Using the weighted constrained equal award rule to allocate CO_2 emission permits

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Content

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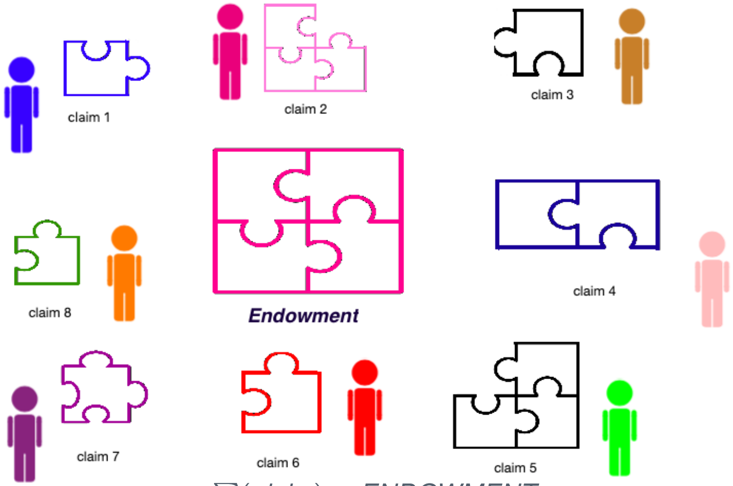
Introduction

Bankruptcy problems



$$\sum(claim) > ENDOWMENT$$

bankruptcy problem

A bankruptcy problem consists in pair $(C, E) \in R_{+}^{N} * R$ with :

- The vector C represents agent's demands vector $c_i \geq 0$
- E the estate that has to be divided among players that is not enough to satisfy all agent's demand C : $0 < E < \sum_{i=1}^{N} c_i = C$
- n indicates the number of players
- N denotes the set of players

bankruptcy problem

- The constraint $E < C$ represents the bankruptcy situation
- If $E \geq C$, all demands can be satisfied
- Inequality $E > 0$ rules out trivial situation ($E = 0$ means that no one gets anything)
- A solution of bankruptcy problems consists in a vector containing the amount of resources that each player receives.

Method

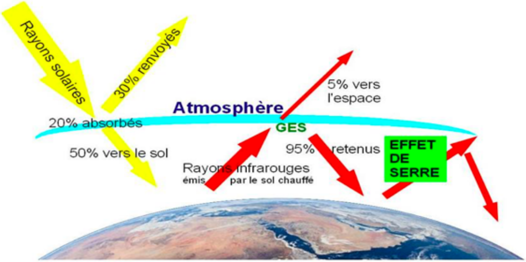
Bankruptcy Method: CEA

The Constrained Equal Awards CEA consists in giving to all the agents the same amount until their demands are not satisfied and the estate is not finished. It favors smaller claims.

$$f_i(C, E) = \min\{c_i, \lambda\} \tag{1}$$

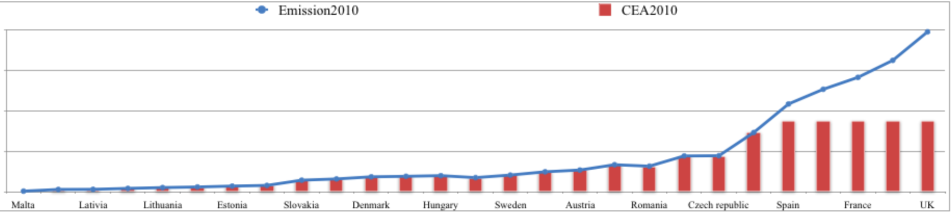
where the parameter λ is such that :

$$\sum_{i \in N} \min\{c_i, \lambda\} = E \tag{2}$$



bankruptcy problem

- Players = EU states
- Claims = EU states demands
- E= $emisions1990 - 22\%$ (kyoto Protocol)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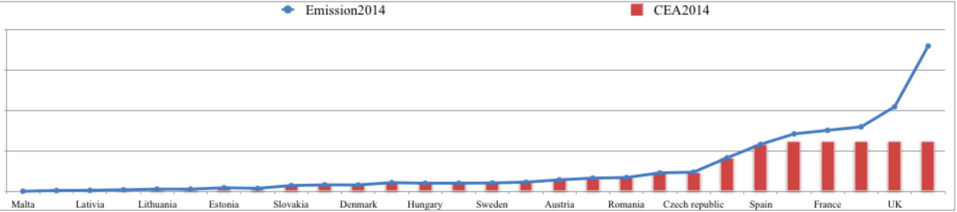
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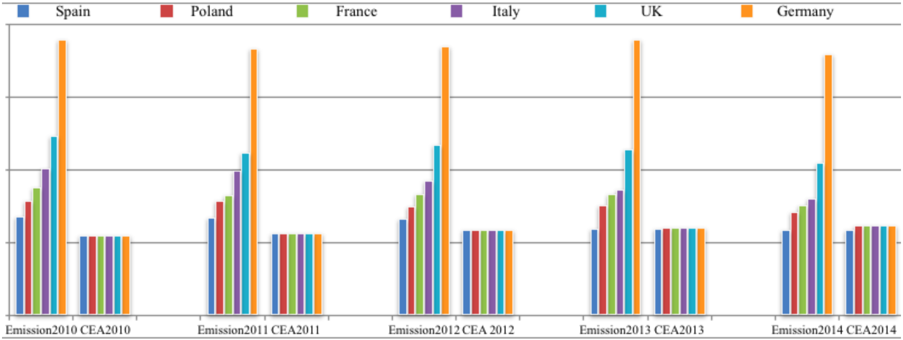
Bankruptcy approach for CO_2 emission permits

Bankruptcy applied to climate change



A set of countries N and a bankruptcy problem with two parameters:

$$(E, (c_i)_{i \in N}, (a_i)_{i \in N}),$$

where

- E is the total amount of emissions that must be shared
- c_i is the claim of emission demanded by country $i \in N$
- a_i is the activity of $country_i$ in terms of the amount of energy, GDP, etc. that a $country_i$ is able to produce under the amount of emission c_i.

Method

Weighted Bankruptcy Method: WCEA

The Weighted Constrained Equal Awards WCEA
a country may claim the need to pollute the amount c_i to realize the activity a_i, but it will never obtain to pollute more than λa_i

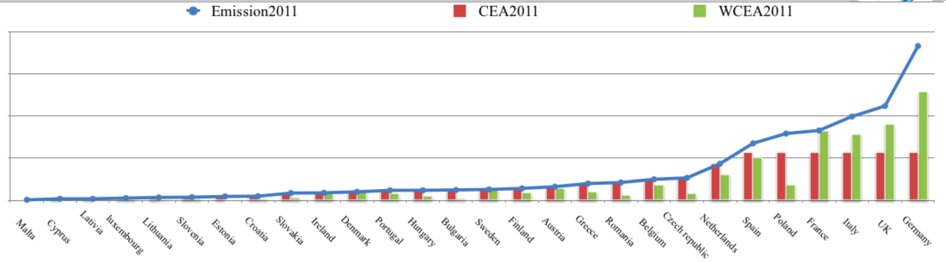
$$x_i = \min\{c_i, \lambda^* a_i\} \tag{3}$$

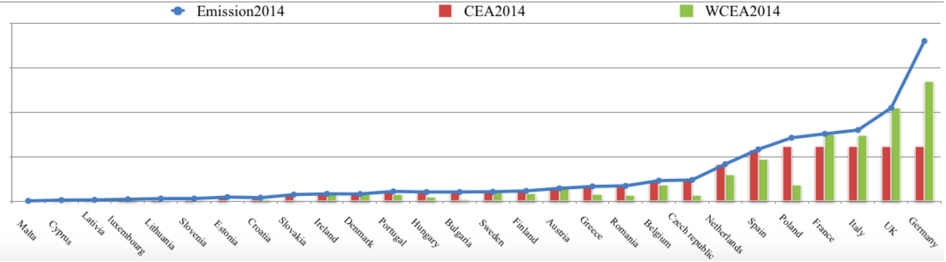
Where

$$\sum_{i \in N} x_i = E \tag{4}$$

Bankruptcy approach for CO_2 emission permits

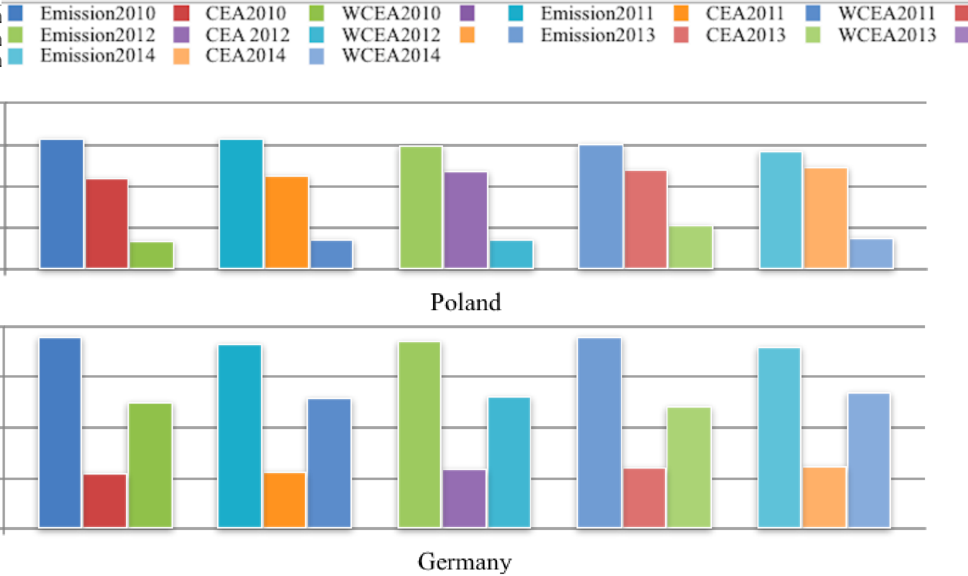
Bankruptcy with two parameters





Bankruptcy approach for CO_2 emission permits

Bankruptcy with two parameters



Technology-Transfer game

- Countries with a high ratio $\frac{c_i}{a_i}$ receive less than their claims, even if their claims are small.
- A low ratio $\frac{c_i}{a_i}$ reflects a good ability to emit low quantities of CO2 for unit of GDP.
- The problem of implementing economic incentives to transfer technology from the most efficient countries to the less efficient ones.
- A TU-game where the profit of each coalition of players (again, the EU-28 countries) is computed as the total profit obtained within the coalition using the highest technological level available for players in the coalition.

Technology-Transfer game

- Let (E, c, a) a weighted bankruptcy situation
- Let $WCEA(E, c, a) = (x_1, x_2, \ldots, x_n)$.
- For each $i \in N$,
  let $\lambda_i = \frac{c_i}{a_i}$
  let $\lambda(S) = \min_{i \in S: x_i - c_i = 0} \lambda_i$ for all $S \subseteq N$ (with the convention that $\lambda(S) = 0$ if $x_i - c_i \neq 0$ for all $i \in S$).
- The corresponding *Technology-Transfer (TT-)game* is defined as the TU-game $(N, \tilde{v})$ such that for all $S \subseteq N$

$$\tilde{v}(S) = \begin{cases} \sum_{i \in S: c_i - x_i > 0} \left(\frac{x_i}{\lambda(S)} - \frac{x_i}{\lambda_i} \right) & \text{if } \lambda(S) \neq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Technology-Transfer game

Let $(N, \tilde{v})$ be the TT-game corresponding to (E, c, a).
Let $i^* \in \arg\min_{i \in N: x_i - c_i = 0} \lambda_i$. Define the allocation $z \in R^N$ such that for each $i \in N$

$$z_i = \begin{cases} \frac{x_i}{\lambda(N \setminus \{i^*\})} - \frac{x_i}{\lambda_i} & \text{if } c_i - x_i > 0, \\ 0 & \text{if } c_i - x_i = 0 \text{ and } i \neq i^*, \\ \tilde{v}(N) - \sum_{i \in N \setminus \{i^*\}} z_i & \text{if } i \neq i^*. \end{cases} \qquad (6)$$

Surplus and technology transfer

Technology-Transfer game

$Player_i$	c_i	a_i	λ_i	X_i	$x_i - c_i$	
1	15	5	3	5	-10	$E = 20$
2	4	8	$\frac{1}{2}$	4	0	$\lambda = 1$
3	10	2	5	2	-8	
4	12	3	4	3	-9	
5	4	12	$\frac{1}{3}$	4	0	

Players	$\tilde{v}(S)$	Computations
1	0	0
2	0	0
...		
1,2	5	$v(1,2) = \frac{5}{\frac{1}{2}} - 5$
1,3	0	0
1,5	10	$v(1,5) = \frac{5}{\frac{1}{3}} - 5$
2,3	2	$v(2,3) = \frac{2}{\frac{1}{2}} - 2$
2,4	3	$v(2,4) = \frac{3}{\frac{1}{2}} - 3$
...		
1,4,5	16	$v(1,4,5) = \frac{5}{\frac{1}{3}} - 5 + \frac{3}{\frac{1}{3}} - 3$
2,4,5	6	$v(2,4,5) = \frac{3}{\frac{1}{3}} - 3$
...		
1,3,4,5	20	$v(1,3,4,5) = \frac{2}{\frac{1}{3}} - 2 + \frac{3}{\frac{1}{3}} - 3 + \frac{5}{\frac{1}{3}} - 5$
1,2,4,5	16	$v(1,2,4,5) = \frac{5}{\frac{1}{3}} - 5 + \frac{3}{\frac{1}{3}} - 3$
1,2,3,4,5	20	$v(1,2,3,4,5) = \frac{2}{\frac{1}{3}} - 2 + \frac{3}{\frac{1}{3}} - 3 + \frac{5}{\frac{1}{3}} - 5$

The game is montonic, superadditive and convex

Monotonicity:
$S, T \subseteq N s.c V(S) \leq V(T)$
$v(S)\{1,2\} = 5$
$v(T)\{123\} = 7$
Superadditivity :
$V(T) + V(S) \leq V(T \cup S)$
$v(S)\{13\} = 0$
$v(T)\{25\} = 0$
$v(S \cup T)\{1235\} = 14$
Core is Non empty

Conclusion and perspectives

Conclusion

- The global indicator, λ , can be considered as a boundary of CO_2 emissions quantities
- TT-game generates a technological transfer among countries.

Perspectives

- In weighted bankruptcy problem, the second parameter can be changed
- Another game where there is a partial or conjoint transfer among players.

Thank you