Fixed Income Derivatives E2025 - Problem Set Week 11

Problem 1

In this problem, we will fit the market using a Hull-White Extended Vasicek model and use Jamshidian decomposition to compute prices of interest rate swaptions on an underlying 5Y receiver swap paying an annual fixed coupon rate against 6M Euribor and investigate what the Black implied volatility surface looks like for these swaptions. Assume that the 6M Euribor has just been announced and that you have the following market data available for FRA's and interest rate swaps paying a fixed

Table 1: Euribor fixing, FRA and swap market data

EURIBOR	Fixing	FRA	Midquote	IRS	Midquote
6M	0.04098	1X7	0.04178	2Y	0.04674
		2X8	0.04255	3Y	0.04890
		3X9	0.04327	4Y	0.05041
		4X10	0.04396	5Y	0.05150
		5X11	0.04461	7Y	0.05291
		6X12	0.04523	10Y	0.05406
		7X13	0.04581	15Y	0.05496
		8X14	0.04637	20Y	0.05540
		9X15	0.0469	30Y	0.05580

a) Fit a term structure of continuously compounded spot rates to the market data in Table 1 and plot the term structures of continuously compounded spot rates, forward rates and the original market data in a plot.

b) Fit a Hull-White Extended Vasicek Model to the term structures from a) in which you set $a = 0.5$ and $\sigma = 0.025$. Then plot f^*, f_T^* and $\Theta(t)$ corresponding to your fit.

c) Compute 5Y forward par swap rates for 5Y interest rates swap beginning at times $T_n \in [0.5, 1, 2, 3, 5]$ and report these in a table.

d) For the following choices of strike offsets in bps from the 5Y forward par swap rates you found in c): $[-100, -75, -50, -25, 0, 25, 50, 75, 100]$, compute swaption prices per unit of principal (that is in bps) for all exercise times $T_n \in [0.5, 1, 2, 3, 5]$ and report these values in a table. In a second table, report the value of r^* used when computing each of the swaption prices and in a third table, report the strike of each of the swaptions in terms of a forward par swap rate and not just the offset from the ATM value of the underlying par swap rate.

e) Give an interpretation of r^* and relate this value to the strikes in terms of the forward par swap rates, you found in d).

f) Find Black implied volatilities corresponding to each of the swaption prices in d) and plot these implied volatilities as a function of the strike offset for each swaption exercise time.

g) Interpret the plot of implied volatilities from e). Are swaption prices consistent with an assumption that 5Y forward par swap rates follow a Geometric Brownian motion? If not, what can then be said about the distribution of the underlying forward par swap rates in the Hull-White Extended Vasicek Model? How do you think the shape of the implied volatility surface depends on the parameters a and σ?

Problem 2

In this problem, we will assume the market data given used in Problem 1 and given in Table 1, and we will use the Hull-White Extended Vasicek Model we fitted in Problem 1b. In addition, we will consider a 10Y fixed rate bullet bond paying an annual simple coupon rate of 6 % semi-annually. This bond has just been issued and for simplicity, we will assume that it has a principal of 1.

a) Compute the price of the 10Y fixed rate bullet bond at present time $t = 0$.

b) Compute the DV01 for a 1 bps increase in each of the 2Y, 4Y, 6Y, 8Y and 10Y continuously compounded spot rates. Also compute the DV01 if the entire continuously compounded spot rate curve were to increase by one bps.

c) Find an expression for the return to the 10Y fixed rate bond in exactly one year from now right after the second coupon has been paid. In doing so, you can assume that the coupon paid in six months is invested at present time $t = 0$.

d) Using simulation and at least $M = 1000$ steps, compute the $\alpha = 0.95$ Value-at-Risk in exactly one year assuming the assumptions from above. Report your answer both in terms of a monetary value and in terms of a loss in percent.

e) Using what we know about the relationship between the value of a fixed rate bond and the short rate in a Hull-White Extended Vasicek Model, compute the $\alpha = 0.95$ VaR from d) explicitly both in monetary units and in terms of a loss in percent.

f) Give an interpretation of the VaR numbers in d) and e) and explain why it can be very problematic to compute VaR, especially for α close to 1. using simulation. Also explain in which cases you are forced to use simulation, both in the single asset and portfolio case.