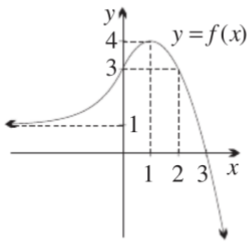
Homework 24-10
2U: Graphing Techniques

15 A SUMMARY OF SHIFTING AND REFLECTING

Transformation	By replacement	By function rule
Shift horizontally right	Replace x by $x - h$	$y = f(x) \to y = f(x - h)$
Shift vertically up	Replace y by $y - k$	$y = f(x) \to y = f(x) + k$
Reflect in the y-axis	Replace x by $-x$	$y = f(x) \to y = f(-x)$
Reflect in the x-axis	Replace y by $-y$	$y = f(x) \to y = -f(x)$
Rotate 180° about O	Replace x by $-x$, y by $-y$	$y = f(x) \to y = -f(-x)$

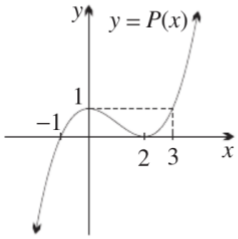
5 In each case an unknown function has been drawn. Draw the functions specified below it.

a



i $y = f(x - 2)$ **ii** $y = f(x) - 2$

b



i $y = P(x + 1)$ **ii** $y = P(x) + 1$

8 In each part explain how the graph of each subsequent equation is a transformation of the first graph (there may be more than one answer), then sketch each curve.

a From $y = -x$: **i** $y = -x + 2$ **ii** $y = -x - 2$ **iii** $y = x + 4$

b From $y = x^2$: **i** $y = (x + 1)^2$ **ii** $y = -(x + 1)^2$ **iii** $y = (x + 1)^2 - 1$

c From $y = \sqrt{x}$: **i** $y = \sqrt{x + 4}$ **ii** $y = -\sqrt{x}$ **iii** $y = -\sqrt{x + 4}$

d From $y = \frac{2}{x}$: **i** $y = \frac{2}{x} - 1$ **ii** $y = \frac{2}{x + 2} - 1$ **iii** $y = -\frac{2}{x}$

e From $y = \sin x$: **i** $y = \sin x - 1$ **ii** $y = \sin\left(x - \frac{\pi}{4}\right) - 1$ **iii** $y = -\sin x$

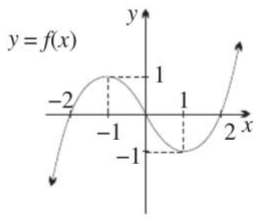
In general...

Horizontal dilation. *If x is replaced by x/A in a formula and $A > 1$, then the effect on the graph is to expand it by a factor of A in the x-direction (away from the y-axis).* If A is between 0 and 1 then the effect on the graph is to contract by a factor of $1/A$ (towards the y-axis). We use the word "dilate" to mean expand or contract.

Vertical dilation. *If y is replaced by y/B in a formula and $B > 0$, then the effect on the graph is to dilate it by a factor of B in the vertical direction.* As before, this is an expansion or contraction depending on whether B is larger or smaller than one. Note that if we have a function $y = f(x)$, replacing y by y/B is equivalent to multiplying the function on the right by B: $y = Bf(x)$. The effect on the graph is to expand the picture away from the x-axis by a factor of B if $B > 1$, to contract it toward the x-axis by a factor of $1/B$ if $0 < B < 1$, and to dilate by $|B|$ and then flip about the x-axis if B is negative.

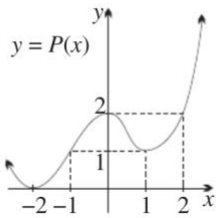
In each case an unknown function has been drawn. Use dilations to draw the new functions indicated below it.

a $y = f(x)$



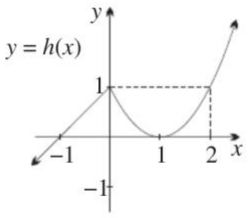
i $y = f(2x)$ **ii** $y = 2f(x)$

b $y = P(x)$



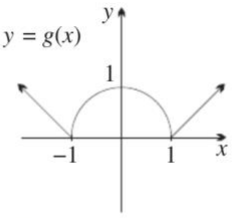
i $y = P\left(\frac{x}{2}\right)$ **ii** $y = \frac{1}{2}P(x)$

c $y = h(x)$



i $\frac{y}{2} = h(x)$ **ii** $y = h\left(\frac{x}{2}\right)$

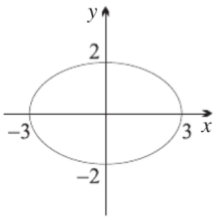
d $y = g(x)$



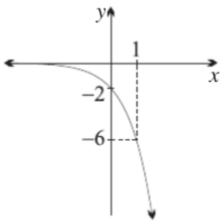
i $2y = g(x)$ **ii** $y = g(2x)$

14 Describe each graph below as a standard curve transformed by dilations, and hence write down its equation.

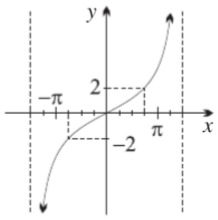
a



b



c



15 **a** For each pair of curves, suggest two simple and distinct transformations by which the second equation may be obtained from the first.

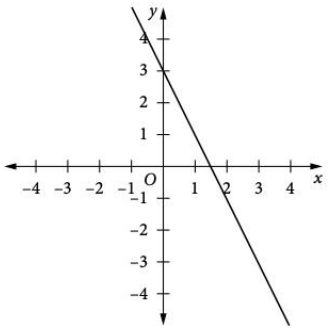
i $y = 2^x$, $y = 2^{x+1}$ **ii** $y = \frac{1}{x}$, $y = \frac{k^2}{x}$ **iii** $y = 3^x$, $y = 3^{-x}$

b Investigate other combinations of curves and transformations with similar ambiguity.

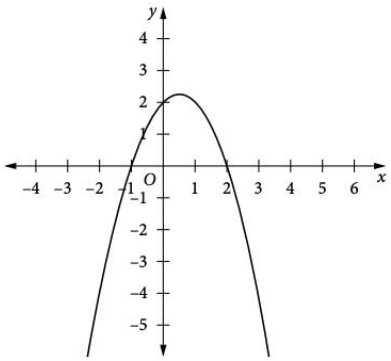
3U: Further Functions

1. Complete the following...

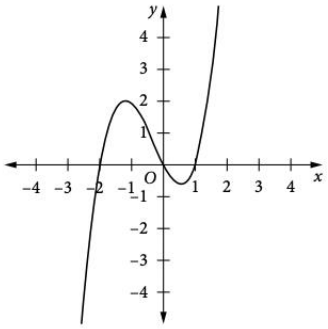
4 Given the graph of $y = 3 - 2x$, draw the graph of $y = \dfrac{1}{3-2x}$.



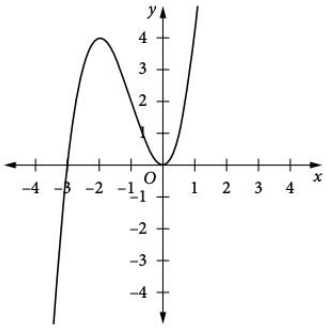
5 Given the graph of $y = (x+1)(2-x)$, draw the graph of $y = \dfrac{1}{(x+1)(2-x)}$.



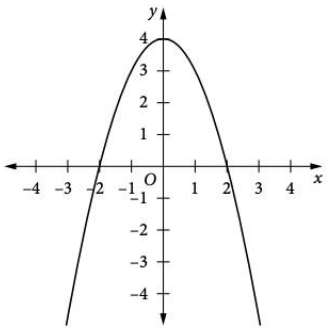
6 Given the graph of $y = x(x-1)(x+2)$, draw the graph of $y = \dfrac{1}{x(x-1)(x+2)}$.



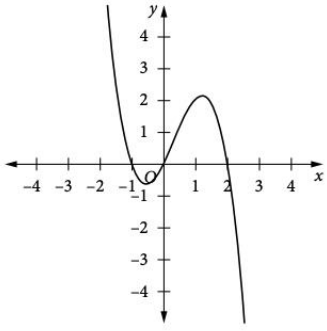
7 Given the graph of $y = x^3 + 3x^2$, draw the graph of $y = \dfrac{1}{x^3+3x^2}$.



8 Given the graph of $y = 4 - x^2$, draw the graph of $y = \dfrac{1}{4-x^2}$.



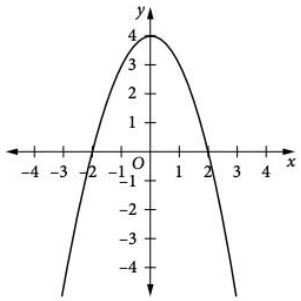
9 Given the graph of $y = x(x+1)(2-x)$, draw the graph of $y = \dfrac{1}{x(x+1)(2-x)}$.



Note! $y^2 = f(x)$ is the same as $y = \pm\sqrt{f(x)}$...

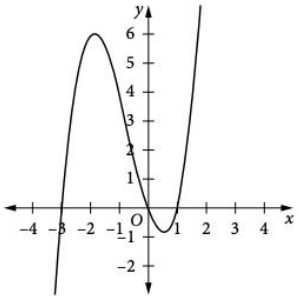
6 Given the graph of $y = (x+2)(2-x)$, draw:

(a) $y = \sqrt{(x+2)(2-x)}$ **(b)** $y^2 = (x+2)(2-x)$



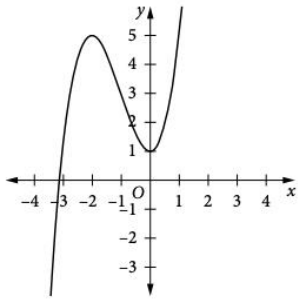
7 Given the graph of $y = x(x-1)(x+3)$, draw:

(a) $y = \sqrt{x(x-1)(x+3)}$ **(b)** $y^2 = x(x-1)(x+3)$



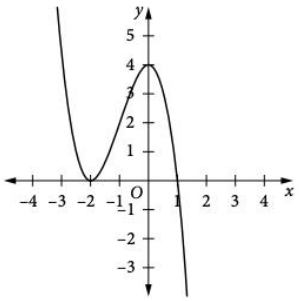
8 Given the graph of $y = x^3 + 3x^2 + 1$, draw:

(a) $y = \sqrt{x^3 + 3x^2 + 1}$ **(b)** $y^2 = x^3 + 3x^2 + 1$



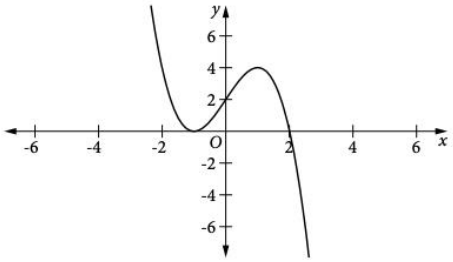
9 Given the graph of $y = (1-x)(x+2)^2$, draw:

(a) $y = \sqrt{(1-x)(x+2)^2}$ **(b)** $y^2 = (1-x)(x+2)^2$



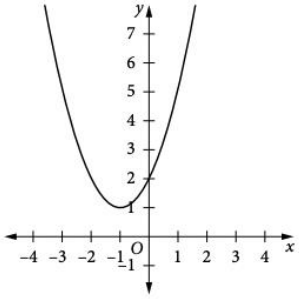
10 Given the graph of $y = 2 + 3x - x^3$, draw:

(a) $y = \sqrt{2 + 3x - x^3}$ **(b)** $y^2 = 2 + 3x - x^3$



11 Given the graph of $y = x^2 + 2x + 2$, draw:

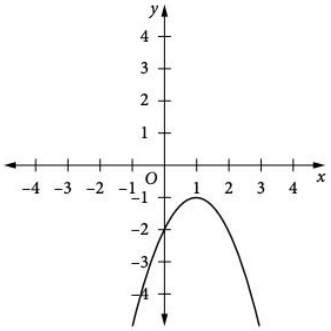
(a) $y = \sqrt{x^2 + 2x + 2}$ **(b)** $y^2 = x^2 + 2x + 2$



12 Given the graph of $y = -x^2 + 2x - 2$, draw:

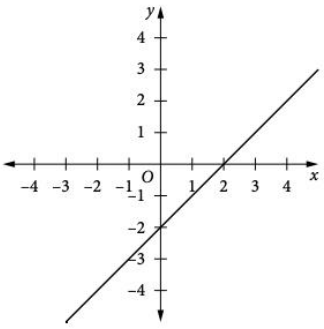
(a) $y = \sqrt{-x^2 + 2x - 2}$

(b) $y = -x^2 + 2x - 2$



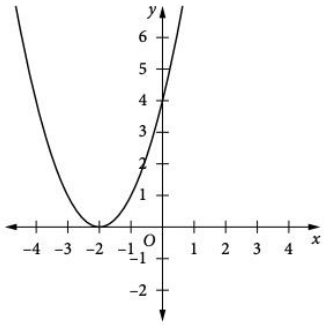
4 Given the graph of $y = x - 2$, draw:

(a) $y = |x - 2|$ **(b)** $y = |x| - 2$



5 Given the graph of $y = (x + 2)^2$, draw:

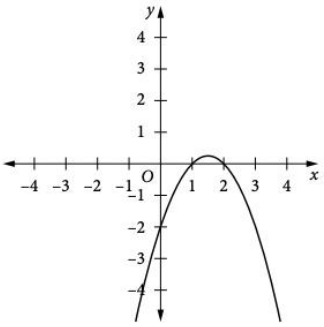
(a) $y = \left|(x + 2)^2\right|$ **(b)** $y = \left(|x| + 2\right)^2$



6 Given the graph of $y = (x - 1)(2 - x)$, draw:

(a) $y = |(x - 1)(2 - x)|$

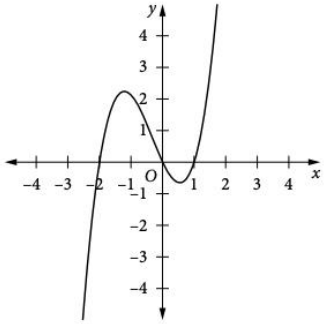
(b) $y = (|x| - 1)(2 - |x|)$



7 Given the graph of $y = x(x - 1)(x + 2)$, draw:

(a) $y = |x(x - 1)(x + 2)|$

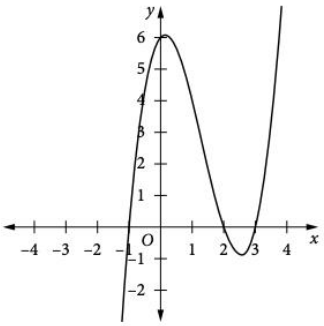
(b) $y = |x|(|x| - 1)(|x| + 2)$



8 Given the graph of $y = (x - 3)(x - 2)(x + 1)$, draw:

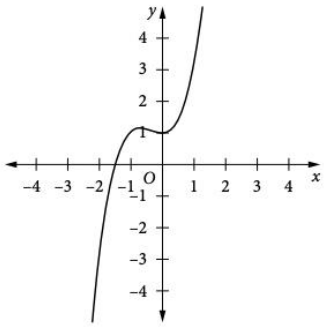
(a) $y = |(x - 3)(x - 2)(x + 1)|$

(b) $y = (|x| - 3)(|x| - 2)(|x| + 1)$

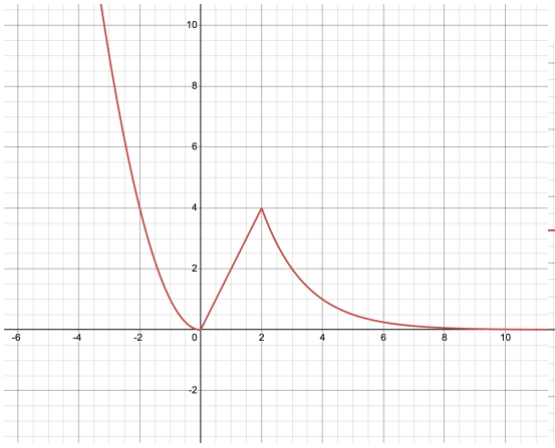


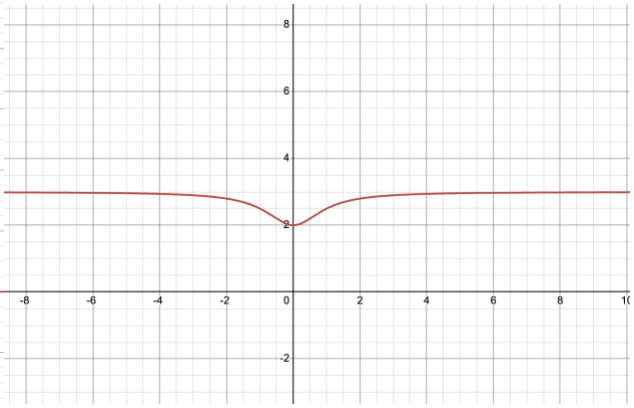
9 Given the graph of $y = x^3 + x^2 + 1$, draw:

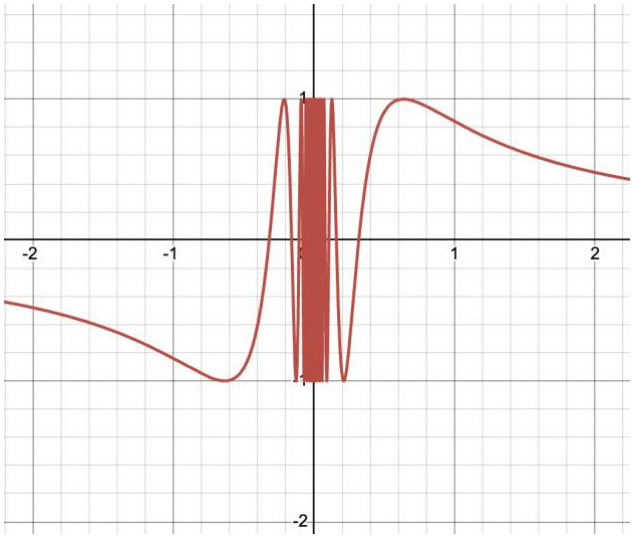
(a) $y = \left|x^3 + x^2 + 1\right|$ **(b)** $y = |x|^3 + |x|^2 + 1$



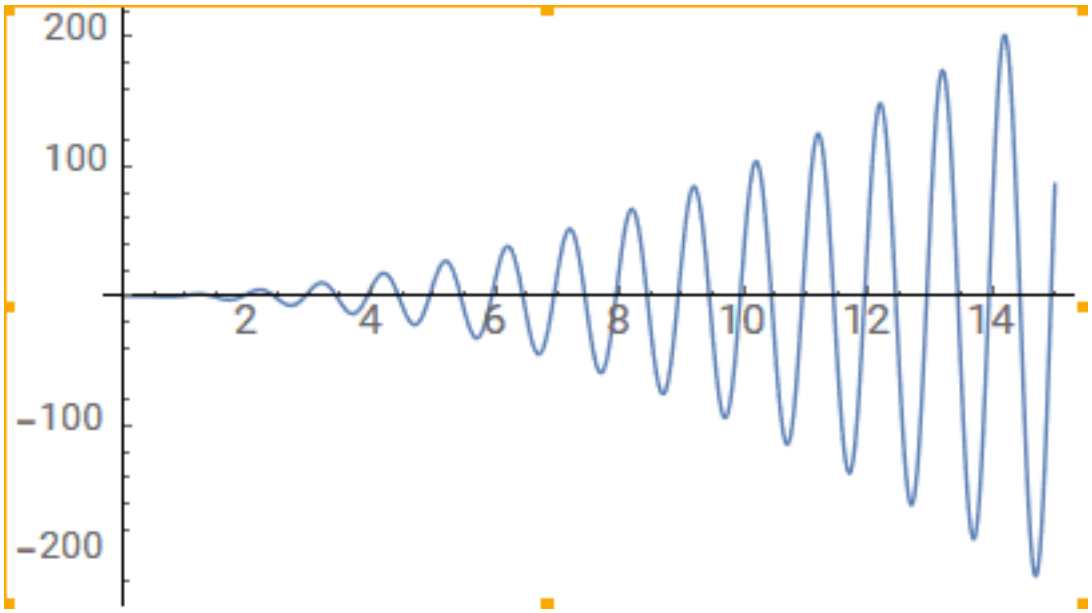
Up for a challenge? Try out the ones on the following page...







(if you zoom in on this one, its like a sin graph – it is oscillating really fast!!)



2. Complete the following exercise on Desmos to understand Arithmetic and Product of Graphs… I WILL POST THIS SOON!!