Risk Analytics

Time Series: summary and practice

Frédéric Aviolat & Juraj Bodik

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ARMA processes

- AR(p):

$$X_t = \sum_{j=1}^{p} \phi_j X_{t-j} + \varepsilon_t, \quad t \in \mathbb{Z};$$

- MA(q):

$$X_t = \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t, \quad t \in \mathbb{Z};$$

- ARMA(p, q):

$$X_t = \sum_{j=1}^{p} \phi_j X_{t-j} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t, \quad t \in \mathbb{Z},$$

where (ε_t) is a white noise process.

ARIMA

- ARIMA is an extension to ARMA, with differencing. (I stands for Integrated.)
- Instead of using series (y_t), one can use
  - first-order differencing: $y'_t = y_t - y_{t-1}$;
  - second-order differencing: $y''_t = y'_t - y'_{t-1}$;
  - $\ldots$
  - seasonal differencing: $y^m_t = y_t - y_{t-m}$, where m is the seasonality period.
- ARIMA(p, d, q) is ARMA(p, q) on the d^{th}-order differences.
- ARMA(p, q) $\equiv$ ARIMA(p, 0, q)

ARMA/ARIMA in `R`

In library `fpp2` (see Hyndman and Athanasopoulos [2018]):

- function `Arima`(p, d, q) specifies the orders of the AR, differencing and MA components.
- function `auto.arima` fits best ARIMA model.

In library `fpp3` (see Hyndman and Athanasopoulos [2021]):

- use function `ARIMA`, with parameters (p, d, q) as above and (P, D, Q) for seasonal components (see doc).

ARCH/GARCH

- General form:

$$X_t = \sigma_t Z_t, \quad t \in \mathbb{Z};$$

- ARCH(p):

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_j X_{t-j}^2, \quad \alpha_j > 0.$$

- GARCH(p, q) process:

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_j X_{t-j}^2 + \sum_{k=1}^{q} \beta_k \sigma_{t-k}^2, \quad \alpha_j, \beta_k > 0.$$

GARCH in `R`

- Use library `fGarch`
- See code

References

Rob J Hyndman and George Athanasopoulos. Forecasting: Principles and Practice, 2nd edition. OTexts, https://otexts.com/fpp2/, 2018.

Rob J Hyndman and George Athanasopoulos. Forecasting: Principles and Practice, 3rd edition. OTexts, https://otexts.com/fpp3/, 2021.