Modeling Schistosomiasis Transmission in Togo's Ogou District

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Here I apply a set of ordinary differential equations adapted from Soklow et al. 2017 to which I have added (ι,y,z) to reflect local dynamics observed during fieldwork contributing to Schistosomiasis transmission in Ogou. I also developed an equation to describe the likelihood of mass drug administration (MDA) success as a function of the variables f, a, p, *unknown* affecting treatment compliance.

presence and awake, history of adverse reactions, having eaten something on any given distribution day treatable population is defined as modified by incorporating (iota and ?) to fit the particular setting in Togo's Ogou district. I incorporated ... term into Eq (1) to reflect a consistent number of Ghanian immigrants belonging to the () culture who may have joined the community without having received preventative chemotherapy, increasing the susceptible human population phenomenon observed during fieldwork. Adapted a nonlinear model accounting for snail dispersal

The first equation (1) describes the rate of human population change in endemic villages. the per capita natality rate μ_H represents population growth, to which I incorporated a variable ι representing net migration given the importance of an influx of untreated persons to the transmission dynamics offset by parasite-incduced death given α parasitic pathogenicity. including the mating pair dynamics (?) and the impact of poor sanitation (variable) This N_v is the symbol for the population...

$$\dot{N}_v = \mu_H \iota (H_v - N_v) - \alpha P_v \tag{1}$$

$$\dot{P}_v = F_v N_v - (\mu_H + \mu_P + \alpha) P_v - \alpha \frac{k+1}{k} \frac{P_v^2}{N_v} \tag{2}$$

$$\dot{S}_w = \upsilon S_w [1 - \gamma_w (S_w + E_w + I_w)] - \mu_S S_w - \rho M_w S_w + D_w^s \tag{3}$$

$$\dot{E}_w = \rho M_w S_w - (\mu_s + \eta) I_w + D_w^E \tag{4}$$

$$\dot{I}_w = \delta E_w - (\mu_s + \eta) I_w + D_w^I \tag{5}$$

$$\dot{C}_w = \zeta I_w - \mu_c C_w + L_w^I \tag{6}$$

$$\dot{M}_w = \varrho M_w + L_w^M \tag{7}$$

$$F_v = \beta \sum_{j=1}^{n_w} \Omega_v w C_w \tag{8}$$

$$F_v = \beta \sum_{j=1}^{n_w} \Omega_v w C_w \tag{9}$$

n_v villages
n_w water points
N_v human population size in each village
P_v total number of parasites in human hosts in each village
S_w susceptible snails in freshwater source
E_w exposed snails in freshwater source
I_w infected snails in freshwater source
C_w concentration of cercariae in freshwater source
M_w concentration of miracidia in freshwater source
P_v total number of adult parasites
μ_H natural human host mortality
μ_P per capita parasite mortality
v intrinsic natality rate
γ_w site-specific effect of density dependence
μ_s mortality rate of susceptible snails
: human contamination rate

$$\rho \cap \alpha \tag{10}$$

1 Equation Example

A typical Linear-Time invariant (LTI) system can be described by the equation

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \tag{11}$$

in which $\mathbf{x}(t)$ represents the system's state vector, $\mathbf{A}$ denotes the system matrix that models the interaction between variables, $\mathbf{B}$ the way in which the inputs affect the different states, and $\mathbf{u}(t)$ the chosen control action at time t.

This way of expressing an LTI system yields a deterministic equation. Since I am interested in working with stochastic processes, I chose to add an additional term to this equation. The modified equation looks like

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \underbrace{\mathbf{B}_w\mathbf{w}(t)}_{\text{random noise variable}}, \tag{12}$$

in which the additional term is a factor of two variables:

$$\mathbf{w}(t) \in \mathbb{R}^n : \text{white uncorrelated random noise variable}$$
$$\mathbf{B}_w \in \mathbb{R}^{m \times n} : \text{matrix that denotes which states are corrupted by noise}$$

The reason behind the addition of the variables $\mathbf{w}(t)$ is that in real life scenarios, our sensors are not able to tell us with 100% accuracy what the actual state of our variables are. Ancillary, if we are controling a system, it is unavoidable that the control input commanded from our computers — e.g., the force or torque to be produced at a certain axis — will not be exactly the same as the one produced by our actuators, and there is bound to be some uncertainty there as well. By including this random variable in the equation, and giving it the appropriate noise characteristics and behaviour that accuratelly describes the statistical behavior of our system, we can obtain a stochastic model.