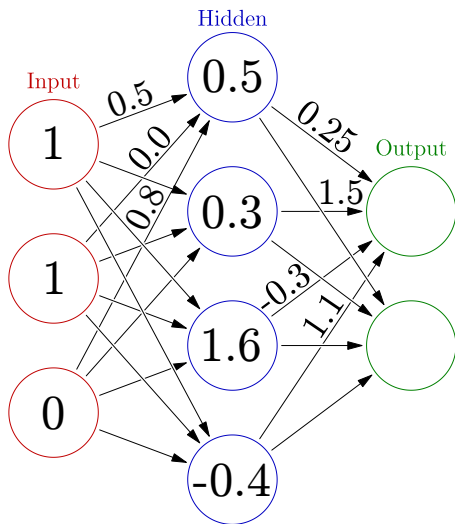


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$$\begin{aligned} 0.5 \times 0.25 &= 0.125 \\ 0.3 \times 1.5 &= 0.45 \\ 1.6 \times -0.3 &= -0.48 \\ -0.4 \times 1.1 &= -0.44 \end{aligned}$$

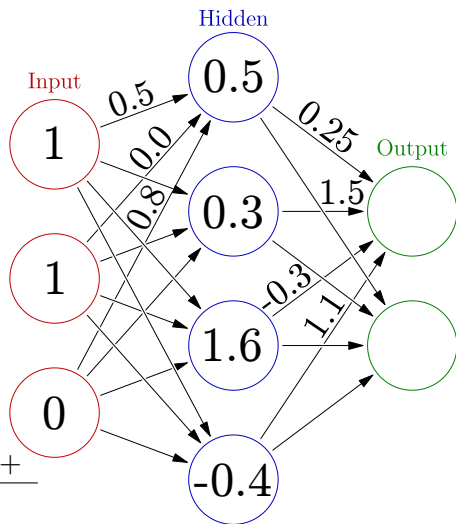


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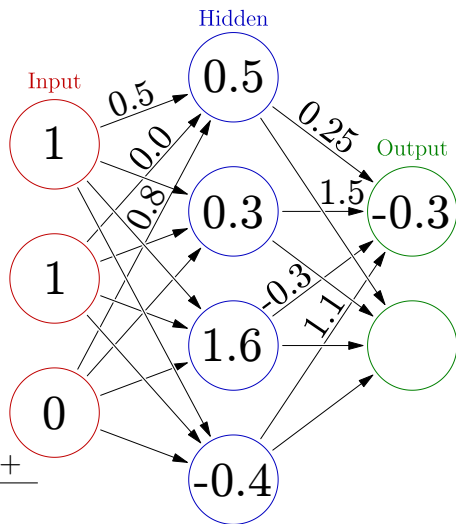


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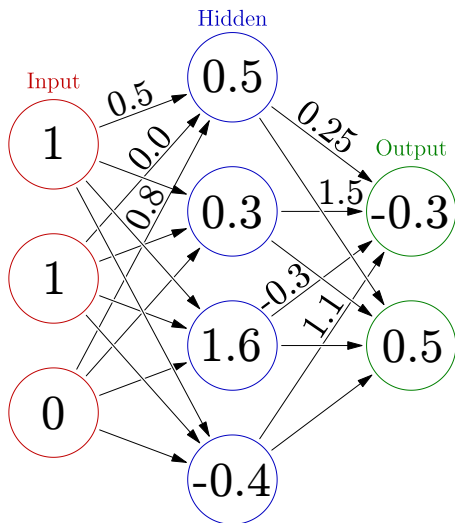
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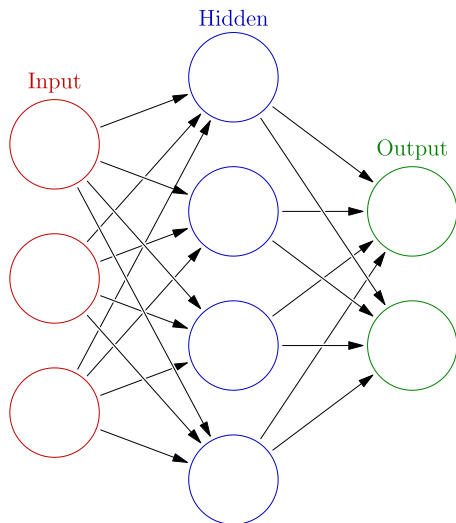
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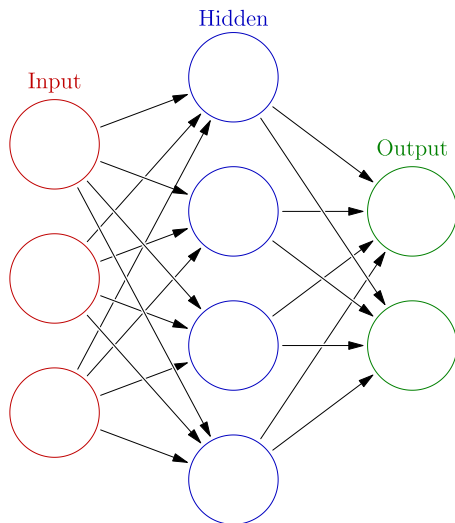
- ▶ But programmers are *lazy*!



How does an artificial neural network run?

By using maths, predictably!

- General names save time

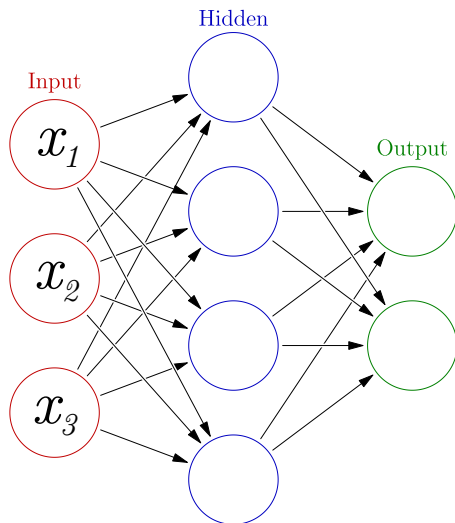


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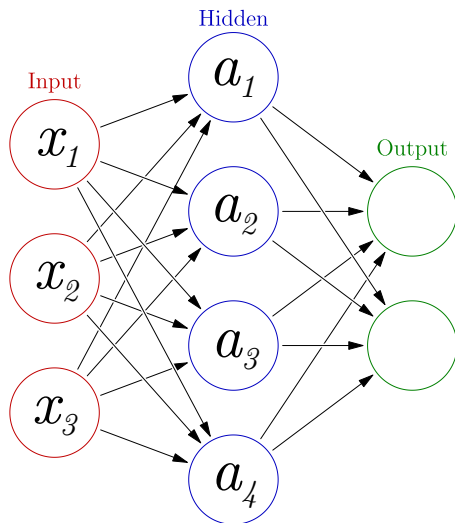
- ▶ **input units:** x_i



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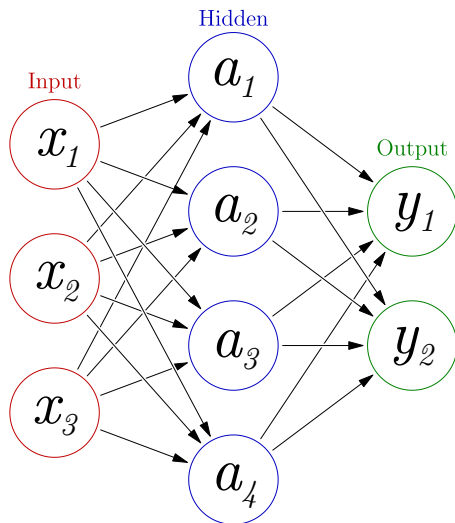
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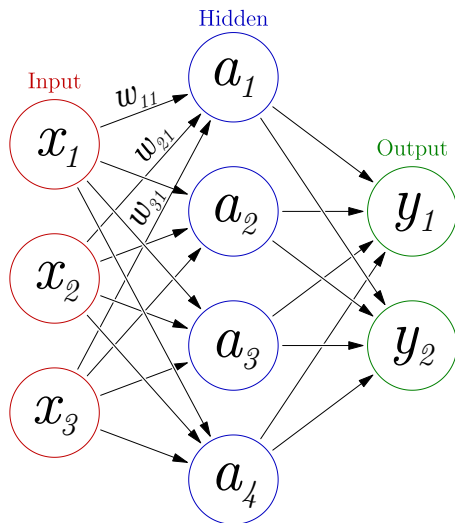
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- ▶ connection weights: w_{ij}



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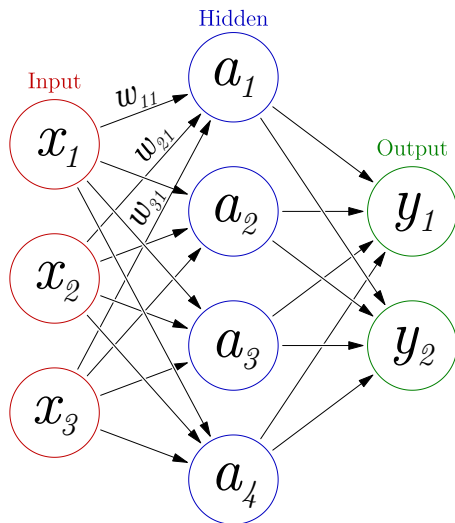
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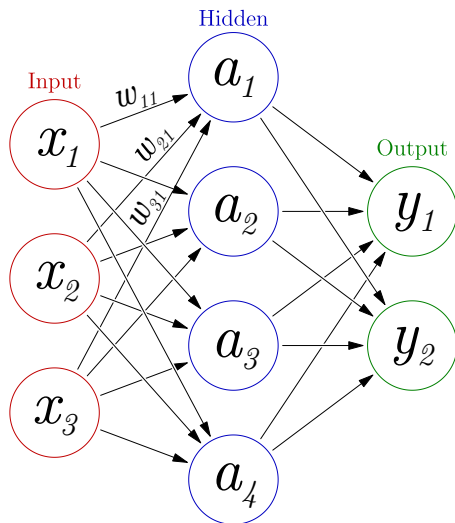
- ▶ subscripts
general: $ijklm\dots$
specific: 12345...



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We use general names to write a general equation:

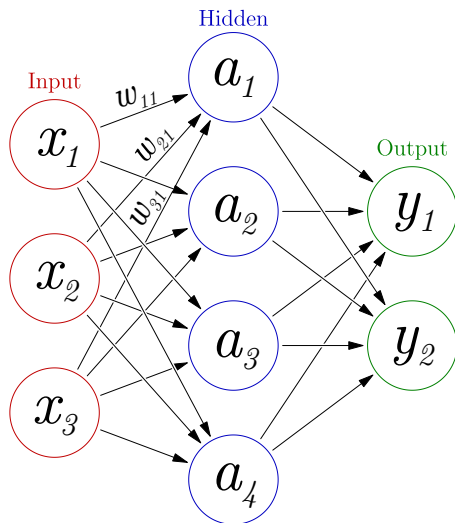


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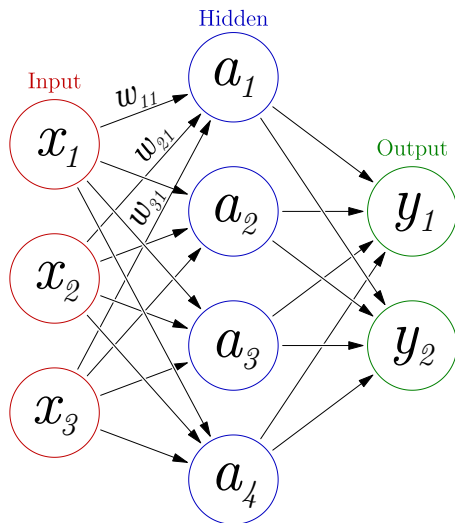


How does an artificial neural network run?

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We use general names to write a general equation:

$$a_i = x_j \times w_{ji}$$

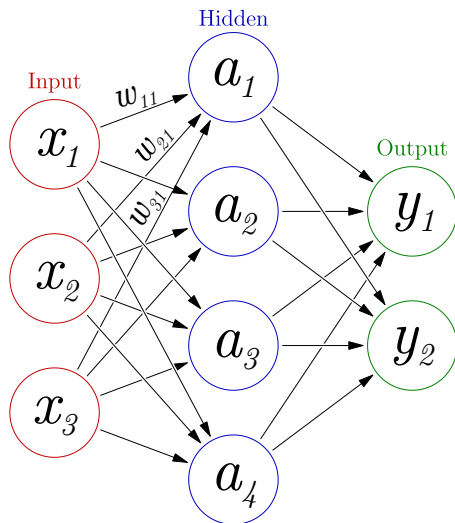


How does an artificial neural network run?

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We use general names to write a general equation:

$$a_i = \sum_{j=1}^N x_j \times w_{ji}$$

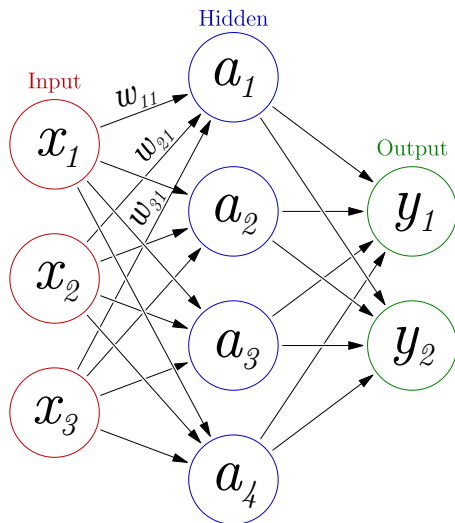


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We use general names to write a general equation:

$$a_i = f \left(\sum_{j=1}^N x_j \times w_{ji} \right)$$



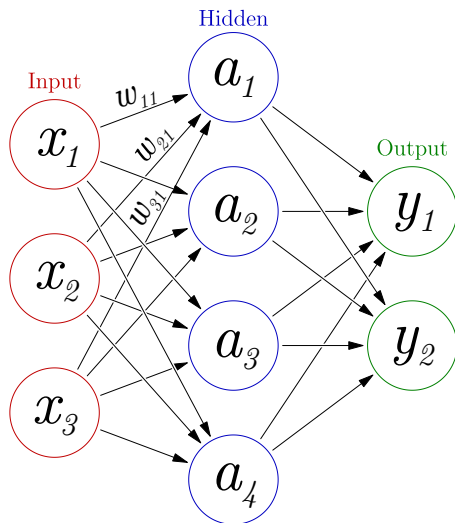
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where a_i is the unit whose state we want to calculate, N is the number of units on the previous layer, w_{ji} is the weight on the connection between i and j , and f is a function that the unit applies.

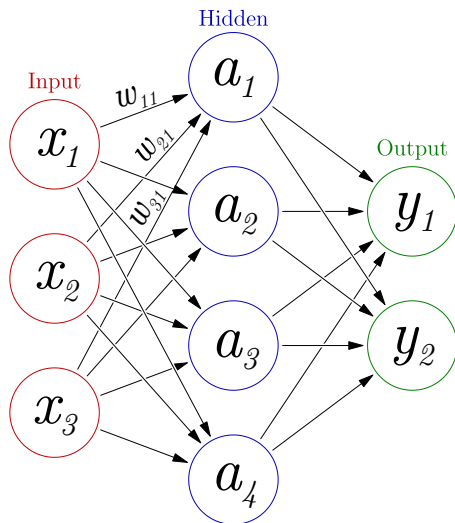


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We use this equation by replacing *iterators* i and j :

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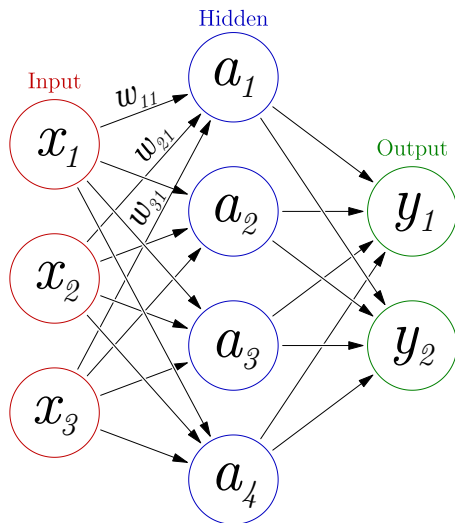
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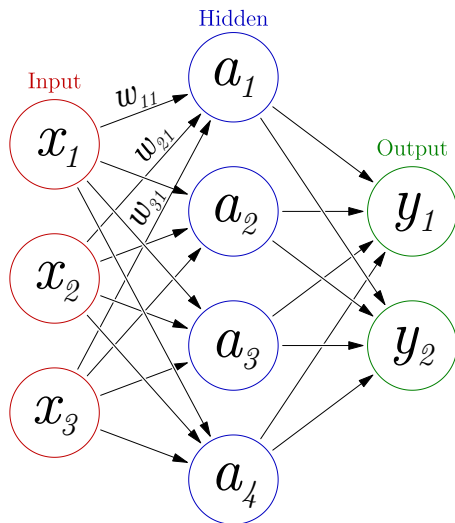
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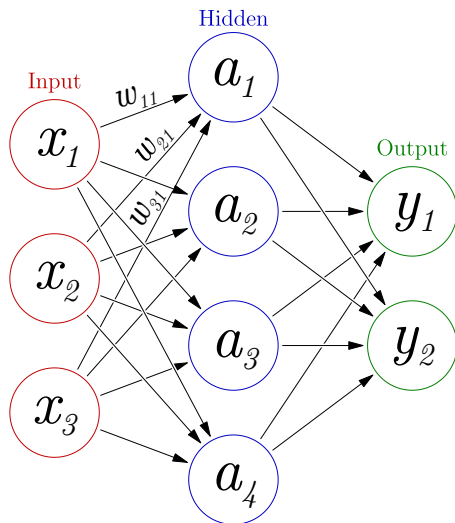
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$$x_{\textcolor{red}{1}} \times w_{\textcolor{blue}{1}\textcolor{blue}{1}}$$



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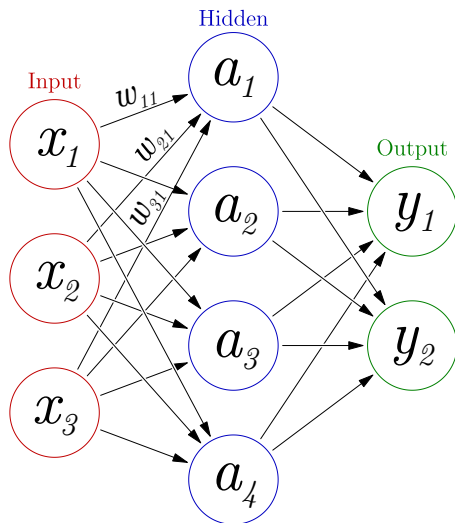
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$$x_{\textcolor{red}{1}} \times w_{\textcolor{red}{1}\textcolor{blue}{1}} + x_{\textcolor{red}{2}} \times w_{\textcolor{red}{2}\textcolor{blue}{1}}$$



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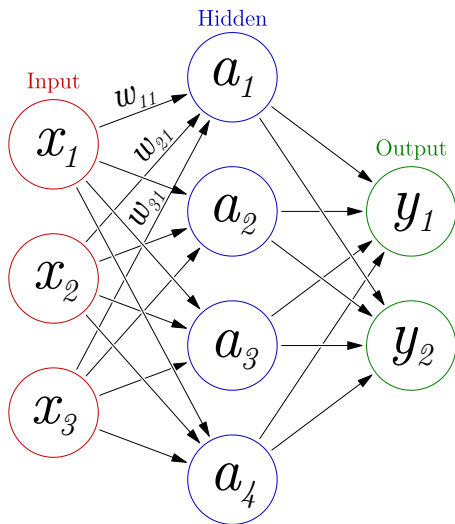
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$$x_1 \times w_{11} + x_2 \times w_{21} + x_3 \times w_{31}$$



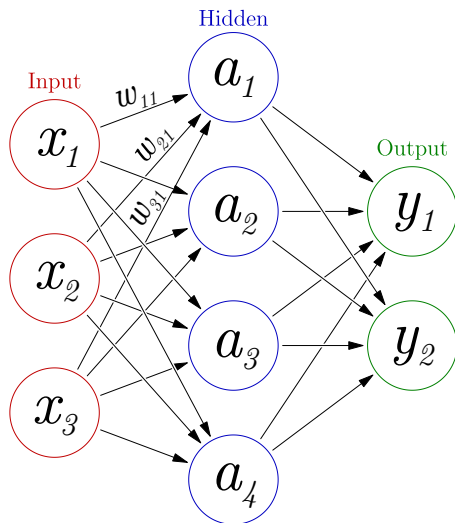
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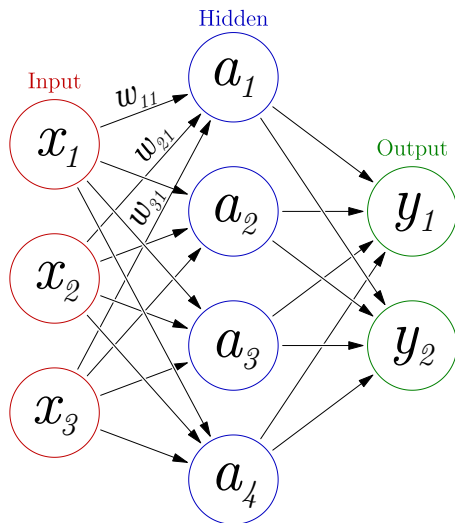
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$$x_{\textcolor{red}{1}} \times w_{\textcolor{blue}{12}}$$



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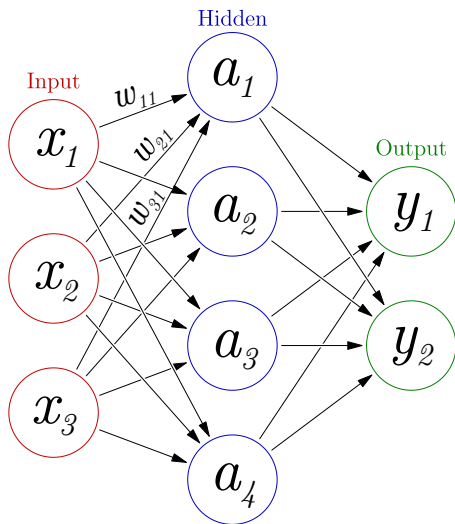
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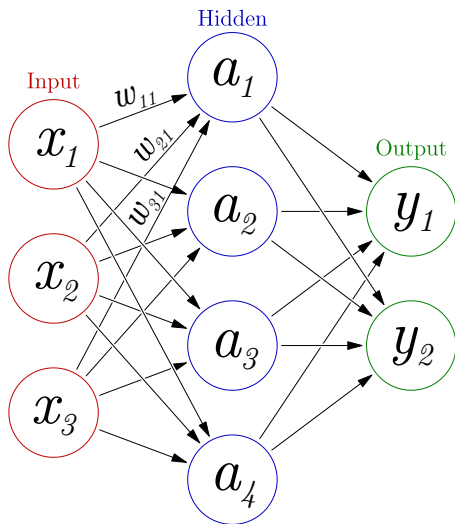
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How do networks learn?

Cunning!

- ▶ Many options: Hebbian learning, back-propagation of error, Boltzmann machine learning, self-organising map algorithm, etc.

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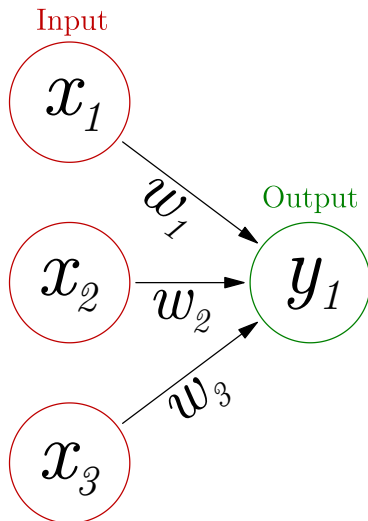
- ▶ Many options: Hebbian learning, back-propagation of error, Boltzmann machine learning, self-organising map algorithm, etc.
- ▶ All learning algorithms work by changing the connection weights
- ▶ Learning can be divided into *supervised*, *unsupervised*, and *reinforcement*

Hebbian learning

A very simple learning rule

“Cells that fire together, wire together”

— Carla Shatz



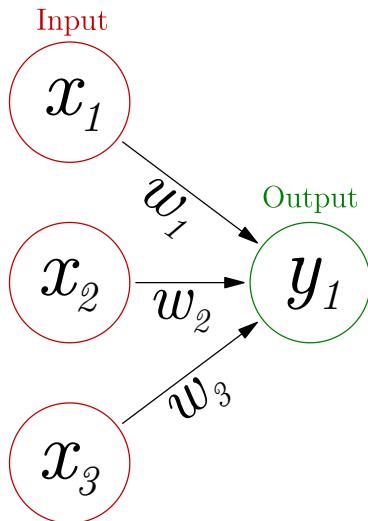
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$$w_i =$$



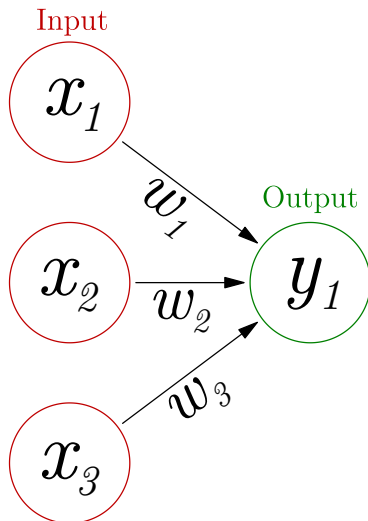
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$$w_i = x_i \times y_j$$



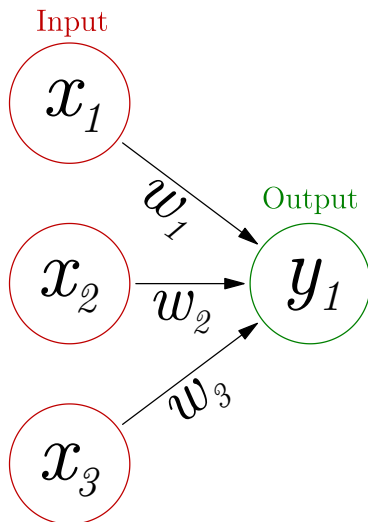
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$$w_i = \eta \times x_i \times y_j$$



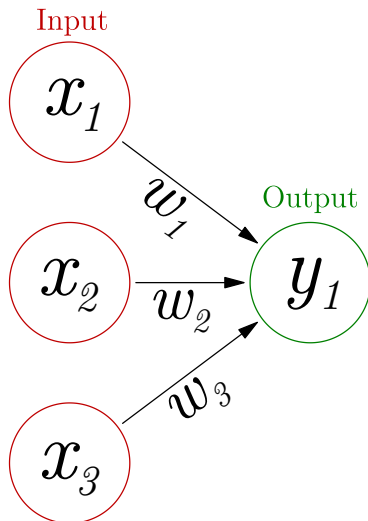
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$$\Delta w_i = \eta \times x_i \times y_j$$



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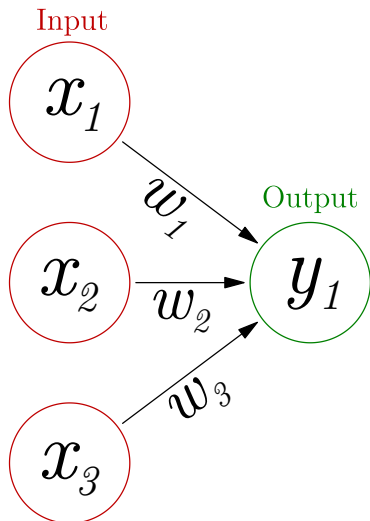
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$$\Delta w_i = \eta \times \textcolor{red}{x}_i \times \textcolor{green}{y}_j$$

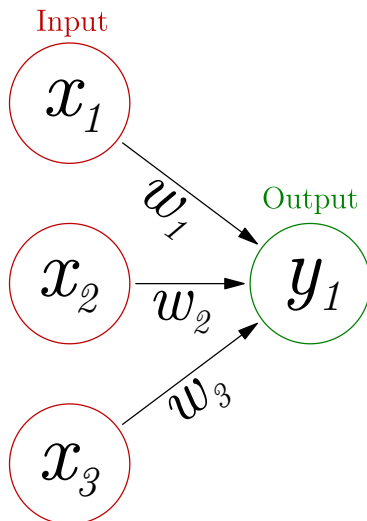
which means each weight is changed by a small in/decrement for every pattern



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Hebb's rule is simple, but very *unstable*!

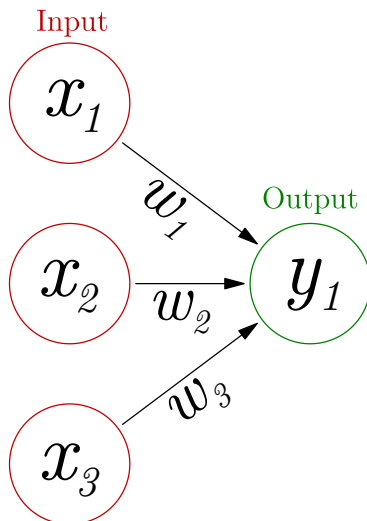


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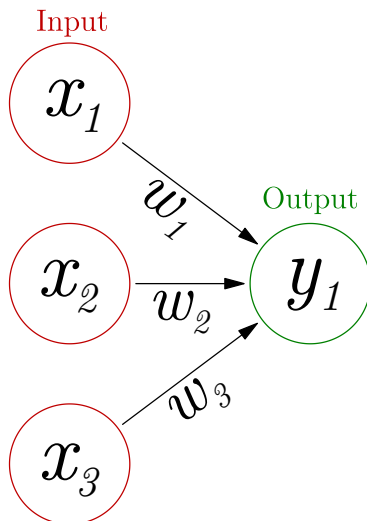
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$$\Delta w_i = \eta \times x_i \times y_j$$

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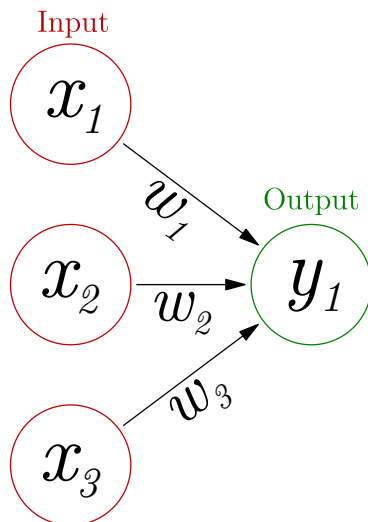
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$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.5 \times x_i \times y_j$$



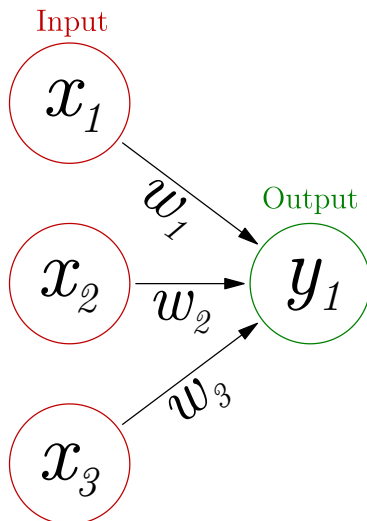
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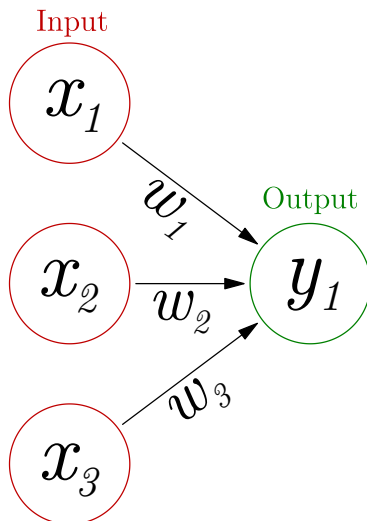
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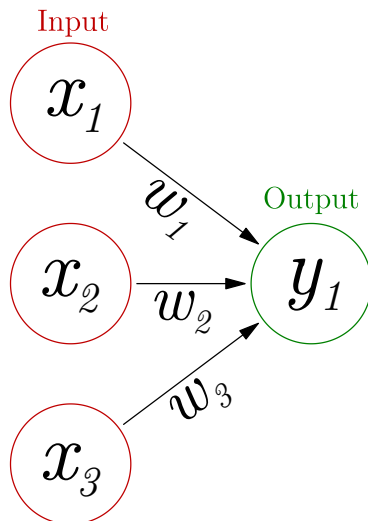
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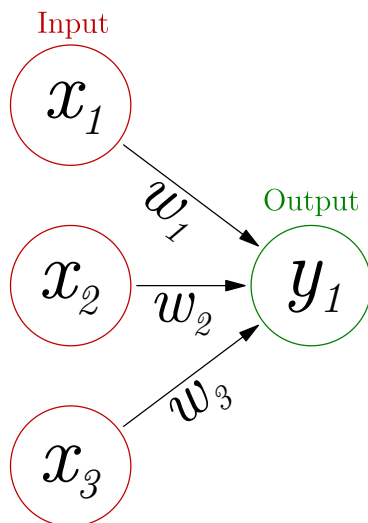
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$$\Delta w_1 = 0.5 \times 1.0 \times 0.3$$



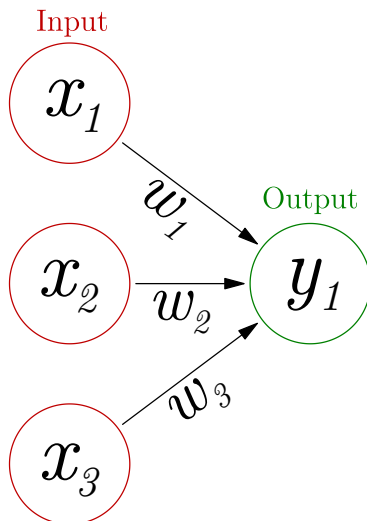
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Hebb's rule is simple, but *very unstable!*

$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$



Hebbian learning

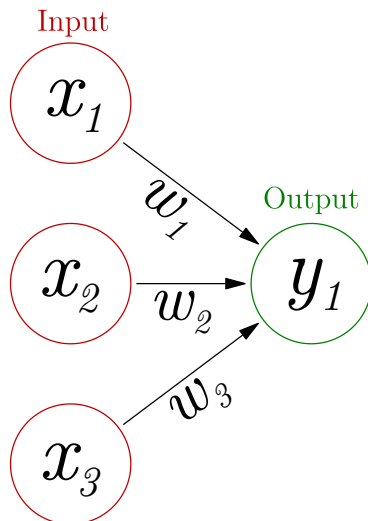
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Hebb's rule is simple, but *very unstable!*

$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

$$\mathbf{new} \ w_1 = \mathbf{old} \ w_1 + \Delta w_1$$



Hebbian learning

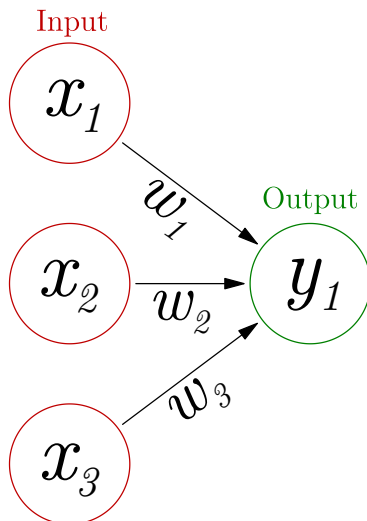
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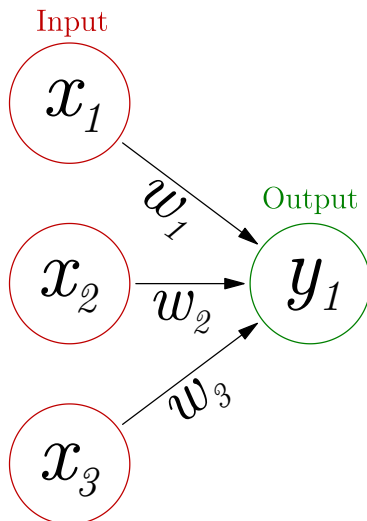
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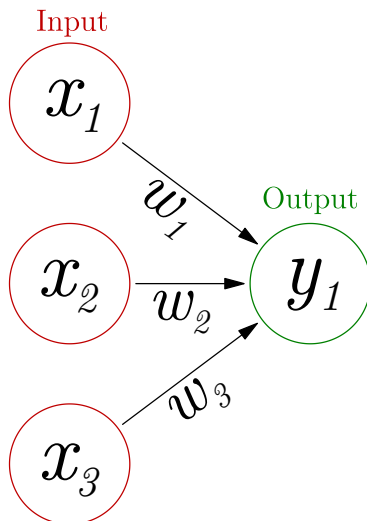
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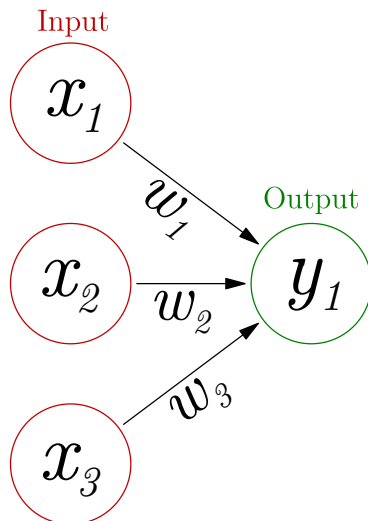
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$$w_1 = 0.15$$



Hebbian learning

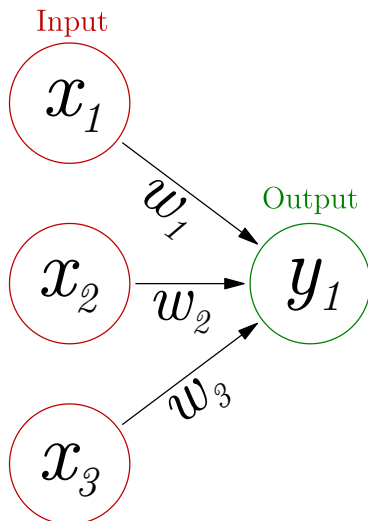
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Hebb's rule is simple, but *very unstable!*

$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

$$w_1 = 0.15 + \text{something positive}$$



Hebbian learning

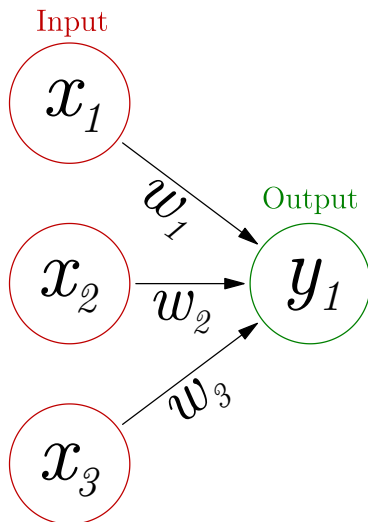
“Cells that fire together, wire together” — Carla Shatz

Hebb's rule is simple, but *very unstable!*

$$\Delta w_i = \eta \times x_i \times y_j$$

$$\Delta w_1 = 0.15$$

$w_1 = 0.15 + \text{something}$
positive + something else
positive +



Hebbian learning

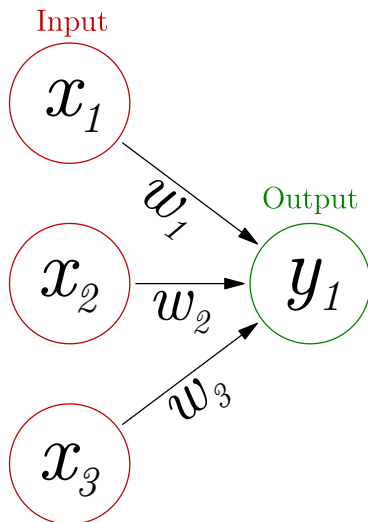
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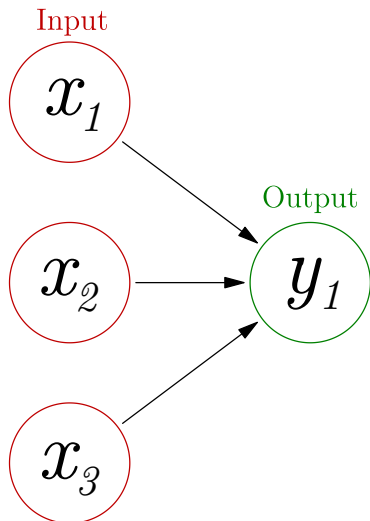
$$\Delta w_1 = 0.15$$

$w_1 = 0.15 + \text{something}$
positive + something else
positive + another positive
value + ...



The perceptron

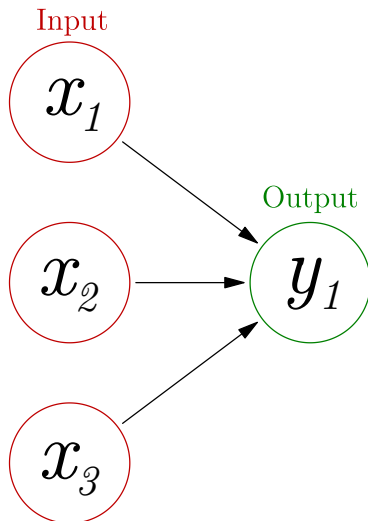
A simple classifier



The perceptron

A simple classifier

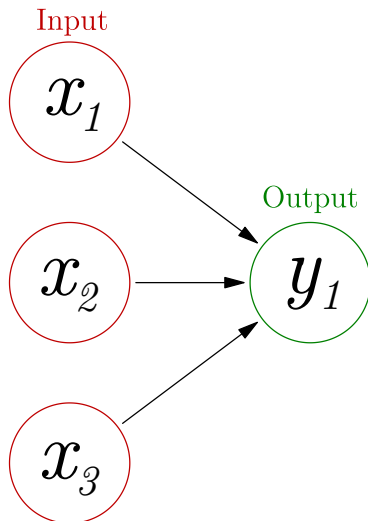
- Created in 1957 at the Cornell Aeronautical Laboratory by Frank Rosenblatt



The perceptron

A simple classifier

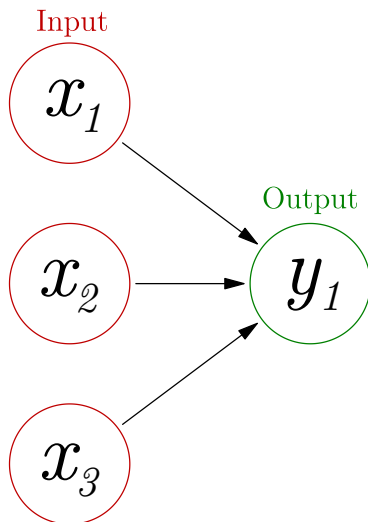
- ▶ Created in 1957 at the Cornell Aeronautical Laboratory by Frank Rosenblatt
- ▶ Linear classifier



The perceptron

A simple classifier

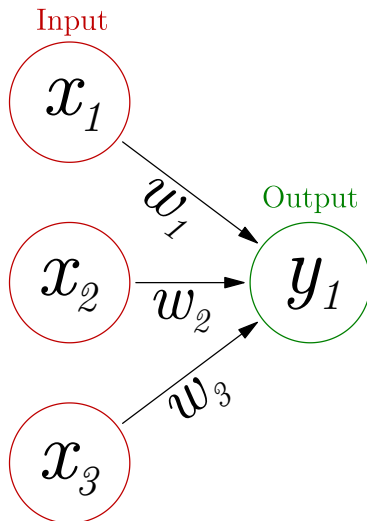
- ▶ Created in 1957 at the Cornell Aeronautical Laboratory by Frank Rosenblatt
- ▶ Linear classifier
- ▶ Simplest form of feedforward network



How does the perceptron learn?

Maths again!

1. Initialise weights



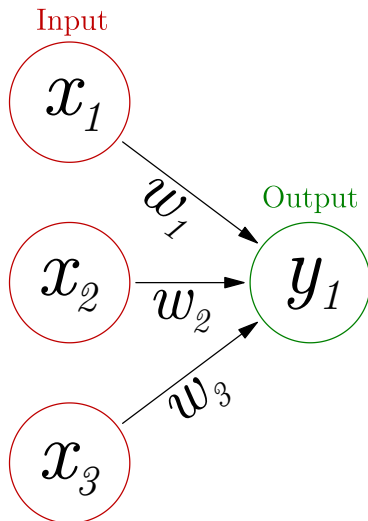
How does the perceptron learn?

Maths again!

1. Initialise weights
2. Run network using:

$$y_j = f\left(\sum_1^N w_i \times x_i\right)$$

same as always!

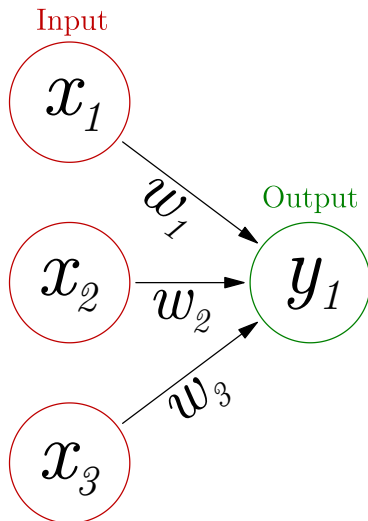


How does the perceptron learn?

Maths again!

1. Initialise weights
2. Run network
3. Update weights using:

$$\Delta w_i = \eta \quad y_j \times x_i$$

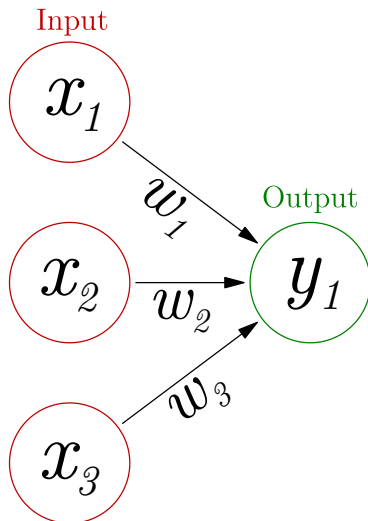


How does the perceptron learn?

Maths again!

1. Initialise weights
2. Run network
3. Update weights using:

$$\Delta w_i = \eta (d_j - y_j) \times x_i$$



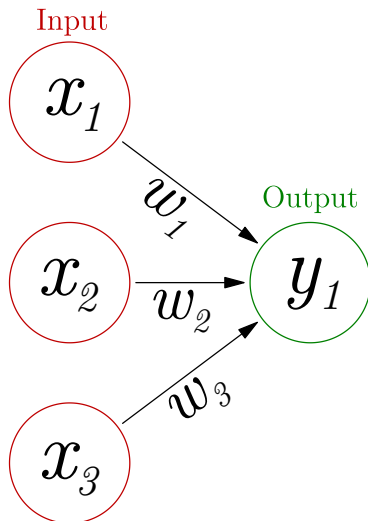
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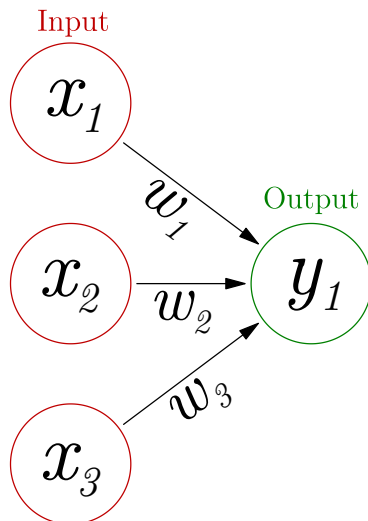
where d is what we want y to be given x , and η is the learning rate.



How does the perceptron learn?

Maths again!

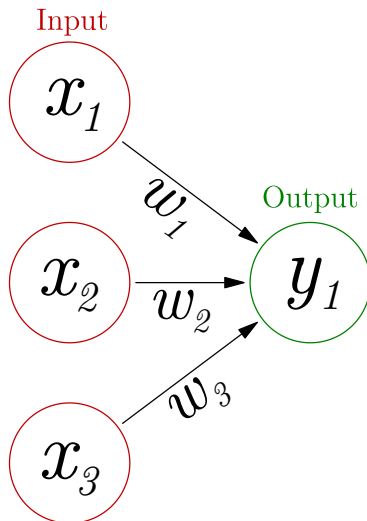
1. Initialise weights
2. Run network
3. Update weights
4. Repeat 2 and 3



How does the perceptron learn?

Maths again!

1. Initialise weights
2. Run network
3. Update weights
4. Repeat 2 and 3
5. When do we stop?



Time to program a perceptron!

Oh, and join our mailing list so we can send you stuff:

<https://groups.google.com/d/forum/introcompcog>

Training the network

```
def Train(self):
```

```
    w[i] += h * error * x[i]  
    self.Draw()
```

Training the network

```
def Train(self):  
    x = self.input_units  
    w = self.weights  
    h = self.learning_rate  
  
    for n in range(100):  
        for p in range(len(self.patterns)):  
            for i in range(len(self.patterns[p])):  
                x[i] = self.patterns[p][i]  
            y = 0  
            for i in range(1+len(self.patterns[p])):  
                y += x[i] * w[i]  
            self.output_units[0] = f(y)  
            error = self.targets[p][0] - self.output_units[0]  
  
            w[i] += h * error * x[i]  
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