Project 2

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Bond Risks

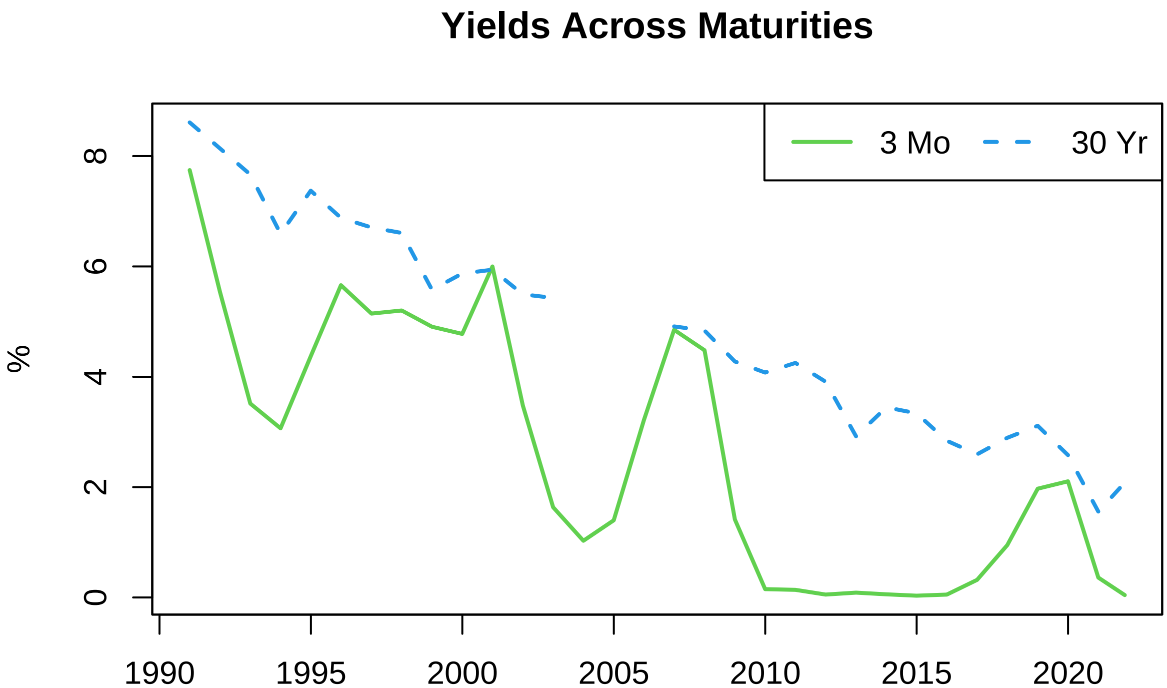
Yield curves track the relationship between interest rates and yields based on the time to maturity. A typical yield curve may measure a series of maturity lengths- ranging from three month treasury bills to thirty year treasury bonds. A positive yield curve will have an upward sloping curve since short term bonds have lower yields than long term bonds. The main risk of a yield curve is due to the fact that bond prices and interest rates have an inverse relationship- meaning that as the interest rate fluctuates, the yield curve will shift. This shift will cause the price of the bond to change as well.

An example that might be able to demonstrate this occurrence of a shifting yield curve would be changes in inflationary expectations - leading to changes in inflation and thus effecting interest rates. An occurrence like this could cause the yield curve to shift upward.

Long term bonds are more sensitive to interest rate changes than short term bonds. This is because there is a greater probability that interest rates will rise (negatively effecting the bonds price) within a longer time period than a shorter one. So, a shift in the yield curve could have varying effects on both long term and short term bond prices.

In the graph shown below, it displays the difference between 3 month Treasury Bonds vs. 30 year Treasury Bonds.

```r
library("readr")
library("xts")
library("knitr")
yd <- read_csv("yieldcurve.csv", na = "N/A")
yd$Date <- as.Date(yd$Date, format = "%y-%d-%m")
yd <- xts(yd[, -1], order.by = yd$Date)
yd_yearly <- apply.yearly(yd, FUN = "mean", na.rm = TRUE)
# Plot of yields:
plot.zoo(yd_yearly[, c(3,12)],
         plot.type = "single",
         col = c(3,12),
         lwd = 2,
         lty = c(1, 2),
         xlab = "",
         ylab = "%",
         main = "Yields Across Maturities")
legend("topright", col= c(3,12), legend = colnames(yd_yearly[, c(3,12)]),
       lty = c(1, 2), lwd = 2, ncol = 2)
```



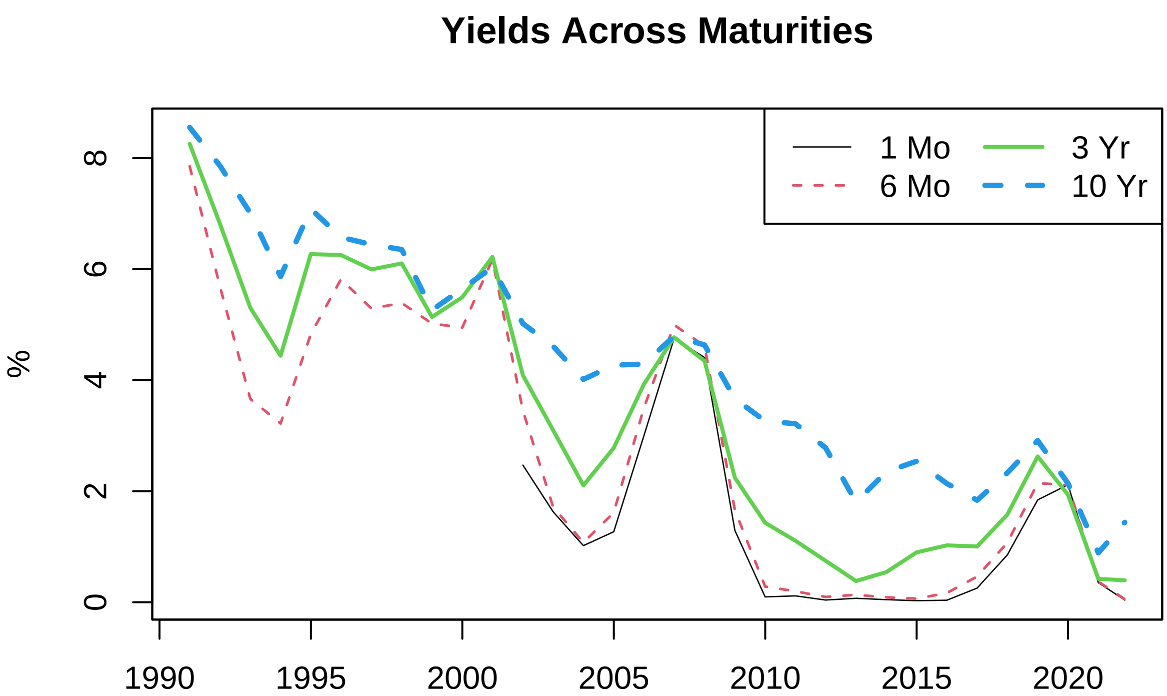
As it can be seen, the yield curve of the short term bonds is much more volatile than that of the long term bonds. This is because when the fed raises short term rates, long term rates must increase in order to reflect increased expectations of short term bond rates. This increase, however, is thwarted by lowered inflation expectations since higher short term rates suggest lower inflation. This cycle also effects bond prices, making them much more volatile for the 30 year treasury bonds as compared to the 3 month bonds due to the longer time to maturity coupled with looming uncertainties of inflation/interest rates.

In order help solve this issue pertaining to bond risks, there exists something known as a liquidity premium. This premium is used to help investors whom are invested in certain things like long term bonds which expose investors to multiple risks. Due to these risks, investors will demand higher returns on their investments- also known as a liquidity premium. Short term bond holders typically require a much smaller liquidity premium as there expectations of yields are far more certain than investors in possession of a 30 year bond.

Evidence on Liquidity Premium

In order to go more in depth about the liquidity premium and its benefits to long term investors, it would be prudent to show this in action. By gathering daily yield curve data for 12 different investment options and examining interest rates, we will be able to get a better understanding of the liquidity premium.

```
# Plot of yields:
plot.zoo(yd_yearly[, seq(1, 12, 3)], plot.type = "single", col = 1:12, lwd = (1:12)/1.5,
         lty = c(1, 2, 1, 2), xlab = "", ylab = "%",
         main = "Yields Across Maturities")
legend("topright", col = 1:12, legend = colnames(yd_yearly[, seq(1, 12, 3)]),
       lty = c(1, 2, 1, 2), lwd = (1:12)/1.5, ncol = 2)
```



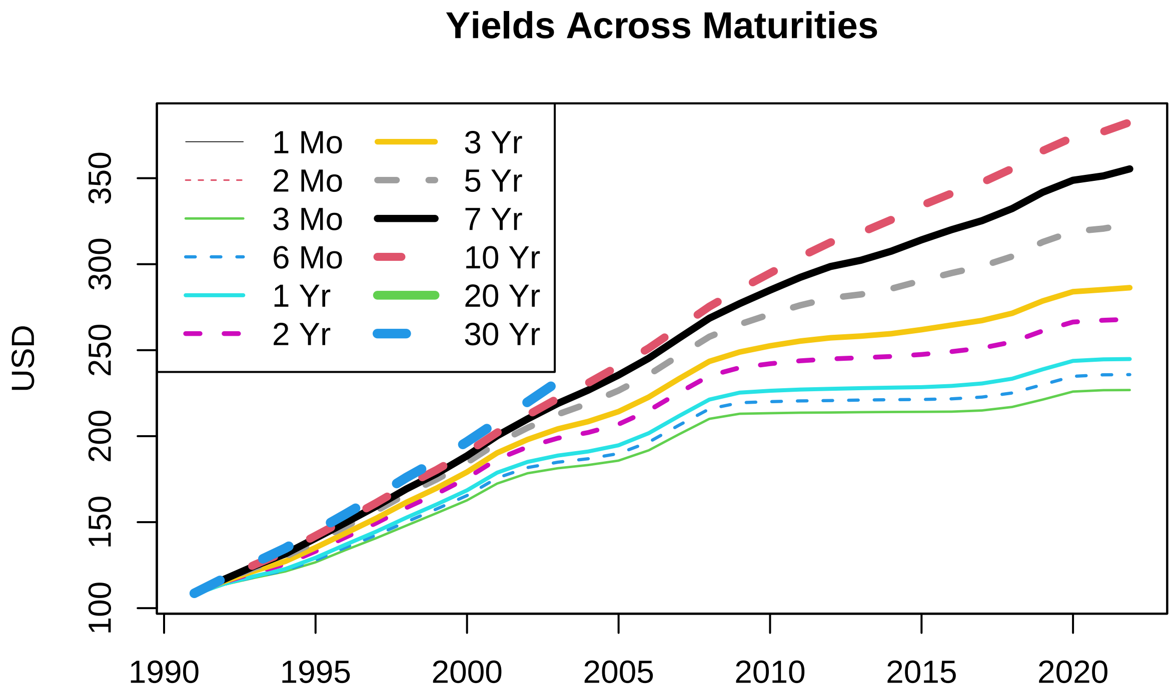
When looking at the graph, there are several observations for each instrument:

- *1 Month*: This instrument has the smallest yield. Since this instrument is an extremely short term bond with only one month until maturity, the liquidity premium received is smaller.
- *6 Months*: This instrument has the second smallest yield. Like the 1 month bond, this instrument still has a relatively small yield since it is still quite short term and only has months until its maturity date.
- *3 Years*: This instrument has the second largest yield. Since this instrument is spanned for 3 years, there is more uncertainty in what rates and inflation will be- leading this bond to be relatively more risky than the 1 month and 6 month bonds. Thus, the liquidity premium is higher.
- *10 years*: This instrument has the largest yield. This bond also has the longest time to maturity, spanning 10 years. Due to this, there is a more extreme level of uncertainty centered around interest rates - hence making this the most risky of all 4 instruments to invest in. Because of this extreme risk, the liquidity premium is much great in order to allow investors to compensate for their risks.

Plot of Investment Stratgey:

In order to further show this relationship between risk and time to maturity, we can create a hypothetical scenario where an investor invested $100 into short, medium and long term bonds.

```
# Plot of investment strategy:
pv_strategy <- 100 * cumprod(1 + yd_yearly / 100)
plot.zoo(pv_strategy, plot.type = "single", col = 1:12, lwd = (1:12)/2.5,
         lty = rep(c(1, 2), 10), xlab = "", ylab = "USD",
         main = "Yields Across Maturities")
legend("topleft", col = 1:12, legend = colnames(pv_strategy),
       lty = rep(c(1, 2), 10), lwd = (1:12)/2.5, ncol = 2)
```



When looking at the graph above, it is quite evident that long term bonds still yield the greatest amount of profits.

- The 30 year, 10 year and 7 year bonds can seen at the top of the graph. At the beginning of the graph, each yield curve is pretty clustered together with a relatively small difference is yield amount. However, as the years go on, the gap gets significantly larger. In 2020, the difference between the 10 yer bonds and 3 month bonds was about $200- a difference that was significantly smaller in 1995.

Medium term bonds yield more than short term bonds but less than long term bonds.

- The 5 year, 3 year, and 2 year bonds all lie without the medium term bond category. The gap between these and both the short and long term bonds is not as extreme as was the case with the 10 year and 3 month bonds. However, the difference between the 5 year bond yield vs the 3 month bond yield was still quite large.

This leaves short term bonds- yielding the smallest amounts.

- The last yield curves to examine that fall within the short term bonds are the 1 year, 6 month and 3 month bonds. These are all relatively small and yield significantly less than the long term bonds.

This can also be represented numerically by examining the data.

- If you had invested 100 USD in 1990, you would have 226.83 USD if you had purchased 3-month Treasury bills, 244.85 USD if invested in 1-year government bonds, and 382.53 USD if invested in 10-year government bonds.
- This further illustrates that liquidity premiums play a big role in how much a bond yields since long term bonds have consistently yielded higher amounts. It has also consistently showed short term bonds yielding the smallest amounts, even further proving this.

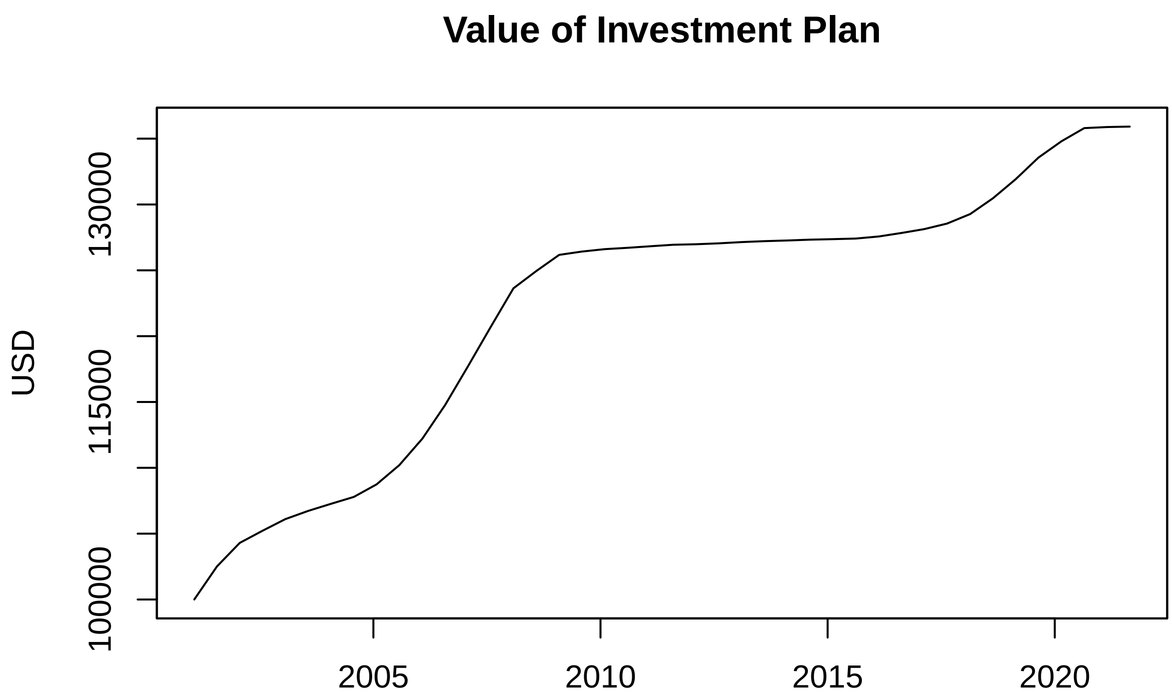
Value of T-Bill Investment Plan

In order to help further illustrate the relationship between bond yields and time to maturity, lets set up a hypothetical investment plan. This plan will center around creating a pension fund for the last two decades. We will invest $100,000 in 6-Month Securities the day we were born. For this analysis, we will use my birthday, January 22, 2001. On this day, the interest rate was 5.11%.

```
yc6m <- yd$`6 Mo`
t_initial <- as.Date("2001-01-22") # Replace "1990-04-13" with your date of birth
min_trade_cycle <- 183 # This is the minimum number of days between purchasing bonds
yd <- na.omit(yc6m) # yc6m is the 6-Mo T-Bill rate from the U.S. Treasury as xts object
# while-loop: purchase next bond at least one day after previous bond matures:
t_trade <- start(yd[index(yd) >= t_initial,])
while (tail(t_trade,1) + min_trade_cycle <= end(yd)) {
    t_trade <- c(t_trade, start(yd[index(yd) >= tail(t_trade,1) + min_trade_cycle,]))
}

ycplan <- yc6m[t_trade]
colnames(ycplan) <- "yc"
ycplan$rc <- (1 + ycplan$yc/100)^(182/365)
ycplan$fc <- cumprod(ycplan$rc)
ycplan$pc <- lag.xts(ycplan$fc)
ycplan$pc[1] <- 1
ycplan$pc <- 100000 * ycplan$pc
ycplan$fc <- 100000 * ycplan$fc

plot.zoo(ycplan$pc, xlab = "", ylab = "USD", main = "Value of Investment Plan")
```



When looking at the graph, it can be seen that the value rose, but not at a constant rate as there are areas in which the value stayed relatively the same for a number of years, like in 2010 where the value stayed at about $122,500 for almost 10 years before the value spiked up to above $130,000. The total value of the pension fund would have 135,915.64 USD today. Had the security type been a longer term type of bond, the pension value might have been even more substantial.

Market Value of T-Bill Investment Plan

Now, lets look at what happens to the value of our investment plan if we decided to sell bonds before their maturity date. In order to do this, we need to calculate both the market and book value of our plan.

- Book value is the amount you paid for an asset minus depreciation, or an asset's reduced value due to

time
- The market value is what an asset would sell for in the current market.

These calculations are important because they help to paint a picture of the overall financial strength - something that is important to know about our investment plan.

Using the formulas below:

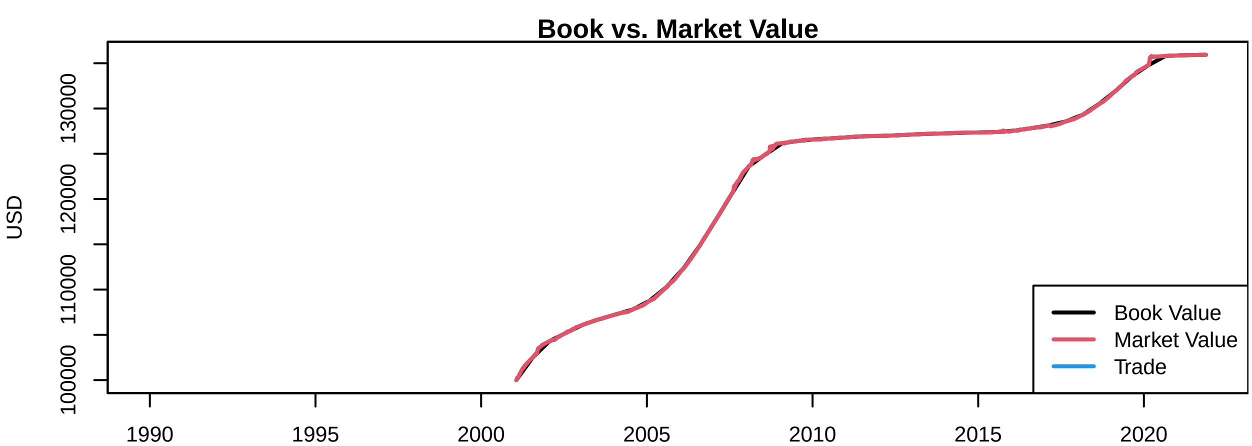
$$BV_t = \frac{FV_t}{(1+i_t^c)^{\frac{m_t}{365}}}$$

$$MV_t = \frac{FV_t}{(1+i_t^c)^{\frac{m_t}{365}}}$$

We can calculate the book value (BV) and market value (MV).

```
ycplan$tm <- xts(index(ycplan), order.by = index(ycplan))
colnames(yc6m) <- "i"
ycdaily <- merge(ycplan, yc6m)
regt <- xts(NULL, order.by = seq(start(ycdaily), end(ycdaily), "1 day"))
ycdaily <- merge(ycdaily, regt)
ycdaily <- na.locf(ycdaily)
ycdaily$m <- as.Date(coredata(ycdaily$tm)) + 182 - index(ycdaily)
ycdaily$m[ycdaily$m < 0] <- 0

# Get market value (mv): -> Mishkin p. 74: PV = FV/(1+i)^n
ycdaily$mv <- ycdaily$fc/(1 + ycdaily$i / 100)^(ycdaily$m / 365)
# Get book value (bv):
ycdaily$bv <- ycdaily$fc/(1 + ycdaily$yc / 100)^(ycdaily$m / 365)
# Subset of periods:
periods_subset <- seq(as.Date("2007-11-01"),as.Date("2009-05-31"),by="day")
# Plot MV and BV:
par(mfrow = c(2,1), mar = c(2, 4, 1, 0), cex = .7)
plot.zoo(ycdaily["/", c("bv", "mv")],
         plot.type = "single", col = 1:2, lwd = 2, xlab = "", ylab = "USD",
         main = "Book vs. Market Value")
legend(x = "bottomright", legend = c("Book Value", "Market Value", "Trade"),
       col=c(1:2,4), lwd = c(2,2,2), bg = "white")
```



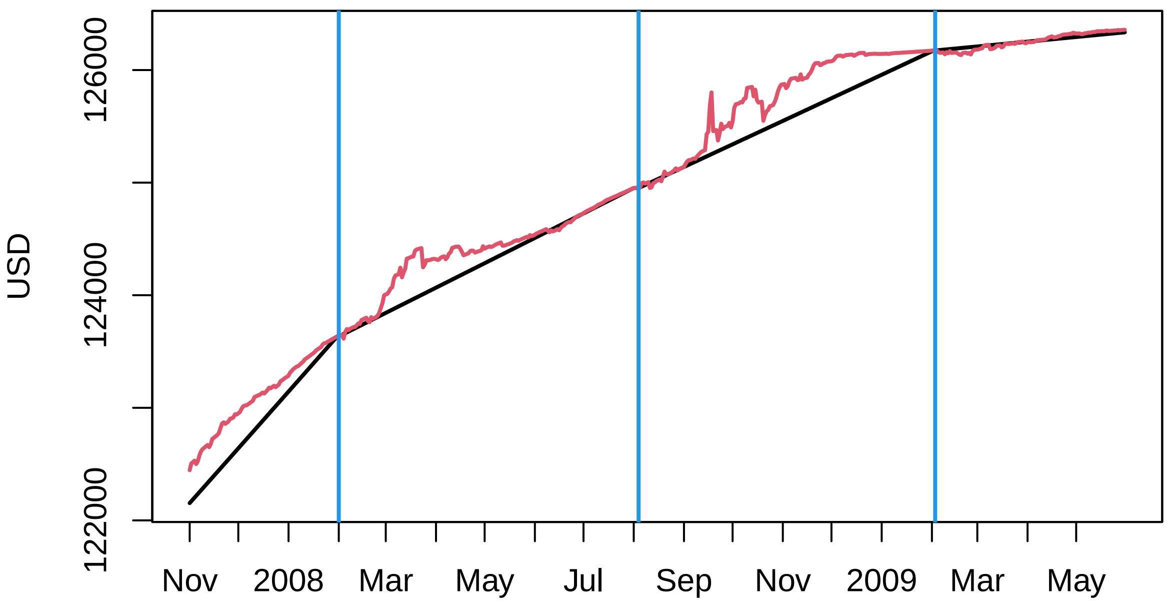
How do we interpret this data?

There are three possible outcomes from these calculations:

1. The book value is higher than market value: This means that asset would sell for less than what it was bought for.
2. The market value is higher than book value: This means that the asset would sell for more than what it was bought for.
3. They are equal: This means the market sees the value to be the same as what you paid for.

Now, lets look at the graph above housing the book value and market value that we just calculated. When analyzing this graph, it can be seen that both the book value and the market value lines lay on top of each other. So, in the long run, the value of these are the same. This proves that our investment is not risky since we know that if they are equal, the value of our asset is equal to market value.

```
plot.zoo(ycdaily[periods_subset, c("bv", "mv")], plot.type = "single",
         col = 1:2, lwd = 2, xlab = "", ylab = "USD", xaxt = 'n')
axat <- index(ycdaily[periods_subset, c("bv", "mv")])
axat <- na.omit(axat[format(axat, "%d") == "01"])
axlabel <- format(axat, "%b")
axlabel[format(axat, "%m") == "01"] <- format(axat[format(axat, "%m") == "01"], "%Y")
axis(side = 1, at = axat, label = axlabel)
abline(v = t_trade, col = 4, lwd = 2)
```



Now, lets look at this figure above which plots the values over a subset of periods between November 01, 2007, and May 31, 2009. Now it is apparent that the market value can deviate significantly from the book value in the short-run. Hence, in the short run, a cash emergency can cause a significant drop or spike in book value. The market value can very widely in a cash crunch so there is risk in the short run while there is very little risk in the long term.

Returns of T-Bill Investment Plan

Next, we now need to calculate the daily book value returns and market returns of the investment plan in Basis Points (BPS).

- Basis points are important because they provide a more precise measure of the amounts in which interest rates change. This will be very important for our analysis as well as knowledge as to what type of security one should invest in.

In order to calculate the basis points of our book and market value, we can use the equations below:

$$BR_t = 100^2 * \frac{BV_t - BV_{t-1}}{BV_{t-1}}$$

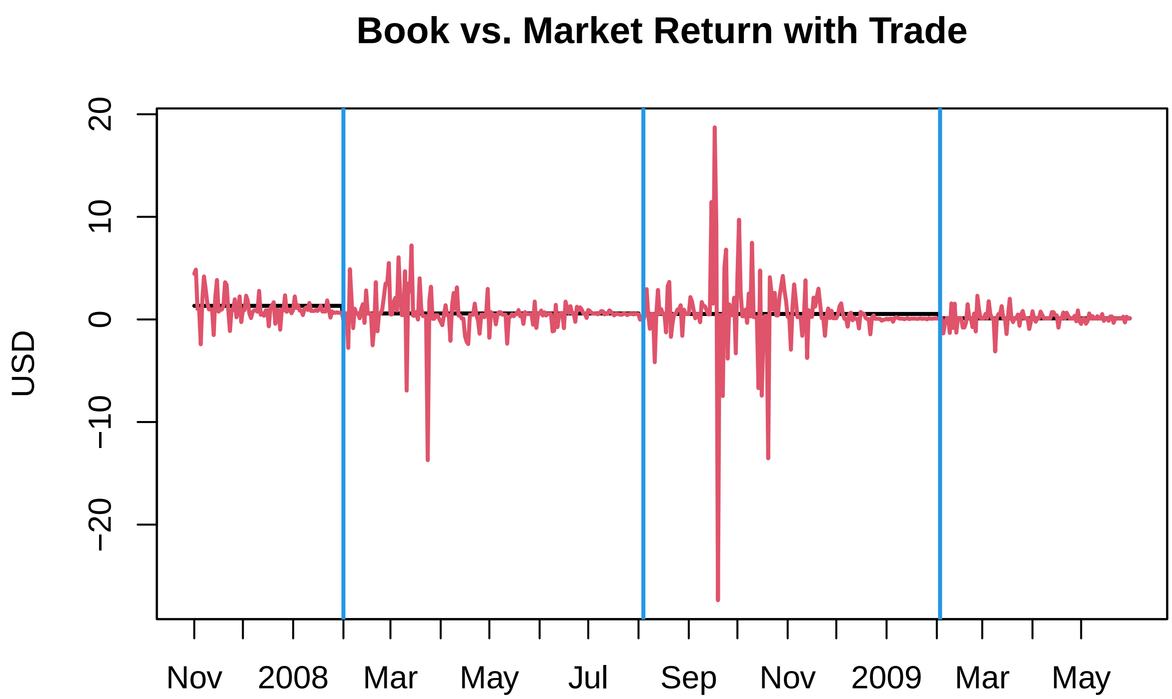
$$MR_t = 100^2 * \frac{MV_t - MV_{t-1}}{MV_{t-1}}$$

Then from these equations, we can plot the results as seen in the table below:

```
#Calculate Daily Book Returns and market returns:
# 1 basis point is 0.01 percentage
# realize risk is much longer with ttm
ycdaily$mr <- 100^2 * (ycdaily$mv - lag.xts(ycdaily$mv))/ lag.xts(ycdaily$mv)
ycdaily$br <- 100^2 * (ycdaily$bv - lag.xts(ycdaily$bv))/ lag.xts(ycdaily$bv)
# Plot MR and BR:
par(mfrow = c(2,1), mar = c(2, 4, 1, 0), cex = .7)
plot.zoo(ycdaily["/", c("br", "mr")],
         plot.type = "single", col = 1:2, lwd = 2, xlab = "", ylab = "USD",
         main = "Book vs. Market Return")
legend(x = "bottomright", legend = c("Book Return", "Market Return", "Trade"),
       col=c(1:2,4), lwd = c(2,2,2), bg = "white")
```

As it can be seen, the market return is very volatile compared to the book return. This is very important to note since basis points can express an assets change in value. So, this graph gives us a better depiction and more precise measure of the book value compared to the market value. By looking at the volatility, we can now see that this may now be riskier.

```
plot.zoo(ycdaily[periods_subset, c("br", "mr")], plot.type = "single",
         col = 1:2, lwd = 2, xlab = "", ylab = "USD", xaxt = 'n', main = "Book vs. Market Return with T
axat <- index(ycdaily[periods_subset, c("br", "mr")])
axat <- na.omit(axat[format(axat, "%d") == "01"])
axlabel <- format(axat, "%b")
axlabel[format(axat, "%m") == "01"] <- format(axat[format(axat, "%m") == "01"], "%Y")
axis(side = 1, at = axat, label = axlabel)
abline(v = t_trade, col = 4, lwd = 2)
```



This graph above again illustrates the book return vs market return except for the fact that it now also displays the days in which the 6 month bonds are traded. As it can be seen, the market value is still exceedingly volatile- except for the days in which the bonds are traded. In all of these instances, the book return and market return lines merge.

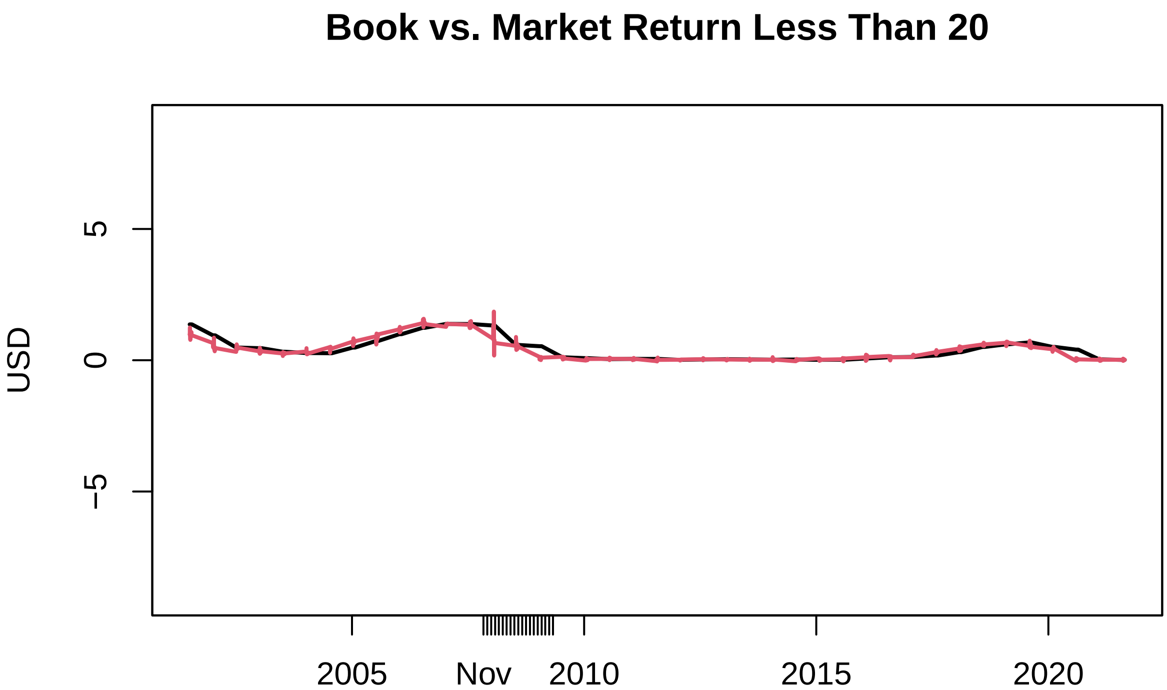
Now we will look at MR and BR with different maturities:

```
# Plot MR and BR different maturities:
par(mfrow = c(2,1), mar = c(2, 4, 1, 0), cex = .7)
plot.zoo(ycdaily[ycdaily$m > 162 & ycdaily$m < 182, c("br", "mr")],
         plot.type = "single", col = 1:2, lwd = 2, xlab = "", ylab = "USD",
         main = "Book vs. Market Return Greater Than 162", ylim = c(-9, 9))
legend(x = "bottomright", legend = c("Book Return", "Market Return", "Trade"),
       col=c(1:2,4), lwd = c(2,2,2), bg = "white")
```

In this graph, we are looking at days where the time to maturity is larger than 162 days but not equal to 182.

```
plot.zoo(ycdaily[ycdaily$m < 20 & ycdaily$m > 0, c("br", "mr")],
         plot.type = "single", col = 1:2, lwd = 2, xlab = "", ylab = "USD",
```

```
         main = "Book vs. Market Return Less Than 20", ylim = c(-9, 9))
axat <- index(ycdaily[periods_subset, c("br", "mr")])
axat <- na.omit(axat[format(axat, "%d") == "01"])
axlabel <- format(axat, "%b")
axlabel[format(axat, "%m") == "01"] <- format(axat[format(axat, "%m") == "01"], "%Y")
axis(side = 1, at = axat, label = axlabel)
```



In this graph, we are looking at days where the time to maturity is smaller than 20 but not equal to 0.

After analyzing both of these graphs, it can be seen that bonds with a longer time to maturity are more volatile and thus more risky than that of those whom have a smaller time to maturity. The graph plotting days with a time to maturity less than 20 have less volatile movements and the lines are essentially on top of each other through the whole graph except for in certain sections. However, in the graph where to time to maturity is larger than 162 days, the graph lines are extremely volatile and never really lay flat on top of each other. These two graphs lead us to the fact that longer maturity bonds are relatively more risky than shorter term bonds.

Value at Risk of T-Bill Investment Plan

To get a better idea of the value at risk, we will examine the five largest positive and negative market returns.

```
ycdaily$di <- ycdaily$i - lag.xts(ycdaily$i)
ycdf <- na.omit(as.data.frame(ycdaily))
ycdf$index <- rownames(ycdf)
ycdf_extreme <- ycdf[order(ycdf$mr), ][c(1:5, (nrow(ycdf) - 4):nrow(ycdf)), ]
kable(ycdf_extreme[, c("yc", "i", "di", "m", "mr")], format = "simple")
```

	yc	i	di	m	mr
2008-09-19	1.97	1.54	0.75	136	-27.37041
2007-08-27	4.95	4.69	0.37	157	-14.06041
2008-03-24	2.15	1.60	0.40	130	-13.71367
2008-10-20	1.97	1.74	0.49	105	-13.53877
2007-09-04	4.95	4.52	0.31	149	-10.98968

	yc	i	di	m	mr
2008-09-15	1.97	1.55	-0.29	140	11.44397
2007-08-16	4.95	4.22	-0.26	168	12.67699
2007-08-15	4.95	4.48	-0.33	169	15.90096
2001-09-13	3.52	2.75	-0.48	131	17.61372
2008-09-17	1.97	1.03	-0.49	138	18.72364

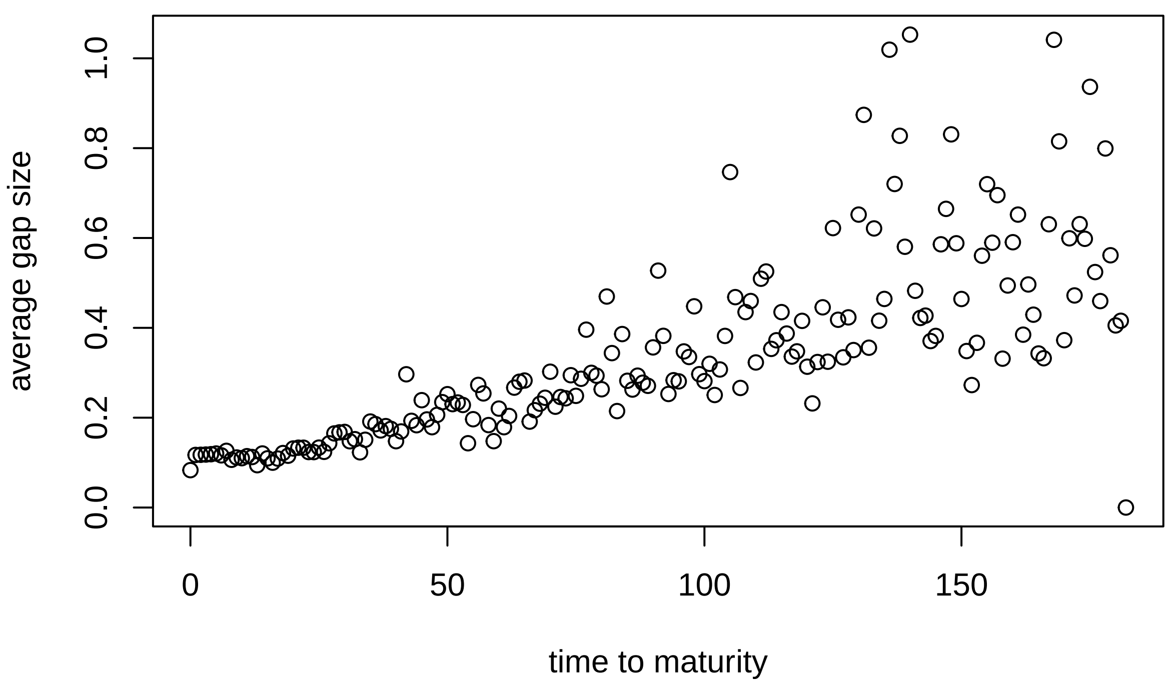
The table above houses the contract and market interest rate, change in interest rate, time to maturity and market return. When analyzing the results, it can be seen that for periods when the market interest was at its highest, the interest rate had fallen by various amounts for each. Thus, these results prove that returns and interest rates have an inverse relationship.

The Drivers of Interest Rate Risks

To tie all of our observations and findings that we have made together, we will show that the time to maturity and interest rates are what causes our investment plan to have risks.

In order to do this, we will need to find the gap size between market and book returns across the time to maturity. The gap size is important because it shows where a securities price either rises or falls from the previous days price. This will help to show price volatility.

```
ycdaily$gap <- ycdaily$mr - ycdaily$br
ycdaily$gapsize <- abs(ycdaily$mr - ycdaily$br)
ycdf <- na.omit(as.data.frame(ycdaily))
gapsize_over_m <- aggregate(ycdf$gapsize, by = list(ycdf$m), FUN = "mean")
plot(x = gapsize_over_m$Group.1, y = gapsize_over_m$x, xlab= "time to maturity", ylab="average gap size
```



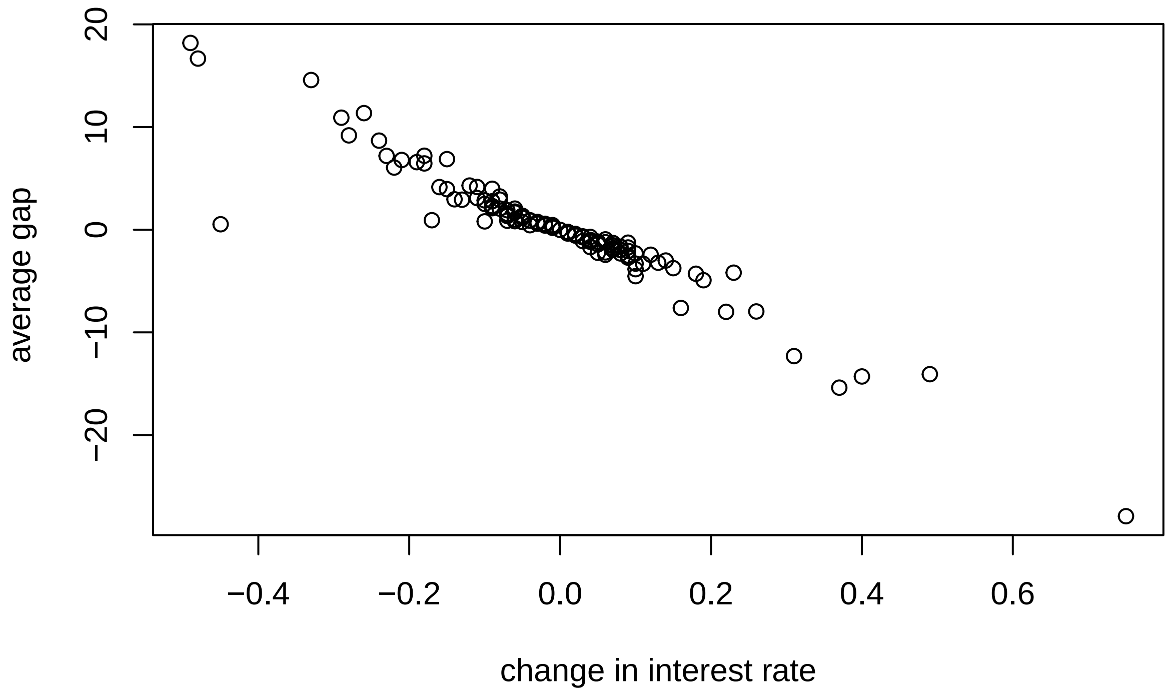

```
cor(gapsize_over_m$Group.1, gapsize_over_m$x)
```

```
## [1] 0.7652735
```

```
gap_over_di <- aggregate(ycdf$gap, by = list(ycdf$di), FUN = "mean")
```

The graph above calculates average gap size vs time to maturity. The correlation between these is 0.7652735.

```
plot(x = gap_over_di$Group.1, y = gap_over_di$x, xlab="change in interest rate", ylab=" average gap")
```




```
cor(gap_over_di$Group.1, gap_over_di$x)
```

```
## [1] -0.9506541
```

The graph above calculates average gap for each change in the interest rate. The correlation is -0.9506541.

After analyzing the outcomes of both of the graphs above, the outcomes of the first gap shows that the gap size is very correlated with the time to maturity- meaning that price volatility is positively correlated with an increase in the time to maturity. The outcomes of the second graph show that the average gap size is negatively correlated to changes interest - meaning that prices and inversely related to interest rate changes. So, the following are proved true:

- Time to maturity has a big effect on market value if it is for long term bonds
- Time to maturity has a small effect on short term bonds
- Interest rates have an inverse relationship with bond prices

So, the outcomes of this analysis point towards interest rates and the time maturity as being the main culprits for risk in the investment plan.