Social ranking under uncertainty

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Introduction

Social rankings

In situations where the only available information is over subgroups (coalitions) of a population :
$\rightarrow$ performance of teams
$\rightarrow$ individuals defined by different criteria...

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In situations where the only available information is over subgroups (coalitions) of a population :
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$\triangleright$ Given a decider's ordinal preference ranking over coalitions, how to determine a preference ranking over the individuals within these coalitions ?

Lexicographic excellence

▷ Introduced by Bernardi, Luchetti & Moretti in 2019 (1)

Let X={1, 2, 3, 4},

$$34 \succ 123 \sim 24 \succ 134 \sim 124 \succ 13 \sim 23 \sim 1234 \succ 12 \succ 234 \sim 14$$

θ_1 = (0, 1, 2, 2, 1, 1)
θ_2 = (0, 2, 1, 2, 1, 1)
θ_3 = (1, 1, 1, 3, 0, 1)
θ_4 = (1, 1, 2, 1, 0, 2)

By simple lexicographic comparison of the θ_i vectors, we find that

$$4 \succ 3 \succ 2 \succ 1$$

CP-majority

▷ Introduced by Haret, Khani, Moretti & Ôzturk in 2018 (2)

Let X={1, 2, 3, 4},

$34 \succ 123 \sim 24 \succ 134 \sim 124 \succ 13 \sim 23 \sim 1234 \succ 12 \succ 234 \sim 14$

$d_{12} = 1$ ($134 \succ 234$), $d_{21} = 1$ ($24 \succ 14$)
$d_{13} = 1$, $d_{31} = 2$
$d_{14} = 1$, $d_{41} = 2$
$d_{23} = 0$, $d_{32} = 2$
$d_{24} = 2$, $d_{42} = 1$
$d_{34} = 2$, $d_{43} = 1$

By pairwise comparison, we find that $1 \sim 2$, $3 \succ 1$, $4 \succ 1$, $3 \succ 2$, $2 \succ 4$ and $3 \succ 4$.

Ordinal Banzhaf

▷ Introduced by Khani, Moretti & Öztürk in 2019 (3)

Let X={1, 2, 3, 4},

$34 \succ 24 \succ 134 \sim 124 \succ 13 \sim 23 \sim 234 \succ 12 \sim 123 \succ 1234 \sim 14$

$u_1^{+,\succeq} = 0$, $u_1^{-,\succeq} = 4$ *(23, 24, 34, 234)*, $s_1^{\succeq} = -4$;
$u_2^{+,\succeq} = 2$ *(13, 14)*, $u_2^{-,\succeq} = 2$ *(34, 134)*, $s_2^{\succeq} = 0$;
$u_3^{+,\succeq} = 1$ *(14)*, $u_3^{-,\succeq} = 2$ *(24, 124)*, $s_3^{\succeq} = -1$;
$u_4^{+,\succeq} = 2$ *(12, 13)*, $u_4^{-,\succeq} = 1$ *(123)*, $s_4^{\succeq} = 1$;

By comparison of ordinal Banzhaf scores $s_i^{\succeq}$, we find that

$$4 \succ 2 \succ 3 \succ 1$$

Types of preference uncertainties

Piecemeal information

All coalitions are present within subrankings.

▷ Incomparable alternatives

▷ Lack of information on pairwise comparisons

Let $X=\{1, 2, 3, 4\}$,

$$12 \succ 13 \succ 234$$
$$124 \succ 24 \succ 34$$
$$23 \succ 123$$
$$14 \succ 1234 \succ 134$$

Piecemeal information

Quasi-totality

$\forall S \subseteq X, \exists T \subseteq X, S \neq T, S \succeq T$ or $T \succeq S$

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Transitivity

$\forall S, T, V \subseteq X, (S \succeq T$ and $T \succeq V) \Rightarrow S \succeq V$

Piecemeal information

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$\forall S \subseteq X, \exists T \subseteq X, S \neq T, S \succeq T$ or $T \succeq S$

Transitivity

$\forall S, T, V \subseteq X, (S \succeq T$ and $T \succeq V) \Rightarrow S \succeq V$

Condition of presence

$\forall x \in X, \exists S \subseteq X, x \in S$ and $\exists T \subseteq X, T \neq S, S \succeq T$ or $T \succeq S$

Partial information

Not all coalitions are present in the preference profile.

▷ Partial elicitation of preferences

▷ Unrealizable coalitions

Let X={1, 2, 3, 4},

$$14 \succ 134 \succ 24 \succ 124 \succ 23$$

Partial information

- ▷ Totality and quasi-totality are not verified.

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Necessary and possible winners

Necessary winner

▷ Introduced by Konczak and Lang in 2005 (4)

Voting procedures for situations where voters' preferences consist of partial orders.
→ Study extensions of these partial orders.

Necessary winner

An alternative *a* is a *necessary winner* if it is a winner for every extension of the voters' profiles.

Possible winner

An alternative *a* is a *necessary winner* if it is a winner for at least one extension of the voters' profiles.

Adapting necessary winners to social ranking

▷ A necessary winner - if there is one - can be found in polynomial time if the social ranking rule is itself polynomial ; determining a possible winner is an NP-complete problem

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▷ Repeating the search for a necessary winner : if none exists, how to use the concept of possible winner ?

▷ Properties to reduce the number of considered extensions based on the given preference order

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