Referee report on "Two principles for two-person social choice" (SCWE-D-23-00280)

Summary

In the classical two-person social choice problem with a finite set of m alternatives, this paper presents two principles for studying social choice rules: the *minimal Rawlsian* principle (MR), which demands the least happy individual to obtain a welfare within the best half of his preference; and the *equal loss* principle (EL), by which selected outcomes make both agents concede "as equally as possible" from their top alternative. Several variants of EL are analyzed: *equal loss compatibility*, which is incompatible with *Pareto efficiency*; *Paretian equal loss compatibility* (PEL-comp); and the two stronger requirements of *Paretian equal loss* (PEL) and *minimal dispersion* (MD). For most values of m, MD is incompatible with MR. To reconcile the two principles, *Rawlsian minimal dispersion* (RMD) requires minimal dispersion over alternatives that are minimally Rawlsian.

These principles are then used to assess several two-person social choice rules, namely: the *unanimity compromise* (UC) rule, the *veto-rank* (VR) rule, the *shortlisting* (SL) rule, and the family of *Pareto-and-veto* (PV) rules. Each PV rule is parameterized by two numbers, v_1 and v_2, where v_i is the number of alternatives vetoed by agent i. An important member of this class is the PV rule that gives the highest equal veto power to both individuals, $PV^=$, for which $v_1 = v_2 = \lfloor \frac{m-1}{2} \rfloor$. Rule $PV^=$ is (essentially) the only rule in the PV family that satisfies MR, whereas UC, VR, and SL rules all satisfy MR (because the three are sub-correspondences of $PV^=$). VR and SL rules are not PEL-comp, and a PV rule satisfies this property if and only if $\max v_i \leq \frac{m}{2}$. Among the rules studied, UC is the only rule that satisfies PEL (and therefore is also PEL-comp). By the incompatibility between MD and MR, rules UC, VR, and SL do not satisfy MD; whereas a PV rule fulfills the property if and only if $\max v_i \leq \frac{m+1}{3}$. Furthermore, rules UC, VR, and SL do not satisfy RMD; whereas the only PV rule that fulfills the property is $PV^=$.

Evaluation

The paper is well-written and the results seem correct. The two principles studied are interesting and relevant, and they help to organize some rules already identified in the literature.

Overall, the paper aligns nicely with the scope of *Social Choice and Welfare*. It is a good work that offers valuable insights. However, I recommend that the authors carefully address the comments I have outlined for revision.

Comments

(1) I feel that the Introduction needs to be reorganized. There is a very long literature review at the beginning of the section. Part of that review should be postponed. The main goals

of the paper should appear earlier in the text.

(2) Does any of the results obtained in the paper still hold when more than two agents are involved? Maybe a final remark on this matter could be interesting.

(3) The same mathematical object, a SCR, is treated as a function, a (binary) relation, and a (sub-)correspondence in just a few lines (lines 174 to 178). Although this is correct, it may lead to confusion. Why not just simply present a SCR as a *correspondence* $f : \mathscr{L}(\mathcal{A})^N \to \mathscr{P}^*(\mathcal{A})$ that, to each profile $\boldsymbol{P} \in \mathscr{L}(\mathcal{A})^N$, assigns a non-empty subset $f(\boldsymbol{P}) \in \mathscr{P}^*(\mathcal{A})$? With this definition, the ideas of "refining" and "proper sub-correspondence" are straightforward.

(4) Is the notation $[\![p, q]\!]$ really necessary? It seems that, in the paper, p always equals 0. If this is the case, given $n \in \mathbb{N}$, why not just define $[\![n]\!] = \{0, 1, 2, \ldots, n\}$?

(5) In the proof of the "only if" part of Proposition 1: shouldn't the inverse ordering be associated with agent $\bar{i}$, i.e., shouldn't $\succ_{\bar{i}}$ equal $\succ_i^{-1}$? Is "$\min\max\lambda_{\mathbf{P}}$" defined previously in the paper? I could only find the definition of $\min\max\lambda_{\mathbf{P}}(x)$.

(6) Line 228: Maybe a footnote remembering the definition of a *Borda winner* could be helpful to the reader.

(7) Remark 3: When displaying the profile of preferences it would be helpful to indicate the preference of each agent to avoid confusion: Consider profile $\mathbf{P} = (\succ_1, \succ_2)$ such that

$$\succ_1\colon\ a\ b\ c\ d\ldots$$

$$\succ_2\colon\ g\ h\ i\ d\ldots$$

This applies to all the examples in the paper.

(8) Theorem 1: For each $\mathbf{P} \to$ For each $\mathbf{P} \in \mathscr{L}(\mathcal{A})^N$.

(9) I don't think Definition 1 needs to be in a Definition environment. It could be discussed in the text. The only notion of k-Rawlsianism used in the paper is MR. Similarly, I don't think Definition 4 needs to be in a Definition environment either. Moreover, all the discussion about the *equal loss* property could be placed in a footnote.

(10) Both Tables 1 and 2 look weird with blank entries, they seem incomplete. Why not add "X" in all blank entries? This is standard.

(11) In Table 1, it is said that results hold for "any $m \geq 4$ and n". For any n? By Corollary 2 we know that UC, VR, and SL satisfy MR. But how can we assure that they are not MD when $m = 5$, for example? The incompatibility between MD and MR occurs, according to Theorem 5, when $m = 7$ or $m > 8$. What am I missing?

(12) Table 2: I think that all results are covered assuming $m \geq 4$. Is this correct? This is not made explicit.

(13) Theorem 4: To be consistent, I would write (as in Proposition 10) "$\mathcal{MR} \cap \mathcal{MD} \neq \emptyset$" instead of "$\exists f \in \mathcal{MR} \cap \mathcal{MD}$".

(14) In the definition of rule SRMD (line 623) the quantifier "for each $\mathbf{P} \in \mathscr{L}(\mathcal{A})^N$" is needed.

Other comments

Most of the following comments are mainly expositional, but I think that the paper can be improved if the authors pay attention to these recommendations.

(a) Line 109: . . . it can be expressed *as* the union *of* all Pareto. . .

(b) Line 241: The *proposition* is proven → The *remark* is proven.

(c) Whenever possible, I try to avoid the symbols ∀ and ∃. I would recommend not to use them. But in case you keep them, some consistency is in order. In Theorem 1 you write "For each $\mathbf{P}$ . . .". However, in Definition 3, you write "$\forall \mathbf{P}$". Please, choose one notation and stick to it thoroughly along the paper.

(d) In Table 2, in the MR column, to be consistent I would write "$\min v_i \geq \lfloor \frac{m-1}{2} \rfloor$" instead of $v_1, v_2 \geq \lfloor \frac{m-1}{2} \rfloor$.

(e) Definition 2: (Minimal Rawlsianism (MR)) → (Minimal Rawlsianism, MR)

(f) Line 320: $\mathbf{P} = (\succ_i, \succ_i^{-1})$ looks weird. Agent i appears twice in the profile? I would prefer $\mathbf{P} = (\succ_1, \succ_2)$ where $\succ_2$ equals $\succ_1^{-1}$.

(g) Definition 3: (Equal loss compatibility (ELC)) → (Equal loss compatibility, ELC)

(h) Definition 8: (Rawlsian minimal dispersion) → (Rawlsian minimal dispersion, RMD)

(i) Definition 9: (Strong Rawlsian minimal dispersion) → (Strong Rawlsian minimal dispersion, SRMD)