Reasons and Means to Model Preferences as Incomplete

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6th October, 2017

https://github.com/oliviercailloux/Survey-pref-models-pres

Outline

1. Preference models: what, why?
2. Reasons for incomplete models
3. Means
4. Conclusion and further remarks

Overview

- Why model preferences?
- What is a model of preferences?
- Then: Not the usual tutorial talk!
- Incomplete models in practice: a topic for research!
- Motivate: reasons
- Sketch means
- Illustrate on recommender systems
- Detour via: psychology, economy…

Disclaimer: I am *not* an expert of all these fields.

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Models basics

- A set of alternatives $\mathscr{X}$
- A user
- A binary relation of preference over $\mathscr{X}$: $R \subseteq A \times A$
- aRb iff the user perfers a over b
- Strictly prefers: R denoted $\succ$
- Weakly prefers: R denoted $\succeq$

Hotels

- Alternatives are hotels
- User wants to choose an hotel

Extensions, remarks

- Preference modeling interested in the *decision* problem (hence vocabulary differences)
- Decision maker / user
- Alternative / object
- Goal is usually to select one alternative
- Sometimes the model includes more than one binary relation
- A model may also represent preference for a against prototypical alternatives (4 stars hotel)

Examples

Movies

- Alternatives are movies
- User wants to choose next movie to see

Students

- Alternatives are students
- User is a teacher ranking students

Production site

- Alternatives are production sites
- User is a CEO, wants to choose where to locate a factory

Recommendation system view

(Disclaimer: Oversimplified view!)

Movies

- Alternatives are movies
- User rates movies she has seen: 1 to 5 stars
- The system learns to predict ratings: unseen movie, what would be its rating?
- The system recommends the next movie to see (one of the predicted 5 stars)

Link with recommendation systems

Link with preference model?

- $m_1 \; R \; m_2$ iff [rating m_1] $\geq$ [rating m_2]
- A weak order with five equivalence classes

Differences?

- (Generally) many users considered simultaneously
- R restricted to max five equivalence classes
- Assumes differences (5 VS 3 $>$ 2 VS 1) are not used; no semantics to categories

Those (important) differences: neglected here

Why model preferences?

Two views

Descriptive

- Preference model describes normal behavior
- Must predict correctly

Prescriptive

- Preference model serves to recommend
- May voluntarily differ from prediction

Do we want to prescribe?

- Recommender system POV: usually mainly descriptive / predictive
- But: User may have "normal" behavior that she rejects when thinking carefully
- Example: user violating dominance (Choose a hotel less good on every aspects)
- Example: user discriminates unwillingly
- Other examples to come

Multiple criteria decision making (MCDM) context

- Alternatives evaluated using a set of criteria $\mathscr{G}$
- Each criterion g: evaluation scale X_g
- $\mathscr{X} = \prod_{g \in \mathscr{G}} X_g$
- Model: R over $\mathscr{X}$

Let's garden a bit

	quantity	taste	supports pollinators	resists to cold
Tomatoes	7	A	A	$--$
Corn	1.5	B	D	$--$
Cabbage	7.5	D	B	$++$
Potatoes	2.5	C	C	$+$
…				

Decision under uncertainty

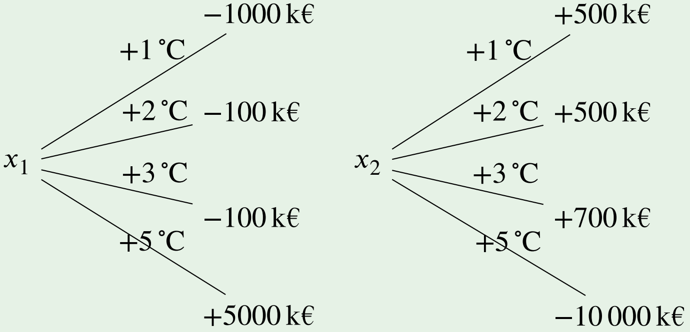
- S possible states of the world
- Consequences C (here, finite set)
- Alt $x : S \to C$
- $x(s)$ = consequence of x under state s
- $\mathscr{X} = C^S$ (the functions from S to C)
- Model: R over $\mathscr{X}$

Benefits of moving production depending on global warming

	+1 °C	+2 °C	+3 °C	+5 °C
Country 1	−1000 k€	−100 k€	−100 k€	+5000 k€
Country 2	+500 k€	+500 k€	+700 k€	−10 000 k€
…				

Decision under uncertainty: graphically

Moving production: graphically



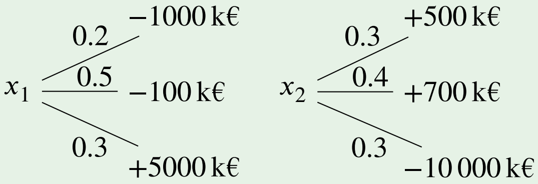
R indicates whether x_1 is preferred to x_2, …

Decision under risk

- "Risk" used when probabilities are known
- Probability measure p over the powerset of S
- $p(s) \in [0, 1]$ (with $s \subseteq S$): probability of occurence of s, $p(S) = 1$

Moving production: with probabilities

$$p(+1\,^{\circ}\mathrm{C}) = 0.2; p(+2\,^{\circ}\mathrm{C}) = 0.1; p(+3\,^{\circ}\mathrm{C}) = 0.4; p(+5\,^{\circ}\mathrm{C}) = 0.3$$



Decision under risk: comparing probabilities

$x \in \mathscr{X}$ can be viewed as a probability mass $p_x : C \to [0, 1]$ over the consequences.

Moving production: comparing probabilities

$$p_{x_1}: \begin{array}{l} -1000\,\text{k€} \mapsto 0.2 \\ -100\,\text{k€} \mapsto 0.5 \\ +5000\,\text{k€} \mapsto 0.3 \end{array} \qquad p_{x_2}: \begin{array}{l} +500\,\text{k€} \mapsto 0.3 \\ +700\,\text{k€} \mapsto 0.4 \\ -10\,000\,\text{k€} \mapsto 0.3 \end{array}$$

- Alternative also called a *lottery*
- R indicates whether p_{x_1} is preferred to p_{x_2}, …

Weak order

Weak order

$\geq$ is a *weak order* iff it is:

Transitive $x \geq y \geq z \Rightarrow x \geq z$

Connected $\forall x \neq y$, $x \geq y$ or $y \geq x$

Reflexive $x \geq x$

- Complete $\Leftrightarrow$ connected and reflexive
- $\geq$ defines ordered equivalence classes

Numerical representation of a weak order

Theorem (Weak order and utility [Fishburn, 1970])

Given a binary relation $\succeq$ on a set $\mathcal{X}$, assume $\mathcal{X}$ is finite. Then, these two conditions are equivalent.

- *$\succeq$ is a weak order*
- *There exists a $u : \mathcal{X} \to \mathbb{R}$ such that*

$$u(x) \geq u(y) \Leftrightarrow x \succeq y \qquad (u \text{ represents } \succeq)$$

(Citations do *not* indicate paternity.)

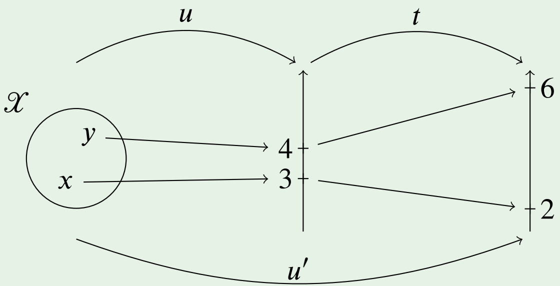
- The theorem goes through with $\mathcal{X}$ infinite and $(\mathcal{X}, \succeq)$ satisfying some denseness condition
- u is called a *utility* (or *value*) function

Significance of the scale

Any increasing transformation of u also represents $\succeq$

- If u represents $\succeq$
- Define $u' = t \circ u$ with $r \leq s \Rightarrow t(r) \leq t(s)$
- Then u' also represents $\succeq$

Equivalent utility functions



Numerics and risk

Consider a context of decision under risk.

Theorem

$\succeq$ *satisfies:*

Order $\succeq$ *is a weak order*

Independence *Given* $x \succeq y$, *any* $z \in \mathscr{X}$, *and* $0 < \alpha < 1$:
$\alpha p_x + (1-\alpha)p_z \succeq \alpha p_y + (1-\alpha)p_z$

Continuity *Given* $x \succeq y \succeq z$, *for some* $0 < \alpha, \beta < 1$:
$\alpha p_x + (1-\alpha)p_z \succeq p_y \succeq \beta p_x + (1-\beta)p_z$

iff $\exists u : \mathscr{X} \to \mathbb{R}$ *and* $u' : C \to \mathbb{R}$ *such that*

$$u(x) \geq u(y) \Leftrightarrow x \succeq y, \qquad (u \text{ represents } \succeq)$$

$$u(x) = \sum_c u'(c) p_x(c). \qquad (u \text{ is an expectation})$$

Numerics and risk: comments

- u is now determined up to affine transformations only
- $u(p_c) = u'(c)$, with p_c being c with probability one

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Two interpretations of incomplete models

- Epistemic incompleteness: incomplete model because the modeler ignores parts of it
- Ontologic incompleteness: incomplete model even when the modeler knows everything

I will talk about *ontological* incompleteness

Arguments for ontological incompleteness

Epistemological	A better representation of reality
Ethical	May permit to give better grounded recommandations (avoid recommending debatable behavior)
Practical	Useful for building consensus in group decision making

Disclaimer: this part presents personal opinions

Rebutting completeness

Completeness sometimes considered true by definition.

An argument for completeness

Offer the choice between two alternatives. The user picks either x or y or is indifferent. Iterate. Obtain a complete model.

This argument has two weaknesses

1. We might want to study (more) *stable* preference
2. We might want to model (more) *reflexive* preference

Local VS stable preferences

- Argument for completeness considers "preference" as a set of time- and place- located events
- What if we are interested in her stable preference over some time span?

Tversky [1969] on incompleteness of stable preferences

- Users "are not perfectly consistent in their choices"
- "often choose x in some instances and y in others"
- Inconsistencies observed "even in the absence of systematic changes in (...) taste (...) due to learning or sequential effects"
- Inconsistencies thus seem to "reflect inherent variability or momentary fluctuation in the evaluative process"

Intuitive preference

von Neumann and Morgenstern (vNM) use a notion of "intuitive" preference

Preference in the vNM sense [Fishburn, 1989]

The "immediate sensation of preference" provides the basis for the measurement of utility

Completeness in the vNM sense

For any two objects, the user "possesses a clear intuition of preference". "[W]e expect him, for any two [alternatives] which are put before him as possibilities, to be able to tell which of the two he prefers".

Arguments for reflexive preference

Reflexive preference: those you stick to after having thought about it

- When recommending, we want to help decide
- Protection against (obvious) “bad behavior” might be useful
- Is the recommendation system falling under manipulative actions by vendors?
- Intuitive preference is not transitive [Mandler, 2001]

Ethical argument for incomplete models

- Can we assume that preferences are complete as an approximation?
- Psychology shows that intuitive behavior can be non reflexive
- Ethical argument: recommendation systems may protect against (worst forms of) such behavior
- Does not apply to all recommendation systems!

Experimental psychology and MCDA

An example of effects known in experimental psychology

- Two criteria
- Ask questions to obtain preference model R
- Assume R satisfies dominance and transitivity
- Trade-offs differ depending on the way questions are asked
- By choosing the kind of questioning strategy, you choose whether to accentuate the most salient criterion

Two kinds of trade-off questions

Binary choice

Choose one alternative

	Price	Distance from center
Hotel a	86	16
Hotel b	68	31

Matching

Make the alternatives equally attractive

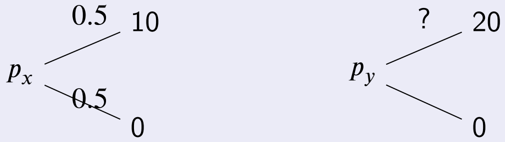
	Price	Distance from center
Hotel x	86	16
Hotel y	?	31

Then we deduce whether $a = (86, 16) \geq b = (68, 31)$.

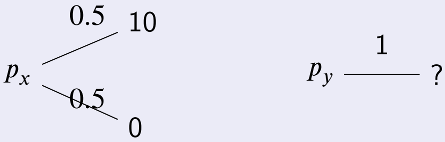
Experimental psychology and decision under risk

Similar effects exist under the “risk” context

Using probability equivalents



Using certainty equivalents



Remarks about preference reversal effects

- Effects called preference reversal (by procedure or description variance)
- Existence of those effects is consensual
- Interpretation is not
- Psychologist's conclusion: frame must be fixed
- Other possible conclusion: reflexive preferences may be incomplete
- More research needed

Practical argument for incomplete models

- The model may reflect what's truly preferable for the user
- The model may have more degrees of freedom

Illustration: group decision making

- In group decision approach: more possibilities for consensus
- Build a second model lexicographically
- First order: what the user prefers for intrinsic reasons
- Second order: what the user allows for reasons of being nice to others, …

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In this subsection

- Outranking approach
- (More) classical economists / psychologists approach

Disclaimer: more research needed for application to recommender systems!

Outranking approach

- Context: MCDM
- Outranking approach is an alternative to value (utility) theory
- Relaxes assumptions
- Intuition 1: compare alternatives pairwise
- Intuition 2: several points of view may conflict
- From *one* POV, x is better than y

Illustration with Electre III

(Method is much simplified here)

- General idea: build two relations C, D
- Define $xRy \Leftrightarrow xCy$ and not xDy
- C for concordance: reasons in favor
- D for discordance: reasons against
- $x\ C\ y$ iff x is sufficiently better than y for being preferred
- For C, only criteria in favor of x are considered
- $x\ D\ y$ iff some strong reasons oppose x being preferred to y
- D only looks for reasons against

Example

Example relations

- xCy iff x is better than y (or equal) for at least two criteria
- xDy iff $\exists g \in \mathcal{G} \mid y_g - x_g \geq 3$

	quantity	taste	resists to cold
Tomatoes	4	4	1
Cabbage	4	1	5
Potatoes	2	2	4

- Cabbage $\succeq$ Potatoes
- Tomatoes incomparable to Cabbage and Potatoes

Another case of incomparability

- x indistingushable to y and y to z
- But $x \succeq z$
- Luce's "grain of sugar"
- How to represent this numerically?
- Which kinds of structure does this correspond to?

Semiorder

Semiorder [Fishburn, 1970]

A binary relation R is a *semiorder* iff it is irreflexive, transitive, and satisfies C1 and C2

C1

$$
\begin{tikzcd}
x \arrow[d, dash] & \sim & z \arrow[d, dash] \\
y & \sim & w
\end{tikzcd}
\Rightarrow
\begin{tikzcd}
x \arrow[dr, dash] & z \\
y & w
\end{tikzcd}
\text{ or }
\begin{tikzcd}
x & z \arrow[dl, dash] \\
y & w
\end{tikzcd}
$$

C2

$$
\begin{tikzcd}
x \arrow[d, dash] & & \\
y \arrow[d, dash] & \sim & w \\
z & &
\end{tikzcd}
\Rightarrow
\begin{tikzcd}
x \arrow[dr, dash] & \\
y & w \\
z &
\end{tikzcd}
\text{ or }
\begin{tikzcd}
x & \\
y & w \arrow[dl, dash] \\
z &
\end{tikzcd}
$$

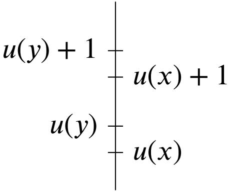
Representation

Theorem (Luce 1956, Scott and Suppes 1958)

Given a semiorder R on $\mathscr{X}$ finite, there exists $u : \mathscr{X} \to \mathbb{R}$ such that:

$$xRy \Leftrightarrow u(x) + 1 < u(y)$$

Example incomparability:



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A normative property of decision

If you prefer x to y, you ought to not reverse your preference in presence of z (says the normative property)

Counter-example

- "Would you like chocolate cake or coconut cake?"
- "I prefer the chocolate cake!"
- "Oh, by the way, there's also ice cream"
- "Mmh, give me the coconut cake"

Odd?

Sen on the act of choice

- Sen [1997] warns against too simple usage of this norm
- You are invited at a garden party
- Out of apple, banana, you would pick apple
- Out of apple, mango, banana, you would pick banana

Possible explanation?

Sen on the act of choice

- Sen [1997] warns against too simple usage of this norm
- You are invited at a garden party
- Out of apple, banana, you would pick apple
- Out of apple, mango, banana, you would pick banana

Possible explanation?

- You prefer mango to banana to apple
- You suspect everyone else has the same preference
- You want to be polite and take the second best

⇒ Reasonableness of norms are relative to a model

Conclusion

- Movie recommendation system that predicts [1, 5] stars: might be less determined than usually believed?
- More research needed!
- Pay attention to the model
- Majority of the literature is about complete models; but more and more analysis about incompleteness
- My interpretation may not be consensual! (But the displayed knowledge is)
- There are also valid reasons for complete models (but think about it!)

Thank you for your attention!

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