1. $m(a+bX) = a + b \cdot m(x)$

$$m(a+bx) = \frac{1}{N}\sum_{i=1}^{N}(a+bx)$$

$$= \frac{1}{N}\left(\underbrace{\sum_{i=1}^{N} a}_{\text{the mean of a constant is that constant}} + b\sum_{i=1}^{N} x\right)$$

$$= a + b\underbrace{\frac{1}{n}\sum_{i=1}^{N} x}_{m(x)}$$

$$\boxed{m(a+bx) = a + b\,m(x)}$$

2. $\text{cov}(x, x) = s^2$

$$\text{cov}(x,y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(Y))$$

$$s^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2$$

$$\text{cov}(x,x) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(x_i - m(x))$$

$$= \underbrace{\frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2}_{s^2}$$

$$\boxed{\text{cov}(x,x) = s^2}$$

3. $\text{cov}(x, a+bY) = b \cdot \text{cov}(x, Y)$

$$\text{cov}(x,y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(Y))$$

$$\text{cov}(x, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))\left((a+by_i) - \underbrace{m(a+bY)}_{a+bm(Y)}\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(\cancel{a} + by_i - \cancel{a} - bm(Y))$$

$$= \underbrace{\frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))\,b\,(y_i - m(Y))}_{\text{cov}(x,Y)}$$

$$\boxed{\text{cov}(x, a+bY) = b\,\text{cov}(x,Y)}$$

4. $\mathrm{cov}(a+bX,\ a+bY) = b^2\,\mathrm{cov}(X,Y)$

$$= \frac{1}{N}\sum_{i=1}^{N}\left((a+bx_i) - m(a+bX)\right)\left((a+by_i) - m(a+bY)\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left((\cancel{a}+\underline{b}x_i) - (\cancel{a}+\underline{b}m(x))\right)\left((\cancel{a}+\underline{b}y_i) - (\cancel{a}+\underline{b}m(Y))\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(b(x_i - m(x))\right)\left(b(y_i - m(Y))\right)$$

$$\boxed{\mathrm{cov}(a+bx,\ a+by) = b^2\,\mathrm{cov}(X,Y)}$$

5. Is $\mathrm{med}(a+bX)$ equal to $a + b\,\mathrm{med}(x)$? → Yes

Applying the transformation $a+bx$ will not change the order of the data, so the order will stay the same and thus the median will stay the same as well. The actual numerical value of the median will be multiplied by b and increased by a in both cases, so that remains the same as well.

Is the IQR of $a+bX$ equal to $a + b\,\mathrm{IQR}(x)$?

Yes. The order of the data don't change, and everything is scaled the same in both cases. The IQR will scale with b.

6. $X = \{1, 9\}$

$$m(x^2) = \frac{1+81}{2} = 41$$

$$(m(x))^2 = \left(\frac{1+9}{2}\right)^2 = 25$$

$41 \neq 25 \quad \therefore \quad m(x^2) \neq (m(x))^2$

$$m(\sqrt{x}) = \frac{1+3}{2} = 2$$

$$\sqrt{m(x)} = \sqrt{\frac{1+9}{2}} = \sqrt{5}$$

$2 \neq \sqrt{5} \quad \therefore \quad m(\sqrt{x}) \neq \sqrt{m(x)}$