Preferences elicitation under incomplete knowledge

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LAMSADE
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Classical setting

Agents = {  }, Altern. = {  }, Chair =  $\Rightarrow$ Voting Rule

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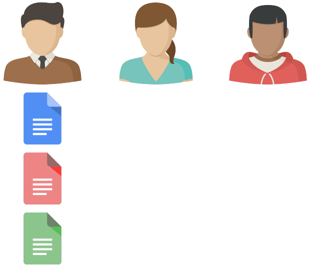

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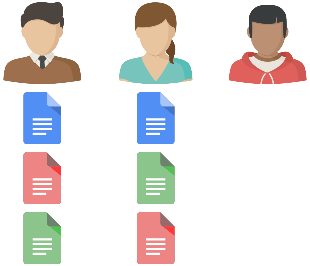

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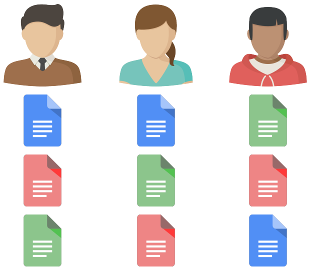

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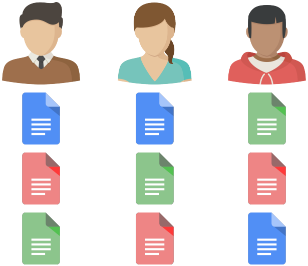

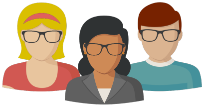

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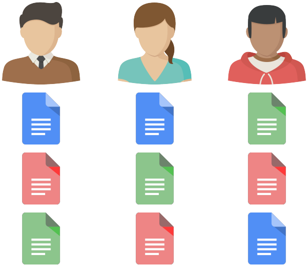

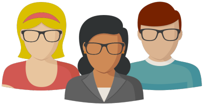

Borda

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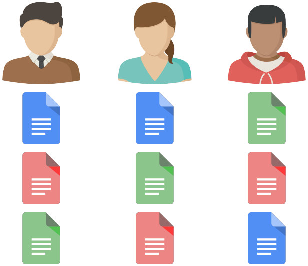

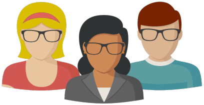

Borda

winner: 

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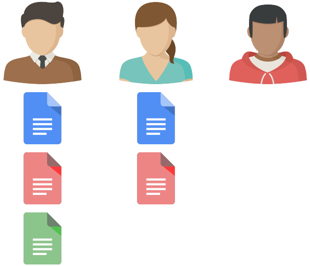

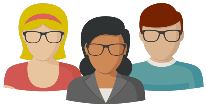

Borda

winner: ?

Related Works

Incomplete profile

- and known rule: Minimax regret to produce a robust winner approximation (*Lu and Boutilier 2011*, [4]; *Boutilier et al. 2006*, [3])

Incomplete knowledge: voting rule

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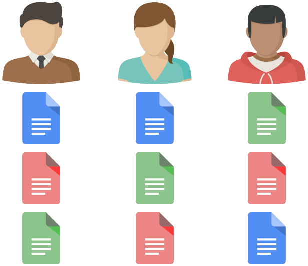

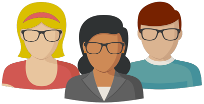

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Related Works

Incomplete profile

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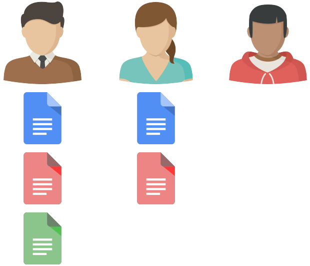
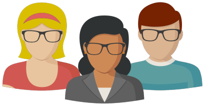
Uncertain rule

- and complete profile: dominance relations derived to eliminate alternatives always less preferred than others (*Stein et al. 1994*, [6])
- considering positional scoring rules (*Viappiani 2018*, [7])

Incomplete knowledge: profile and voting rule

Agents = $\{$, , $\}$, Altern. = $\{$, , $\}$, Chair = $\Rightarrow$ Voting Rule





?

winner: ?

Simultaneous Elicitation of Scoring Rule and Agent Preferences for Robust Winner Determination

Setting: Incompletely specified preferences and social choice rule

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Goal: Reduce uncertainty, inferring (*eliciting*) incrementally and simultaneously the true preferences of agents and chair to quickly converge to an optimal or a near-optimal alternative

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Approach:

Beatrice Napolitano, Olivier Cailloux, and Paolo Viappiani. Simultaneous elicitation of scoring rule and agent preferences for robust winner determination.
In *Proceedings of Algorithmic Decision Theory - 7th International Conference, ADT 2021*

- Develop query strategies that interleave questions to the chair and to the agents
- Use *Minimax regret* to measure the quality of those strategies

Notation

A	alternatives, $\|A\| = m$
N	agents (*voters*)
$P = (\succ_j,\ j \in N),\ P \in \mathcal{P}$	complete preferences profile
$W = (W_r,\ 1 \leq r \leq m),\ W \in \mathcal{W}$	**convex** scoring vector that the chair has in mind

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W defines a **Positional Scoring Rule** $f_W(P) \subseteq A$ using scores $s^{W,P}(a),\ \forall\ a \in A$

P and W exist in the minds of agents and chair but unknown to us

Questions

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Questions to the agents

Comparison queries that ask a particular agent j to compare two alternatives $a, b \in \mathcal{A}$

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Comparison queries that ask a particular agent j to compare two alternatives $a, b \in \mathcal{A}$

Questions to the chair

Queries relating the difference between the importance of consecutive ranks from r to $r+2$

The answers to these questions define $\mathbf{C_P}$ and $\mathbf{C_W}$ that is our knowledge about P and W

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

$$\mathrm{PMR}^{C_P,C_W}(a,b) = \max_{P \in C_P, W \in C_W} s^{P,W}(b) - s^{P,W}(a)$$

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We care about the worst case loss: *maximum regret* between a chosen alternative a and best real alternative b

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We select the alternative that *minimizes* the maximum regret

Elicitation strategies

At each step, the strategy selects a question to ask either to one of the agents about her preferences or to the chair about the voting rule

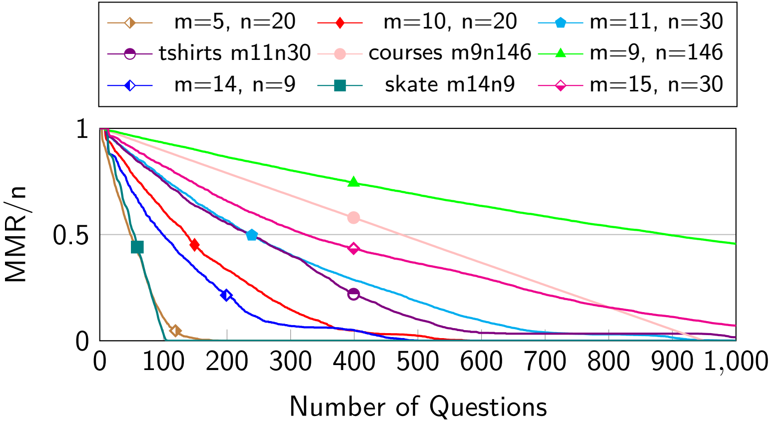
The termination condition could be:

- when the minimax regret is lower than a threshold
- when the minimax regret is zero

- It selects first $n + (m - 2)$ candidate questions: one per each agent and one per each rank (excluding the first and the last one which are known)

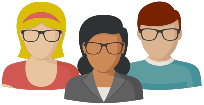
- It selects the question that leads to minimal regret in the worst case from the set candidate questions

Figure: Average MMR (normalized by n) after k questions with Pessimistic strategy for different datasets.



Incomplete knowledge: profile

Agents $=$ { , ,  }, Altern. $=$ { , ,  }, Chair $=$  $\Rightarrow$ Voting Rule



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Majority Judgment

Majority Judgment

Voters judges candidates assigning grades from an ordinal scale. The winner is the candidate with the highest median of the grades received. (Balinski and Laraki 2011, [2])

Excellent	Very good	Good	Average	Mediocre	Inadequate	To be Rejected

Majority Judgment

Agents = { , , }, Altern. = { , , }, Chair = ⇒ Majority Judgment

	Excellent	Average	Mediocre
	Good	Mediocre	Good
	Inadequate	Excellent	Very good

Majority Judgment

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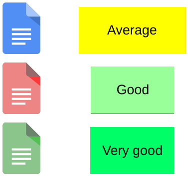
Median



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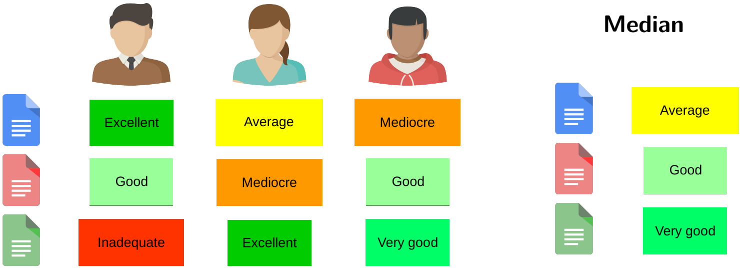

winner: 

Majority Judgment: Incomplete Knowledge

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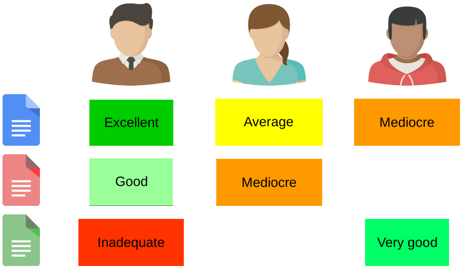

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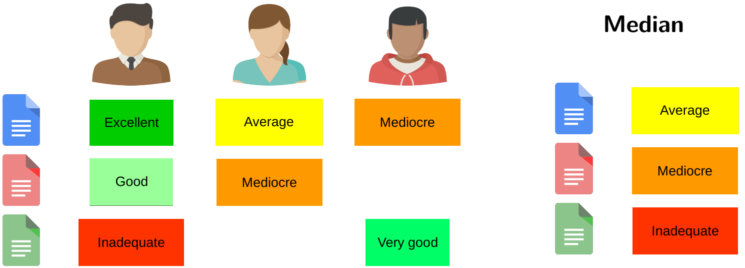

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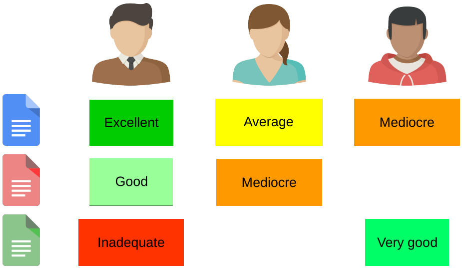

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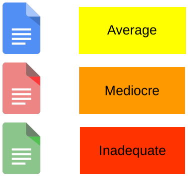


winner: 

In the last few years MJ has being adopted by a progressively larger number of french political parties including: Le Parti Pirate, Génération(s), LaPrimaire.org, France Insoumise and La République en Marche. [1]

LaPrimaire.org is a french political initiative whose goal is to select an independent candidate for the french presidential election using MJ as voting rule.

The procedure consists of two rounds:

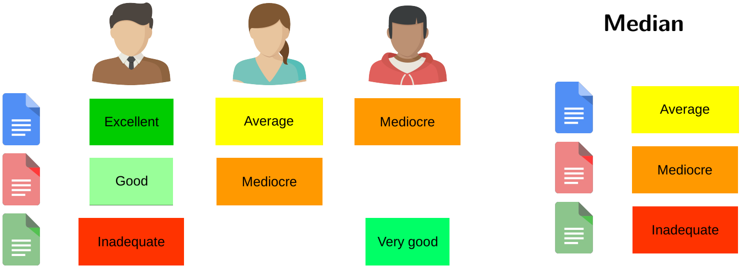
1: each voter expresses her judgment on five random candidates. The five ones with the highest medians qualify for the second round

2: each voter expresses her judgment on all the five finalists. The one with the best median is the winner

Majority Judgment

LaPrimaire.org

Agents $= \{$, , $\}$, Altern. $= \{$, , $\}$, Chair $=$ $\Rightarrow$ Majority Judgment



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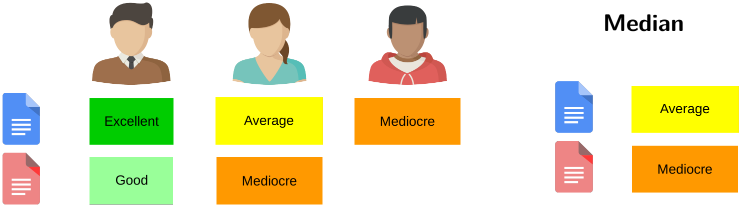

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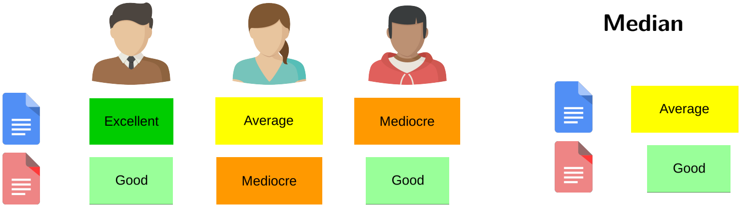

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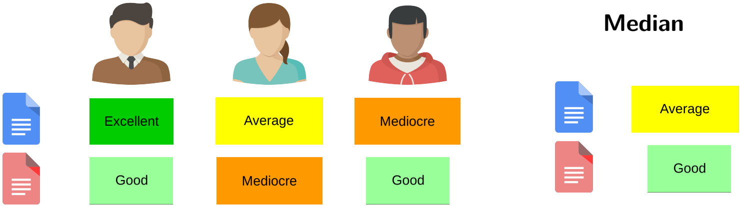

winner: 

Research Questions

- What is the probability of selecting a winner different from the one selected in case of complete knowledge?
- Can we elicit voters preferences using a minimax regret notion?
- What is the best trade-off between communication cost and optimal result?
- What is the voting rule applied on the resulting incomplete profile? What are its properties?

Thank You!

Icon made by Eucalyp from www.flaticon.com

Mieux voter.

Michel Balinski and Rida Laraki.
Majority Judgment: Measuring, Ranking, and Electing.
The MIT Press, 01 2011.

Craig Boutilier, Relu Patrascu, Pascal Poupart, and Dale Schuurmans.
Constraint-based optimization and utility elicitation using the minimax decision criterion.
Artificial Intelligence, 2006.

Tyler Lu and Craig Boutilier.
Robust approximation and incremental elicitation in voting protocols.
In *Proc. of IJCAI'11*, 2011.

Beatrice Napolitano, Olivier Cailloux, and Paolo Viappiani.
Simultaneous elicitation of scoring rule and agent preferences for robust winner determination.
In *Proceedings of Algorithmic Decision Theory - 7th International Conference, ADT 2021*.

William E. Stein, Philip J. Mizzi, and Roger C. Pfaffenberger.
A stochastic dominance analysis of ranked voting systems with scoring.
EJOR, 1994.

Paolo Viappiani.
Positional scoring rules with uncertain weights.
In *Scalable Uncertainty Management*, 2018.

Empirical Evaluation

Pessimistic reaching "low enough" regret

Table: Questions asked by Pessimistic strategy on several datasets to reach $\frac{n}{10}$ regret, columns 4 and 5, and zero regret, last two columns.

dataset	m	n	$q_c^{MMR \leq n/10}$	$q_a^{MMR \leq n/10}$	$q_c^{MMR=0}$	$q_a^{MMR=0}$
m5n20	5	20	0.0	[4.3 | 5.0 | 5.8]	5.3	[5.4 | 6.2 | 7.2]
m10n20	10	20	0.0	[13.9 | 16.1 | 18.4]	32.0	[19.7 | 21.8 | 24.7]
m11n30	11	30	0.0	[16.6 | 19.0 | 22.3]	45.2	[23.1 | 25.7 | 28.9]
tshirts	11	30	0.0	[13.1 | 16.6 | 19.6]	43.2	[28.2 | 32.0 | 35.6]
courses	9	146	0.0	[6.0 | 7.0 | 7.0]	0.0	[6.8 | 7.0 | 7.0]
m14n9	14	9	5.4	[30.3 | 33.5 | 36.7]	64.1	[37.6 | 40.5 | 44.3]
skate	14	9	0.0	[11.4 | 11.6 | 12.3]	0.0	[11.5 | 11.8 | 12.8]
m15n30	15	30	0.0	[25.0 | 29.5 | 33.7]		

Empirical Evaluation

Pessimistic chair first and then agents (and vice-versa)

Table: Average MMR in problems of size $(10, 20)$ after 500 questions, among which q_c to the chair.

q_c	ca $\pm$ sd	ac $\pm$ sd
0	0.6 ± 0.5	0.6 ± 0.5
15	0.5 ± 0.5	0.5 ± 0.5
30	0.3 ± 0.5	0.3 ± 0.4
50	0.0 ± 0.1	0.0 ± 0.1
100	0.1 ± 0.2	0.1 ± 0.1
200	2.3 ± 1.4	2.1 ± 1.8
300	5.2 ± 2.4	6.8 ± 0.6
400	10.9 ± 0.9	12.2 ± 1.0
500	20.0 ± 0.0	20.0 ± 0.0

Building concrete questions for the chair

Queries relating the difference between the importance of consecutive ranks

$$W_r - W_{r+1} \geq \lambda(W_{r+1} - W_{r+2}) \quad ?$$

Building concrete questions for the chair

Queries relating the difference between the importance of consecutive ranks

$$W_2 - W_3 \geq 2\,(W_3 - W_4) \quad ?$$

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Abstract queries are difficult to answer

$$W_2 \; - \; W_3 \geq 2\; W_3 - 2\; W_4$$
$$W_2 + 2\; W_4 \geq 3\; W_3$$

Building concrete questions for the chair

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$$\begin{aligned} W_2 \;-\; W_3 &\geq 2\ W_3 - 2\ W_4 \\ W_2 + 2\ W_4 &\geq 3\ W_3 \\ s(a) &\geq s(b) \end{aligned}$$

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#1	**#2**
–	–
a	–
b	**b**
–	**a**

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#**1**	#**2**	#**3**	#**3**
c	*d*	*a*	*b*
a	*c*	*b*	*a*
b	**b**	*c*	*d*
d	**a**	*d*	*c*