Stable Matching with Efforts

Felipe Garrido Lucero

Journée du LAMSADE, April 18th 2019

Stable Matching

D. Gale and L.S. Shapley, "*College Admissions and the Stability of Marriage*" in 1962

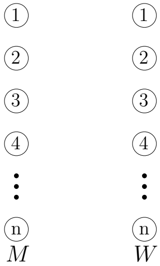


Figure: Stable Matching

Stable Matching

D. Gale and L.S. Shapley, "*College Admissions and the Stability of Marriage*" in 1962

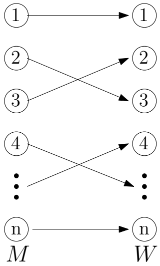
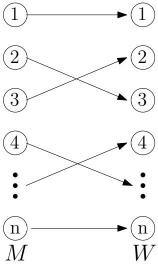


Figure: Stable Matching

Stable Matching

D. Gale and L.S. Shapley, "*College Admissions and the Stability of Marriage*" in 1962



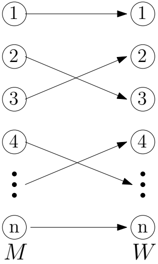
$$M = \begin{bmatrix} M_{1,1} & M_{1,2} & ... & M_{1,n} \\ M_{2,1} & M_{2,2} & ... & M_{2,n} \\ ... & ... & ... & ... \\ M_{n,1} & M_{n,2} & ... & M_{n,n} \end{bmatrix}$$

$$W = \begin{bmatrix} W_{1,1} & W_{1,2} & ... & W_{1,n} \\ W_{2,1} & W_{2,2} & ... & W_{2,n} \\ ... & ... & ... & ... \\ W_{n,1} & W_{n,2} & ... & W_{n,n} \end{bmatrix}$$

Figure: Stable Matching

Stable Matching

D. Gale and L.S. Shapley, "*College Admissions and the Stability of Marriage*" in 1962



$$M = \begin{bmatrix} M_{1,1} & M_{1,2} & ... & M_{1,n} \\ M_{2,1} & M_{2,2} & ... & M_{2,n} \\ ... & ... & ... & ... \\ M_{n,1} & M_{n,2} & ... & M_{n,n} \end{bmatrix}$$

$$W = \begin{bmatrix} W_{1,1} & W_{1,2} & ... & W_{1,n} \\ W_{2,1} & W_{2,2} & ... & W_{2,n} \\ ... & ... & ... & ... \\ W_{n,1} & W_{n,2} & ... & W_{n,n} \end{bmatrix}$$

Figure: Stable Matching

Definition

We say that a matching μ is **stable** *if it does not exist a pair $i, j \in M \times W$ s.t.*

1. *i and j are not a couple,*
2. $M_{i,j} > M_{i,\mu_i}$ *and* $W_{i,j} > W_{\mu'_j,j}$

Stable Matching with Efforts

We introduce the extension when agents can play efforts to increase their *attractiveness* to the other set, such that if agent i plays effort $e_i \geq 0$,

$$W = \begin{bmatrix} W_{1,1} & W_{1,2} & \dots & W_{1,n} \\ W_{2,1} & W_{2,2} & \dots & W_{2,n} \\ \dots & \dots & \dots & \dots \\ W_{i,1} & W_{i,2} & \dots & W_{i,n} \\ \dots & \dots & \dots & \dots \\ W_{n,1} & W_{n,2} & \dots & W_{n,n} \end{bmatrix} \longrightarrow W = \begin{bmatrix} W_{1,1} & W_{1,2} & \dots & W_{1,n} \\ W_{2,1} & W_{2,2} & \dots & W_{2,n} \\ \dots & \dots & \dots & \dots \\ W_{i,1}+e_i & W_{i,2}+e_i & \dots & W_{i,n}+e_i \\ \dots & \dots & \dots & \dots \\ W_{n,1} & W_{n,2} & \dots & W_{n,n} \end{bmatrix}$$

Figure: Efforts

Stable Matching with Efforts

We introduce the extension when agents can play efforts to increase their *attractiveness* to the other set, such that if agent i plays effort $e_i \geq 0$,

$$W = \begin{bmatrix} W_{1,1} & W_{1,2} & ... & W_{1,n} \\ W_{2,1} & W_{2,2} & ... & W_{2,n} \\ ... & ... & ... & ... \\ W_{i,1} & W_{i,2} & ... & W_{i,n} \\ ... & ... & ... & ... \\ W_{n,1} & W_{n,2} & ... & W_{n,n} \end{bmatrix} \longrightarrow W = \begin{bmatrix} W_{1,1} & W_{1,2} & ... & W_{1,n} \\ W_{2,1} & W_{2,2} & ... & W_{2,n} \\ ... & ... & ... & ... \\ W_{i,1}+e_i & W_{i,2}+e_i & ... & W_{i,n}+e_i \\ ... & ... & ... & ... \\ W_{n,1} & W_{n,2} & ... & W_{n,n} \end{bmatrix}$$

Figure: Efforts

- Solution to one-sided case with positive efforts, algorithm finds an ε-solution

Stable Matching with Efforts

We introduce the extension when agents can play efforts to increase their *attractiveness* to the other set, such that if agent i plays effort $e_i \geq 0$,

$$W = \begin{bmatrix} W_{1,1} & W_{1,2} & ... & W_{1,n} \\ W_{2,1} & W_{2,2} & ... & W_{2,n} \\ ... & ... & ... & ... \\ W_{i,1} & W_{i,2} & ... & W_{i,n} \\ ... & ... & ... & ... \\ W_{n,1} & W_{n,2} & ... & W_{n,n} \end{bmatrix} \longrightarrow W = \begin{bmatrix} W_{1,1} & W_{1,2} & ... & W_{1,n} \\ W_{2,1} & W_{2,2} & ... & W_{2,n} \\ ... & ... & ... & ... \\ W_{i,1}+e_i & W_{i,2}+e_i & ... & W_{i,n}+e_i \\ ... & ... & ... & ... \\ W_{n,1} & W_{n,2} & ... & W_{n,n} \end{bmatrix}$$

Figure: Efforts

- Solution to one-sided case with positive efforts, algorithm finds an ε-solution
- Existence of stable matching for $\varepsilon = 0$

Stable Matching with Efforts

We introduce the extension when agents can play efforts to increase their *attractiveness* to the other set, such that if agent i plays effort $e_i \geq 0$,

$$W = \begin{bmatrix} W_{1,1} & W_{1,2} & \dots & W_{1,n} \\ W_{2,1} & W_{2,2} & \dots & W_{2,n} \\ \dots & \dots & \dots & \dots \\ W_{i,1} & W_{i,2} & \dots & W_{i,n} \\ \dots & \dots & \dots & \dots \\ W_{n,1} & W_{n,2} & \dots & W_{n,n} \end{bmatrix} \longrightarrow W = \begin{bmatrix} W_{1,1} & W_{1,2} & \dots & W_{1,n} \\ W_{2,1} & W_{2,2} & \dots & W_{2,n} \\ \dots & \dots & \dots & \dots \\ W_{i,1}+e_i & W_{i,2}+e_i & \dots & W_{i,n}+e_i \\ \dots & \dots & \dots & \dots \\ W_{n,1} & W_{n,2} & \dots & W_{n,n} \end{bmatrix}$$

Figure: Efforts

- Solution to one-sided case with positive efforts, algorithm finds an ε-solution
- Existence of stable matching for $\varepsilon = 0$
- Solving case two-sided with efforts