Test 4 – Challenge Test

Question 1

Find $\frac{d}{dx}\sin\left(\ln\left(\tan^2 x\right)\right)$. 2 marks

Question 2

Find $\frac{d}{dx}\int_{x^2}^{x^3} \sin^3 t\, dt$. 2 marks

Question 3 Beware ⚠

Let a be a fixed constant such that $0 < a < 3$. With the aid of a diagram, show that

$$\int_0^a \sqrt{9-x^2}\, dx = \frac{9}{2}\left(\frac{\pi}{2} - \cos^{-1}\frac{a}{3}\right) + \frac{1}{2}a\sqrt{9-a^2}.$$

3 marks

Question 4

Assume that the running cost of flying an aircraft can be modelled by

$$c = A + Bs^2$$

dollars, where s is the speed the aircraft is travelling at measured in km h^{-1}. When the aircraft is stationary the running cost is constant at \$120, and at a speed of 300 km h^{-1} the running cost is \$250. At what speed should the aircraft run at to minimise its cost for a 2,000 km trip? Round your answer to 2 decimal places. 4 marks

Question 5

Show that $a^{\ln b} = b^{\ln a}$ and hence find $\int 3^{\ln x}\,dx$. 2 marks

Question 6

(a) Sketch $y = e^x$ over the domain $[-1, 1]$. 1 mark

(b) Use the trapezoidal rule with two subintervals to approximate $\int_{-1}^{1} e^x\,dx$. Leave your answer in exact form. **2 marks**

(c) Find the exact value of $\int_{-1}^{1} e^x\,dx$. **1 mark**

(d) Use your graph to justify why the trapezoidal rule overestimates $\int_{-1}^{1} e^x\,dx$. **1 mark**

(e) Hence, show algebraically that $e < 2 + \frac{3}{e}$. **2 marks**

END OF CALCULUS TEST 4