HOMEWORK – SCHEME I

22. (append (append xs ys) zs) == (append xs (append ys zs))

Case1. null case

xs = '()

(append (append xs ys) zs) = {by assumption, xs = '()}

(append (append '() ys) zs) = {append-empty law}

(append ys zs) = {by assumption, ys = '()}

ys = '()

(append '() zs) = {append-empty law}

zs = (cons w ws)

(cons w ws)

Case2. single-element list

xs = (cons w ws)
ws = '()

(append (append xs ys) zs) = {by assumption, xs = cons w ws, append-list law}

(append (cons w (append ws ys)) zs) = {by assumption, ws = '()}

(append (cons w (append '() ys)) zs) = {append-empty law}

(append (cons w ys) zs) = {append-list law}

(cons w (append ys zs)) = {by assumption, ys = cons v vs, append-list law}

ys = (cons v vs)
vs = '()

(cons w (append (cons v vs) zs) = {append-list law}

(cons w (cons v (append vs zs))) = {by assumption, vs = '()}

(cons w (cons v (append '() zs))) = {append-empty law} (cons w (cons v zs))

Case1. null case

ys = '()

(append xs (append ys zs)) = {by assumption, ys = '()}

(append xs (append '() zs)) = {append-empty law}

(append xs zs) = {by assumption, xs = '()}

xs = '()

(append '() zs) = {append-empty law}

zs = (cons w ws)

(cons w ws)

Case2. single-element list

ys = (cons v vs)
vs = '()

(append xs (append ys zs)) = {by assumption, ys = cons vs, append-list law}

(append xs (cons v (append vs zs))) = {by assumption, vs = '()}

(append xs (cons v (append '() zs))) = {append-empty law}

(append xs (cons v zs)) = {append-list law}

xs = (cons w ws)
ws = '()

(append (cons w ws) (cons v zs)) = {by assumption, xs = cons w ws, append-list law}

(cons w (append ws (cons v zs))) = {append-list law}

(cons w (append '() (cons v zs))) = {by assumption, ws = '()}

(cons w (cons v zs)) = {append-empty law}

4. (cdr (cons x xs)) = = xs

a) $\langle e, \rho, \sigma_0 \rangle \Downarrow \langle \text{PRIMITIVE (cdr)}, \sigma_1 \rangle$

$$\frac{\langle e_1, \rho, \sigma_1 \rangle \Downarrow \langle \text{PAIR}\langle \ell_1, \ell_2 \rangle, \sigma_2 \rangle}{\langle \text{APPLY}(e, e_1), \rho, \sigma_0 \rangle \Downarrow \langle \sigma_2(\ell_2), \sigma_2 \rangle} \text{ CDR}$$

v of CDR is represented by $\sigma_2(\ell_2)$, which maps to the same ℓ_2 as $\langle \text{PAIR}\langle \ell_1, \ell_2 \rangle, \sigma_2 \rangle$ in the initial evaluation.

Thus, (cdr (cons x xs)) = xs

b) e_1 = ((val x 2) (val y 1) x)

e_2 = (let* [x 3] [y 4] [x y]) x)

ρ = <PAIR <ℓ_1, ℓ_2>>

$\sigma = \{\ell_1 \mapsto 2, \ell_2 \mapsto 4\}$

(cdr (cons e_1 e_2)) $\neq e_2$