## *The Scots College Research Department*

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### Examining Methods used to measure the Refractive and Group Indexes of Soda-lime Glass


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Produced by: The Scots College Research Department for the HSC Science Extension Curriculum


Please note that due to the inability to obtain certain primary data points, caused by the recent Coronavirus epidemic, some experimental data displayed as results was obtained from an archived source at Sydney University. This will be further elaborated on in the discussion.

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**Abstract**

The past century has seen sustained and significant advances in our ability to measure the properties of electromagnetic radiation. Through the development of new mechanical and digital technology, there are now many more ways in which the properties of electromagnetic radiation can be experimentally measured. This report examines the few ways in which technological and non-technological solutions can be used to measure the refractive and group indexes of red-wavelength and near-infrared radiation respectively, comparing their efficiency and accuracy to background information about the properties of soda-lime glass, allowing for an evaluation of each method and the contexts they may be used in. The two primary methods used in this investigation are the use of an interferometric optical autocorrelator and a paper and pen method. Both results had no significant difference. The conclusion assessed the usefulness of each method, determining that technological solutions be used in situations where accuracy and precision are required.


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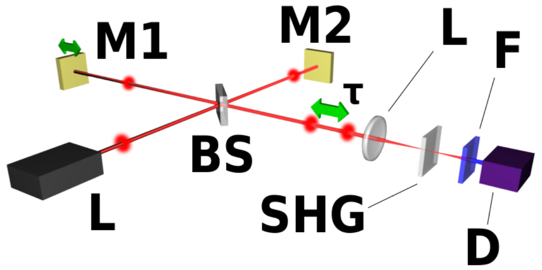
**Introduction**

The concept of refraction has been developed over the course of a long period of time, beginning with Newton's corpuscular theory of light, laid out in his book, *Opticks*. (Bechler, 1973) At the same time, Huygens developed his own wave model of light, which was shown to be more accurate than Newton by Young's double slit experiment (Kushawaha et. al., 2013; Menzel et. al., 2012). Although the wave model is now insufficient in explaining more complex phenomena such as the photoelectric effect or the emission spectra of certain elements, it is useful primarily in explaining the more well-observed properties of light. Among these properties is refraction. Huygens' treatise on light drew on the work of fellow Dutch physicist Snell, showing geometrically how wavefronts act as a useful model to explain the process of refraction. (Shapiro, 1973). Snell's work also found its way into Newton's *Opticks* (Bechler, 1973).

Other related phenomena, such as the relationship between the wavelength of light and the process of refraction came in the development of the Cauchy-Sellmeier equations, which asserted that the refractive index was somewhat inversely proportional to wavelength (Schmid et. al., 2019). A major development in measuring properties of light came in the optical autocorrelator device, which can measure the group index of light using ultrashort pulses. (Cong et. al., 2018; Cong et. al., 2016; Spielmann et. al., 1997). Refraction is a result of the apparent speed of light slowing down. Of course, from Einstein, we know that the speed of light in a vacuum is an absolute constant. However, what occurs when light travels through a certain medium is that each individual wavefront will cause the electrons in the atoms to oscillate, producing their own electromagnetic waves of the same wavelength that interferes with the original light waves. This leads to an overall perception that the light has slowed - its 'apparent speed' (Golowich et. al., 2002; Paschotta, n.d.). This is different from the group index, which is concerned with the movement of the envelope of light slowing down, a sum total of all wavelengths in the material (Bor et. al., 1990; Brillouin 1960; Humanetics Group, n.d.; Paschotta, n.d.).

**Figure 1.1:** Image of apparatus for experiment one[1]




Experiment one will act as the technological solution to measure the effect of refraction. The apparatus that will be used is an interferometric optical autocorrelator, as seen in figure 1.1. A less than 200 femtosecond pulse of near-infrared radiation leaving the laser (L) will interact with the beam splitter (BS) which will send the pulse in two different directions, one towards an adjustable mirror on a guide rail (M1) and the other, a fixed mirror (M2). The two pulses of radiation will merge together again at a lens (L) and the single electromagnetic wave will travel to a photodiode detector (D) which will measure the voltage created by the electromagnetic wave. Over the course of 50 seconds, the pulses of radiation will continue in rapid succession.

This device can be used to measure the time delay experienced by radiation passing through glass and then comparing it with the time delay through air. This can be determined by mechanically moving M1 toward BS at a constant velocity. When the split-up radiation pulses combine at the lens, and onto a crystal, SHG, which has nonlinear optical properties, the intensity of the electrical field is determined by the square of the incoming intensities: $E = k(E_{M1} + E_{M2})^2$ (Franken et. al., 1961; Kemnitz et. al., 1986; Paschotta, n.d.), where all three variables described are functions of time over the course of the 50 second time interval. Hence, when the position of M1 changes, the delay experienced, $\tau = \frac{2z}{c}$ (z is the position of M1), will alter the electrical field function picked up by the detector: $E = k(E_{M1}(t) + E_{M2}(t - \tau))^2$.

By replacing the space between BS and M1 with another material, such as glass, the light pulse will take a longer amount of time to reach the lens. This would make the 'apparent' path length for the light to travel through longer, which means that the value of $\tau$ where $E$ will reach maximum intensity will change, thus enabling the measurement of this new value by changing the position of M1.

Mathematically, this can be used to calculate the group index. First, let $t_{M1}$ and $t_{M2}$ be the total amount of time it takes for the pulse to go from the laser, through either M1 or M2 respectively, to the detector. Therefore, we can put both in terms of $l_{M1}$, $l_{M2}$ (the optical path length from the laser to the detector through either M1 or M2), $T$ (twice the thickness of the glass - due to the ray passing through the glass twice), and $v$, the apparent velocity of light in glass:

$$t_{M1} = \frac{1}{c}(l_{M1} - T) + \frac{1}{v}T$$

$$t_{M2} = \frac{1}{c}l_{M2}$$

When both pulses coincide to reach a maximum electric field, $t_{M1} = t_{M2}$, and using the identity of $v = \frac{c}{n}$, where $n$ is the group index:

$$\therefore l_{M2} = l_{M1} - T + nT$$

There are two values for $T$ that will be measured, one where $T = 0$, creating a peak when the mirror is at position B:

[1] *Note*. From Optical interferometric autocorrelation setup [Diagram], by Fgabolde, 2007, Wikipedia, (https://en.wikipedia.org/w/index.php?curid=1298880). Own work, CC BY-SA 3.0.

$l_{M2} = l_B$ (1)

And $T = 0.95$mm, creating a peak when the mirror is at position A:

$l_{M2} = l_A - T + nT$ (2)

Substituting (1) into (2):

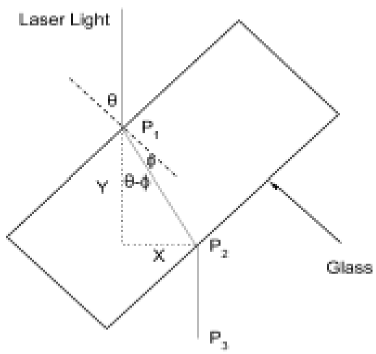
$l_B - l_A = T(n - 1)$

$l_B - l_A$ is positive, as the mirror decreases the total optical distance travelled over time.

$\therefore n = \frac{l_A - l_B}{T} + 1$

The time at which each spike occurs will be recorded. Hence, the velocity of the mirror must be known to calculate $l_A - l_B$. The mirror will move forward by 1.5 mm over the course of 50 seconds. Hence, the optical path of light will decrease by 3 mm. From further research, the refractive index for 650 nm light is 1.521 (4 s.f.), and the group index for 1320 nm, 1.525 (4 s.f.) (Polyanskiy, n.d.; Rubin, 1985). This figure will be used as a comparison point in the analysis.
For experiment two, the use of paper is required to analyse the direction of the three points (entering glass, exiting glass, after exiting glass). Figure 1.2 demonstrates this procedure.

**Figure 1.2:** Diagram of light path in experiment two




When the refraction of light is recorded, two points ($P_1$ and $P_3$), and the angle of incidence (θ) will be known (the angle of incidence being the independent variable). In order to find the angle of refraction (ϕ), the position of $P_2$ relative to $P_1$ needs to be found. Through the use of vertically opposite angles:

$\tan(\theta - \varphi) = \frac{X}{Y}$

Measuring the value of X can be done using a ruler, as the horizontal displacement from $P_2$ to $P_1$ is the same as from $P_3$ to $P_1$. By asserting that $P_1$ is the origin of a cartesian plane and defining the equation of the two sides of the glass block, a value for Y can be calculated as a function of X.

For the glass side that $P_1$ lies on: $y = x\tan\theta$

For the glass side that $P_2$ lies on: $\frac{|y - x\tan\theta|}{\sqrt{1+\tan^2\theta}} = l$, where $l$ is the width of the glass. This describes a locus of two lines, of which the absolute value is not necessary when calculating the value of Y, as whether or not Y - Xtanθ > 0 is irrelevant for θ < 60˚.[2] This is only if Y remains positive for use in the tangent function, as only the 1st quadrant value of φ is needed.
Simplifying the equation, it is then showed that for $P_2$: $Y = X\tan\theta - l\sec\theta$. It is important to note that there will be no issues with the domains of the functions used as $0^\circ \leq \theta \leq 60^\circ$. Once the angle of refraction calculated, a simple application of Snell's law will produce the refractive index: $\sin\theta = n\sin\varphi$ hence, the refractive index will be the gradient between the sine ratios of the incidence and refraction angles.
Before a comparison can be made between the two experiments, it is necessary to consider that both experiments used different wavelengths of radiation (experiment one using 1320 nm infrared radiation and experiment two using 650 nm red light). Thus a conversion must be made using a simplified Cauchy equation (Ghosh, 1997; Schmid et. al., 2019):

$n = A + \frac{B}{\lambda^2}$ (A, B are both constants)

Hence, $\lambda_1^2(n_1 - A) = \lambda_2^2(n_2 - A)$.

$\therefore n_2 = \frac{\lambda_1^2}{\lambda_2^2}(n_1 - A) + A$.

The Cauchy equation is not as accurate for wavelengths in the infrared spectrum (Paschotta,

[2] See Appendix II

n.d.). However, it is useful as there is a direct relationship between the wavelengths and indexes being described in the equation. Note that the determined group index from experiment one will be converted into its equivalent refractive index using a 650 nm laser. This is due to experiment two measuring the angles of refraction as its dependent variable. The constant $A$ is dependent on the material the radiation passes through. The material used in both experiments is soda-lime glass, giving that $A \approx 1.527$ (4 s.f.).[3] This value was calculated from the theoretical indexes of both experiments, as the group index will be treated as a refractive index.

The statistical test that will be used is a chi-squared test due to the given results coming in the form of individual angles of refraction, where the 'expected' field will represent the predicted angles coming from the optical autocorrelator and the 'observed' result will come from direct measurement. The alpha value will be 0.05. This is a standard value that determines whether the difference between the two methods is significant enough. A regression analysis will be performed on the encountered relations in experiment two, allowing for the establishment of a strong correlation. It is presumed that there will be a causation determined by the wave model of light, however the $R^2$ value will simply be used as a metric to determine precision.

The relevant literature on measuring light refraction includes advanced concepts beyond the scope of interpretation in this paper. This can make it difficult for the research to reflect, and develop a weight of knowledge, of how to conduct and analyse the experimental results. This has resulted in the research of less relevant sources, pulling from subject areas outside of physics and optical technology, or researching papers that may be investigating unrelated concepts to that of this investigation. Along with this, the broadened scope has also required that a large portion of the research be historical. Access to academic sources that discuss the necessary equations was limited, requiring the inclusion of sources not just coming from research papers, but also from online encyclopedias, which may be less trustworthy or credible.

Many sources also employ the use of much older physical models concerning classical electromagnetism. However credible, models developed by Huygens and Maxwell may or may not be able to reflect modern representations of physics, although these more modern representations are not necessary to conduct the experiment per se.

**Hypothesis**

Both methods used should yield a refractive index of red-wavelength light that is 1.52, as per the background research. It should be expected that when comparing the angles of refraction between experiment one and two, there is no significant difference between the two results, generating a chi-squared value less than 11.1.

**Method**

Experiment one (Refer to figure 1.1):

First, the optical autocorrelator apparatus was set up according to figure 1.1, ensuring that the mirror M1 is able to automatically move forward by 1.5 mm from an equal-distance position between M2 and BS. M1 started at the equal-distance position (0 mm). [1] The laser was prepared, ensuring that it was capable of firing ultrashort pulses of 1320 nm infrared radiation, then applying the necessary protection equipment i.e. 1320 nm laser goggles, warning labels in and outside the room the experiment was conducted in, etc. The laser was turned on, checked to confirm that a reading appeared on the data logger. The automatic movement of M1 was activated using the necessary software, recording the displacement of M1 from the equal-distance position and the voltage produced in the detector, then allowing M1 to slide forward 1.5 mm over the course of 50 seconds. Once M1 reached a displacement of 1.5 mm from its origin, the laser was turned off and the movement of M1 was stopped. [2] The experimental apparatus was reset according to [1] and a 0.95 mm slide of soda-lime glass was placed in between M1 and BS, repeating the steps from [1] to [2].

Experiment two (Refer to figure 1.2):

Firstly, a sheet of paper was placed on an elevated platform, marking the position of $P_1$ on the paper, then the midpoint of the soda-lime glass block was placed on the point $P_1$. The laser was set up such that the path of the beam was slightly above the page and directly perpendicular to the face of the block that touched point $P_1$, ensuring that the path length of the light in the soda-lime is 61 mm, that entails that the width of the glass block be 61 mm. A line was drawn on the paper that is perpendicular to the oncoming laser at point $P_1$ - being deemed the 'horizontal'. [3]

[3] See Appendix III for derivation

A protractor was used to angle the glass block at 10° to the horizontal, ensuring that the same side of glass is touching point $P_1$, then recording a point $P_3$ on the path of the light beam after leaving the glass, using an opaque object to determine the path of the beam. The horizontal displacement from $P_3$ to $P_1$ was measured. [4] Finally, the steps from [3] to [4] were repeated, each time changing the angle at which the glass lies to the horizontal. This angle was complementary to the angle of incidence.

**Risks and Variables**

Overall, the risk of experiment one is high, requiring proper education and training before conducting the experiment. It is clear that a strict standard of safety must be kept when conducting experiment one to ensure the safety of all involved. This involves physical protection such as the use of laser goggles or lab coats to ensure eye and skin protection. People also need to be advised to keep clear of the area in use. The laser light that will be used in recording this experiment is a class 4 laser, meaning that it is the most susceptible to damaging a person's skin or eyesight, especially considering the nature of near-infrared laser radiation.

The laser used in experiment two is a 1 $Wm^{-2}$ red-wavelength laser, hence no safety equipment is necessary, however it is important to notify people of the presence of laser light in the area. For both lasers, it is important to ensure that there is no ability for the light to reflect off certain surfaces.

For experiment one, there is no independent variable, because only one data point could be recorded. Experiment one acts as a measurement of a constant value, thus no independent variable is necessary. Experiment two will change the angle of incidence the laser takes when entering into the glass

For experiment one, the magnitude of the voltage produced by the electric field of the wave when it reaches the end of the autocorrelator will be recorded directly, and from there, the data will be interpreted into providing a group index for glass based on the delay from each glass thickness. For experiment two, the horizontal displacement between points $P_1$ and $P_3$ will be measured, allowing for a derivation of the angle of refraction.

The variables that will be controlled in experiment one are: the speed at which the autocorrelator stage moves; what type of material is going to be present in the path of the radiation and its orientation (straight, to eliminate angled refraction); the provided electrical energy going towards producing the radiation. Experiment two will control: the position of point $P_1$; the direction of the laser, along with its elevation; the point at which the light enters into the glass (at point $P_1$). Both experiments will control the thickness of the glass used, i.e. 0.95 mm for one, and 61 mm for two.

**Results**[4]

**Figure 2.1:** Recorded voltage over time for experiment one

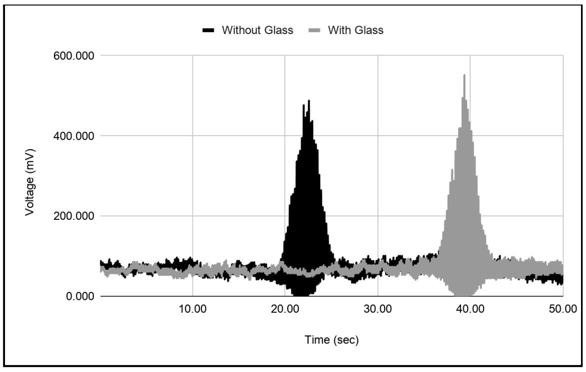



**Figure 2.2:** Sine ratios of incidence and refraction angles for experiment two

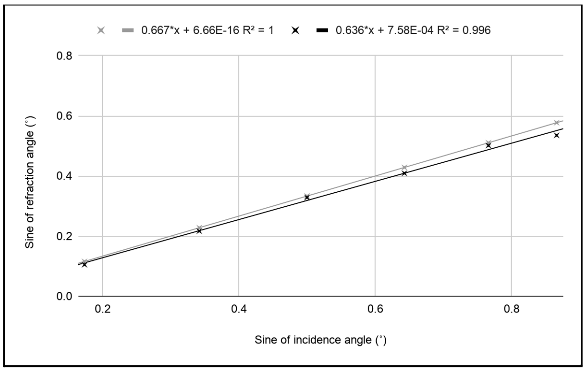



Figure 2.1 is the recorded voltage for experiment one with glass and without glass. Using this graph and the formula developed in the background research, $n = \frac{l_B - l_A}{T} + 1$, the group index was measured to be $n = 1.53$. The determined sine ratio of the incidence versus the refraction angle is shown in figure 2.2. Using the gradient of the black line of best fit (experimental), the refractive index of light for a 650 nm ray is $n = 1.53$. This creates a 0.376% error with the expected value of 1.521.

[4] For directly measured results, see Appendix IV

**Figure 2.3:** Determined angles of refraction from both experiments

| Incidence Angle | Expected | Observed | Chi-Squared Term |
|---|---|---|---|
| 10 | 6.47 | 6.03 | 0.03 |
| 20 | 12.83 | 12.52 | 0.01 |
| 30 | 18.95 | 19.17 | 0.00 |
| 40 | 24.67 | 24.16 | 0.01 |
| 50 | 29.83 | 30.10 | 0.00 |
| 60 | 34.22 | 32.36 | 0.10 |
| | | | 0.16 |

**Figure 2.4:** Refractive indexes and errors in both experiments

| | *Experiment 1* | *Experiment 2* |
|---|---|---|
| Wavelength (nm) | 1320 | 650 |
| RI (Experimental) | 1.53*/1.54 | 1.53 |
| RI (Theoretical) | 1.53* | 1.52 |
| Error (%x) | 0.301*/1.25 | 0.376 |

*The value is for 1320 nm radiation

**Figure 2.5:** Results from the Chi-Squared analysis

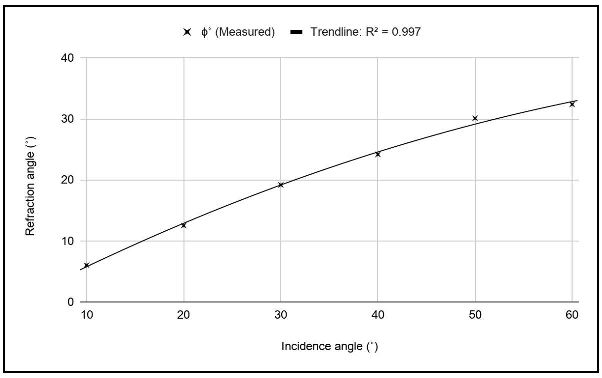
| | |
|---|---|
| χ-value | 0.16 |
| df | 5 |
| α-value | 0.05 |
| Critical value (X) | 11.1 |
| p > α ? | No |

These results suggest that both forms of measurement are accurate, compared to one another and to the theoretical value. Although, it is important to note that the directly measured refractive index of experiment two is exactly that of experiment one, leading to a wavelength-adjusted value of $n = 1.54$. However, this error is not enough that the null hypothesis must be rejected, since The chi-squared test (Figure 2.5) shows that there is no significant difference between the two methods.

Using a linear regression analysis, the precision of experiment two can be determined, resulting in an $R^2$ value of 0.996, in terms of the sine ratios (figure 2.2), and a value of 0.997 in terms of a direct comparison between the angle of incidence and refraction (figure 2.6). This demonstrates that the collected results for experiment two were both precise and accurate. The shape of the graph in figure 2.6 was expected to be non-linear, as $\sin^{-1}(\frac{1}{1.5}\sin\varphi)$ will result in a sinusoid, thus a regression analysis can be performed on a polynomial of degree two.

One notable effect found in experiment one (figure 2.1) was that the peak of the 'with glass' test was much higher than that of the 'no glass' test. This is most likely due to the limit of reading placed on the apparatus.

**Figure 2.6:** Incidence Angle versus Refraction Angle




**Discussion**

There was a substantial difficulty in recording large amounts of primary data for the investigation. Due to the recent Coronavirus epidemic, the original experiment had to be postponed as there was no way to conduct the experiment in person. Instead, the experiment relied on gathering a single data point of secondary data done previously. This has ramifications for both the reliability and the validity of the experiment as the research question was potentially unable to be answered in a proper way.

In terms of the assumptions made when conducting both experiments, it is presumed that the glass material is internally consistent and that it will not produce any angled refraction in the case of experiment one, or internal reflection in the case of experiment two. It is assumed that the air surrounding the radiation will be constant in its pressure, density and temperature. Due to the use of three significant figures being used as the basis, it is unlikely that these assumptions affected the results in any way.

Mathematically, there were also multiple assumptions made, such as that the light intensity or the operation of any electrical circuits would not affect the value of the group index. It is worth noting that certain aspects of the mathematical derivation were performed after obtaining the results, this may lead to theory-dependence, which may also harm the validity of the results.

In terms of the limitations placed on the experiment, there was a significance error concerning the detector's inability to record information for time intervals shorter than 20 milliseconds, meaning that the data may not be as precise as desired. This is demonstrated in figure 2.1, where there was a difference between the recorded voltages. This is likely due to how well the laser is aligned with the second harmonic crystal and diode rather than electric interference from other equipment, leading to imperfections in the recorded data. The air still had an effect on the path that the light travelled in both experiments. It is noted that for experiment two there was a large amount of diffraction and internal reflection occurring from the laser beam. This is an equipment limitation for the experiment.

The substantial error in experiment one was expected, most likely being caused by two different possibilities: a systematic error occurred when performing the mathematics, or that the composition of the glass used to obtain the theoretical value was different from the one used in experiment one. This systematic error came from the fact that the Cauchy model is less accurate for infrared light and that using the group index in such an equation is not particularly valid. This is contrasted with experiment two, which should have been the more inaccurate method, due to human error, however it seems that the experiment was conducted in a way that would eliminate any certain variables. This still has the implication that this method should only be used as a demonstration in a low-level environment, such as a classroom environment, even if it produces precise and accurate results. However, a reliable value for the refractive index can still be determined because this method allows for collecting multiple data points.

In comparison with experiment two, the bulky nature of the apparatus in experiment one, and the limited accessibility of it, makes it difficult to use the apparatus to collect multiple data points. Thus this method remains as unreliable concerning the ability to retest the experiment in a given timeframe or even reproduce the experiment elsewhere. This also means that the precision of the method is reliant on the precision of the equipment.

For experiment one, each interval of time recorded was 0.02 seconds, hence the limit of reading for the displacement of M1 was $\pm\ 6 \times 10^{-4}$mm. The limit of reading for the ruler measure used was $\pm\ 5 \times 10^{-3}$mm, for experiment one, and $\pm\ 0.5$mm for experiment two, the protractor used in experiment two had a limit of reading of $\pm\ 0.5^{\circ}$. Figure 3.1 demonstrates the total limit of readings for both experiments, along with their relative uncertainties, demonstrating that the measured values are fairly precise. This precision, as well, relates to the regression analysis being above a standard 0.95, demonstrating a significantly strong correlation between the sines of the angles of incidence and refraction. This was not used to directly analyse the accuracy of the results, as it was already given and predicted that there is an existing causation due to Snell's law.

**Figure 3.1:** Limits of reading involved in both experiments

| | *Experiment 1* | *Experiment 2* |
|---|---|---|
| Limit of Reading (Δx) | $3.42 \times 10^{-3}$ | $1.55$ |
| Relative Uncertainty (%x) | $0.224$ | $9.50$ |

Experiment one remains as a valid method, as it was able to control all of the necessary variables, however, due to it being obtained from an archive at Sydney University, it is not as valid as it is a secondary source of data. This is especially true when considering that only one data point was collected, it is unsure truly if it could control the correct variables and thus be completely valid.

Experiment two is valid due to the ability to control the several random errors, producing an unexpected

accuracy. These errors indicate that the experiment was able to control all of the variables necessary.
Despite their flaws, both experiments were successful in providing a response to the research question and the hypothesis, providing a relatively accurate and precise quantitative measurement, making them at least somewhat valid.

## Conclusion

In conclusion, the chi-squared test demonstrated that there is no significant difference between the two methods. Experiment one remains as a much more precise method in determining the group index, having a relatively small limit of reading error, leading to a small relative uncertainty. This, however, may not be as true for experiment two, which has a much larger relative uncertainty, although still precise as $R^2 > 0.95$.
In terms of the ways in which the different methods can be used, it seems that an optical autocorrelator device would be much more useful for making a single constant measurement, allowing for an accurate reading of the group index. Using a low-tech paper and pencil solution was much better in terms of its repeatability, thus making it more beneficial in a classroom environment where there are less safety protocols involved and any inaccuracies in results are not too costly.
Hence, there are many different ways in which the same measurement can be achieved, however each method should be used considerably in different contexts and environments.
Further research may involve conducting experiment one again, to ensure that the collected results are properly reliable and valid. Experiment two could also be further refined so as to limit human errors. More extensive research and analysis into a larger and broader range of methods is the best next step forward, allowing for a much more substantial comparison of the techniques used to measure the refractive and group indexes.


## Acknowledgements

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## Appendix I: Reference List


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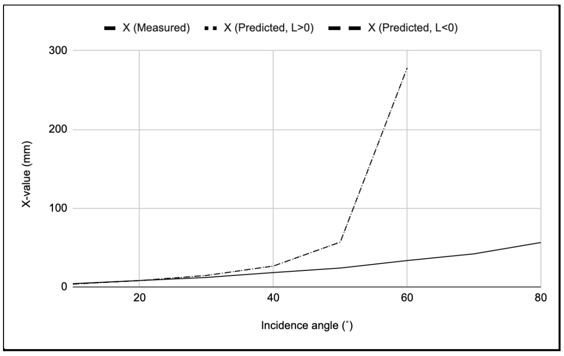
**Appendix II:** Horizontal Displacements Compared to Differing Angles of Incidence




*Note that $L = y - x\tan\theta$

**Appendix III:** Calculation of the A-Constant

In order to calculate the A-Constant in the Cauchy-Sellmeier equation, the following formula, rearranged from the relationship described in the introduction, is:

$$A = \frac{\lambda_1^2 n_1 - \lambda_2^2 n_2}{\lambda_1^2 - \lambda_2^2}$$

Hence, deriving the A-Constant from theoretical values for $n_1$ and $n_2$.

**Appendix IV:** Directly Recorded Results from Experiment 3

| *Angle of Incidence* | *Horizontal Displacement* | *Angle of Refraction* |
|---|---|---|
| 10 | 4.25 | 6.03 |
| 20 | 8.13 | 12.5 |
| 30 | 12.1 | 19.2 |
| 40 | 18.3 | 24.2 |
| 50 | 24.0 | 30.1 |
| 60 | 33.5 | 32.4 |

*Multiple tests were conducted for $\theta \leq 50^{\circ}$, these results were averaged