Axiomatics, then what?

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Axiomatics

> *Rather than dream up a multitude of arbitration schemes and determine whether or not each withstands the best of plausibility in a host of special cases, let us invert the procedure. Let us examine our subjective intuition of fairness and formulate this as a set of precise desiderata that any acceptable arbitration scheme must fulfil. Once these desiderata are formalized as axioms, then the problem is reduced to a mathematical investigation of the existence of and characterization of arbitration schemes which satisfy the axioms.*

Luce and Raiffa [1957, p. 121]

Beyond axioms?

Possible view (that I want to counter):

- Mathematically capture the behavior of a rule (with axioms…)
- Left out for the user: confront their intuition about fairness with the axioms
- Scientific approach stops at the first step
- The rest is ultimately subjective

I propose a different view:

- Confronting our subjective intuition of fairness with axioms is hard
- Nobody can do this
- Because we can't compute implications
- We can help doing this scientifically

Problem reduced?

- Formulate our subjective intuition of fairness as axioms?
- *Excluding* impossibilities?
- Making sure no hidden axiom is left out?
- Accept Arrow's axioms?
- Who's subjective intuition?

Understand the rule?

- When do you *understand* a voting rule?
- Borda rule: I know how to count scores
- Is that all?
- Recall we want to capture our (?) idea of fairness

What do axioms say?

- Humans have limited deductive power
- Hence, knowing the definition of a rule does not determine whether we accept it
- Knowing the *axiomatization* of a rule does not determine whether we accept it
- Wanted: equilibrium between principles and case-based, concrete intuitions [Goodman, 1983, Rawls, 1999]

What do axioms say? (Illustrations)

Example (Borda)

- We know Borda is the rule that satisfies neutrality, reinforcement, faithfulness, cancellation
- This does not obviously say that Borda fails on Condorcet

Example (Dictatorship)

- One may want Arrow's axioms
- But fail to see what it implies: dictatorship
- Once the implications are understood, one does not want all of Arrow's axioms any more

Fishburn-against-Condorcet

Fishburn [1974, p. 544] argument against the Condorcet principle (see also http://rangevoting.org/FishburnAntiC.html).

Condorcet winner

w VS $\mu, \mu \in \{a, \dots, h\}$?

	nb voters					
	31	19	10	10	10	21
1	a	a	f	g	h	h
2	b	b	w	w	w	g
3	c	c	a	a	a	f
4	d	d	h	h	f	w
5	e	e	g	f	g	a
6	w	f	e	e	e	e
7	g	g	d	d	d	d
8	h	h	c	c	c	c
9	f	w	b	b	b	b

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Condorcet winner

w VS $\mu, \mu \in \{a, \ldots, h\}$? 51/101

	nb voters					
	31	19	10	10	10	21
1	a	a	f	g	h	h
2	b	b	w	w	w	g
3	c	c	a	a	a	f
4	d	d	h	h	f	w
5	e	e	g	f	g	a
6	w	f	e	e	e	e
7	g	g	d	d	d	d
8	h	h	c	c	c	c
9	f	w	b	b	b	b

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Condorcet winner

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nb voters

	31	19	10	10	10	21
1	a	a	f	g	h	h
2	b	b	w	w	w	g
3	c	c	a	a	a	f
4	d	d	h	h	f	w
5	e	e	g	f	g	a
6	w	f	e	e	e	e
7	g	g	d	d	d	d
8	h	h	c	c	c	c
9	f	w	b	b	b	b

ranks

	1	≤ 2	≤ 3	≤ 4	≤ 5	≤ 6	≤ 7	≤ 8	≤ 9
w	0	30	30	51	51	82	82	82	101
a	50	50	80	80	101	101	101	101	101

Possible approach

- To know whether one accepts a voting rule, we have to check whether one accepts the implications of the voting rule
- Can't be done exhaustively
- Can be done using known possible problematic cases
- Axiomatics can help in providing reasonings
- Different axiomatics may have different convincing power
- Acceptance of people in concrete cases can in principle be studied empirically [Gaertner and Schokkaert, 2012]

Proposition

- Proposition: propose a research program aiming at such a study / or
- Propose first steps (a framework?)

Key element

- Obviously, in many cases, no single decisive rule is the most appropriate
- We have to allow for incompleteness
- E.g. Impossible to completely rank all universities by "quality"
- Possible approach: search for non decisive rules (possible winners...)

Thank you for your attention!

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Definition

- Voting rule: a systematic way of aggregating different opinions and decide
- Multiple reasonable ways of doing this
- Different voting rules have different interesting properties
- None satisfy all desirable properties

Voting rule

Alternatives $\mathscr{A} = \{ a, b, c, d, \dots \}$

Voters $\mathscr{N} = \{ 1, 2, \dots \}$

Profile function $\boldsymbol{R}$ from $\mathscr{N}$ to linear orders on $\mathscr{A}$.

Voting rule function f mapping each $\boldsymbol{R}$ to winners $\emptyset \subset A \subseteq \mathscr{A}$.

$$\underbrace{\begin{array}{cc} \boldsymbol{R}(1) & \boldsymbol{R}(2) \\ \hline a & b \\ b & a \\ c & c \end{array}}_{\boldsymbol{R}} \xrightarrow{f} A = \{ a, b \}$$

Example of a profile

	nb voters					
	33	16	3	8	18	22
1	a	b	c	c	d	e
2	b	d	d	e	e	c
3	c	c	b	b	c	b
4	d	e	a	d	b	d
5	e	a	e	a	a	a

Who wins?

- Most top-1: a
- c is in the top 3 for everybody
- delete worst first, lowest nb of pref: c, b, e, $a \Rightarrow d$
- delete worst first, from bottom: a, e, d, $b \Rightarrow c$
- Borda: b
- Condorcet: c

Thanks to Jérôme Lang

Borda

Given a profile $\boldsymbol{R}$:

- score of $a \in \mathscr{A}$: number of alternatives it beats
- the highest scores win

$$\boldsymbol{R} = \begin{array}{ccccc} a & a & a & b & b \\ b & b & b & c & c \\ c & c & c & a & a \end{array}$$

- score a is…?

Borda

Given a profile $\boldsymbol{R}$:

- score of $a \in \mathscr{A}$: number of alternatives it beats
- the highest scores win

$$\boldsymbol{R} = \begin{matrix} a & a & a & b & b \\ b & b & b & c & c \\ c & c & c & a & a \end{matrix}$$

- score a is...? $2+2+2=6$
- score b is $1+1+1+2+2=7$
- score c is $1+1=2$

Winner: b.

Condorcet's principle

Condorcet's principle

We ought to take the Condorcet winner as sole winner if it exists.

- a *beats* b iff more than half the voters prefer a to b.
- a is a *Condorcet winner* iff a beats every other alternatives.

$$\boldsymbol{R} = \begin{array}{ccccc} a & a & a & b & b \\ b & b & b & c & c \\ c & c & c & a & a \end{array}$$

Who wins?

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$$\boldsymbol{R} = \begin{array}{ccccc} a & a & a & b & b \\ b & b & b & c & c \\ c & c & c & a & a \end{array}$$

Who wins? a

Condorcet's principle and a voting rule

- Condorcet's principle does not define a voting rule. Why?

Condorcet's principle and a voting rule

- Condorcet's principle does not define a voting rule. Why?
- No winner is defined when no Condorcet winner

$$\boldsymbol{R} = \begin{array}{ccc} b & c & d \\ c & d & b \\ a & b & a \\ d & a & c \end{array}$$

Condorcet's principle and a voting rule

- Condorcet's principle does not define a voting rule. Why?
- No winner is defined when no Condorcet winner

$$\boldsymbol{R} = \begin{array}{ccc} b & c & d \\ c & d & b \\ a & b & a \\ d & a & c \end{array}$$

a loses against b; b against d; c against b; d against c

- Dodgson's method (1876): candidates "closest" to being Condorcet winners (in nb of swaps)