Eliciting a suitable voting rule via examples

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https://github.com/oliviercailloux/eliciting-voting-rules-pres-short

Introduction

Context

- A *committee* (a group of decision makers)
  - a panel attributing a research price
  - a management committee
- Recurring decisions
- A decision is taken using a voting rule
- Voting rule: a systematic way of aggregating different opinions and decide

Our goal

We want to help the committee choose a suitable voting rule.

Voting rule

Input

- A set of possible alternatives (options) $\mathscr{A}$
- Each voter $i \in N$ has a linear order of preference over $\mathscr{A}$
- A profile $\mathbf{R}$ associates each i to such an order.

Voting rule

Associates to each profile $\mathbf{R}$ winning alternatives $A \subseteq \mathscr{A}$.

$$\underbrace{\begin{array}{|cc|} \multicolumn{1}{c}{R_1} & \multicolumn{1}{c}{R_2} \\ \hline a & b \\ b & a \\ c & c \\ \hline \end{array}}_{\mathbf{R}} \xrightarrow{f} \{\, a, b \,\}$$

Our goal

Making decisions involves two steps.

1. Establish a constitution: choose a voting rule.
2. Solve a decision problem: apply the voting rule.

Our goal

We focus on step 1: help the committee choose a voting rule.

- Class of functions $\mathscr{F}$ (the set of all voting rules)
- Preference elicitation in order to choose a function $f \in \mathscr{F}$.
- We want to ask *simple* questions: example-based.

A naïve attempt

A first attempt

Simply give a profile $\mathbf{R}$ and ask for $f(\mathbf{R})$. Then iterate.

$$\underbrace{\begin{array}{c} R_1 \quad R_2 \\ \boxed{\begin{array}{cc} a & c \\ b & b \\ c & a \end{array}} \end{array}}_{\mathbf{R}} \xrightarrow{f} \{\,?\,\}$$

- Completely general: all functions in $\mathscr{F}$ can be reached.

But…

- One question brings very little information.
- Questions may be difficult to answer.

General idea

- Ask *good* (informative, example-based) questions.
- Restrict the class of a-priori acceptable functions to $\mathscr{F}' \subset \mathscr{F}$.

Outline

1. Context
2. Asking good questions
3. Restrict the class of functions
4. Which questions to ask?
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A different view of a profile

- We want to ask more informative questions about f.
- We look at profiles under a different angle.
- A *rank-vector* maps voters to ranks, $x : N \to [1, m]$.
- All rank vectors: $[1, m]^N$.

$$\underbrace{\begin{array}{cc} R_1 & R_2 \\ a & a \\ b & c \\ c & b \end{array}}_{\mathbf{R}}
\qquad
\mathscr{A}\ \underbrace{\begin{array}{c|cc|l} & \multicolumn{2}{c}{N} & \\ & 1 & 2 & \\ \hline a & 1 & 1 & \leftarrow \mathbf{x_R}(a) \in [1, m]^N \\ \hline b & 2 & 3 & \leftarrow \mathbf{x_R}(b) \in [1, m]^N \\ \hline c & 3 & 2 & \leftarrow \mathbf{x_R}(c) \in [1, m]^N \\ \hline \end{array}}_{\mathbf{x_R}}$$

To each profile $\mathbf{R}$ corresponds a *rank-profile* $\mathbf{x_R}$.

Representing the preferences of the committee

- We can now ask for the preference status of, e.g., $\boxed{1 \quad 3}$ versus $\boxed{2 \quad 2}$.
- Series of such questions permit to identify a voting rule.

Weak-order based rules

- $\geq$ a weak-order (transitive, reflexive, connected) over $[1, m]^N$.
- Having a profile $\mathbf{R}$, look at the maximal rank-vectors in it according to $\geq$.
- The rule $f_\geq$, at $\mathbf{R}$, selects those alternatives having maximal rank-vectors in $\mathbf{R}$.

A rule f is *weak-order based* if there exists $\geq$ st $f = f_\geq$.

Incomplete question sets

- We do not want to ask every possible questions!
- Can we get away with only *some* answers?

Robust rules

- $\succsim$ a *preorder* (transitive, reflexive) over $[1, m]^N$.
- Look at all weak-orders $\succeq$ extending $\succsim$.
- The *robust* rule $F_{\succsim}$, at $\mathbf{R}$, selects those alternatives winning in *some* $f_{\succeq}$ (for some $\succeq$ extension of $\succsim$).

A rule f is *robust* if there exists $\succsim$ st $f = F_{\succsim}$.

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The WOB class

Not every rule is weak-order based.

- Bad news: we are not fully general any more.
- Good news: we have restricted our class of functions. $WOB = \left\{ f_{\geq}, \geq \text{ a weak-order over } [1, m]^N \right\}$ instead of $\mathscr{F}$.

How does WOB compare to other known classes of rules?

- Every scoring rule (e.g. Borda) is a WOB rule.
- Some WOB rules are not scoring rules. E.g. Bucklin.
- Many Condorcet rules are not WOB rules.

Some relationships between classes of rules

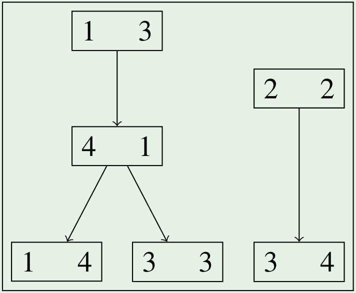
Scoring ⊂ WOB; Condorcet ∩ WOB = ∅ (for $n = 3k, m \geq 4$).

The class of robust rules

Some robust rules are not WOB rules.

Example (A robust rule)

$\succsim =$



	$\mathbf{R}_1$	$F_{\succsim}(\mathbf{R}_1)$	$\mathbf{R}_2$	$F_{\succsim}(\mathbf{R}_2)$
a	1 3	✓	1 4	
b	2 2	✓	2 2	✓
c	3 4		3 3	
d	4 1		4 1	✓

Some relationships between classes of rules

Scoring ⊂ WOB ⊂ Robust.

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Which questions to ask?

- Assume the committee has a weak-order $\succeq$ in mind.
- We want to discover much information using few questions.
- Different questions bring different amount of information.

Elicitation strategy

An elicitation strategy tells us which question should be asked considering our current knowledge.

A strategy:

1. computes the fitness of asking about a pair of rank-vectors, for each pair;
2. chooses the fittest pair.

We ask q questions, then compare our approximation $\boldsymbol{F}_{\succsim}$ to $f_{\succeq}$.

Which strategy?

We tested three strategies.

optimistic fitness of (x, y) proportional to the number of rank-vectors dominated by x or y, but not both;

pessimistic a variant of the previous strategy, using the min operator rather than the sum;

likelihood fitness proportional to the likelihood of a profile occurring where both rank-vectors are possible winners.

We assume pareto-dominance and indifference to permutations.

Comparison of strategies

- Optimistic not better than random!
- Likelihood much better than pessimistic.

Number of questions

How many questions must be asked for a useful approximation?

- Our approximation has all the true winners: $f_{\geq}(\mathbf{R}) \subseteq F_{\succsim}(\mathbf{R})$.
- But it may have supplementary winners.
- We are interested in the ratio of approximated VS true winners: $\frac{|F_{\succsim}(\mathbf{R})|}{|f_{\geq}(\mathbf{R})|}$.
- We average it over all profiles: $\frac{1}{|\mathscr{R}|} \sum_{\mathbf{R} \in \mathscr{R}} \frac{|F_{\succsim}(\mathbf{R})|}{|f_{\geq}(\mathbf{R})|}$.

For 6 voters, 6 alternatives, using the likelihood strategy:

	Target rule	
nb q	Borda	Random $\geq$
0	1.9	2.2
25	1.3	1.7
99	1.0	1.3

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Conclusion

We propose to help a committee choose a voting rule.

- We introduce a different look at a profile (see also Sen, 1977).
- We use it to ask simple questions to elicit preferences.
- We analyse the class of rules reachable by our questioning process.
- A robust voting rule may be defined to give all possible winners (inspired by Dias et al., 2002).
- We compare and analyse several elicitation strategies.

Thank you for your attention!

Bibliography

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