

Reasons and Means to Model Preferences as Incomplete

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<https://github.com/oliviercailloux/Survey-pref-models-pres>

Outline

- 1 Preference models: what, why?
- 2 Reasons for incomplete models
- 3 Means
- 4 Conclusion and further remarks

Overview

- Why model preferences?
- What is a model of preferences?
- Then: Not the usual tutorial talk!
- Incomplete models in practice: a topic for research!
- Motivate: reasons
- Sketch means
- Illustrate on recommender systems
- Detour via: psychology, economy...

Disclaimer: I am *not* an expert of all these fields.

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Models basics

- A set of alternatives \mathcal{X}
- A user
- A binary relation of preference over \mathcal{X} : $R \subseteq A \times A$
- aRb iff the user prefers a over b
- Strictly prefers: R denoted \succ
- Weakly prefers: R denoted \succeq

Hotels

- Alternatives are hotels
- User wants to choose an hotel

Extensions, remarks

- Preference modeling interested in the *decision* problem (hence vocabulary differences)
- Decision maker / user
- Alternative / object
- Goal is usually to select one alternative
- Sometimes the model includes more than one binary relation
- A model may also represent preference for a against prototypical alternatives (4 stars hotel)

Examples

Movies

- Alternatives are movies
- User wants to choose next movie to see

Students

- Alternatives are students
- User is a teacher ranking students

Production site

- Alternatives are production sites
- User is a CEO, wants to choose where to locate a factory

Recommendation system view

(Disclaimer: Oversimplified view!)

Movies

- Alternatives are movies
- User rates movies she has seen: 1 to 5 stars
- The system learns to predict ratings: unseen movie, what would be its rating?
- The system recommends the next movie to see (one of the predicted 5 stars)

Link with recommendation systems

Link with preference model?

- $m_1 R m_2$ iff $[\text{rating } m_1] \geq [\text{rating } m_2]$
- A weak order with five equivalence classes

Differences?

- (Generally) many users considered simultaneously
- R restricted to max five equivalence classes
- Assumes differences (5 VS 3 > 2 VS 1) are not used; no semantics to categories

Those (important) differences: neglected here

Why model preferences?

Two views

Descriptive

- Preference model describes normal behavior
- Must predict correctly

Prescriptive

- Preference model serves to recommend
- May voluntarily differ from prediction

Do we want to prescribe?

- Recommender system POV: usually mainly descriptive / predictive
- But: User may have “normal” behavior that she rejects when thinking carefully
- Extreme case: user violating dominance (Choose a hotel less good on every aspects)
- Realistic case: user discriminates unwillingly
- Other examples to come

Multiple criteria decision making (MCDM) context

- Alternatives evaluated using a set of criteria \mathcal{G}
- Each criterion g : evaluation scale X_g
- $\mathcal{X} = \prod_{g \in \mathcal{G}} X_g$
- Model: R over \mathcal{X}

	quantity	taste	supports pollinators	resists to cold
Tomatoes	7	A	A	--
Corn	1.5	B	D	--
Cabbage	7.5	D	B	++
Potatoes	2.5	C	C	+
...				

Decision under uncertainty

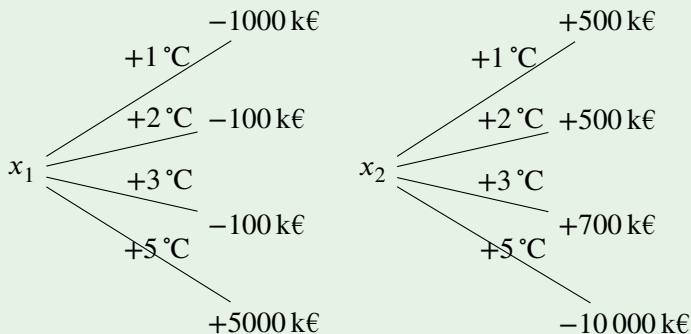
- S possible states of the world
- Consequences C (here, finite set)
- Alt $x : S \rightarrow C$
- $x(s)$ = consequence of x under state s
- $\mathcal{X} = C^S$ (the functions from S to C)
- Model: R over \mathcal{X}

Benefits of moving production depending on global warming

	+1 °C	+2 °C	+3 °C	+5 °C
Country 1	-1000 k€	-100 k€	-100 k€	+5000 k€
Country 2	+500 k€	+500 k€	+700 k€	-10 000 k€
...				

Decision under uncertainty: graphically

Moving production: graphically



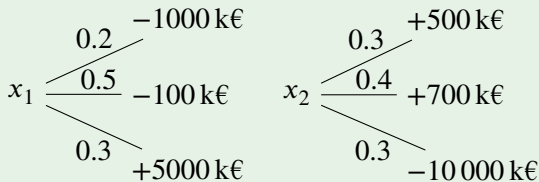
R indicates whether x_1 is preferred to x_2 , ...

Decision under risk

- “Risk” used when probabilities are known
- Probability measure p over the powerset of S
- $p(s) \in [0, 1]$ (with $s \subseteq S$): probability of occurrence of s ,
 $p(S) = 1$

Moving production: with probabilities

$$p(+1^\circ\text{C}) = 0.2; p(+2^\circ\text{C}) = 0.1; p(+3^\circ\text{C}) = 0.4; p(+5^\circ\text{C}) = 0.3$$



Decision under risk: comparing probabilities

$x \in \mathcal{X}$ can be viewed as a probability mass $p_x : C \rightarrow [0, 1]$ over the consequences.

Moving production: comparing probabilities

$$-1000 \text{ k€} \mapsto 0.2$$

$$+500 \text{ k€} \mapsto 0.3$$

$$p_{x_1}: -100 \text{ k€} \mapsto 0.5$$

$$p_{x_2}: +700 \text{ k€} \mapsto 0.4$$

$$+5000 \text{ k€} \mapsto 0.3$$

$$-10\,000 \text{ k€} \mapsto 0.3$$

- Alternative also called a *lottery*
- R indicates whether p_{x_1} is preferred to p_{x_2} , ...

Weak order

Weak order

\succeq is a *weak order* iff it is:

Transitive $x \succeq y \succeq z \Rightarrow x \succeq z$

Connected $\forall x \neq y, x \succeq y \text{ or } y \succeq x$

Reflexive $x \succeq x$

- Complete \Leftrightarrow connected and reflexive
- \succeq defines ordered equivalence classes

Numerical representation of a weak order

Theorem (Weak order and utility [Fishburn, 1970])

Given a binary relation \succeq on a set \mathcal{X} , assume \mathcal{X} is finite. Then, these two conditions are equivalent.

- \succeq is a weak order
- There exists a $u : \mathcal{X} \rightarrow \mathbb{R}$ such that

$$u(x) \geq u(y) \Leftrightarrow x \succeq y \quad (u \text{ represents } \succeq)$$

(Citations do *not* indicate paternity.)

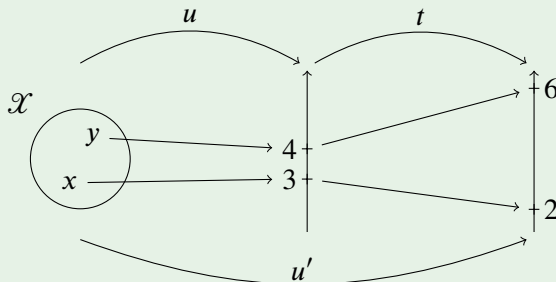
- The theorem goes through with \mathcal{X} infinite and (\mathcal{X}, \succeq) satisfying some denseness condition
- u is called a *utility* (or *value*) function

Significance of the scale

Any increasing transformation of u also represents \succeq

- If u represents \succeq
- Define $u' = t \circ u$ with $r \leq s \Rightarrow t(r) \leq t(s)$
- Then u' also represents \succeq

Equivalent utility functions



Numerics and risk

Consider a context of decision under risk.

Theorem

\succeq satisfies:

Order \succeq is a weak order

Independence Given $x \succeq y$, any $z \in \mathcal{X}$, and $0 < \alpha < 1$:

$$\alpha p_x + (1 - \alpha)p_z \succeq \alpha p_y + (1 - \alpha)p_z$$

Continuity Given $x \succeq y \succeq z$, for some $0 < \alpha, \beta < 1$:

$$\alpha p_x + (1 - \alpha)p_z \succeq p_y \succeq \beta p_x + (1 - \beta)p_z$$

iff $\exists u : \mathcal{X} \rightarrow \mathbb{R}$ and $u' : C \rightarrow \mathbb{R}$ such that

$$u(x) \geq u(y) \Leftrightarrow x \succeq y, \quad (u \text{ represents } \succeq)$$

$$u(x) = \sum_c u'(c)p_x(c). \quad (u \text{ is an expectation})$$

Numerics and risk: comments

- u is now determined up to affine transformations only
- $u(p_c) = u'(c)$, with p_c being c with probability one

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Two interpretations of incomplete models

- Epistemic incompleteness: incomplete model because the modeler ignores parts of it
- Ontologic incompleteness: incomplete model even when the modeler knows everything

I will talk about *ontological* incompleteness

Arguments for ontological incompleteness

Epistemological A better representation of reality

Ethical May permit to give better grounded recommendations (avoid recommending debatable behavior)

Practical Useful for building consensus in group decision making

Disclaimer: this part presents personal opinions

Rebutting completeness

Completeness sometimes considered true by definition.

An argument for completeness

Offer the choice between two alternatives. The user picks either x or y or is indifferent. Iterate. Obtain a complete model.

This argument has two weaknesses

- 1 We might want to study (more) *stable* preference
- 2 We might want to model (more) *reflexive* preference

Local VS stable preferences

- Argument for completeness considers “preference” as a set of time- and place- located events
- What if we are interested in her stable preference over some time span?

Tversky [1969] on incompleteness of stable preferences

- Users “are not perfectly consistent in their choices”
- “often choose x in some instances and y in others”
- Inconsistencies observed “even in the absence of systematic changes in (...) taste (...) due to learning or sequential effects”
- Inconsistencies thus seem to “reflect inherent variability or momentary fluctuation in the evaluative process”

Intuitive preference

von Neumann and Morgenstern (vNM) use a notion of “intuitive” preference

Preference in the vNM sense [Fishburn, 1989]

The “immediate sensation of preference” provides the basis for the measurement of utility

Completeness in the vNM sense

For any two objects, the user “possesses a clear intuition of preference”. “[W]e expect him, for any two [alternatives] which are put before him as possibilities, to be able to tell which of the two he prefers”.

Arguments for reflexive preference

Reflexive preference: those you stick to after having thought about it

- When recommending, we want to help decide
- Protection against (obvious) “bad behavior” might be useful
- Is the recommendation system falling under manipulative actions by vendors?
- Intuitive preference is not transitive [Mandler, 2001]

Ethical argument for incomplete models

- Can we assume that preferences are complete as an approximation?
- Psychology shows that intuitive behavior can be non reflexive
- Ethical argument: recommendation systems may protect against (worst forms of) such behavior
- Does not apply to all recommendation systems!

Experimental psychology and MCDA

An example of effects known in experimental psychology

- Two criteria
- Ask questions to obtain preference model R
- Assume R satisfies dominance and transitivity
- Trade-offs differ depending on the way questions are asked
- By choosing the kind of questioning strategy, you choose whether to accentuate the most salient criterion

Two kinds of trade-off questions

Binary choice

Choose one alternative

	Price	Distance from center
Hotel <i>a</i>	86	16
Hotel <i>b</i>	68	31

Matching

Make the alternatives equally attractive

	Price	Distance from center
Hotel <i>x</i>	86	16
Hotel <i>y</i>	?	31

Then we deduce whether $a = (86, 16) \succeq b = (68, 31)$.

Experimental psychology and decision under risk

- Similar effects exist under the “risk” context
- Ask using probability equivalents: Prefer $p_x = (0.5, 10VS0)$ to $p_y = (?, 20VS0)$?
- Or ask using certainty equivalents: Prefer $p_x = (0.5, 10VS0)$ to $p_y = (1, ?)$?

Remarks about preference reversal effects

- Effects called preference reversal (by procedure or description variance)
- Existence of those effects is consensual
- Interpretation is not
- Psychologist's conclusion: frame must be fixed
- Other possible conclusion: reflexive preferences may be incomplete
- More research needed

Practical argument for incomplete models

- The model may reflect what's truly preferable for the user
- The model may have more degrees of freedom

Illustration: group decision making

- In group decision approach: more possibilities for consensus
- Build a second model lexicographically
- First order: what the user prefers for intrinsic reasons
- Second order: what the user allows for reasons of being nice to others, ...

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In this subsection

- Outranking approach
- (More) classical economists / psychologists approach

Disclaimer: more research needed for application to recommender systems!

Outranking approach

- Context: MCDM
- Outranking approach is an alternative to value (utility) theory
- Relaxes assumptions
- Intuition 1: compare alternatives pairwise
- Intuition 2: several points of view may conflict
- From *one* POV, x is better than y

Illustration with Electre III

(Method is much simplified here)

- General idea: build two relations C, D
- Define $xRy \Leftrightarrow xCy$ and not xDy
- C for concordance: reasons in favor
- D for discordance: reasons against
- $x C y$ iff x is sufficiently better than y for being preferred
- For C , only criteria in favor of x are considered
- $x D y$ iff some strong reasons oppose x being preferred to y
- D only looks for reasons against

Example

Example relations

- xCy iff x is better than y (or equal) for at least two criteria
- xDy iff $\exists g \in \mathcal{G} \mid y_g - x_g \geq 3$

	quantity	taste	resists to cold
Tomatoes	4	4	1
Cabbage	4	1	5
Potatoes	2	2	4

- Cabbage \succeq Potatoes
- Tomatoes incomparable to Cabbage and Potatoes

Another case of incomparability

- x indistinguishable to y and y to z
- But $x \succeq z$
- Luce's "grain of sugar"
- How to represent this numerically?
- Which kinds of structure does this correspond to?

Semiorder

Semiorder [Fishburn, 1970]

A binary relation R is a *semiorder* iff it is irreflexive, transitive, and satisfies C1 and C2

C1

$$\begin{array}{c} x \\ | \\ y \end{array} \sim \begin{array}{c} z \\ | \\ w \end{array} \Rightarrow \begin{array}{c} x \\ \backslash \\ y \end{array} \begin{array}{c} z \\ / \\ w \end{array} \text{ or } \begin{array}{c} x \\ / \\ y \end{array} \begin{array}{c} z \\ \backslash \\ w \end{array}$$

C2

$$\begin{array}{c} x \\ | \\ y \\ | \\ z \end{array} \sim w \Rightarrow \begin{array}{c} x \\ \backslash \\ y \\ / \\ z \end{array} \begin{array}{c} w \\ / \\ z \end{array} \text{ or } \begin{array}{c} x \\ / \\ y \\ \backslash \\ z \end{array} \begin{array}{c} w \\ \backslash \\ z \end{array}$$

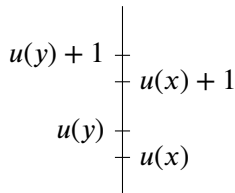
Representation

Theorem (Luce 1956, Scott and Suppes 1958)

Given a semiorder R on \mathcal{X} finite, there exists $u : \mathcal{X} \rightarrow \mathbb{R}$ such that:

$$xRy \Leftrightarrow u(x) + 1 < u(y)$$

Example incomparability:



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A normative property of decision

If you prefer x to y , you ought to not reverse your preference in presence of z (says the normative property)

Counter-example

- “Would you like chocolate cake or coconut cake?”
- “I prefer the chocolate cake!”
- “Oh, by the way, there’s also ice cream”
- “Mmh, give me the coconut cake”

Odd?

Sen on the act of choice

- Sen [1997] warns against too simple usage of this norm
- You are invited at a garden party
- Out of apple, banana, you would pick apple
- Out of apple, mango, banana, you would pick banana

Possible explanation?

Sen on the act of choice

- Sen [1997] warns against too simple usage of this norm
- You are invited at a garden party
- Out of apple, banana, you would pick apple
- Out of apple, mango, banana, you would pick banana

Possible explanation?

- You prefer mango to banana to apple
- You suspect everyone else has the same preference
- You want to be polite and take the second best

⇒ Reasonableness of norms are relative to a model

Conclusion

- Movie recommendation system that predicts $[1, 5]$ stars: might be less determined than usually believed?
- More research needed!
- Pay attention to the model
- Majority of the literature is about complete models; but more and more analysis about incompleteness
- My interpretation may not be consensual! (But the displayed knowledge is)
- There are also valid reasons for complete models (but think about it!)

Thank you for your attention!

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