Homework 16-02

2U – Curve-sketching

1. Complete the following questions from the Grind section of Curve Sketching: 3-9

3U – Projectile Motion

2. Complete the below questions.

$\therefore 0.29 \le \tan\alpha \le ...$

$\therefore 16°10' \le \alpha \le 76°38'$, since $\tan\alpha$ is a continuous and increasing function for $0° \le \alpha < 90°$.

$\therefore$ To clear the fence the particle must be projected at an angle of projection α such that $16°10' \le \alpha \le 76°38'$.

Exercise 8.3

Take $g = 10\ \text{ms}^{-2}$ for questions 1 to 5. Take $g = 9.8\ \text{ms}^{-2}$ for questions 6 to 11.

1 A golfer hits a golf ball from his tee with a speed of 20 m/s at an angle of 30° with the ground.
(a) Derive the parametric equations of motion of the golf ball.
(b) Find the time taken to reach the ground.
(c) Find the horizontal range travelled by the ball.

2 A particle is projected from the top of a cliff, 240 m high, above the sea level with initial velocity of 40 m/s that makes an angle of projection of $\tan^{-1}\frac{3}{4}$ with the horizontal.
(a) Considering the foot of the cliff to be the origin derive the parametric equations of the path of the particle.
(b) Find when and where the particle strikes the sea.

3 A tennis ball is smashed at a point 1.8 m high with a speed of 65 m/s at an angle of −20° with the horizontal.
(a) Considering the point of release to be the origin derive the parametric equations of the path of the ball.
(b) Find the position of the ball when it strikes the ground.

4 A ball is projected horizontally from the top of a building 12 m high to hit a point 20 m horizontally on the ground.
(a) Considering the base of the building to be the origin derive the parametric equations of the motion of the ball.
(b) Find the time when it hits the ground.
(c) Find the initial speed.
(d) Find the velocity (both the magnitude and direction) with which the particle hits the ground.

5 A plane flying horizontally at 160 m/s releases a bomb to hit a target which is on the ground and at an angle of depression of 32° as seen from the plane.
The bomb reaches the ground in 20 seconds.
You are given these equations of motion of the bomb: $x = 160t, y = -5t^2$.
(a) Find the height of the plane.
(b) Determine whether the target is hit.
(c) Find the magnitude and the direction of the velocity of the bomb when it hits the ground.

6 A stone is projected horizontally with initial speed U from the top of a cliff 50 m above the sea level. Its equations of motion are $x = Ut, y = -4.9t^2 + 50$.
At the same instant a ball is thrown from the bottom of the cliff in the same vertical plane as the stone with initial speed V and inclines an angle of 34° with the horizontal.
(a) Derive the parametric equations of the motion of the ball.

(b) If the stone and the ball collide in mid-air after 1.5 seconds, find their initial speeds and the coordinates of the point of collision.

7 A stone is projected from the ground with initial speed of 20 m/s which makes an angle of 30° with the ground. Its equations of motion are $x = 10\sqrt{3}t, y = -4.9t^2 + 10t$.
At the same instant a ball is thrown from the ground at a point 50 m away in the same vertical plane as the stone with the initial speed of 34 m/s but inclines at an angle of 162°53' with the horizontal.
(a) Derive the parametric equations of the motion of the ball.
(b) The stone and the ball collide in mid-air. Find the time of collision and the co-ordinates of the point of collision.

In questions 8 to 13, you may use these parametric equations $x = Ut\cos\alpha, y = -\frac{1}{2}gt^2 + Ut\sin\alpha$.

8 A mortar bomb is fired at 50 m/s from the ground to hit a point on the ground 200 m away.
(a) Derive the Cartesian equation of the motion of the bomb.
(b) Find the two possible angles that the bomb must be fired at to hit the target.

9 A particle is thrown at an angle of 40° has a horizontal range of 100 m.
Find the initial velocity.

10 A particle is projected to clear a fence 2 m high, and 100 m away (i.e. $x = 100, y \geq 2$). The particle's initial velocity is 45 m/s.
(a) Derive the Cartesian equation of the path of the particle.
(b) Calculate the range of angles of projection to clear the fence to the nearest degree.

11 A fire hose is placed on the ground. It is aimed at the face of a building 7 m high, and 40 m away.
(a) Derive the Cartesian equation of the path of the particle.
(b) When the angle of projection is 20°, the water just reaches the foot of the building. Find the value of the initial velocity.
(c) Calculate the range of angles of projection to wet the face of the building to the nearest degree.

12 A particle is projected from the ground at variable angles of projection α with initial velocity V inside a tunnel of height h. Find the maximum horizontal range.

13 A particle is projected from the origin at an angle of projection α to the horizontal. After a time t less than $\frac{V\sin\alpha}{g}$ (i.e. when the particle has not reached its maximum height), the angle of elevation of the particle measured from the origin is β. If γ is the angle that the velocity of the particle inclines with the horizontal at time t, show that $2\tan\beta = \tan\alpha + \tan\gamma$.

14 A particle is projected with initial velocity of 20 m/s from the top of a cliff 40 m high above a lake. You are given these parametric equations $x = 20t\cos\alpha, y = -5t^2 + 20t\sin\alpha + 40$.
(a) If T is the time that the particle hits the lake, show that $x^2 = -25T^4 + 800T^2 - 1600$, where x is the horizontal range from the foot of the cliff.
(b) Show that $\frac{d}{dT}(x^2) = 0$ corresponds to a maximum value of x, hence, calculate the angle of projection that gives the maximum range.

In questions 15 to 18, you may use this Cartesian equation $y = -\frac{gx^2}{2U^2}\sec^2\alpha + x\tan\alpha$.