Linear Time Membership in a Class of Regular Expressions with Counting, Interleaving and Unordered Concatenation

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joint work with Giorgio Ghelli and Carlo Sartiani

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  - data extraction from text
  - path expressions in query languages over tree- and graph-shaped data

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- Example
  - $T = a \cdot (c + d)^* \cdot (f + \epsilon)$
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- Membership checking: does $w \in L(T)$ hold?
- Polynomial for standard REs [J.E. Hopcroft and J.D. Ullman1979]

Adding counting # and shuffle &

$$T ::= \quad \epsilon \;\mid\; a \;\mid\; T + T \;\mid\; T \cdot T \;\mid\; T\,[n..m] \;\mid\; T \& T \;\mid$$

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- But is polynomial for RE(#) [Kilpeläinen and Tuhkanen2003]

Our results

- Polynomial membership checking for a class of REs including & and #, obtained by means of mild restrictions
- More precisely
  - Quadratic membership-checking algorithm based on *constraints* for REs
  - Characterisation of a stability property, and it use for the design of linear membership algorithms
  - Extension of n-ary REs enriched with unordered concatenation, application to XML schema
  - Extensive experiments

The conflict-free restriction

- Conflict-free REs
  - *counting restricted to symbols*:

$$T ::= \quad \epsilon \;\mid\; a\,[m..n] \;\mid\; T+T \;\mid\; T\cdot T \;\mid\; T\&T$$

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- Used very often in practice, especially in the setting of XML schemas [Barbosa et al.2006, Choi2002].
- Example:

$$T = (a\,[1..3] \cdot b\,[2..2]) + c\,[1..*]$$

- Counterexamples

$$\begin{aligned} T' &= (a\,[1..3] \cdot b\,[2..2])\,[3..4] + c\,[1..*] \\ T'' &= (a\,[1..3] \cdot b\,[2..2]) + a\,[5..*] \end{aligned}$$

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- Benefits:
  - inclusion $T \leq U$ over CF-REs can be decided in polynomial time [Colazzo et al.2009b]
  - even in the mixed case $T \leq U$ where only U is CF [Colazzo et al.2009a, Colazzo et al.2013b, Colazzo et al.2013a].
  - Quasi linear membership [Ghelli et al.2008] - extended abstract of this paper.

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  - Quasi linear membership [Ghelli et al.2008] - extended abstract of this paper.
- Main ingredient of our approaches: constraint-based characterisation of CF expressions [Colazzo et al.2009b].

The constraints

- To illustrate the intuition behind our constraints, consider again

$$T = ((a\,[1..3] \cdot b\,[2..2]) + c\,[1..*])$$

- It can be fully represented by the *conjunction* of the following constraints:
- Flat constraints $\mathcal{FC}(T)$:
  - Lower-bound: abc^+
  - Upper-bound: $\mathrm{upper}(abc)$
  - Cardinality: $a?[1..3] \wedge b?[2..2] \wedge c?[1..*]$
- Nested constraints $\mathcal{NC}(T)$:
  - Co-occurrence : $a^+ \Rrightarrow b^+ \wedge b^+ \Rrightarrow a^+$
  - Order: $a \prec b$
  - Mutual exclusion: $(a \prec c \wedge c \prec a) \wedge (b \prec c \wedge c \prec b)$

Syntax and semantics

$$F \ ::= \quad A^+ \ \mid \ A^+ \Rrightarrow B^+ \ \mid \ a?[m..n] \ \mid \ \text{upper}(A) \ \mid \ a \prec b \ \mid \ F \wedge F' \ \mid \ \mathbf{true}$$

$$\begin{array}{rcl}
w \models A^+ & \Leftrightarrow & (S(w) \cap A) \neq \emptyset, \text{ i.e., some } a \in A \text{ appears in } w \\
w \models A^+ \Rrightarrow B^+ & \Leftrightarrow & w \not\models A^+ \text{ or } w \models B^+ \\
w \models a?[m..*] & \Leftrightarrow & \text{if } a \text{ appears in } w\text{, then it appears at least } m \text{ times} \\
w \models a?[m..n] & \Leftrightarrow & \text{if } a \text{ appears in } w\text{, then it appears at least } m \text{ times} \\
(n \neq *) & & \text{and at most } n \text{ times} \\
w \models a \prec b & \Leftrightarrow & \text{there is no occurrence of } a \text{ in } w \text{ that follows an occurrence} \\
& & \text{of } b \text{ in } w \text{ (hence, both } a \text{ and } b \text{ may be missing)} \\
w \models \text{upper}(A) & \Leftrightarrow & S(w) \subseteq A \\
w \models F_1 \wedge F_2 & \Leftrightarrow & w \models F_1 \text{ and } w \models F_2 \\
w \models \mathbf{true} & \Leftrightarrow & \text{always}
\end{array}$$

Abbreviations

$$
\begin{aligned}
A^{+} \Leftrightarrow B^{+} \quad &=_{def} \quad A^{+} \Rrightarrow B^{+} \wedge B^{+} \Rrightarrow A^{+} \\
a \prec\succ b \quad &=_{def} \quad (a \prec b) \wedge (b \prec a) \\
A \prec B \quad &=_{def} \quad \bigwedge_{a \in A, b \in B} a \prec b \\
A \prec\succ B \quad &=_{def} \quad \bigwedge_{a \in A, b \in B} a \prec\succ b \\
\mathbf{false} \quad &=_{def} \quad \emptyset^{+} \\
A^{-} \quad &=_{def} \quad A^{+} \Rrightarrow \emptyset^{+}
\end{aligned}
$$

Constraint extraction

- Quadratic time extraction, soundness and completeness [Colazzo et al.2009b]:

$$w \in [\![T]\!] \quad \Leftrightarrow \quad w \models \mathcal{FC}(T) \wedge \mathcal{NC}(T)$$

- An expression T is nullable, noted as $\mathrm{N}(T)$, iff $\epsilon \in [\![T]\!]$
- To illustrate, consider $T = ((a + \epsilon)\&b\,[1..5]) \cdot (c + d\,[1..*])$, note that $\mathrm{N}(a + \epsilon)$
  - CC: $a^+ \Mapsto b^+ \;\wedge\; ab^+ \Leftrightarrow cd^+$
  - OC: $c \prec\succ d \;\wedge\; ab \prec cd$
  - Flat:

$$\begin{array}{ll} abcd^+ \wedge & \text{Lower-bound} \\ a?[1..1] \wedge b?[1..5] \wedge c?[1..1] \wedge d?[1..*] \wedge & \text{Cardinality} \\ \mathrm{upper}(abcd) & \text{Upper-bound} \end{array}$$

Constraint residuation

- $F \xrightarrow{a} F'$ means that F is transformed (or residuated) into F' by parsing the symbol a
- Main cases:

Computing the residual of a nested co-occurrence constraint.

Condition	$a \in A$	$a \in B$	$a \in A$	$a \in B$	$a \in A$
F	$A^+ \Mapsto B^+$	$A^+ \Mapsto B^+$	$A^+ \Leftrightarrow B^+$	$A^+ \Leftrightarrow B^+$	A^+
F'	B^+	**true**	B^+	A^+	**true**

Computing the residual of a nested order constraint.

Condition	$a \in A$	$a \in B$	$a \in A$	$a \in B$	$a \in A$
F	$A \prec B$	$A \prec B$	$A \prec\succ B$	$A \prec\succ B$	A^-
F'	$A \prec B$	A^-	B^-	A^-	**false**

Word membership checking

- Residuation is shifted to words $F \xrightarrow{w}{}^{*} F'$ in the obvious way:

$$
\begin{aligned}
&F \xrightarrow{\epsilon}{}^{*} F \\
&F \xrightarrow{aw}{}^{*} F'' \quad \Leftrightarrow_{\text{def}} \quad F \xrightarrow{a} F' \ \wedge \ F' \xrightarrow{w}{}^{*} F''
\end{aligned}
$$

- Corresponds to constraint semantics

$$w \models F \Leftrightarrow F \downarrow^{w} \textbf{true}$$

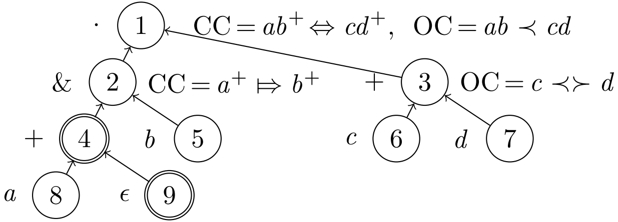
- Example of residuation:

$$ab^{+} \Leftrightarrow cd^{+} \xrightarrow{b} cd^{+} \xrightarrow{a} cd^{+} \xrightarrow{c} \textbf{true}$$

- In order to check $w \in [\![T]\!]$ we can residuate $\mathcal{NC}(T)$
- Complexity: $O(|w| * |\mathcal{NC}(T)|)$:
  - given w, for each constraint F in $\mathcal{NC}(T)$, we need to eventually parse the whole w in order to evaluate $F \downarrow^{w} G$.

We can do much better

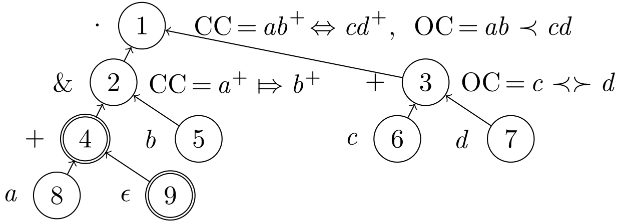
- Keep constraints/residuals implicit in a tree-shaped data structure with size $O(|T|)$
- For $T = ((a+\epsilon)\& b\,[1..5]) \cdot (c + d^+)$ we have:



- The two nullable nodes have double line in the picture.

Residuation algorithm

- For each character a from the input word w:
  - it scans the ancestors of $a\,[m..n]$ in the constraint tree,
  - residuates all the constraints in this branch, and keeps track of all the resulting A^+ constraints.



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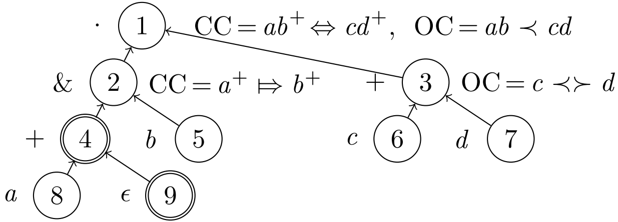
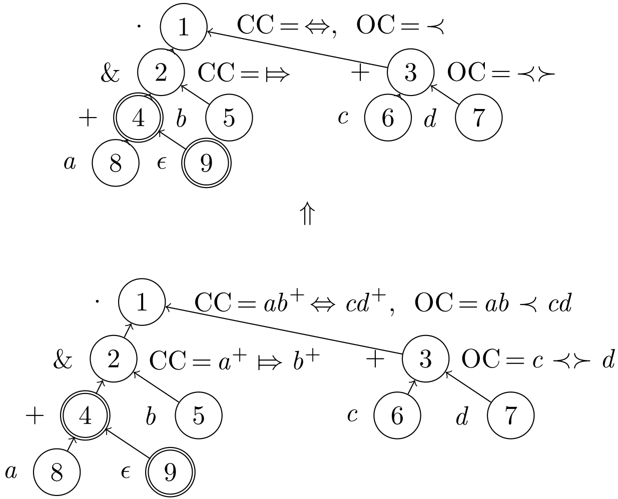
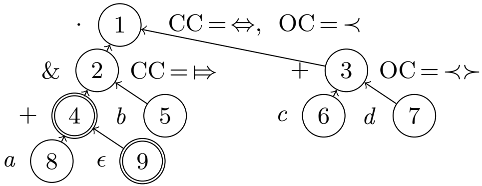
  - residuates all the constraints in this branch, and keeps track of all the resulting A^+ constraints.
- At the end of w, it checks that all the A^+ constraints have been further residuated into **true** – the generation of a **false** causes an immediate failure.
- Flat constrains can be trivially checked in constant time for each symbol.



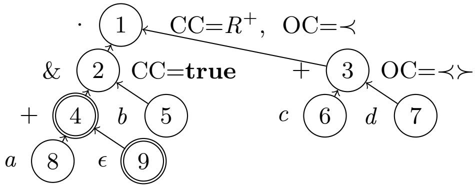
Optimising tree representation



Evolution of the tree while parsing $bbac$ for $T = ((a + \epsilon)\&b\,[1..5])\cdot(c + d^+)$



Initial tree



after $\underline{b}$ and after $b\underline{b}$

$a^+ \Rrightarrow b^+ \xrightarrow{b} \mathbf{true} \xrightarrow{b} \mathbf{true} \quad ab^+ \Leftrightarrow cd^+ \xrightarrow{b} cd^+ \xrightarrow{b} cd^+$

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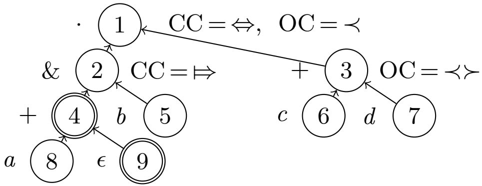

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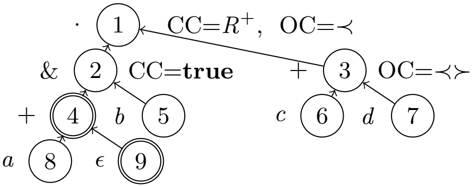

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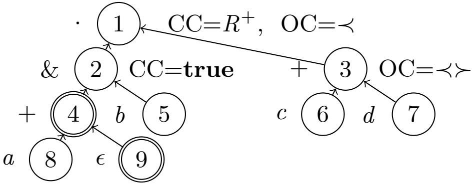
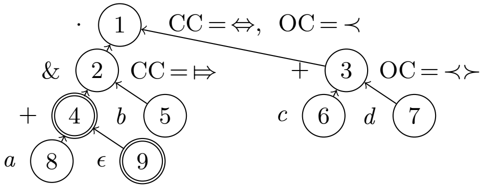


after $bb\underline{a}$
constraints unchanged

The evolution of the tree for $T = ((a+\epsilon)\&b\,[1..5])\cdot(c+d^+)$ while parsing bb

Evolution of the tree while parsing *bbac* for $T = ((a+\epsilon)\&b\,[1..5])\cdot(c+d^{+})$



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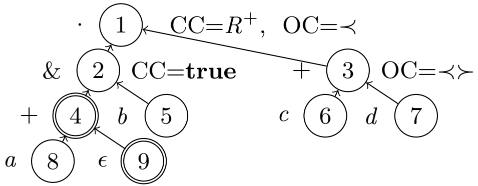
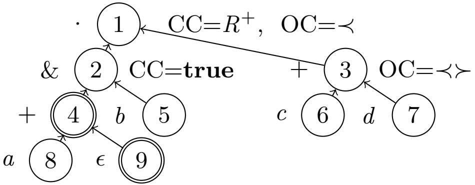


after $\underline{b}$ and after $b\underline{b}$

$a^{+} \Rrightarrow b^{+} \xrightarrow{b}$ **true** $\xrightarrow{b}$ **true** $\quad ab^{+} \Leftrightarrow cd^{+} \xrightarrow{b} cd^{+} \xrightarrow{b} cd^{+}$



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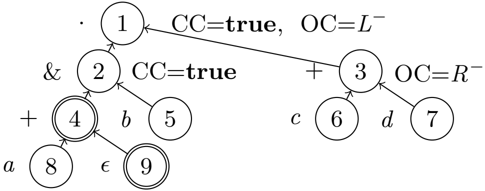

after $bba\underline{c}$

$cd^{+} \xrightarrow{c}$ **true** $\quad ab \prec cd \xrightarrow{c} ab^{-}$

Soundness and completeness, complexity

- *Member(w,T)* yields *true* iff $w \in [\![T]\!]$
  - Follows from the fact the algorithms checks both nested and flat constraints of T.
- *Member(w,T)* runs in time $O(|T| + |w| * \mathit{depth}(T))$
  - The constraint tree can be built in time $O(|T|)$.
  - For every symbol a in w, only visit the path from the leaf node a to the root.
  - For each node in this path, a constant number of unit time operations is executed.
  - The final check on CC constraints scans at most $|T|$ nodes, while flat contraints can be checked in linear time.

Linear membership via stability

- The MEMBER algorithm visits all ancestors of an a-leaf every time a is found in w, which is redundant.
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- We can avoid redundant operations, by relying on the fact that some constraints become stable after residuation (already seen in the example).
- Crucial observation: whenever a node constraint has been residuated because a symbol of A_L has been met, there is *almost* no reason to visit the node again because of a symbol from A_L (and the same holds for A_R)

Linear membership via stability

- More precisely, given the first $a \in A_1$
  - A_1^+ becomes **true**
  - ${A_1}^+ \Mapsto {A_2}^+$ or ${A_1}^+ \Leftrightarrow {A_2}^+$ becomes A_2^+.
  - $A_1 \prec\succ A_2$ becomes ${A_2}^-$.

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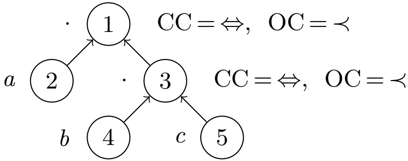
- None of the obtained constraints can be affected by a second symbol from A_1.
- There is only one exception, for the constraint $A_1 \prec A_2$
  - the first $a \in A_1$ does not residuate it
  - but a subsequent symbol in A_2 residuates it into ${A_1}^-$
  - so another $a' \in A_1$ cannot be ignored, as it will cause the algorithm to *end* and yield "false".

Linear mebership

- Our algorithm MEMBERSTAB exploits stability and has $O(|T| + |w|)$ complexity.
- In a nutshell, it 'deactivates' the edges along the paths that are visited, since they do not need to be visited again, unless an $A_1 \prec A_2$ node is residuated to ${A_1}^-$
- In this last case the paths below A_1 are eventually re-activated, but this is done at most once: as soon as one of the re-activated paths is traversed the algorithm stops, and the checking fails.
- So, in MEMBERSTAB each edge is traversed at most three times for any word w, and this yields linearity.
- Let's see un example.

MemberStab example, $T = a \cdot (b \cdot c)$ and $w = bbcb$



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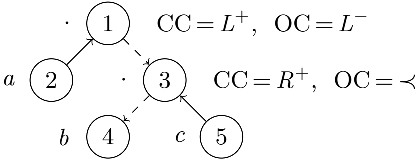
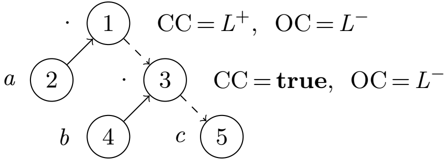


After $\underline{b}$: residuation and edge discarding
After $b\underline{b}$: *no action*



After $bb\underline{c}$: the (4,3) edge is restored
After $bbc\underline{b}$: return **false**

Also in the paper

- Extension with unordered concatenation
- Extension to n-ary types (still linear)
- Extensive experiments.

Thank you !

Adding unordered concatenation

$$w \in [\![T_1 \% T_2 \% \ldots \% T_n]\!] \iff \exists \pi \in S_n.\ w \in [\![T_{\pi(1)} \cdot T_{\pi(2)} \cdot \ldots \cdot T_{\pi(n)}]\!]$$

- Not associative
- So we have considered n-ary expressions of the form

  $$T_1 \otimes \ldots \otimes T_n$$

  with $\otimes$ ranging over $\{\cdot,\ \&,\ \%\}$
- Needed to switch to flat-constraints

  $$F \ ::= \ \ \%\{A_1, \ldots, A_n\} \ \mid A_1 \prec \ldots \prec A_n \mid A_0{}^+ \Mapsto \{A_1{}^+, \ldots, A_n{}^+\} \mid \ \ldots$$

- Residuation for % constraints generates hybrid constraints including $\prec$-constraints.
- To illustrate, let $a \in A_2$

  $$\%\{A_1, A_2, A_3\} \xrightarrow{a} A_2 \prec \%\{A_1, A_3\}$$

MemberFlat and MemberFlatStab

- MemberFlat is a generalisation of Member to n-ary expressions/constrains
- Complexity is $O(|T| + |w| * \mathit{flatdepth}(T))$ where $\mathit{flatdepth}(T)$ is the depth of the type after all operators have been flattened.
- Quasi linear as $\mathit{flatdepth}(T)$ has in practice $\mathit{flatdepth}(T) < 4$
- We have proved that stability transposes to the n-ary case, thus obtaining MemberFlatStab
- Linear complexity $O(|T| + |w|)$
- Also in the paper: multiword checking and application to XML schema membership

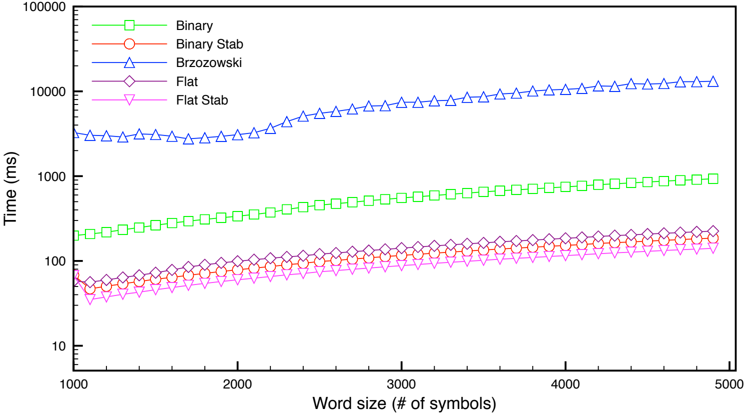
Performance and scalability

- We implemented our algorithms in Java
- To perform experimental analysis we performed comparisons with a baseline approach based on Brzozowski derivatives - as we did not found any competitor.
- Brzozowski derivatives are widely used to check membership, and to define finite state automata for REs.
- Examples:
  - $d_a(a\,[2..4]) = a\,[1..3]$
  - $d_a(d_a(a\,[2..4])) = a\,[0..2]$
  - $d_a(a\,[0..0]) = \emptyset$
  - $d_a(a\,[2..4] + b\,[1..3]) = d_a(a\,[2..4])$
  - $d_a((b\,[1..3] + \epsilon)\cdot\, a\,[2..4])) = d_a(a\,[2..4])$
- Complete formalisation in the paper.
- Soundness and completeness: $w \in [\![T]\!] \Leftrightarrow \epsilon \in [\![d_w(T)]\!]$
- Complexity of the baseline algorithm is $O(|w| * |T|^2)$.

Performance and scalability

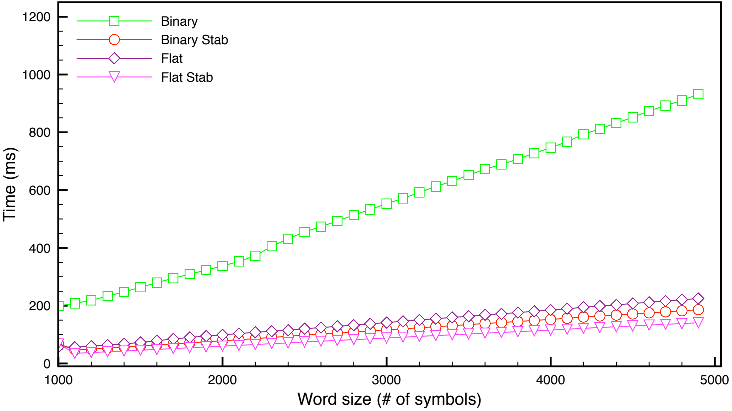
- We implemented a generator of synthetic expressions ensuring that operators have same probability of being generated
- For adopted types binary depth is 15 while flat depth is 4.
- We generated both positive and negative sets of words for each expression.
- The data generation algorithm guarantees that the expression is *equally covered* by the words.
- For each generated T we generated 30000 positive/negative words having from 1000 to 5000 symbols each

Positive experiments



Positive experiments for cf-RE(#, &): logarithmic scale.

Positive experiments



Positive experiments for cf-RE($\#, \&$): binary and flat algorithms.

Positive experiments

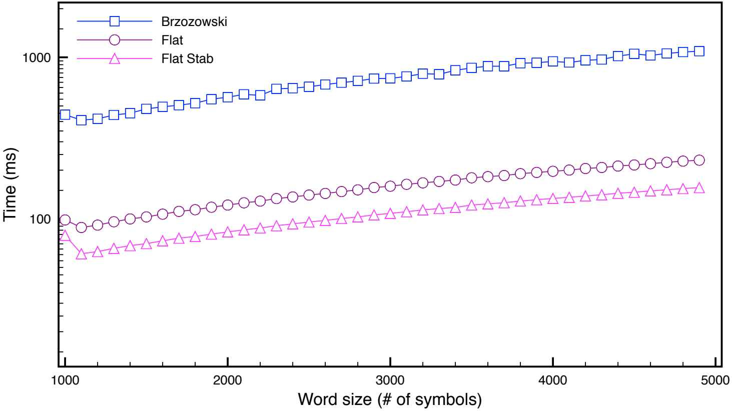


Figure: Positive experiments for cf-RE(#, &, %): logarithmic scale.

Negative experiments

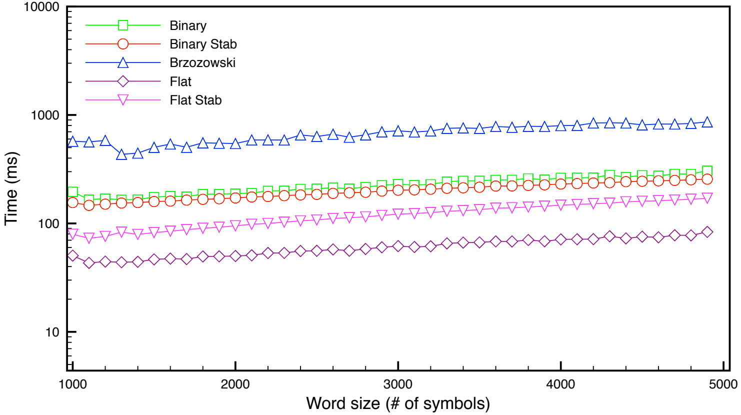


Figure: Negative experiments for cf-RE$(\#, \&)$: logarithmic scale on y-axis.

Negative experiments

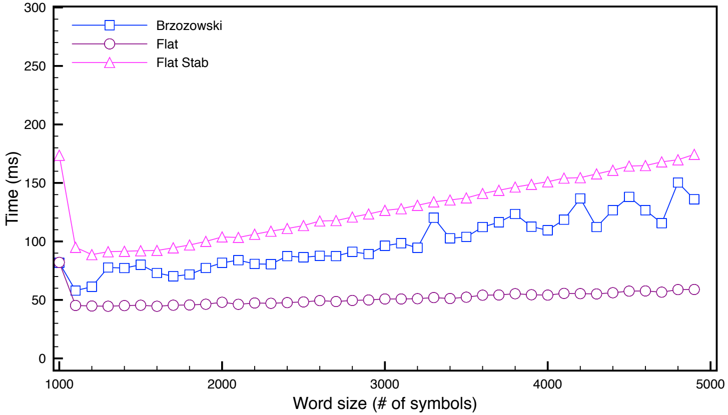


Figure: Negative experiments for cf-RE(#, &, %).

Negative experiments

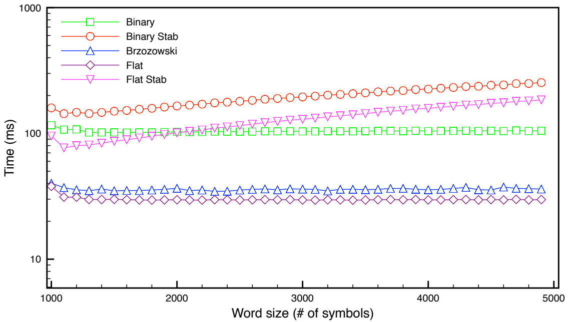


Figure: Negative experiments for cf-RE(#, &): random words.

Negative experiments

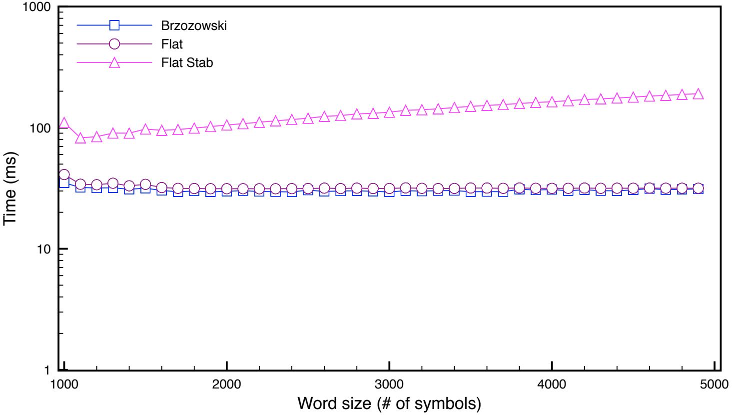


Figure: Negative experiments for cf-RE($\#, \&, \%$): random words.

Related works and conclusion

- Regular expressions have been extensively studied in the past (see the paper for an overview).
- Past studies [Gelade et al.2009, Hovland2012] have focused on extended REs, but no linear algorithm has been provided for a significative restriction.
- We have shown how restrictions and constraints can help in separating main properties that have to be checked during membership checking.
- This has enabled us to devise optimisations for specific aspects like order co-occurrence and order constraints.
- Experiments have validated complexity analysis on positive data sets.
- Surprisingly enough, experiments on negative data sets revealed that in this setting even a baseline algorithm can be competitive.

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