

# University of Delaware

## CISC260 Homework 2 Solution

Jiefu Li

March 19, 2018

### 1 Notation

Here, let me use the following notations for logic:

$\bar{X}$ : NOT X .

$X + Y$ : X or Y.

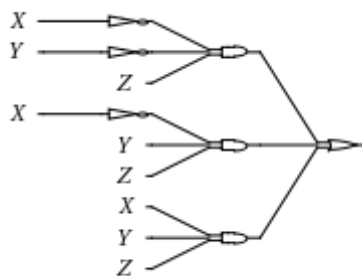
$X * Y$ : X and Y.

### 2 Question 1.

[25 points] Given the following truth table, where X, Y, and Z are input and W is output, write the canonical expression and generate gate-level logical circuit.

X	Y	Z	W
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

**Answer:**  $W = \bar{X} * \bar{Y} * Z + \bar{X} * Y * Z + X * Y * Z$  The circuit is shown as following.



### 3 Question 2.

[25 points] Write the Boolean expression and fill out the truth table for following logical circuit.

### Answer

The expression is:

$$F = A * (B + \bar{C}) + B * (C + \bar{D}) + BD$$

The truth table is:

A	B	C	D	E	F
0	0	0	0	0	0
0	0	0	1		0
0	0	1	0		0
0	0	1	1		0
0	1	0	0		1
0	1	0	1		1
0	1	1	0		1
0	1	1	1		1
1	0	0	0		1
1	0	0	1		1
1	0	1	0		0
1	0	1	1		0
1	1	0	0		1
1	1	0	1		1
1	1	1	0		1
1	1	1	1		1

## 4 Question 3.

[25 points] You are asked to design a circuit to detect if an overflow occurs when adding two integers represented in two's complement:  $Z = X + Y$ . Let  $S_z$ ,  $S_x$ , and  $S_y$  be the sign bit for  $Z$ ,  $X$ , and  $Y$  respectively, and they are fed as input to the circuit. Let  $O$  be the output bit of the circuit, whose value is 1 if an overflow happens, and 0 if otherwise.

### Answer

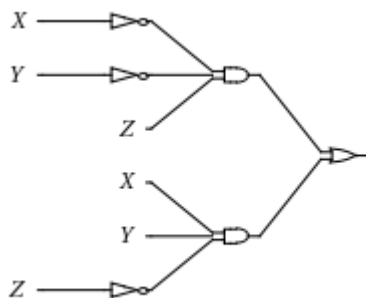
The truth table is:

$S_x$	$S_y$	$S_z$	$O$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

From the above truth table, the expression is:

$$O = \bar{S}_x * \bar{S}_y * S_z + S_x * S_y * \bar{S}_z$$

Then, from the expression, the circuit is the following:



## 5 Question 4.

[25 points] Prove that NOR gate is universal, by showing how AND, OR, and NOT gates can be built by wiring a bunch of NOR gates. Draw the wire diagram for each case.

**Answer**

NOR to NOT:  
 $NOT(x) = (\bar{x}) = \overline{(x + x)} = NOR(x, x)$

NOR to OR:  
 $OR(x, y) = x + y = NOT(\overline{(x + y)}) = NOR(NOR(x, y), NOR(x, y))$

NOR to AND:  
 $AND(x, y) = x * y = NOT(\bar{x} + \bar{y}) = NOR(NOR(x, x), NOR(y, y))$   
Then, Wire diagram is trivial.