# Arguing about Voting Rules

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## Main objects

- $\mathcal{A} = \{a, b, c\}$ a set of *alternatives*
- $N = \{v_1, v_2, v_3\}$ a set of *voters*
- $\boldsymbol{R}$ a *profile*: maps voters to linear orders on $\mathcal{A}$
- *Voting rule*: maps profiles to subsets of alternatives: $[\boldsymbol{R} \mapsto A \subseteq \mathcal{A}]$

## Some example profiles

$$\boldsymbol{R} = \begin{array}{ccc} v_1 & v_2 & v_3 \\ a & a & c \\ b & b & b \\ c & c & a \end{array}, \quad \boldsymbol{R}_1 = \begin{array}{c} v_1 \\ a \\ b \\ c \end{array}, \quad \boldsymbol{R}_2 = \begin{array}{cc} v_2 & v_3 \\ a & c \\ b & b \\ c & a \end{array}$$

## Our objective

Scenario:
- Assume a commitee wants to choose a voting rule
- Many rules
- No best rule

We want to:
- *automatically* explain the outcome of rules
- *based* on their properties
- by referring to *examples*

## Example dialogue

Let's argue that in $\boldsymbol{R}$, $a$ should win, using "unanimity", "symmetry" and "reinforcement"

**System** Consider election $\boldsymbol{R}_1$, involving only voter $v_1$. Do you agree that $a$, enjoying unanimous support, should win this election?

**User** Yes, of course.

**System** Now consider election $\boldsymbol{R}_2$, involving only voters $v_2$ and $v_3$. Do you agree that, for symmetry reasons, the outcome should be a three-way tie?

**User** Yes, that sounds reasonable.

**System** Observe that when we combine $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$, we obtain our election of interest, namely $\boldsymbol{R}$. Do you agree that in this combined election, as there was a three-way tie in $\boldsymbol{R}_2$, $\boldsymbol{R}_1$ should be used to decide the winner?

**User** Yes, I do.

**System** To summarise, you agree that $a$ should win for $\boldsymbol{R}$.

## Language

- A general language $\mathcal{L}$ to talk about properties of voting rules
- Logic-based
- Can represent axioms and specific outcomes (such as $[\boldsymbol{R} \mapsto A]$)
- Permits *arguments*: proof (in the language) from axioms to outcomes

**Theorem 1** (Completeness)**.** *Take some axioms of a voting rule. Encode in our language as $\boldsymbol{J}$. Suppose some outsome $[\boldsymbol{R} \mapsto A]$ holds for every rule $f$ satisfying those axioms. Then there exists a proof of $[\boldsymbol{R} \mapsto A]$ in our logic that is grounded in $\boldsymbol{J}$.*

## Example for axioms in $\mathcal{L}$

**Reinf**: $[\boldsymbol{R}_1 \mapsto A_1] \wedge [\boldsymbol{R}_2 \mapsto A_2] \rightarrow [\boldsymbol{R}_1 \oplus \boldsymbol{R}_2 \mapsto A_1 \cap A_2]$ with $A_1 \cap A_2 \neq \emptyset$

**Canc**: $\boldsymbol{R}$ such that $[\forall(a, b) : a \succ b$ as often as $b \succ a] \rightarrow [\boldsymbol{R} \mapsto \mathcal{A}]$

**Anon**: $[\boldsymbol{R} \mapsto A] \rightarrow [(\boldsymbol{R} \circ \sigma) \mapsto A]$ for any permutation $\sigma$

## Arguing for Borda

We provide a concrete algorithm for explaining the outcomes of the Borda voting rule. Given any profile $\boldsymbol{R}^*$:
- Let $W \subseteq \mathcal{A}$ be the set of Borda winners
- The algorithm generates a proof that $[\boldsymbol{R}^* \mapsto W]$
- Grounded in axioms: Reinf, Canc, Elem, Cycl

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## Example: Arguing for Borda

Given $\boldsymbol{R}^* = \begin{array}{cc} a & c \\ b & b \\ d & a \\ c & d \end{array}$, let's argue that $[\boldsymbol{R}^* \mapsto \{a, b\}]$

### Proof

$$\begin{array}{cc} a & b \\ b & a \\ c & d \\ d & c \end{array} \overset{\text{Elem}}{\longmapsto} \{a, b\} \qquad \begin{array}{cc} a & c \\ b & b \\ c & a \\ d & d \end{array} \overset{\text{Elem}}{\longmapsto} \{a, b, c\} \qquad \begin{array}{cccc} a & d & c & b \\ d & c & b & a \\ c & b & a & d \\ b & a & d & c \end{array} \overset{\text{Cycl}}{\longmapsto} \mathcal{A} \qquad \begin{array}{cccc} a & b & d & c \\ b & d & c & a \\ d & c & a & b \\ c & a & b & d \end{array} \overset{\text{Cycl}}{\longmapsto} \mathcal{A}$$

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$\boldsymbol{R}' \longmapsto \{a, b\}$ — Reinf

$\boldsymbol{R}' \oplus \overline{4\boldsymbol{R}^*} \oplus 4\boldsymbol{R}^* \longmapsto \{a, b\}$ — $\boldsymbol{R}' \oplus \overline{4\boldsymbol{R}^*} \oplus 4\boldsymbol{R}^*$ must have the same winners as $\boldsymbol{R}'$, by Reinf and Canc

$4\boldsymbol{R}^* \longmapsto \{a, b\}$ — $4\boldsymbol{R}^*$ must have the same winners as $\boldsymbol{R}' \oplus \overline{4\boldsymbol{R}^*} \oplus 4\boldsymbol{R}^*$, because $\boldsymbol{R}' \oplus \overline{4\boldsymbol{R}^*}$ cancels

$\boldsymbol{R}^* \longmapsto \{a, b\}$ — Oh yeah