# Eliciting a suitable voting rule via examples

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https://github.com/oliviercailloux/eliciting-voting-rules-pres-short

# Introduction

## Context

- A *committee* (a group of decision makers)
  - a panel attributing a research price
  - a management committee

- Recurring decisions

- A decision is taken using a voting rule

- Voting rule: a systematic way of aggregating different opinions and decide

## Our goal

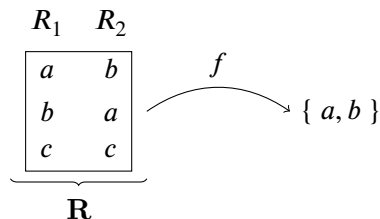We want to help the committee choose a suitable voting rule.

# Voting rule

## Input

- A set of possible alternatives (options) $\mathscr{A}$
- Each voter $i \in N$ has a linear order of preference over $\mathscr{A}$
- A profile $\mathbf{R}$ associates each $i$ to such an order.

## Voting rule

Associates to each profile $\mathbf{R}$ winning alternatives $A \subseteq \mathscr{A}$.

$$
\begin{array}{cc}
R_1 & R_2 \\
\hline
a & b \\
b & a \\
c & c \\
\hline
\end{array}
\quad \xrightarrow{\ f\ } \quad \{\, a, b \,\}
$$

$$\underbrace{\phantom{R_1 \quad R_2}}_{\mathbf{R}}$$

# Our goal

Making decisions involves two steps.

1. Establish a constitution: choose a voting rule.
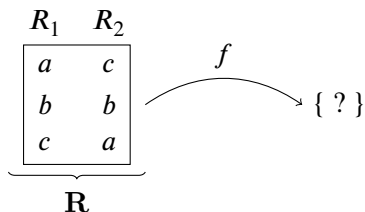2. Solve a decision problem: apply the voting rule.

## Our goal

We focus on step 1: help the committee choose a voting rule.

- Class of functions $\mathscr{F}$ (the set of all voting rules)
- Preference elicitation in order to choose a function $f \in \mathscr{F}$.
- We want to ask *simple* questions: example-based.

# A naïve attempt

## A first attempt

Simply give a profile $\mathbf{R}$ and ask for $f(\mathbf{R})$. Then iterate.



- Completely general: all functions in $\mathscr{F}$ can be reached.

*But...*

- One question brings very little information.
- Questions may be difficult to answer.

Context    Asking good questions    Restrict the class of functions    Which questions to ask?    Conclusion

Our goal        General approach

# General idea

- Ask *good* (informative, example-based) questions.
- Restrict the class of a-priori acceptable functions to $\mathscr{F}' \subset \mathscr{F}$.

Context     Asking good questions     Restrict the class of functions     Which questions to ask?     Conclusion

Our goal          General approach

# Outline

1. Context

2. Asking good questions

3. Restrict the class of functions
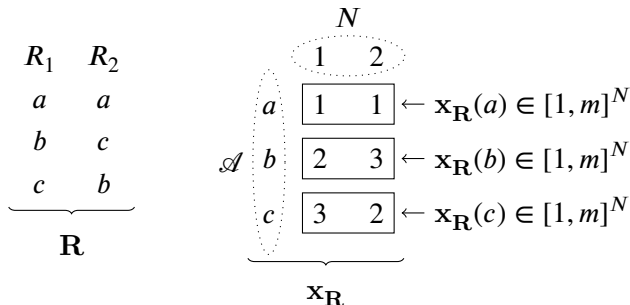
4. Which questions to ask?

5. Conclusion

# Outline

1 Context

2 **Asking good questions**

3 Restrict the class of functions

4 Which questions to ask?

5 Conclusion

# A different view of a profile

- We want to ask more informative questions about $f$.
- We look at profiles under a different angle.
- A *rank-vector* maps voters to ranks, $x : N \to [1, m]$.
- All rank vectors: $[1, m]^N$.

$$
\begin{array}{cc}
R_1 & R_2 \\
a & a \\
b & c \\
c & b
\end{array}
$$

$$\underbrace{\hphantom{R_1 \quad R_2}}_{\mathbf{R}}$$

$N$

$$\mathscr{A} \left\{
\begin{array}{c}
a \\
b \\
c
\end{array}
\right.
\begin{array}{c}
\boxed{1 \quad 1} \leftarrow \mathbf{x_R}(a) \in [1, m]^N \\
\boxed{2 \quad 3} \leftarrow \mathbf{x_R}(b) \in [1, m]^N \\
\boxed{3 \quad 2} \leftarrow \mathbf{x_R}(c) \in [1, m]^N
\end{array}
$$

$$\underbrace{\hphantom{xxxxxxxx}}_{\mathbf{x_R}}$$

To each profile $\mathbf{R}$ corresponds a *rank-profile* $\mathbf{x_R}$.

## Representing the preferences of the committee

- We can now ask for the preference status of, e.g.,

  | 1 | 3 | versus | 2 | 2 |.

- Series of such questions permit to identify a voting rule.

### Weak-order based rules

- $\succeq$ a weak-order (transitive, reflexive, connected) over $[1, m]^N$.
- Having a profile $\mathbf{R}$, look at the maximal rank-vectors in it according to $\succeq$.
- The rule $f_{\succeq}$, at $\mathbf{R}$, selects those alternatives having maximal rank-vectors in $\mathbf{R}$.

A rule $f$ is *weak-order based* if there exists $\succeq$ st $f = f_{\succeq}$.

Context    **Asking good questions**    Restrict the class of functions    Which questions to ask?    Conclusion

Using a weak-order                                              Using preorders

# Incomplete question sets

- We do not want to ask every possible questions!
- Can we get away with only *some* answers?

### Robust rules

- $\succsim$ a *preorder* (transitive, reflexive) over $[1, m]^N$.
- Look at all weak-orders $\succeq$ extending $\succsim$.
- The *robust* rule $F_{\succsim}$, at $\mathbf{R}$, selects those alternatives winning in *some* $f_{\succeq}$ (for some $\succeq$ extension of $\succsim$).

A rule $f$ is *robust* if there exists $\succsim$ st $f = F_{\succsim}$.

# Outline

# The WOB class

Not every rule is weak-order based.

- Bad news: we are not fully general any more.
- Good news: we have restricted our class of functions.
  $WOB = \left\{ f_{\succeq}, \succeq \text{ a weak-order over } [1, m]^N \right\}$ instead of $\mathscr{F}$.

How does WOB compare to other known classes of rules?

- Every scoring rule (e.g. Borda) is a WOB rule.
- Some WOB rules are not scoring rules. E.g. Bucklin.
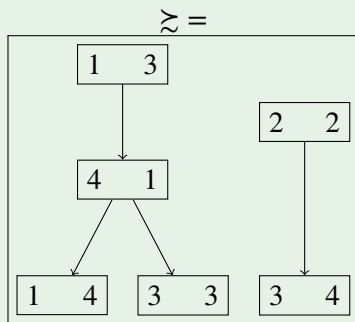- Many Condorcet rules are not WOB rules.

Some relationships between classes of rules

Scoring $\subset$ WOB; Condorcet $\cap$ WOB $= \varnothing$ (for $n = 3k, m \geq 4$).

# The class of robust rules

Some robust rules are not WOB rules.

## Example (A robust rule)



| | $\mathbf{R}_1$ | $F_{\succsim}(\mathbf{R}_1)$ | $\mathbf{R}_2$ | $F_{\succsim}(\mathbf{R}_2)$ |
|---|---|---|---|---|
| $a$ | 1   3 | ✓ | 1   4 | |
| $b$ | 2   2 | ✓ | 2   2 | ✓ |
| $c$ | 3   4 | | 3   3 | |
| $d$ | 4   1 | | 4   1 | ✓ |

## Some relationships between classes of rules

Scoring $\subset$ WOB $\subset$ Robust.

# Outline

# Which questions to ask?

- Assume the committee has a weak-order $\succeq$ in mind.
- We want to discover much information using few questions.
- Different questions bring different amount of information.

### Elicitation strategy

An elicitation strategy tells us which question should be asked
considering our current knowledge.

A strategy:

1. computes the fitness of asking about a pair of rank-vectors,
   for each pair;

2. chooses the fittest pair.

We ask $q$ questions, then compare our approximation $F_{\succsim}$ to $f_{\succeq}$.

# Which strategy?

We tested three strategies.

optimistic fitness of $(x, y)$ proportional to the number of rank-vectors dominated by $x$ or $y$, but not both;

pessimistic a variant of the previous strategy, using the min operator rather than the sum;

likelihood fitness proportional to the likelihood of a profile occurring where both rank-vectors are possible winners.

We assume pareto-dominance and indifference to permutations.

## Comparison of strategies

- Optimistic not better than random!
- Likelihood much better than pessimistic.

## Number of questions

How many questions must be asked for a useful approximation?

- Our approximation has all the true winners: $f_{\succeq}(\mathbf{R}) \subseteq F_{\succeq}(\mathbf{R})$.
- But it may have supplementary winners.
- We are interested in the ratio of approximated VS true winners: $\frac{|F_{\succeq}(\mathbf{R})|}{|f_{\succeq}(\mathbf{R})|}$.
- We average it over all profiles: $\frac{1}{|\mathscr{R}|} \sum_{\mathbf{R} \in \mathscr{R}} \frac{|F_{\succeq}(\mathbf{R})|}{|f_{\succeq}(\mathbf{R})|}$.

For 6 voters, 6 alternatives, using the likelihood strategy:

|        | Target rule | |
|--------|-------|------------|
| nb q   | Borda | Random $\succeq$ |
| 0      | 1.9   | 2.2        |
| 25     | 1.3   | 1.7        |
| 99     | 1.0   | 1.3        |

# Outline

1. Context

2. Asking good questions

3. Restrict the class of functions

4. Which questions to ask?

5. **Conclusion**

# Conclusion

We propose to help a committee choose a voting rule.

- We introduce a different look at a profile (see also Sen, 1977).
- We use it to ask simple questions to elicit preferences.
- We analyse the class of rules reachable by our questioning process.
- A robust voting rule may be defined to give all possible winners (inspired by Dias et al., 2002).
- We compare and analyse several elicitation strategies.

*Thank you for your attention!*

# Bibliography

Dias, L., Mousseau, V., Figueira, J., and Clímaco, J. (2002). An aggregation/disaggregation approach to obtain robust conclusions with ELECTRE TRI. *European Journal of Operational Research*, 138(2):332–348.

Sen, A. (1977). On weights and measures: Informational constraints in social welfare analysis. *Econometrica*, 45(7):1539–1572.