Homework 30-10

2U: Graphing Techniques

1 a Sketch $y = \dfrac{1}{x-1}$ after carrying out the following steps:

i State the natural domain.

ii Find the y-intercept.

iii Explain why $y = 0$ is a horizontal asymptote.

iv Draw up a table of test values to examine the sign.

v Identify any vertical asymptotes, and use the table of signs to write down its behaviour near any vertical asymptotes.

b Repeat the steps of part **a** to sketch $y = \dfrac{2}{3-x}$.

c Likewise sketch $y = -\dfrac{2}{x+2}$.

d Now sketch $y = \dfrac{5}{2x+5}$.

4 Find the horizontal asymptotes of these functions by dividing through by the highest power of the denominator and taking the limit as $x \to \infty$ and as $x \to -\infty$.

a $f(x) = \dfrac{1}{x-2}$

b $f(x) = \dfrac{x-3}{x+4}$

c $f(x) = \dfrac{2x+1}{3-x}$

d $f(x) = \dfrac{5-x}{4-2x}$

e $\dfrac{1}{x^2+1}$

f $\dfrac{x}{x^2+4}$

7 Let $y = \dfrac{2}{x^2+1}$.

a Determine the horizontal asymptote.

b Explain why there are no vertical asymptotes.

c Show that the tangent to the curve is horizontal at the y-intercept.

d Sketch $y = \dfrac{2}{x^2+1}$. Use a table of signs if needed.

e What is the range of the function?

f Is this function one-to-one or many-to-one?

8 Sketch the function $y = \dfrac{x-1}{x+1}$ by carrying out the following steps.

a State the natural domain.

b Find the y-intercept.

c Determine the horizontal asymptote.

d Investigate the vertical asymptote.

e Sketch the function.

f What is its range?

g Is this function one-to-one or many-to-one?

9 Follow the steps of Question 8 to sketch these graphs.

a $y = \dfrac{x}{x+2}$

b $y = \dfrac{x+1}{x-2}$

c $y = \dfrac{2x-1}{x+1}$

3U: Arithmetic and Product of Graphs

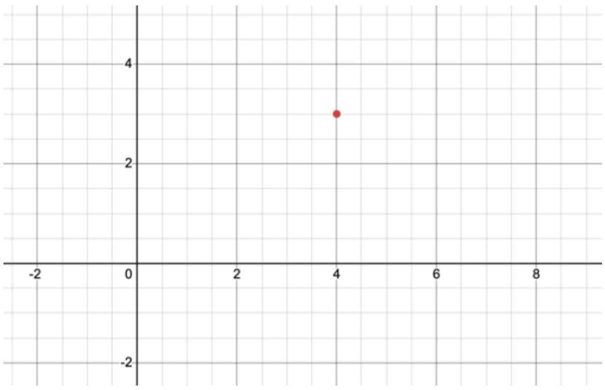
1. Arithmetic: Consider the addition of the two values below:

$$x_1 = 3, x_2 = 7$$
$$\therefore y = x_1 + x_2$$

It gets you 3 + 7 = 10, right? This addition is in 1 dimension. Now, let's look at when we apply this to 2 dimensions...

$$x = 4$$
$$f(4) = 5, g(4) = -2$$
$$\therefore, f(4) + g(4) = 3$$

Note how here, at x = 4, when we add the two functions, we get 3? This means that when I plot the function $f(x) + g(x)$ onto a graph, at x=4, it looks like this:

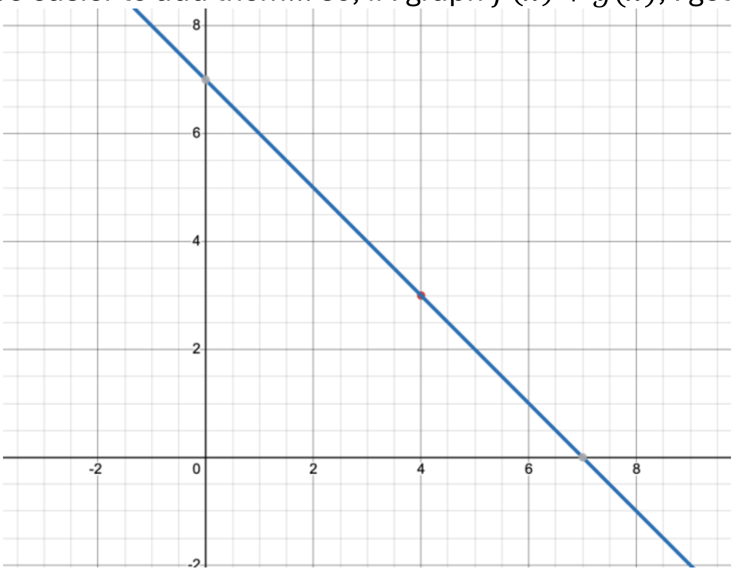


Now, let's do this for all values, if we are given the functions:

$$f(x) = 2x - 3, g(x) = -3x + 10$$

$$\therefore f(x) + g(x) = (2x - 3) + (-3x + 10)$$

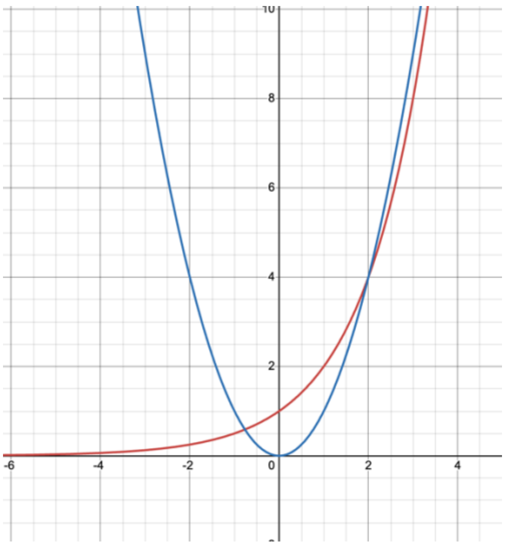
Note how I haven't *actually* added them together? If we do this, for more complex functions, it will be easier to add them... so, if I graph $f(x) + g(x)$, I get this:



Pretty chill, right?

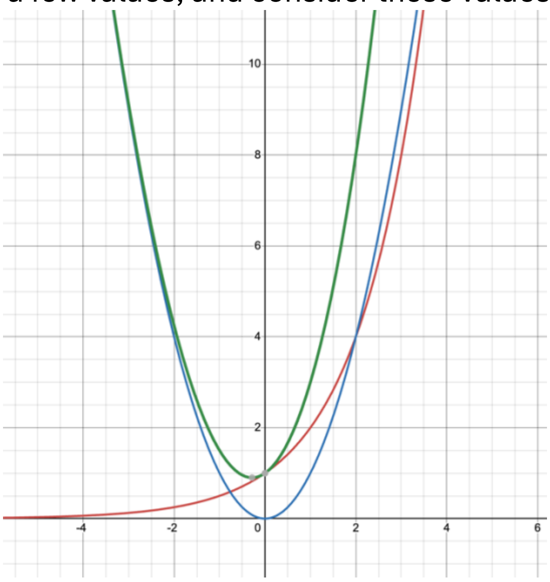
Let's now make it harder:

Graph $y = 2^x + x^2$.
Ooft… let's just stick to graphing them individually first…



Okay. Important take-aways are:
- Intercepts between graphs! These values will double when we add them (e.g. 4 + 4 = 8)
- X – intercepts: means that you have 0 + something…
- Y - intercepts
- Asymptotes: note here how 2^x follows y = 0 as it approaches negative infinity? This means that you are adding to x^2 increasingly small values…

  So, when we test a few values, and consider these values, we end up with:



NOTE: even though it LOOKS like a parabola, it isn't one. On the left hand side, the graph is closely approaching the parabola line as you can see, but isn't ever crossing it. This is because of the sum between the asymptote and parabola! It means that $g(x)$ in this scenario now acts as an asymptote for the sum...

You can probably immediately figure out some features with these graphs, and others are a lot harder. At the end of the day, it's just practise!

Below are some questions to practise. Recommendation: ALWAYS draw them on the same axis, and check your answers with Desmos.

HIGHLIGHT: treat negative subtractions as the following:

$$f(x) - g(x) = f(x) + (-g(x))$$

This just means to flip $g(x)$ over your x-axis, and then sum like usual!

Sketch the following graphs.

1. $y = x + \dfrac{1}{x^2}$

2. $y = x - \dfrac{1}{x^2}$

3. $y = \dfrac{e^x + e^{-x}}{2}$ $(e \approx 2.71828\cdots)$

4. $y = \dfrac{e^x - e^{-x}}{2}$

HARDER:

6. $y = \dfrac{1}{x^2 - \frac{1}{x}}$

7. (a) Show that

$$\frac{x^3 - x^2 + 1}{x - 1} = x^2 + \frac{1}{x - 1}$$

(b) Hence sketch $y = \dfrac{x^3 - x^2 + 1}{x - 1}$.

2. Product of graphs…

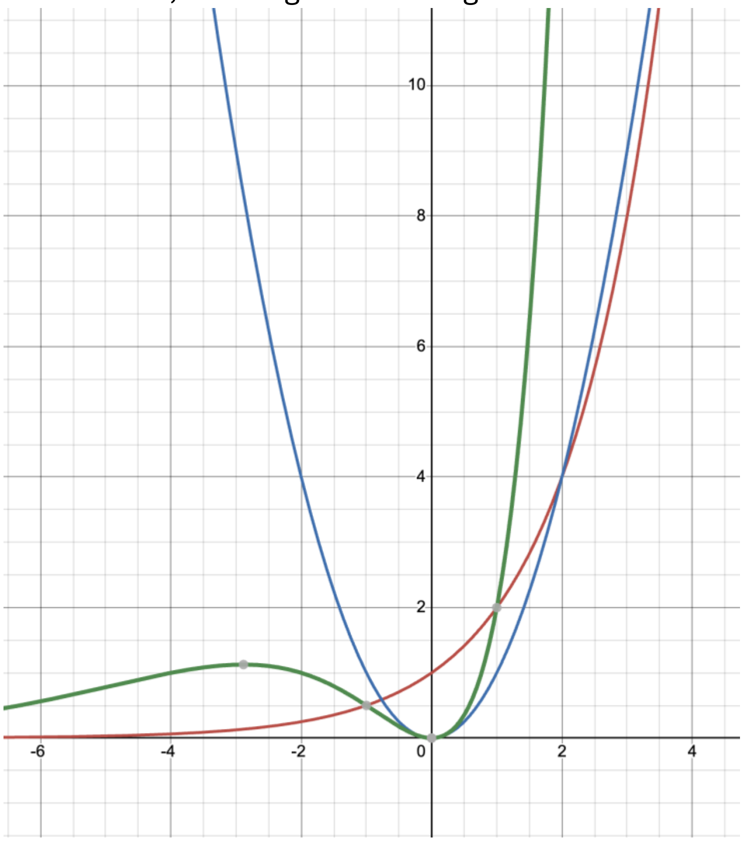
We work in a similar style! Instead, we are looking out for different things…

<u>Graph $y = x^2 * 2^x$</u>

Again, graph them individually. This time, we are looking for the following:

- X-intercepts: 0 times anything is zero, so x-intercepts will stay.
- Asymptote; if y = 0 is an asymptote, it will remain an asymptote. Zeros always carry through!
- If both graphs are positive or negative, then the product will be positive (above x-axis).
- If one graph is negative, then the product will be negative (below x-axis).
- Positive times positive becomes even more positive (a.k.a. bigger).

So, we will get something like this!:



Similar to our arithmetic of graphs, division works in the same way:

$$\frac{f(x)}{g(x)} = f(x) * \frac{1}{g(x)}$$

And you know how to do reciprocal graphs!!!

Complete the following:

Sketch the following graphs.

1. $y = x\left(x^2 - 9\right)$
2. $y = xe^{-x}$
3. $y = \dfrac{x}{x-2}$
4. $y = \dfrac{x}{x+1}$
5. $y = \dfrac{x}{x^2+1}$
6. $y = x\sqrt{1-x}$
7. $y = x\left(e^x - 1\right)$

WARNING: They always ask you multiple choice on the following!!:

- Odd $\times$ Odd $\rightarrow$ Even
- Odd $\times$ Even $\rightarrow$ Odd
- Even $\times$ Even $\rightarrow$ Even

Remember them…