# Mixed Dominating Set

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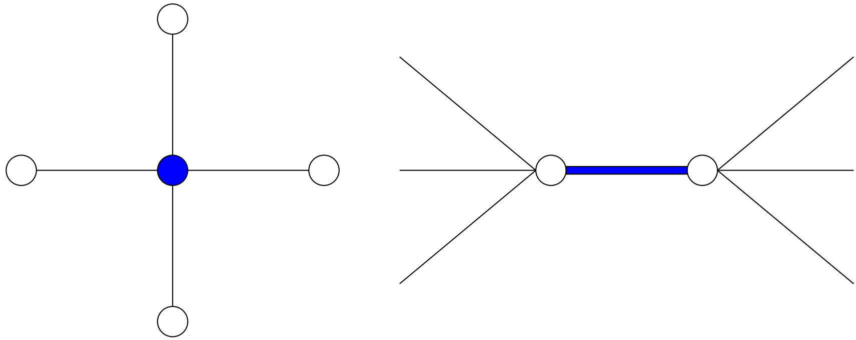
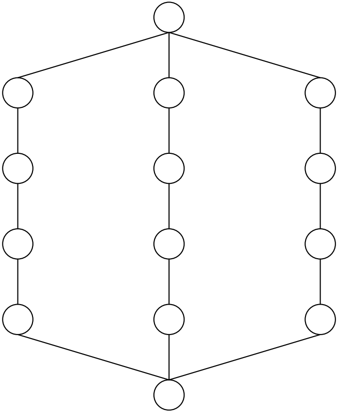
Journée du Lamsade, April 18, 2019

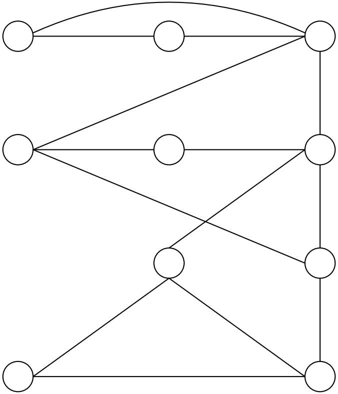
# Sommaire

# Domination

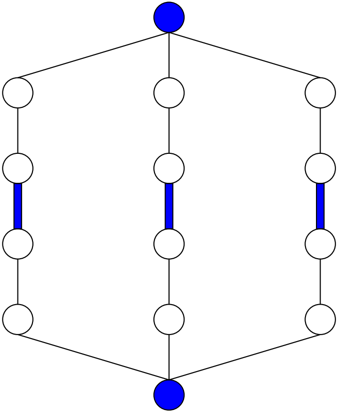


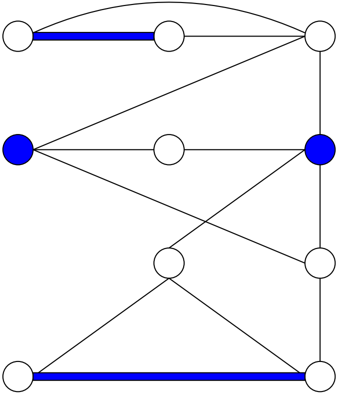
# Mixed Dominating Set



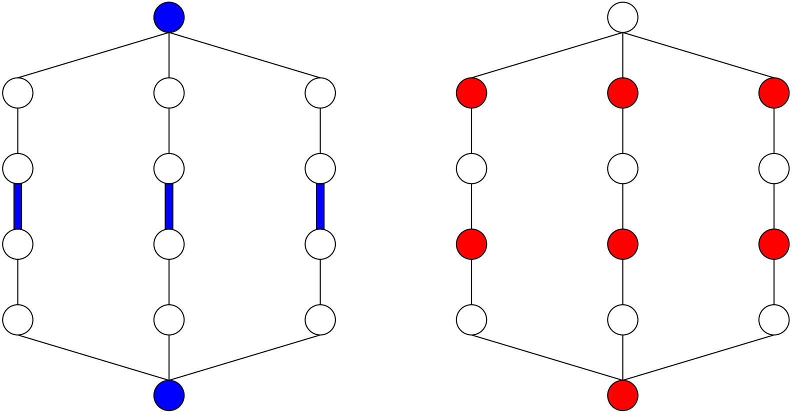


# Mixed Dominating Set

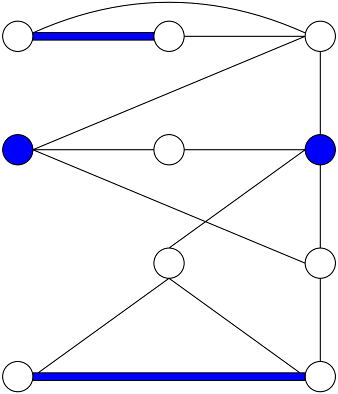


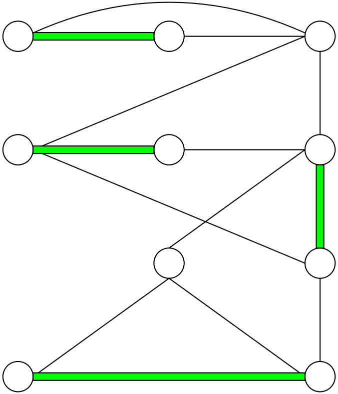


# Mixed Dominating Set $\leq$ Vertex Cover



# Mixed Dominating Set $\geq$ Edge Dominating Set





# State of the art

- NP-complete problem (Majumbar, *Clemson University*, 1992)
- 2-approximation using maximum matching (Hatami, *Discussiones Mathematicae Graph Theory*, 2010)
- Exact algorithm: $O^*(2^n)$
- FPT algorithm parameterized by the solution size: $O^*(7.465^k)$ (Jain, Jayakrishnan, Panolan, Sahu, *International Workshop on Graph-Theoretic Concepts in Computer Science*, 2017)
- FPT algorithm parameterized by the treewidth: $O^*(6^{tw})$ (*ibid.*)

# Our results

- 3-approximation using an improved maximal matching
- 2-inapproximability under Unique Games Conjecture
- Exact algorithm: $O^*(2.1889^n)$
- FPT algorithm parameterized by the solution size: $O^*(4.27^k)$
- FPT algorithm parameterized by the treewidth: $O^*(5^{tw})$
- PTAS in planar graphs: $(1+1/k)$-approximation in time $O(5^{3k+5}n)$
- FPT approximation schema: $(1+\varepsilon)$-approximation in time $O^*(4.27^{(1-\varepsilon)k})$

# Inapproximability

**Definition**

For some NP-hard problem $\Pi$: under a certain conjecture, it is impossible to have approximation ratio better than $r$, for some $r$.

# Unique Games Conjecture (UGC)

**Definition**

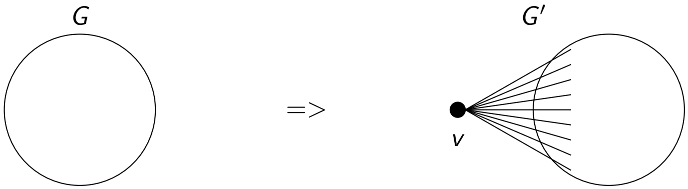
The problem of determining the approximate value of a certain type of game (unique game) has NP-hard complexity.

# Inapproximability of Edge Dominating Set

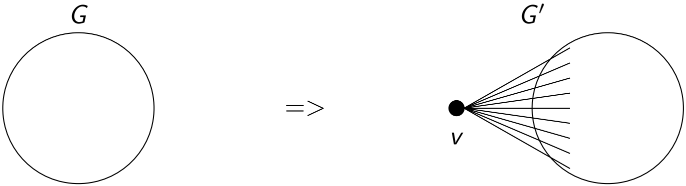
**Theorem (Dudycz, Lewandowski, Marcinkowski, 2018)**

*Assuming UGC, it is NP-hard to approximate Edge Dominating Set with constant ratio better than 2.*

# Reduction EDS $\leq$ MDS



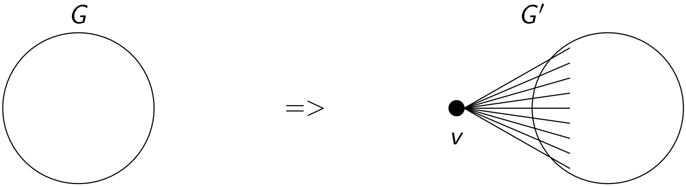
# Reduction EDS $\leq$ MDS




**Lemma**

$alg(G') \leq alg(G) + 1$

# Reduction EDS $\leq$ MDS




**Lemma**

$opt(G') \geq opt(G)$

# Reduction EDS $\leq$ MDS

$$\frac{alg(G)}{opt(G)} \leq r \Leftrightarrow \frac{alg(G')}{opt(G')} \leq \frac{alg(G)+1}{opt(G)} \leq \frac{alg(G)}{opt(G)} + \frac{1}{opt(G)} \leq r + \varepsilon$$

# Inapproximability of Mixed Dominating Set

**Theorem**

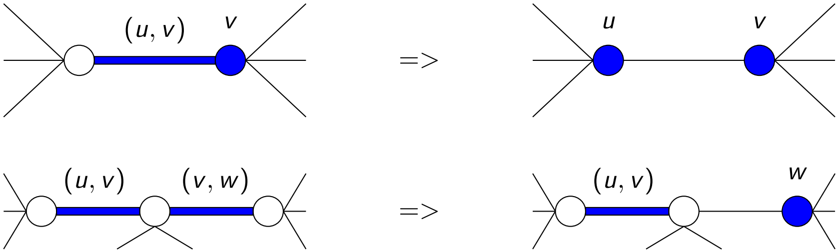
*Assuming UGC, it is NP-hard to approximate Mixed Dominating Set with constant ratio better than 2.*

# Mixed Dominating Set = Matching + Vertex Cover

**Lemma**

*For any graph $G = (V, E)$, $G$ has a mixed dominating set of size $k$ iff $G$ has a mixed dominating set $D \cup M$ of size $k$ where $V(M) \cap D = \emptyset$ and $M$ is a matching.*

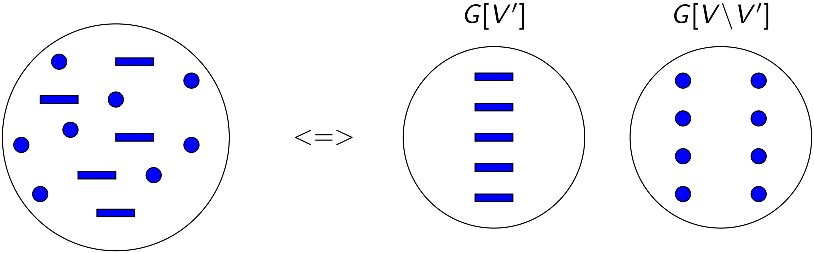
# Mixed Dominating Set = Matching + Vertex Cover



# Mixed Dominating Set = Matching + Vertex Cover

**Lemma**

*The graph $G$ has a mixed dominating set of size $k$ iff there exists $V' \subseteq V$ such that $G[V']$ has a perfect matching and $G[V \backslash V']$ has a vertex cover of size $k - \frac{|V'|}{2}$.*



# Exact algorithm

**Algorithm:**

- For all $V' \subseteq V$ which contains a perfect matching $M$, calculate a vertex cover $D$ on $G[V \backslash V']$.
- Return the union $M \cup D$ of minimum size.

# Complexity

- The Vertex Cover problem can be solved in time $O^*(1.1889^n)$ (Robson, *LaBRI*, 2001)

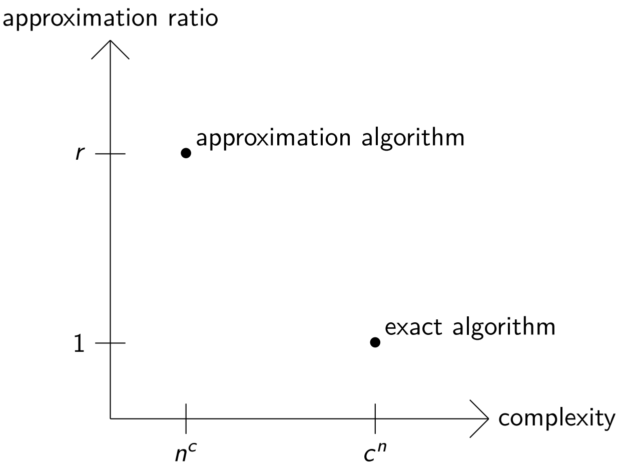
$$\sum_{i=0}^{n/2} \binom{n}{2i} \cdot 1.1889^{n-2i} \leq \sum_{i=0}^{n} \binom{n}{i} \cdot 1.1889^{n-i} = \sum_{i=0}^{n} \binom{n}{i} \cdot 1.1889^{i} = 2.1889^n$$

# Exact algorithm

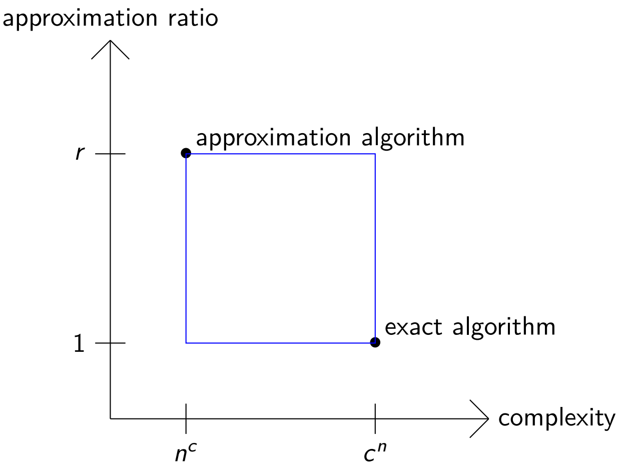
**Theorem**

*The Mixed Dominating Set problem can be solved in time* $O^*(2.1889^n)$.

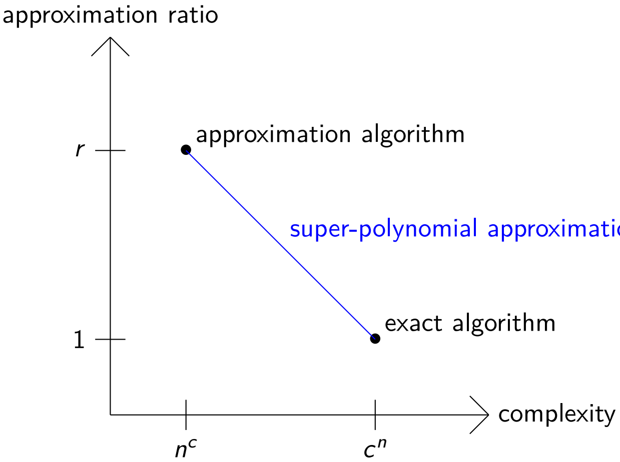
# Classical paradigms



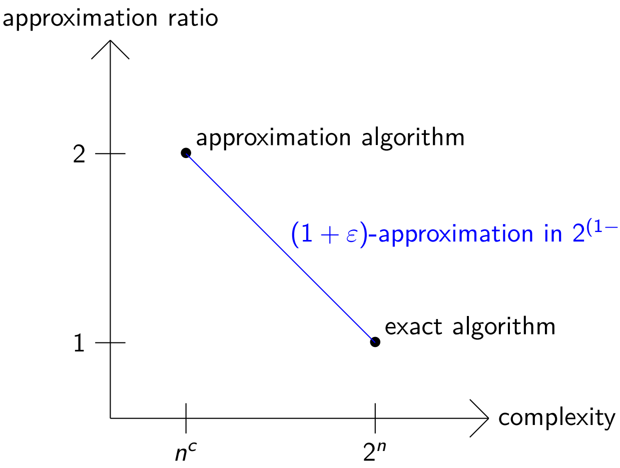
# Classical paradigms



# Super-polynomial approximation



# Super-polynomial approximation of MDS



Merci !