Project 1

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Internship at Finance Company

You are a new intern at Lending Tree, which is a Finance Company that raises funds by selling debt instruments and then lends these funds to consumers, who use them to purchase items such as furniture, automobiles, and home improvements.

Question 1: What is the future value (FV) of 3,100 USD in 17 years with an interest rate of 7.5%?

```
pv <- 3100
ttm <- 17
ir <- 7.5/100
fv <- pv *(1+ir)^ttm
```

The future value of 3,100 USD in 17 years when interest rates are 7.5% is 10,599.99 USD.

Question 2: What is the present value (PV) of 700 USD received in 46 years when the interest rate is 7.5%?

```
fv <- 700
ttm <- 46
ir <- 7.5/100
pv <- fv / (1 + ir)^ttm
```

The present value of 700 USD received in 46 years when interest rates are 7.5% is 25.13672 USD.

Question 3: What is the future value (FV) of 10,300 USD in 16 years and 107 days with an interest rate of 9.0%?

```
pv <- 10300
ir<- 9/100
fv<- pv*(1+ir)^((365*16 +107)/365)
```

The future value of 10,300 USD in 16 years and 107 days when interest rates are 9% is 41,940.42 USD.

Question 4: Calculate the PV of investment with cash flows: 1,600, 700, 1,000, 300, 600, 1,400, 1,400,500, and 1,700 USD received in 2, 3, 4, 7, 10, 14, 17, 18, and 19 years with a 2% discount rate.

```
cf <- c(1600, 700, 1000, 300, 600, 1400, 1400, 500,1700)
year<- c(2, 3, 4, 7, 10, 14, 17, 18, 19)
ir <- 2/100
```

```
PV <- cf/(1+ir)^year
PVI <- sum(PV)
```

The future value of cash flows in years when interest rates are 2% is 7,452.583 USD.

Question 5: Calculate the PV of investment with cash flows: -300, 1,300, -400, -100, 1,900, 1,800, and1,400 USD received in -5, -4, -2, 5, 9, 10, and 17 years with discount rates 13.0%, 12.6%, 10.5%, 12.9%,13.1%, 12.9%, and 10.9%.

```
cf <- c(-300, 1300, -400, -100, 1900, 1800,1400)
years <- c(-5, -4, -2, 5, 9, 10, 17)
ir <- c(13.0/100, 12.6/100, 10.5/100, 12.9/100,13.1/100, 12.9/100, 10.9/100)
pv <- cf/(1+ir)^years
PVI <- sum(pv)
```

The future value of cash flows in a series of years at varying interest rates are is 2,397.698 USD.

Question 6: What is the present value (PV) of a fixed payment loan with yearly fixed payments of 164,000 USD and a time to maturity of 99 years when the interest rate is 18.5%?

```
fp <- c(164000)
ttm <- 99
ir <- 18.5/100

cf <- fp * rep(1, ttm)
pv <-sum(cf / (1 + ir)^seq(1, ttm))
```

The present value of a yearly fixed payment of $164,000 for 99 years with an IR of 18.5% is 886,486.4 USD.

Question 7: What is the yearly fixed payment (FP) of a fixed payment loan with a loan value (LV) of 138,000 USD and a time to maturity of 89 years when the interest rate is 12.0%?

```
lv <- 138000
ttm <- 89
ir <- 12/100
fp <- lv / sum((1 / (1 + ir)^seq(1, ttm)))
```

The yearly fixed payment of a loan with a value of $138,000 and a time to maturity of 89 Years with an IR of 12% is 16,560.69 USD.

Question 8: What is the present value (PV) of a 10.5% coupon bond with face value 194,000 USD and a time to maturity of 58 years when coupon payments are yearly and interest rate is 13.0%?

```
cr <- 10.5/100
fv <- 194000
ttm <- 58
ir <- 13/100

cf <- cr * fv * rep(1, ttm)
cf[length(cf)] <- cf[length(cf)] + fv

pv <- sum(cf / (1 + ir)^seq(1, ttm))
```

The present value of a 10.5% coupon bond with a value of $194,000 and a time to maturity of 58 Years with an IR of 13% is 156,723.4 USD.

Question 9: What is the present value (PV) of a perpetuity with a yearly coupon payment of 19,100 USD when the interest rate is 1.5%?

```
cp <- 19100
ir <- 1.5/100
pv <- cp/ir
```

The present value of a perpetuity with a value of $19,100 and an IR of 1.5% is 1,273,333 USD.

Question 10: What is the present value (PV) of a discount bond with face value 62,000 USD due in 12 years when interest rates are 3.5%?

```
fv <- 62000
ttm <- 12
ir <- 3.5/100
pv <- fv / (1 + ir)^ttm
```

The present value of a discount bond with face value 62,000 USD due in 12 years when interest rates are 3.5% is 41,030.56 USD.

Question 11: What is the yield to maturity (YTM) of a discount bond with face value 190,000 USD due in 26 years sold at 66,340 USD?

```
fv <- 190000
ttm <- 26
p <- 66340
ytm <- (fv / p) ^ (1 / ttm) - 1
```

The yield to maturity of a discount bond with face value 190,000 USD due in 26 years sold at 66,340 is 0.04130051 USD.

Question 12: What is the yield to maturity (YTM) of a perpetuity with a yearly coupon payment of 12,600 USD sold at 65,690 USD?

```
cp <- 12600
p <- 65690
ytm <- cp / p
```

The yield to maturity of a perpetuity with a yearly coupon of 12,600 USD sold at 65,690 is 0.19181 USD.

Question 13: What is the yield to maturity (YTM) of a fixed payment loan with yearly fixed payments of 189,000 USD and a time to maturity of 33 years sold at 1,095,480 USD?

```
fp <- 189000
ttm <- 33
p <- 1095480
pv_minus_p <- function(ytm) sum(189000 / (1 + ytm/100)^seq(1, 33)) - 1095480
ans <- uniroot(pv_minus_p, c(-100,200))$root
```

The yield to maturity of a FP loan with payments of 189,000 and a time to maturity of 33 years sold at 1,095,480 is 17.16 USD.

Question 14: What is the yield to maturity (YTM) of a 0.5% annual coupon bond with face value 106,000 USD and a time to maturity of 87 years sold at 14,450 USD?

```
cf <- (0.5 / 100) * 106000 * rep(1, 87)
cf[length(cf)] <- cf[length(cf)] + 106000
pv_minus_p <- function(ytm) sum(cf / (1 + ytm/100)^seq(1, 87)) - 14450
ans <- uniroot(pv_minus_p, c(-100, 100))$root
```

The yield to maturity of a 0.5% annual coupon bond with a face value of 106,000 USD and a time to maturity of 87 years sold at 14,450 is 4.359242 USD.

Question 15: Calculate one-year returns (R) over 2 years of a 2.5% annual coupon bond with face value 166,000 USD and initial time to maturity of 62 years. The initial interest rate is 12.80%, but it changes to 13.70%, and 10.60% after 1, and 2 years.

```
cr <- 2.5/100
fv <- 166000
cp <- fv * cr
ttm <- 62
ir <- c(12.8/100, 13.7/100, 10.6/100)

cf0 <- cp * rep(1, 62)
ans <- cf0[length(cf0)] <- cf0[length(cf0)] + fv
```

The one-returns over 2 years of 2.5% annual coupon bond with a face value of 166,000 USD and initial time to maturity of 62 years is 170,150 USD.

Prepare a Car Loan

In order to prepare the car loan, you first need to acquire the four-year-car-loan from the St. Louis Federal Reserve FRED database. Then, we need to assign values to variables. The interest rate will be assigned to the value acquired from the FRED. Then we assign the loan value ($83,000) to LV. Lastly, we assign the time to maturity (4) to ttm.

```
donotprint <- getSymbols(Symbols = "TERMCBAUTO48NS", src = "FRED")
ir_car <- tail(na.omit(TERMCBAUTO48NS), 1)
lv <- 83000
ttm <- 4
fp <- lv / sum(c(1 / (1 + coredata(ir_car)/100))^seq(1, ttm))
```

In order to calculate present value (PV) of a fixed payment loan, we will use the formula below:

$$PV = \frac{FP}{(1+i)} + \frac{FP}{(1+i)^2} + \cdots + \frac{FP}{(1+i)^n}$$

The loan is issued at PV so that $LV = PV$, and therefore, we can solve for fixed payments (FP) using the formula below- substituting the LV for $83,000, the i for 5.14%, and the n's for their corresponding years (1-4):

$$
\begin{aligned}
LV &= FP\left(\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \cdots + \frac{1}{(1+i)^n}\right) \\
FP &= \frac{LV}{\left(\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \cdots + \frac{1}{(1+i)^n}\right)} \\
FP &= \frac{83,000\ USD}{\left(\frac{1}{(1+5.14\%)} + \frac{1}{(1+5.14\%)^2} + \frac{1}{(1+5.14\%)^3} + \frac{1}{(1+5.14\%)^4}\right)} \\
FP &= 23,483.15\ USD
\end{aligned}
$$

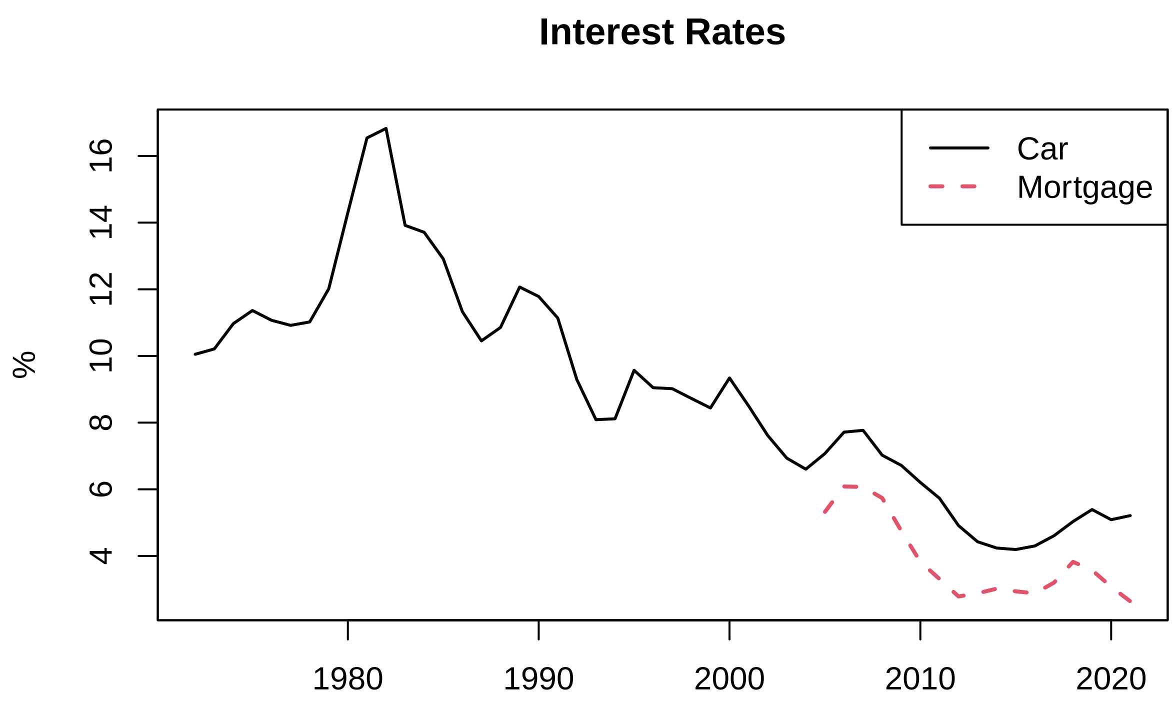
The yearly payment on the car loan is 23,483.15 USD. The total amount the customer pays out on the 83,000 USD. loan is 93,932.6 USD.

Discovery

Part 1

In order to aggregate the interest rates to yearly rates, we need to acquire the interest rates from the FRED for the 5/1-year ARM rate and the four-year-car-loan rate. Then, we will index the rates for each and find the mean. This will give us the average. Afterwards, we will plot the data, giving us the graph below:

```
irs_mortgage <- apply.yearly(MORTGAGE5US, FUN = "mean")
index(irs_mortgage) <- as.Date(format(index(irs_mortgage), "%Y-01-01"))
irs_car <- apply.yearly(TERMCBAUTO48NS, FUN = "mean", na.rm = TRUE)
index(irs_car) <- as.Date(format(index(irs_car), "%Y-01-01"))
irs <- merge(irs_car, irs_mortgage)
plot.zoo(irs, plot.type = "single", col = c(1, 2), lwd = c(1.5, 2), lty = c(1, 2),
         main = "Interest Rates", xlab ="", ylab = "%")
legend("topright", legend = c("Car", "Mortgage"), col = c(1, 2), lwd = c(1.5, 2),
       lty = c(1, 2))
```



After analyzing the graph, it is shown that the ARM Rate is less than the Car Loan Rate. In 2020, the Mortgage rate was running at 4% while the Car Loan rate was running at 6%. This is a 2% difference, with the ARM rate (the rate used to incorrectly calculate car loans) being less. It can also be seen that towards the end of the graph, at year 2020, the lines are veering off in opposite directions - even furthering the gap between the rates.

Part 2

In order to calculate the losses Lending Tree suffered from 2017-2024, we need to again get the yearly rates through the mean.

```
ir_car <- apply.yearly(TERMCBAUTO48NS, FUN = "mean", na.rm = TRUE)
index(ir_car) <- as.Date(format(index(ir_car), "%Y-01-01"))
ir_mortgage <- apply.yearly(MORTGAGE5US, FUN = "mean", na.rm = TRUE)
index(ir_mortgage) <- as.Date(format(index(ir_mortgage), "%Y-01-01"))
ir_both <- merge(ir_car, ir_mortgage)
```

Using the formula below, we can calculate the fixed payments:

$$LV = PV = \frac{FP}{(1+i)} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \frac{FP}{(1+i)^4}$$

$$LV = FP\left(\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \frac{1}{(1+i)^4}\right)$$

$$FP = \frac{LV}{\left(\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \frac{1}{(1+i)^4}\right)}$$

Using this formula, we can calculate the discount factor of both the car loans and the mortgage rates for 2016-2020. Then, we can subtract the total fixed payments of the car loan and the mortgage to get the difference. We can then multiply the default by the difference in order to get the total loss.

```
lv_total <- 2500000
no_default <- .97

# Loss only due to 2016
discount_factor_car_2016 <- 1 / c(1 + coredata(ir_car["2016"]) / 100)^seq(1, 4)
fp_car_2016 <- lv_total / sum(discount_factor_car_2016)
discount_factor_mortgage_2016 <- 1 /
    c(1 + coredata(ir_mortgage["2016"]) / 100)^seq(1, 4)
fp_mortgage_2016 <- lv_total / sum(discount_factor_mortgage_2016)
loss_caused_in_2016 <- fp_mortgage_2016 - fp_car_2016

# Losses caused between 2016 - 2020
irs <- ir_both["2016/2020"]
colnames(irs) <- c("car", "mortgage")
irs <- as.data.table(irs)
irs$fp_car <- NA
irs$fp_mortgage <- NA
irs$index <- as.numeric(format(irs$index, "%Y"))
for (year in irs$index){
    discount_car <- 1 / c(1 + irs$car[irs$index == year] / 100)^seq(1, 4)
    discount_mortgage <- 1 /
        c(1 + irs$mortgage[irs$index == year] / 100)^seq(1, 4)
    irs$fp_car[irs$index == year] <- lv_total / sum(discount_car)
    irs$fp_mortgage[irs$index == year] <- lv_total / sum(discount_mortgage)
}
irs$diff <- irs$fp_mortgage - irs$fp_car
irs$loss <- -no_default * irs$diff
```

The table below utilizes all of the results calculated above and lists the fixed payments calculated with correct and incorrect rates, as well as the resulting difference in future payments received:

```
# Create table
irs_pretty <- copy(irs)
irs_pretty$car <- NULL
irs_pretty$mortgage <- NULL
irs_pretty$fp_car <- format(round(irs_pretty$fp_car, 2), big.mark = ",")
irs_pretty$fp_mortgage <- format(round(irs_pretty$fp_mortgage, 2), big.mark = ",")
irs_pretty$diff <- format(round(irs_pretty$diff, 2), big.mark = ",")
irs_pretty$loss <- format(round(irs_pretty$loss, 2), big.mark = ",")
colnames(irs_pretty) <- c("Time of Mistake", "Correct FP", "Incorrect FP",
                          "Difference", "Yearly Loss")
knitr::kable(irs_pretty)
```

Time of Mistake	Correct FP	Incorrect FP	Difference	Yearly Loss
2016	693,601.1	670,589.3	-23,011.86	22,321.50
2017	698,571.9	675,768.7	-22,803.25	22,119.15
2018	705,561.9	685,852.6	-19,709.32	19,118.04
2019	711,468.8	681,805.9	-29,662.87	28,772.98
2020	706,463.1	673,769.7	-32,693.42	31,712.62

From the table, it can be seen that:

- In 2016, the correct fixed payment was 693,601.1 and the incorrect fixed payment was 670,589.3. The difference between these was 23,011.86, totaling in a loss of 22,321.50
- In 2017, the correct fixed payment was 698,571.9 and the incorrect fixed payment was 675,768.7. The difference between these payments was 22,803.25, totaling in a loss of 22,119.15.
- In 2018, the correct fixed payment was 705,561.9 and the incorrect fixed payment was 685,852.6. The difference between these was 19,709.32, totaling in a loss of 19,118.04.
- In 2019, the correct fixed payment was 711,468.8 and the incorrect fixed payment was 681,805.9. The difference between these payments was 29,662.87, totaling in a loss of 28,772.98.
- In 2020, the correct fixed payment was 706,463.1 and the incorrect fixed payment was 673,769.7. The difference between these was 32,693.42, totaling in a loss of 31,712.62.

We can reorganize the data:

```
# Reorganize
fp_losses <- data.table(year = rep(irs$index, each = 4) + seq(1, 4),
                        losses = rep(irs$loss, each = 4))
fp_losses <- aggregate(list(loss = fp_losses$losses),
                       by = list(year = fp_losses$year), FUN = "sum")
```

The below table lists the losses that occurred and will occur between 2017 and 2024:

```
fp_losses_pretty <- copy(fp_losses)
fp_losses_pretty$loss <- format(round(fp_losses_pretty$loss,2 ), big.mark = ",")
colnames(fp_losses_pretty) <- c("Year of Loss", "Loss")
knitr::kable(fp_losses_pretty, align='lr')
```

Year of Loss	Loss
2017	22,321.50
2018	44,440.65
2019	63,558.70

Year of Loss	Loss
2020	92,331.68
2021	101,722.79
2022	79,603.64
2023	60,485.60
2024	31,712.62

From the table, it can be seen that the total loss increases every year until 2021 where it is at its max at 101,722.79. Then, the losses start to fall afterwards, hitting 31,712.62 in 2024.

Part 3

To calculate the present value of the mistake, we need to use the yearly 1-year Treasury Rate (GSI) from 2017-2020 so that we may be able to compound losses. We will also need to use the 1-Year Treasury Rate of July 2021 to discount future losses.

First, we need to find the yearly rate by indexing the rates and finding the mean. Then, we need to calculate the discount factor in order to get the losses. After getting the losses, we will multiply them by the difference in order to get the present value. Then, we can sum all of these up to get the total present value of the losses.

```
# 1 year interest rates
ir_1year <- apply.yearly(GS1, FUN = "mean", na.rm = TRUE)
index(ir_1year) <- as.Date(format(index(ir_1year), "%Y-01-01"))
ir_future <- GS1["2021-07-01"]
# Include interest rates in table
fp_losses$dr <- c(coredata(ir_1year["2017/2020"]),
                  rep(coredata(ir_future), 4))
# Calculate discount factor
fp_losses$df <- NA
fp_losses$df[fp_losses$year <= 2020] <- rev(
    cumprod(rev(1 + fp_losses$dr[fp_losses$year <= 2020] / 100)))
fp_losses$df[fp_losses$year == 2021] <- 1
fp_losses$df[fp_losses$year > 2021] <- 1/
    cumprod(1 + fp_losses$dr[fp_losses$year > 2021] / 100)
# Calculate present value
fp_losses$pv <- fp_losses$df * fp_losses$loss
# Total loss
pv_total_loss <- sum(fp_losses$pv)
```

Using the information calculated above, we can create a table the lists the present value of past and future losses, as seen below:

```
fp_losses_pretty <- copy(fp_losses)
fp_losses_pretty$loss <- format(round(fp_losses_pretty$loss, 2), big.mark = ",")
fp_losses_pretty$dr <- round(fp_losses_pretty$dr, 2)
fp_losses_pretty$df <- round(fp_losses_pretty$df, 4)
fp_losses_pretty$pv <- format(round(fp_losses_pretty$pv, 2), big.mark = ",")
colnames(fp_losses_pretty) <- c("Year", "Loss", "1-Yr Rate",
                                "Discount Factor", "Present Value")
knitr::kable(fp_losses_pretty, align = "lrllr")
```

Year	Loss	1-Yr Rate	Discount Factor	Present Value
2017	22,321.50	1.20	1.0609	23,680.36
2018	44,440.65	2.33	1.0483	46,586.63

Year	Loss	1-Yr Rate	Discount Factor	Present Value
2019	63,558.70	2.05	1.0244	65,109.71
2020	92,331.68	0.38	1.0038	92,681.77
2021	101,722.79	0.08	1.0000	101,722.79
2022	79,603.64	0.08	0.9992	79,540.01
2023	60,485.60	0.08	0.9984	60,388.94
2024	31,712.62	0.08	0.9976	31,636.63

When looking at the table:

- In 2017, the PV was 23,680.36
- In 2018, the PV was 46,586.63
- In 2019, the PV was 65,109.71
- In 2020, the PV was 92,681.77
- In 2021, the PV was 101,722.79
- In 2022, the PV was 79,540.01
- In 2023, the PV was 60,388.94
- In 2024, the PV was 31,636.63

When summing all of these present values together, we get the total present value of the mistake as 501,347 USD.

Part 4

To calculate the present value of discovering this mistake, we need to assume that all yearly losses of all of the years is equal to 2021 (we calculated it in part 2). Then we can use the perpetuity formula to derive present value.

Then, we can figure out the sunk costs and subtract them from the present value of the perpetuity. This will give us the present value of the discovery.

```
# 30-year rate
ir_perpetuity <- coredata(GS30["2021-07-01"])
cf_perpetuity <- fp_losses$loss[fp_losses$year == 2021]
pv_perpetuity <- cf_perpetuity/(ir_perpetuity / 100)

sunk_cost <- sum(fp_losses$loss[fp_losses$year > 2021] /
                     c(1 + ir_perpetuity/100)^seq(1,3))
pv_discovery <- pv_perpetuity - sunk_cost
```

Present value of perpetuity is found with this formula:

$$PV = \frac{CF}{i} = \frac{101,722.8}{0.0194} = 5,243,443$$

After using this formula in order to get the present value of perpetuity, we subtract the sunk cost - the sum of the discounted losses. Doing this will yield the present value of the discovery which is 5,077,212 USD.