Collaborative learning of models of deliberated preference

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Abstract. Recommender systems aim at recommending some item as most appropriate for the user. This article introduces a new way of measuring appropriateness for the user: an item is among the most appropriate ones if it is among the preferred items of the user when considering all arguments in favor or against all possible items. I describe precisely this goal and describe what a recommender system aiming for that goal could look like, called an Argumentative Recommender. I also provide a way of measuring whether a recommender system has achieved the goal, which can be used to compare such recommender systems, and briefly outline a way of building such a system.

1 Introduction

In a situation where a user has to choose an item among a set of possible items, recommender systems aim at recommending some item as most appropriate for the user. In the collaborative learning community, this is usually taken to mean: recommend an item among the most preferred for the user. In this article, I want to propose another goal for recommender systems. I keep the global aim of recommending some item as most appropriate for a given user, but I propose a different understanding of appropriateness. The goal I propose is to recommend an item among those that are in the Deliberated Preference (DP) of the user, and to prove that it does. The DP of the user captures what she prefers when considering all arguments in favor or against all possible items. This differs from her intuitive preference in that the latter does not consider arguments. Thus, the goal I propose is appropriate when one is interested to help the user form a deliberated judgment about which item is best, rather than predict which item the user intuitively prefers. Because this will be done using systems that argue, I call the kind of system I define here an Argumentative Recommender (AR). Further motivations for this new goal are described in section 2, and it is precisely defined in section 3.

Together with this new goal, it is necessary to propose a corresponding measure of the performance of the recommender system (done in section 4). In the classical approach, measuring the performance of the system amounts to compare its claims (the items it proposes as preferred) to reality (the preference of the user). Because the DP of the user is not directly observable, this direct check of correspondance becomes non applicable. However, as I will show, the problem of evaluating the quality of an AR can be framed as a prediction problem (although involving different objects), thereby recovering the possibility of empirical validation by confrontation between claims of the system and reality.

Section 5 relates our proposal to the literature. Section 6 then sketches one possible way of undertaking this task, applicable in specific contexts: recommend on the basis of explicit principles coming from decision theory.

2 Motivation

This article discusses a notion of preference that possibly changes when confronted to arguments. That is, I assume that the user's knowledge of which item is best for him is not necessarily fixed a priori, and in particular, that the user may change his mind when presented with arguments in favor of, or counter-arguments against, various items.

By contrast, the notion of preference considered by classical recommender systems (usually left implicit) is that the user intuitively knows what he prefers, and that this is the right basis for him to decide. This notion can be related to the conception of preference put forth by von Neumann and Morgenstern [1944, p. 16] in their seminal work. They write that "It is clear that every measurement – or rather every claim of measurability – must ultimately be based on some immediate sensation (. . .). In the case of utility the immediate sensation of preference (. . .) provides this basis." When applied to collaborative filtering, this view suggests that the user does not need help for comparing pairs of item: the job of the system is merely to relieve the user from having to search through the whole set. The user, when presented with a pair of items, knows which one he prefers by introspection, possibly after having tried them both. Similarly, when the user has already provided some comparisons of preference, this is the final word about those pairs.

This notion of preference as "intuitive" preference is not always the best basis to ground recommendation. Indeed, the user might be unwilling to judge what is best for her on the sole basis of her unaided intuition. First, there are cases where the user can't easily try out the items. An example is a non repeatable choice, such as choosing a university to study in. Second, the best choice might be the result of a complex thought process that should better ensure, as much as possible, that all relevant arguments have been considered, rather than purely rely on intuition. Consider the decision of which smartphone,

or which computer, or which house, to buy. This is also easily the case when a notion of fairness is involved, for example when determining who will receive a prize, or how to best distribute revenue in a society, or to which cause I should donate money. Third, the user might appreciate being pointed to some non-salient feature of the items under comparison; for example, a user might feel an intuitive attraction to the plane alternative when chosing a transportation means for holidays, seeing that the flight time is only one hour compared to three hours by train, but change her mind when being reminded that the total travel time should be taken into account. Finally, an argument coming from psychology is that unaided intuition is known to be sensible, in some contexts, to framing effects that would perhaps not be considered relevant (by the user himself) when considering arguments and counter-arguments [Kahneman, 2013]. An example that involves several kinds of the just enumerated concerns is obtaining a decision procedure for credit requests in a bank: the user probably wants to involve fairness considerations in order to avoid (possibly unconscious) appearance of unjust discriminations; and even if only profit considerations are to bear, the user might want the procedure to go beyond reflecting the bare intuition of an expert.

3 Definition of deliberated preference

This section describes the new understanding of preference I propose, called the DP of the user (it is a simplification of the recently proposed concept of deliberated judgment [Cailloux and Meinard, 2018]).

I consider given a set of items $\mathcal{J}$ among which a user i wants to choose. I also consider given a set S^* containing arguments that may possibly help i form a deliberated opinon about which item is best for her. Elements of S^* are not detailed and can be conceived as strings in a natural language. An example of an argument is $s =$ "Item j is better than item j' because j has a good performance on criteria 'price' and 'speed' while item j' has a good performance only on criterion 'aspect', which you do not consider important".

The DP of i is defined as the subset of items that are considered by i as having no item strictly preferred to them, considering all arguments in S^*. To define this properly, I assume we can observe the reaction of i to arguments, as follows.

Define $\triangleright$ as a binary relation over $\mathcal{J} \times S^*$, representing the results of the following experiments. Present (j, s) and (j', s') to i and let her decide, considering the arguments s and s', which item among j and j' she prefers, if any. Define $(j, s) \triangleright (j', s')$ iff i strictly prefers j to j', given s and s'. Note that if no strict preference holds, $\triangleright$ does not relate these two pairs (thus $(j, s) \not\triangleright (j', s')$ and $(j', s') \not\triangleright (j, s)$). The relation $\triangleright$ plays here the role of the basic observable von Neumann and Morgenstern talk about (see section 2).

The DP $J_i \subseteq \mathcal{J}$ of i in that situation contains an item j iff $\forall (j', s') \in \mathcal{J} \times S^*, \exists s \in S^* \mid (j', s') \not\triangleright (j, s)$.

4 Prediction of deliberated preference

An AR should be able, given a user, to single out some items as being in his DP, and some items as being not in his DP, and to prove its claims by arguing correspondingly. Formally, an AR η has a *scope I*, representing a set of users it claims

to be able to predict the DP of, and gives, for a user $i \in I$, a tuple $(J_\eta, f_\eta^{\mathrm{def}}, R_\eta, f_\eta^{\mathrm{att}})$, where $J_\eta \subseteq \mathcal{J}$ is a set of items that η claims are among the DP of i, $f_\eta^{\mathrm{def}} : J_\eta \times \mathcal{J} \to S^*$ is an argumentation strategy used to defend items in J_η, $R_\eta \subseteq \mathcal{J} \times \mathcal{J}$ is a binary relation that contains pairs of items (j, j') such that η claims that i deliberately prefers j to j', and $f_\eta^{\mathrm{att}} : R_\eta \to S^*$ is an argumentation strategy used to support the claims represented by R_η. (Elements of this tuple depend on i, though this is omitted from the notation.)

An AR is valid if its functions f_η^{def} and f_η^{att} indeed justify its claims. Informally, it is required of f_η^{def} that, when given an item j' and given an item $j \in J_\eta$ considered by η as being in the DP of i, f_η^{def} produces an argument that successfully defends j, whatever the argument given in favor of j'. More precisely and completely, an AR η is said to be *correct* about a user i iff $\forall j \in J_\eta, j' \in \mathcal{J}, s' \in S^* : (j', s') \not\triangleright (j, f_\eta^{\mathrm{def}}(j, j'))$ and $\forall (j, j') \in R_\eta, s' \in S^* : (j, f_\eta^{\mathrm{att}}(j, j')) \triangleright (j', s')$. An AR is *valid* iff it is correct about all users in its scope.

Importantly, validity implies the correctness of the claims concerning the DP of i. This fact is stated formally below, with $J_\eta^\downarrow = R_\eta(\mathcal{J}) = \{ j' \in \mathcal{J} \mid (j, j') \in R_\eta \text{ for some } j \in \mathcal{J} \}$. (The proof is omitted.)

Fact 1. *If an AR η is valid, then $\forall i \in I : J_\eta \subseteq J_i$ and $J_\eta^\downarrow \subseteq \mathcal{J} \setminus J_i$.*

This notion of validity can be used to *compare* ARs, in the following way. Consider two ARs η and η' dealing with the same set of items $\mathcal{J}$ and having overlapping scopes (so that the intersection of the users in their scopes is not null). Such ARs could for example have been built by different research teams studing the same recommendation setting. Considering a user i in both their scopes, obtain two tuples, $(J_\eta, f_\eta^{\mathrm{def}}, R_\eta, f_\eta^{\mathrm{att}})$ and $(J_{\eta'}, f_{\eta'}^{\mathrm{def}}, R_{\eta'}, f_{\eta'}^{\mathrm{att}})$. Assume they strongly disagree on the prediction of the DP of i, meaning that for some item $j \in \mathcal{J}$, the first system claims that j is in the DP of i, thus $j \in J_\eta$, and the second system claims it is not, thus $\exists j' \mid (j', j) \in R_{\eta'}$. Suffices now to let the two systems play against each other and use i as a judge. That is, we obtain an argument from the first system in defense of its claim, $s = f_\eta^{\mathrm{def}}(j, j')$, and an argument from the second system, $s' = f_{\eta'}^{\mathrm{att}}(j', j)$. We present (j, s) and (j', s') to i and accordingly obtain $(j', s') \triangleright (j, s)$ or $(j', s') \not\triangleright (j, s)$. In the first case, the first system is invalidated, in the second one, the second system is invalidated.

5 Relation to existing literature

Approaches exist which propose to estimate a decision-theoretic model that best approximates a user's observed behavior [Greco et al., 2010, Sobrie et al., 2018]. Furthermore, numerous approaches have been proposed for modeling preferences using machine learning methods [Fürnkranz and Hüllermeier, 2010]. Yet other approaches propose to use argumentation theory to enhance recommender systems [Chesñevar et al., 2009, Rago et al., 2018]. These trends constitute promising areas of investigation for building ARs. Such articles typically are focused towards producing recommender systems on the basis of some principles chosen by the authors. By comparison, this article proposes a precise definition of a new goal for similar argumentative systems (which may or may not be similar to the goal the authors of the cited articles

have in mind), and a way of comparing ARs experimentally when their predictions disagree.

The idea of using a human as a judge to compare argumentative systems also appears in the paper of Irving et al. [2018]. There are also articles that are interested in using collaborative filtering on the basis of implicit feedback [Rendle et al., 2009, Hu et al., 2008], an idea somewhat related to ours in the sense that the observed data is not directly the one the model tries to predict. Also somewhat related are articles interested in collecting preference data for improving the learning [Sepliarskaia et al., 2018]. An important literature analyzes necessary and sufficient conditions for the existence of various decision-theoretic models [Krantz et al., 1971, Gonzales, 1996, Bouyssou and Pirlot, 2015]. Finally, this proposal is intimately related to the important literature on explainable AI [Zhang and Chen, 2018].

Johnson [2014] also uses logits for matrix factorization. Chen et al. [2018] review techniques that make use of supplementary data in collaborative filtering tasks.

Todo. Relate to work by Passerini et alia [Teso et al., 2016, 2017, Dragone et al., 2018b, Erculiani et al., 2018, Dragone et al., 2018a].

6 Recommend on the basis of explicit principles

Section 3 described a new kind of recommender system, that consider the recommendation task under a different light, and section 4 described how to evaluate such ARs. But such ARs do not exist yet. One way of building such systems consists in adapting existing works (related references are given in section 5). Another way is to design an AR from the ground up specifically for tackling the goal described here. Although it is not the main point of this conceptual article, roughly sketching a possible way of doing this could help make this proposal more concrete. This section first briefly describes such a possible way in the context of collaborative settings, then indicates some conditions on the context that must be fulfilled in order to make the proposed approach applicable.

Consider a collaborative learning setting, where we have collected ratings from users on items, assumed to represent their intuitive rather than deliberated preference. A simple approach to build a proof-of-concept AR in such a context is to represent the DP of i as a weak order $\succeq \subseteq \mathcal{J} \times \mathcal{J}$ corresponding to R_η and whose maximal elements correspond to J_η (though the definition tolerates other possibilities). Assume that the DP of the user can be represented by a decision-theoretic model: because such models are conceived for grounding decisions in sound principles, they might be adequate to model deliberated preferences. For concreteness, assume it is Multiple Attribute Value Theory (MAVT) [Keeney and Raiffa, 1993], a set of principles for choice well-known in decision theory.

Existing works [Carenini and Moore, 2006, Labreuche, 2011] describe how to generate arguments in natural language that explain, on the basis of an MAVT model, why an alternative is preferred to another one. This provides a starting point for the step of generating arguments. More generally, by picking functions supported by well-studied decision principles, it is made possible to build arguments on the basis of those same principles, and it might be considered that such arguments will have a fighting chance against counter-arguments.

Finally, assume the intuitive behavior of the user is a noisy version of her DP. (More elaborated developments would incorporate knowledge from experimental psychology about differences between the usual behavior of users and the one dictated by decision-theoretic models [Kahneman, 2013].) Then, our task becomes close to one of classical collaborative learning, as MAVT defines the class of functions among which to select an optimal one during learning. Future work will attempt to draw on this approach to effectively propose an MAVT-based AR.

The principles on which MAVT rests are applicable in specific contexts: those in which the items (among which the user aims to choose) can be exhaustively described by a set of objective descriptors of the items, known a priori, and which each evaluate the desirability of the items on some aspect. Such aspects are called criteria. Exhaustively described means that the user considers nothing else than the performance of the items on the criteria as relevant for the recommendation. Thus, consider a set of criteria $\mathcal{C}$ is known, together with corresponding scales $X_c, c \in \mathcal{C}$, and descriptors $b_c : \mathcal{J} \to X_c, c \in \mathcal{C}$, that each describe the items on a specific criterion c.

In such a context, an MAVT model representing $\succeq$ is a set of functions $v_c : X_c \to \mathbb{R}$ (one for each criterion) such that $j \succeq j'$ iff $\sum_{c \in C} v_c(b_c(j)) \geq \sum_{c \in C} v_c(b_c(j'))$.

Consider as an example the choice of an apartment to rent for holidays. The set of criteria might be: surface of the living room, number of beds, distance to city center, modernity of equipment, presence of a washing machine, price. Other examples where building an AR on the basis of an MAVT model may be appropriate include: buying a smartphone, buying a computer, choosing a flight to reach a given place, choosing who to give a "best student" award among a given classroom, locating a new factory, or choosing a university to study at.

By contrast, a recommendation task is not suitable for the approach proposed in this section if no set of criteria is known that fully describe the desirability of an item. For example, a choice of movie to watch this evening.

7 Estimating an MAVT model

Assume we have a set of users I of size $m \in \mathbb{N}$, a set of items J of size $n \in \mathbb{N}$, a set of observed pairs $O \subseteq I \times J$ and a relation r mapping those observed pairs $(i, j) \in O$ to some rating r_{ij}, an integer in $[\![1, 5]\!]$ (throughout the article, this notation designates intervals in the integers). We also have descriptions of the users in a vector space A of dimension $d_A \in \mathbb{N}$: user i is described by $a_i \in A$. The items are described by a set of descriptors whose indices are in $\mathcal{C} = [\![1, \ldots, c, \ldots, d]\!]$. The descriptor $B_c : J \to X_c, B_c \in \mathcal{B}$ evaluates each item according to a scale X_c. Let $X = \prod_{c \in \mathcal{B}}$ denote the cardinal product space where descriptions of items lie, and let $b_j \in X$ denote the description of an item j.

7.1 Classical collaborative learning

In classical collaborative learning, we wish to obtain compressed descriptions of the users and the items, $\{u_i, i \in I\}$ and $\{v_j, j \in J\}$, with all u_i and v_j being vectors of some size $k \in \mathbb{N}$. We write U the matrix of size (m, k) that has all u_i^T as rows, and V the matrix of size (k, n) that has all v_j

as columns. The compressed descriptions should be such that $r_{ij} \approx u_i^T v_j$.

7.2 Using given knowledge

However, we also wish that our compressed descriptions u_i be in a simple relationship with the provided descriptions a_i, thus, we search for $u_i \approx f(a_i)$, and similarly, for $v_j \approx g(b_j)$, with f and g "simple". More precisely, we search for a matrix W_A of size (k, d_A) such that $W_A a_i \approx u_i$, and W_B of size (k, d_B) such that $W_B b_j \approx v_j$. We call $W_A a_i$ and $W_B b_j$ our explainable representation of the users and the items.

We could define $u_i = W_A a_i$ and $v_j = W_B b_j$ and obtain $r_{ij} \approx a_i^T (W_A^T W_B) b_j$, thus we could optimize by searching for the best $(W_A^T W_B)$. But as this could be too constrained, we will also try to relax these constraints and search for a representation that takes into account that u_i should be close to $W_A a_i$, but not necessarily equal.

7.3 Predict ratings with logits

Assume we are given a loss function L that compares the ratings r_{ij} to our approximations of the ratings, for $(i, j) \in O$.

Define $\sigma(\alpha, \beta, x) = \frac{1}{1 + \alpha e^{-\beta x}}$.

We want to find $\{w_i^c, i \in I, c \in \mathcal{C}\}, \{\alpha^c, c \in \mathcal{C}\}, \{\beta^c, c \in \mathcal{C}\}$ that optimize the following objective:

$$\min_{w, \alpha, \beta} \sum_{(i,j) \in O} L(r_{ij}, \sum_{c \in \mathcal{C}} w_i^c \sigma(\alpha^c, \beta^c, b_j^c)) + \lambda \sum_{i,c} \|w_i^c\|_2^2.$$

We force w positive. And α is forced positive to respect the preference direction: we assume the data is encoded so that higher b_j^c means preferred performance. Where b_j^c is the performance of item b_j on criterion c.

We considered the ratings over $J_i, i \in I$, and $\mathcal{C} = \{\imath style\jmath, \imath majorgroup\jmath, \imath heaviness\jmath, \imath frequency\jmath, \imath price\jmath, \imath popularity\jmath\}$, $|\mathcal{C}| = 6$. We obtain an RMSE of about 1.3. With Jill-Jen's bilinear model (with biases), we obtained 1.2106. With only bias for user and bias for item, we obtain 1.1670. The RMSE of a constant model (that always predicts the observed average) is 1.2729. With SVD++ (bilinear and bias, rank=3): 1.1685.

See `https://github.com/caserec/Datasets-for-Recommnederi-Systems`

7.4 Predict ratings

We could want to check whether we are able to predict the right ratings. In that case, we would assume we are given loss functions L, that compare the ratings r_{ij} to our approximations of the ratings, for $(i, j) \in O$; L_I, that compare the vectors u_i to our explainable representation of them; and L_J, that compare the vectors v_j to our explainable representation of them.

We would want to find matrices U, V, W_A, W_B that optimize the following objective:

$$\min_{U, V, W_A, W_B} \sum_{(i,j) \in O} L(r_{ij}, u_i^T v_j) + \sum_{i \in I} L_I(u_i, W_A a_i) + \sum_{j \in J} L_J(v_j, W_B b_j)$$

7.5 Predict orders

We are interested in comparing our predictions to orderings (because that's what we have underhand).

We assume we are given loss function L, that compares preorders (transitive and reflexive binary relations), and L_I and L_J, as above. We assume we have a preorder $\geq_i$ for each user indicating her true preference over some pairs of items among J.

We want to find matrices U, V, W_A, W_B that optimize the following objective:

$$\min_{U, V, W_A, W_B} \sum_{i \in i} L(\geq_i, \{(v_j, v_{j'}) \mid u_i^T v_j \geq u_i^T v_{j'}\}) + \sum_{i \in I} L_I(u_i, W_A a_i) + \sum_{j \in J}$$

7.6 With artificial data

Let us consider that users share a personally-scaled and noisy version of an additive utility representation. Define the domain of each criterion as $[0, 1]$, and the partial value function v_c on criterion c as a piecewise linear value function with discontinuity points at x_1 and x_2. User i has discontinuity points $x_1 + \alpha_{i,c}, x_2 + \beta_{i,c}$ with $\alpha_{i,c}, \beta_{i,c}$ taken from some probability distribution. User i has value function $v_i = \sum_{c \in \mathcal{C}} w_i v_i^c$. Furthermore, the user reports a preference that is a noisy version of her deliberated preference, up to some random error parameter. We generate datasets using those artificial value functions, and we check to which extend we can predict her deliberated preference.

7.7 Data set: Sushi

We experiment using the Sushi data set (ref. . .)

Here J is a set of 100 sushi types [1]. The original authors collected the sushi from the menu of 25 restaurants, computed the frequency of each sushi type occurring on a menu, and took the 100 most frequent. Write P_J the frequency (thus a multiple of $1/25$).

Write $J_{\text{top}} \subset J$ the 10 most frequent sushi types [2].

The set I has cardinal $m = 5000$. The authors drew randomly a set of 10 sushi types among J, for each user i, according to the distribution P_J. Let us denote this by $J_i \subset J$ [3].

The authors obtained, for each user i in I, three preference relations, defined as strict partial orders (transitive, asymmetric binary relations). First, they collected a preference ordering over the sushi types from J_{top} from most to least preferred, with no tie allowed. Let $>_i^{\text{top}}$ denote that preference ordering [4] ("top" designates the fact that the ordering is over the 10 most frequent sushi types). Second, they collected a rating, in $[\![1, 5]\!]$, for each sushi type in J_i. Define O as the set of pairs (i, j) such that $j \in J_i$, and let r_{ij} denote that rating. Third, after two supplementary questions asking how oily the user think each sushi type is and how frequently they eat them, they collected a preference ordering over J_i, that we write $>_i^{\text{indiv}}$ [5] ("indiv" designates the fact that this relation

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[1] $X^* = X_B$ in the original notation.
[2] X_A
[3] X_i^B
[4] O_i^A
[5] O_i^B

orders a set of sushi that has been drawn specifically for that user).

Users are described in a space A comprising the attributes {gender, age category, origin-prefecture, origin-region, origin-east/west (binary), current-region, current-east/west (binary), origin-prefecture}. Items are described in a space B comprising the attributes {group, heaviness/oiliness in taste (in $[0,4]$), frequency of eating (in $[0,3]$), normalized price (in $[0,1]$), $P_J(j)$}.

We summarize the preference of i as follows. The ratings over J_i are transformed to preorders $\geq_i^{\text{rating}}$ that tolerate ex-æquos, in the obvious way (better ratings indicate preference). We then merge, for each user i, all preferential information, considering contradictions as indicating indifference, and obtain a preorder $\geq_i = [\geq_i^{\text{rating}} \cup >_i^{\text{top}} \cup >_i^{\text{indiv}}]$, where the brackets designate taking the transitive closure and adding the identity relation (to obtain reflexivity). For example, if $>_i^{\text{top}}$ represents the ordering $(j_1 > j_2 > j_3)$ and $>_i^{\text{indiv}}$ represents $(j_6 > j_3 > j_4 > j_2)$ and $\geq_i^{\text{rating}} = \emptyset$, then $\geq_i$ considers j_1 and j_6 as strictly preferred to j_2 to j_4, j_2 to j_4 ex-æquos, and j_1 incomparable to j_6.

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A Sufficient conditions for an MAVT representation

Condition 1: the descriptors $\{b_c, c \in \mathcal{C}\}$ are such that whether $j \succeq j'$ depend only on the descriptions on the items according to $\{b_c, c \in \mathcal{C}\}$.

In the sequel we treat $\succeq$ as a relation over $X = \prod_{c \in \mathcal{C}} X_c$, which we can do thanks to condition 1. Define $\sim$ as the symmetric part of $\succeq$ and $\succ$ its asymmetric part. Thus, $\sim$ is an equivalence relation.

Our second condition mandates that comparisons between two items that have the same value on some descriptor do not change if that value changes equally for both items (thus, the comparison must be independent of that value provided it is equal on both items). For example, if the user deliberately prefers a flat that is 50 meters square and has a washing machine to a flat that is 60 meters square but has no washing machine, given that both flats are priced 500 € (and assuming only these three criteria matter), then the user has the same deliberated preference ordering for two flats that have the same characteristics except that they are both priced 800 €.

With $C = \mathcal{C} \setminus \{c\}$ for some $c \in \mathcal{C}$, thus C contains all criteria but one, let $X_C = \prod_{d \in C} X_d$, and let $(x_C, w_c) \in X$ represent the performance of an item $j \in \mathcal{J}$ where $x_C \in X_C$ describes the item j on the criteria C and $w_c \in X_c$ is the remaining description.

Condition 2: $\forall C = \mathcal{C} \setminus \{c\}, w_c, z_c \in X_c, x_C, y_C \in X_C$: $(x_C, w_c) \succ (y_C, w_c) \Rightarrow (x_C, z_c) \succ (y_C, z_c)$.

Our last two conditions are more technical. First, the set of possible items must be rich enough. We say that a criterion c is solvable iff, with $C = \mathcal{C} \setminus \{c\}$, for all $x \in \mathcal{J}$ and $y_C \in X_C$, there exists $z_c \in X_c$ such that $x \sim (y_C, z_c)$.

Condition 3: there are at least three solvable criteria.

In the example of a choice of a flat, three solvable criteria might be the size of the flat in meter squares, its price, and its distance to the city center. The presence of a washing machine is a non solvable criterion.

The second technical and last condition is an archimedian requirement. In loose terms, we want to forbid that some item be infinitely far away, in the sense that the item would be preferred to an infinite sequence of equally spaced items. Given a criterion c, with $C = \mathcal{C} \setminus \{c\}$, given $y_C, z_C \in X_C$, we say that y_C and z_C are not indifferent iff there exists some $x_c \in X_c$ such that not $(x_c, y_C) \sim (x_c, z_C)$ (note that by Condition 2, this "exists" is equivalent to a "for all"). Given $c \in \mathcal{C}, C = \mathcal{C} \setminus \{c\}$, we say that an infinite sequence of values in X_c is equally spaced iff there exists $y_C, z_C \in X_C$ not indifferent such that for any two consecutive elements x_c, x'_c in the sequence, $(x_c, y_C) \sim (x'_c, z_C)$. Furthermore, such a sequence

is bounded iff there exists an item that is strictly deliberately preferred to (x_c, y_C) for every x_c in the sequence.

Condition 4: There exists a solvable criterion c such that there is no infinite sequence of values in X_c that is equally spaced and bounded.

Theorem 1. *Given a set of items $\mathcal{J}$, a set of criteria $\mathcal{C}$, scales X_c and descriptors $b_c : \mathcal{J} \to X_c, c \in \mathcal{C}$, and $\succeq$ a binary relation over $\mathcal{J}$, if the four conditions above are satisfied, then an MAVT model representing $\succeq$ exists.*

B Double-strategy

In the double-strategy, we use different dimensions: $k = d_{A'} + d_U = d_{B'} + d_V$. The matrix W_A has size $(d_{A'}, d_A)$ and U has size (m, d_U). W_B has size $(d_{B'}, d_B)$ and V has size (d_V, n). $r_{ij} = \mu_i^T \nu_j$ where μ_i is the k-dimensional vector composed of $W_A a_i$ then u_i and ν_j is composed of $W_B b_j$ then v_j.

We should compare this approach to the more classical one.

C Using raw item descriptions

The matrix W_B could be constrained to having mostly zeroes, to make sure that indeed the attributes of the items (b_j) are used in their μ-vectorial description. We could even take $d_{B'} = d_B$ and take W_B equal to identity. (Is there any benefit not to do so?)

This suggests the following simple approach: concatenate b_j to v_j to obtain ν_j. Don't use W_B. Obtain μ_i of size $k = d_B + d_V$, using $W_A a_i$ as its first components.

D Factorization machines

We should compare (theoretically or experimentally) our approach to factorization machines.