Available online at www.sciencedirect.com

ScienceDirect

International Journal of Approximate Reasoning
45 (2007) 321–340

INTERNATIONAL JOURNAL OF APPROXIMATE REASONING

www.elsevier.com/locate/ijar

An argumentation framework for merging conflicting knowledge bases ☆

Leila Amgoud [a,*], Souhila Kaci [b]

[a] *Institut de Recherche en Informatique de Toulouse (IRIT), 118 route de Narbonne, 31062 Toulouse Cedex 4, France*

[b] *Centre de Recherche en Informatique de Lens (CRIL), Rue de l'Université SP 16, 62307 Lens Cedex, France*

Received 4 January 2006; received in revised form 14 June 2006; accepted 30 June 2006
Available online 14 August 2006

Abstract

The problem of merging multiple sources of information is central in many information processing areas such as databases integrating problems, multiple criteria decision making, etc. To solve this problem, two kinds of approaches have been proposed. The first category of approaches *merges* the different bases into a unique consistent base, and the second category, such as argumentation, accepts inconsistency and copes with it.

It is well known that priorities are crucial to solve conflicts. Recently, powerful approaches have been proposed to merge multiple sources information where priorities are either explicitly or implicitly associated to information [L. Cholvy, Reasoning about merging information, Handbook of Defeasible Reasoning and Uncertainty Management Systems, vol. 3, 1998, pp. 233–263; S. Konieczny, R. Pino Pérez, On the logic of merging, in: Proceedings of the 6th International Conference on Principles of Knowledge Representation and Reasoning (KR'98), Trento, 1998, pp. 488–498; J. Lin, Integration of weighted knowledge bases, Artificial Intelligence 83 (1996) 363–378; J. Lin, A. Mendelzon, Merging databases under constraints, International Journal of Cooperative Information Systems 7(1) (1998) 55–76; N. Rescher, R. Manor, On inference from inconsistent premises, Theory and Decision 1 (1970) 179–219; P.Z. Revesz, On the semantics of theory change: arbitration between old and new information, in: 12th ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Databases, 1993, pp. 71–92; S. Benferhat, D. Dubois, S. Kaci, H. Prade, Possibilistic merging and

☆ This paper is an extended version of a conference paper: L. Amgoud, S. Kaci. An argumentation framework for merging conflicting knowledge bases: the prioritized case. in: Proceedings of the 8th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU'05), 2005, pp. 527–538.

* Corresponding author. Tel.: +33 5 61 55 64 19; fax: +33 5 61 55 62 39.
E-mail address: amgoud@irit.fr (L. Amgoud).

0888-613X/$ - see front matter
doi:10.1016/j.ijar.2006.06.014

distance-based fusion of propositional information, Annals of Mathematics and Artificial Intelligence, 34(1–3) (2002) 217–252; S. Benferhat, D. Dubois, H. Prade, M. Williams, A practical approach to fusing and revising prioritized belief bases, in: Proceedings of the 9th Portuguese Conference on Artificial Intelligence (EPIA'99), 1999, pp. 222–236; S. Kaci, Connaissances et Préférences: Représentation et fusion en logique possibiliste, Thèse de doctorat, Université Paul Sabatier, Toulouse, 2002]. In this paper, we present an argumentation framework for solving conflicts which could be applied to conflicts arising between agents in a multi-agent system. We suppose that each agent is represented by a knowledge base and that the different agents are conflicting. We show that the argumentation framework retrieves the results of the merging approaches. Moreover, an argumentation-based approach palliates the limits, due to the *drowning* problem, of the merging operator when information is pervaded with explicit priorities.

Keywords: Argumentation; Information merging

1. Introduction

In many areas such as cooperative information systems, multi-databases, multi-agents reasoning systems, GroupWare, distributed expert systems, information comes from multiple sources. The multiplicity of sources providing information often makes that information is contradictory. For example, in a distributed medical expert system, different experts often disagree on the diagnosis of patients' diseases. In a multi-database system two component databases may record the same data item but give it different values because of incomplete updates, system error, or differences in underlying semantics.

Two approaches to deal with contradictory information coming from multiple sources are distinguished:

- The first approach consists of *merging* these items of information and constructing a *consistent* set of information which represents the result of merging [6,7,12,17–20,24,26,9]. In other words, starting from different bases $B_1, \ldots, B_n$ which are conflicting, these works return a *unique consistent base*.
- The second approach consists of solving the conflicts without merging the bases. *Argumentation* is one of the most promising of these approaches [15,2,1,10,22]. It is based on the construction of arguments and counter-arguments (defeaters) and the selection of the most acceptable of these arguments. Then inferences are drawn from acceptable arguments.

Besides, the notion of priority plays a crucial role in the study of knowledge-based systems. When priorities attached to pieces of knowledge are available, the task of coping with inconsistency is greatly simplified, since conflicts have a better chance to be resolved. Two kinds of priorities can be distinguished: *implicit* priorities that are extracted from knowledge bases, and *explicit* priorities that are specified outside the logical theory to which they apply.

Priorities have been considered in the two above approaches, and several priority-based *operators* have been proposed for merging multiple sources of information. When information is modelled in propositional logic, existing approaches [18–20,24,26] define implicit priorities based on a *distance*, generally *Hamming's distance* [13]. In [6,7,17], other merging operators have been proposed using explicit priorities. In those works, possibilistic bases are considered where prioritized information are encoded by means of weighted propositional formulas.

The aim of this paper is to establish the relationship between argumentation theory and information merging when priorities are either implicitly or explicitly expressed. Inspired by the work presented in [2], we present a preference-based argumentation framework for reasoning with conflicting knowledge bases where each base could be part of a separate agent. This framework uses preference relations between arguments in order to determine the acceptable ones. We show that by selecting an appropriate preference relation between arguments, the preference-based argumentation framework can be used to merging conflicting bases in the sense that it recovers the results of fusion operators defined in [11,18–20,24,25,6,7].

The remainder of this paper is organized as follows. After presenting the language in the next section, Section 3 recalls the merging process when information is based on implicit or explicit priorities. In Section 4, a general preference-based argumentation framework is presented. Section 5 first recalls the connection between argumentation framework and merging approaches [3] based on implicit priorities presented in Section 3.1. Then it presents the result of the present paper which consists of connecting argumentation framework to merging approaches based on explicit priorities presented in Section 3.2. Section 6 is devoted to concluding remarks.

2. Logical language

Let us consider a propositional language $\mathscr{L}$ over a finite alphabet $\mathscr{P}$ of atoms. Ω denotes the set of all the interpretations. Logical equivalence is denoted by $\equiv$ and classical conjunction and disjunction are respectively denoted by $\wedge$ and $\vee$. $\vdash$ denotes classical inference. The notation $\omega \models \phi$ means that the interpretation ω is a model of (or satisfies) the formula ϕ. Mod(K) denotes the set of models of a propositional formulas base K.

A preference relation on a set $\mathscr{M} \subseteq \Omega$ is a (total or partial) preorder such that $\forall \omega, \omega' \in \mathscr{M}$, $\omega \succeq \omega'$ stands for ω is at least as preferred as ω'. $\succ$ denotes the strict order associated to $\succeq$. Preferred (called also minimal) elements of $\mathscr{M}$ w.r.t. $\succeq$, denoted $\min(\mathscr{M}, \succeq)$, are those which are not dominated by any other element of $\mathscr{M}$. Formally, we write

$$\min(\mathscr{M}, \succeq) = \{\omega : \omega \in \mathscr{M} \text{ and } \nexists \omega' \in \mathscr{M} \text{ s.t. } \omega' \succ \omega\}.$$

3. Merging multiple sources information

We present in this section some merging operators defined on the basis of priorities. As said before, two kinds of priorities can be distinguished: *implicit* priorities which are extracted from a knowledge base, and *explicit* priorities which are given in terms of weights associated to each piece of information in a knowledge base, as it is the case with possibilistic logic bases, or given in terms of a total or partial pre-order on a knowledge base.

3.1. Merging propositional information: use of implicit priorities

Let $E = \{K_1, \ldots, K_n\}$ be a set of n propositional bases to be merged. Merge(E) will denote the result of merging the bases of E. In [18–20,26,27] implicit priorities are assumed. These last rely on a *distance* between interpretations and the bases to be merged. The three basic steps followed for defining this distance-based merging are:

(1) Rank-order the set of interpretations Ω w.r.t each propositional base K_i by computing a local distance, denoted $d(\omega, K_i)$, between ω and each K_i in E. The local distance is based on Hamming's distance [13]. The distance between an interpretation ω and a propositional base K_i is the number of atoms on which this interpretation differs from some model of the propositional base. Formally, $d(\omega, K_i) = \min\{\text{dist}(\omega, \omega') \mid \omega' \in \text{Mod}(K_i)\}$ where $\text{dist}(\omega, \omega')$ is the number of atoms whose valuations differ in the two interpretations.

Example 1. Let us consider the three following bases: $K_1 = \{a\}$, $K_2 = \{a \to b\}$ and $K_3 = \{\neg b\}$. $\Omega = \{\omega_0, \omega_1, \omega_2, \omega_3\}$ where $\omega_0 = \neg a \neg b$, $\omega_1 = \neg ab$, $\omega_2 = a \neg b$ and $\omega_3 = ab$. Table 1 gives local distances between the interpretations and the bases.

(2) Rank-order the set of interpretations Ω w.r.t all the propositional bases. This leads to the overall distance obtained from the aggregation of local distances using a merging operator denoted Δ. The resulting distance is denoted $d_\Delta(\omega, E)$. On the basis of the global distance, an ordering relation $\succeq_\Delta$ between the interpretations is defined as follows:

$$\omega \succeq_\Delta \omega' \quad \text{iff } d_\Delta(\omega, E) \leqslant d_\Delta(\omega', E).$$

Several methods have been proposed in order to aggregate the local distances $d(w, K_i)$ according to whether the bases have the same weight or not. In particular the following operators have been proposed:

- The *sum* operator [20], denoted $\mathscr{SUM}$, defined by

$$d_{\mathscr{SUM}}(\omega, E) = \sum_{i=1}^{n} d(\omega, K_i).$$

This operator follows the point of view of the majority of bases [20].

- The weighted sum operator [19], denoted $\mathscr{WS}$, defined by

$$d_{\mathscr{WS}}(\omega, E) = \sum_{i=1}^{n} d(\omega, K_i) \times \alpha_i,$$

where α_i is a positive integer representing the weight associated with the base K_i.

- The max operator [26,27], denoted $\mathscr{MAX}$, defined by

$$d_{\mathscr{MAX}}(\omega, E) = \max\{d(\omega, K_i) \mid i = 1, \ldots, n\}.$$

Table 1
Local distances

ω	$d(\omega, K_1)$	$d(\omega, K_2)$	$d(\omega, K_3)$
ω_0	1	0	0
ω_1	1	0	1
ω_2	0	1	0
ω_3	0	0	1

This operator tries to satisfy all the bases [26,27].

Example 2 (*continued*). Table 2 gives the global distances w.r.t. the merging operators given above. Let $\alpha_1 = \alpha_3 = 1$ and $\alpha_2 = 3$ be the weights associated to the bases for $\mathscr{WS}$ operator.

(3) Lastly the result of merging $\text{Merge}_\Delta(E)$ is defined by being such that its models are minimal with respect to $\succeq_\Delta$, namely

$$\text{Mod}(\text{Merge}_\Delta(E)) = \min(\Omega, \succeq_\Delta).$$

Example 3 (*continued*). Minimal models are

(1) $\text{Mod}(\text{Merge}_{\mathscr{SUM}}(E)) = \{\omega_0, \omega_2, \omega_3\}$,
(2) $\text{Mod}(\text{Merge}_{\mathscr{WS}}(E)) = \{\omega_0, \omega_3\}$,
(3) $\text{Mod}(\text{Merge}_{\mathscr{MAX}}(E)) = \top$.

3.2. Merging prioritized information in possibilistic logic

Before presenting merging approaches when explicit priorities are used, let us give necessary background on possibilistic logic, an appropriate logic for modeling such priorities.

Prioritized information is represented in possibilistic logic at both semantic and syntactic levels. At the semantic level, possibilistic logic is based on the notion of a possibility distribution [28], denoted by π, which is a mapping from Ω to $[0, 1]$ representing the available information. $\pi(\omega)$ represents the degree of compatibility of the interpretation ω with the available beliefs about the real world if we are representing uncertain pieces of knowledge (or the degree of satisfaction of reaching a state ω if we are modeling preferences). By convention, $\pi(\omega) = 1$ means that it is totally possible for ω to be the real world (or that ω is fully satisfactory), $1 > \pi(\omega) > 0$ means that ω is only somewhat possible (or satisfactory), while $\pi(\omega) = 0$ means that ω is certainly not the real world (or not satisfactory at all). Associated with a possibility distribution π is the necessity degree of any formula $\phi : N(\phi) = 1 - \Pi(\neg\phi)$ which evaluates to what extent ϕ is entailed by the available beliefs, and defined from the consistency degree of a formula ϕ w.r.t. the available information, $\Pi(\phi) = \max\{\pi(\omega) \mid \omega \models \Omega \text{ and } \omega \models \phi\}$.

Note that the mapping N reverses the scale on which π is ranging, and that $N(\phi) = 1$ means that ϕ is a totally certain piece of knowledge or a compulsory goal, while $N(\phi) = 0$ expresses the complete lack of knowledge or of priority about ϕ, but does not mean that

Table 2
Global distances

ω	$d_{\mathscr{SUM}}(\omega, E)$	$d_{\mathscr{WS}}(\omega, E)$	$d_{\mathscr{MAX}}(\omega, E)$
ω_0	1	1	1
ω_1	2	2	1
ω_2	1	3	1
ω_3	1	1	1

ϕ is or should be false. Moreover, the duality equation $N(\phi) = 1 - \Pi(\neg\phi)$ extends the existing one in classical logic, where a formula is entailed from a set of classical formulas if and only if its negation is consistent with this set.

At the syntactic level, prioritized items of information are represented by means of *a possibilistic knowledge base* (or *a possibilistic base* for short) which is a set of weighted formulas of the form $B = \{(\phi_i, a_i) \mid i = 1, \ldots, n\}$, where ϕ_i is a propositional formula and a_i belongs to a totally ordered scale such as the unit interval $[0, 1]$. The pair (ϕ_i, a_i) means that the certainty (or priority) degree of ϕ_i is at least equal to $a_i(N(\phi_i) \geqslant a_i)$. We denote by B^* the propositional base associated with B obtained from B by forgetting the weights of formulas. A possibilistic base B is consistent if and only if its associated propositional base B^* is consistent.

Given a possibilistic base B, we can generate a unique possibility distribution, denoted by π_B, such that all the interpretations satisfying all the formulas in B will have the highest possibility degree, namely 1, and the other interpretations will be ranked w.r.t. the highest formula that they falsify, namely we get [14].

Definition 1. $\forall \omega \in \Omega$,

$$\pi_B(\omega) = \begin{cases} 1 & \text{if } \forall (\phi_i, a_i) \in B, \;\; \omega \models \phi_i, \\ 1 - \max\{a_i | (\phi_i, a_i) \in B \text{ and } \omega \not\models \phi_i\} & \text{otherwise.} \end{cases}$$

Example 4. Let $B = \{(\neg p \vee \neg q, .7); (p, .6)\}$ be a knowledge base. Its associated possibility distribution is: $\pi_B(p\neg q) = 1$; $\pi_B(\neg p \neg q) = \pi_B(\neg pq) = .4$ and $\pi_B(pq) = .3$.

The interpretation $p\neg q$ is the most preferred since it satisfies all the formulas in B. The interpretations $\neg p \neg q$ and $\neg pq$ are more preferred than pq since the highest formula falsified by $\neg p \neg q$ and $\neg pq$ (i.e., $(p,.6)$) is less certain (or less prioritized) than the highest formula falsified by pq (i.e., $(\neg p \vee \neg q, .7)$).

In the following, we give some definitions useful for the rest of the paper [7]:

Definition 2 (*Equivalence*). Let B_1 and B_2 be two possibilistic bases. B_1 and B_2 are said to be *equivalent* iff $\pi_{B_1} = \pi_{B_2}$.

Definition 3 (*a-cut and strict a-cut*). Let B be a possibilistic knowledge base, and $\mathbf{a} \in [0, 1]$. We call the **a**-cut (resp. strict **a**-cut) of B, denoted by $B_{\geqslant \mathbf{a}}$ (resp. $B_{> \mathbf{a}}$), the set of propositional formulas in B having a certainty degree at least equal to **a** (resp. strictly greater than **a**).

Definition 4 (*Inconsistency degree*). The *inconsistency degree* of a possibilistic base B is

$$\mathrm{Inc}(B) = \max\{a_i | B_{\geqslant a_i} \text{ is inconsistent}\}$$

with $\mathrm{Inc}(B) = 0$ when B^* is consistent.

Definition 5 (*Subsumption*). Let (ϕ, a) be a formula in B. (ϕ, a) is said to be *subsumed* in B if

$$(B - \{(\phi, a)\})_{\geqslant a} \vdash \phi$$

and (ϕ, a) is said to be strictly subsumed in B if $B_{>a} \vdash \phi$.

Subsumed formulas are in some sense redundant formulas as it is shown by the following lemma [7]:

Lemma 1. *Let (ϕ, a) be a subsumed formula in B. Then B and $B' = B - \{(\phi, a)\}$ are equivalent.*

Lastly, weights are propagated out in the inference process in the following way:

Definition 6 (*Plausible inference*). Let B be a possibilistic base. The formula ϕ is a *plausible consequence* of B iff

$$B_{>\mathrm{Inc}(B)} \vdash \phi.$$

Definition 7 (*Possibilistic inference*). Let B be a possibilistic base. The formula (ϕ, a) is a *possibilistic consequence* of B, denoted $B \vdash_\pi (\phi, a)$, iff

- $B_{>\mathrm{Inc}(B)} \vdash \phi$,
- $a > \mathrm{Inc}(B)$ and $\forall b > a$, $B_{>b} \nvdash \phi$.

Now that we have given necessary background on possibilistic logic, we recall the merging process of information provided with explicit priorities encoded in that framework. It is a two step process

(1) From a set of possibilistic bases,[1] computing a new possibilistic base, called the *aggregated base*, which is generally inconsistent [7].
(2) Inferring conclusions from the new base.

A possibilistic merging operator, denoted by $\oplus$, is a function from $[0,1]^n$ to $[0,1]$. $\oplus$ is used to aggregate the certainty degrees associated with pieces of information provided by different sources. Formally, let $\mathscr{B} = \{B_1, \ldots, B_n\}$ be a set of n (possibly inconsistent) possibilistic bases. The result of merging the bases of $\mathscr{B}$ using $\oplus$, denoted by $\mathscr{B}_\oplus$, is defined as follows [6]:

Definition 8 (*Aggregated base*). Let $\mathscr{B} = \{B_1, \ldots, B_n\}$ be a set of possibilistic bases and $\oplus$ a merging operator. The result of merging $\mathscr{B}$ with $\oplus$ is defined by

$$\mathscr{B}_\oplus = \{(D_j, \oplus(x_1, \ldots, x_n)) \mid j = 1, \ldots, n\},$$

where D_j are disjunctions of size j among formulas taken from different B_i's $(i = 1, \ldots, n)$ and x_i is either equal to a_i or to 0 depending respectively on whether ϕ_i belongs to D_j or not.

Two properties for $\oplus$ are assumed in this definition [8,7]

[1] These bases may be individually inconsistent.

(1) $\oplus(0,\ldots,0) = 0$,
(2) If $a_i \geqslant b_i$ for all $i = 1,\ldots,n$ then $\oplus(a_1,\ldots,a_n) \geqslant \oplus(b_1,\ldots,b_n)$.

The first property says that if a formula does not explicitly appear in any base, then it should not appear explicitly in the result of merging. The second property is simply the unanimity property (called also monotonicity property) which means that if all the sources say that a formula ϕ is more plausible than (or preferred to) another formula ψ, then the result of merging should confirm this preference.

Example 5. Let $B_1 = \{(\phi \vee \psi, .9), (\neg\phi, .8), (\xi, .1)\}$ and $B_2 = \{(\neg\psi, .7), (\phi, .6)\}$. Let $\oplus$ be the probabilistic sum defined by $\oplus(a,b) = a + b - a * b$. Following Definition 8, we get: $\mathscr{B}_\oplus = \{(\phi \vee \psi, .9), (\neg\phi, .8), (\xi, .1)\} \cup \{(\neg\psi, .7), (\phi, .6)\} \cup \{(\phi \vee \psi, .96), (\neg\phi \vee \neg\psi, .94), (\xi \vee \neg\psi, .73), (\xi \vee \phi, .64)\}$ which is equivalent to $\{(\phi \vee \psi, .96), (\neg\phi \vee \neg\psi, .94), (\neg\phi, .8), (\xi \vee \neg\psi, .73), (\neg\psi, .7), (\xi \vee \phi, .64), (\phi, .6), (\xi, .1)\}$.

Lemma 2 gives a rewriting of $\mathscr{B}_\oplus$ given in Definition 8 which will be useful in the rest of the paper, but first let us give the following definition:

Definition 9 (*Existential consequence*). Let B be a possibilistic base. The formula (ϕ, a) is an *existential consequence* of B, denoted by $B \Vdash (\phi, a)$, iff

- $\exists B' \subseteq B$ s.t. $B' \vdash_\pi (\phi, a)$,
- B' is consistent,
- $a = \min\{a_i | (\phi_i, a_i) \in B'\}$,
- B' is a minimal for set inclusion,
- $\nexists B'' \subseteq B$ satisfying the above conditions with $B'' \vdash_\pi (\phi, b)$ and $b > a$.

This definition focuses on the subbases containing the most prioritized formulas.

Example 6. Let $B = \{(\phi \vee \psi, .9), (\neg\phi, .7), (\xi \vee \psi, .6), (\neg\xi, .5)\}$. Then $B \Vdash (\phi \vee \psi, .9)$, $B \Vdash (\neg\phi, .7)$ and $B \Vdash (\psi, .7)$ however $B \Vdash (\neg\psi, 0)$.

Lemma 2. *Let $\mathscr{B}_\oplus$ be the result of merging $B_1,\ldots,B_n$ with $\oplus$. Then, $\mathscr{B}_\oplus$ is equivalent to*

$$\{(\phi, \oplus(a_1,\ldots,a_n)) \mid \phi \in \mathscr{L} \text{ and } B_i \Vdash (\phi, a_i)\}.$$

Now that the base $\mathscr{B}_\oplus$ is defined, we are ready to define the result of merging. This corresponds to the possibilistic consequences of $\mathscr{B}_\oplus$. Formally:

Definition 10 (*Result of merging*). Let $\mathscr{B}_\oplus$ be the result of merging n possibilistic bases $B_1,\ldots,B_n$ using a possibilistic merging operator $\oplus$. The *result of merging* is

$$\mathscr{T} = \{(\phi_i, a_i) \mid \mathscr{B}_\oplus \vdash_\pi (\phi_i, a_i)\}.$$

Example 7. Let us consider again the base $\mathscr{B}_\oplus$ obtained in Example 5. We have $\mathscr{B}_\oplus = \{\phi \vee \psi, .96), (\neg\phi \vee \neg\psi, .94), (\neg\phi, .8), (\xi \vee \neg\psi, .73), (\neg\psi, .7), (\xi \vee \phi, .64), (\phi, .6), (\xi, .1)\}$. Then $\mathscr{T}$ is equivalent to $\{(\phi \vee \psi, .96), (\neg\phi \vee \neg\psi, .94), (\neg\phi, .8), (\psi, .8), (\xi, .73)\}$. Here $\mathscr{T}$ is the minimal result of merging; we did not give subsumed formulas, for e.g. $(\neg\phi \vee \psi, a)$ with $a \leqslant .8$.

4. Basic argumentation framework

Argumentation is a reasoning model based on the construction and the comparison of arguments. Argumentation frameworks have been developed for decision making under uncertainty [4], and others [1,21] for handling inconsistency in knowledge bases where each conclusion is justified by arguments. Arguments represent the reasons to believe in a fact. An argumentation process follows the five following steps:

(1) Constructing *arguments* (in *favor* of/*against* a "statement") from bases.
(2) Defining the *strengths* of those arguments.
(3) Determining the different *conflicts* between the arguments.
(4) Evaluating the *acceptability* of the different arguments.
(5) Concluding or defining the *justified conclusions*.

Indeed, argumentation systems are built around an underlying logical language $\mathscr{L}$ and an associated notion of logical consequence, defining the notion of argument. The argument construction is a monotonic process: new knowledge cannot rule out an argument but only gives rise to new arguments which may interact with the first argument. Since the knowledge bases may be inconsistent, the arguments may be conflicting too. Consequently, it is important to determine among all the available arguments, the ones which will be *justified*. In what follows, we present the *general* argumentation framework proposed in [2] which is an extension of the famous framework presented by Dung in [15].

Definition 11 (*Argumentation framework*). An *argumentation framework* (AF) is a triple $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$. $\mathscr{A}$ is a set of arguments, $\mathscr{R}$ is a binary relation representing defeat relationship between arguments. $\succeq$ is a (partial or complete) pre-order on $\mathscr{A} \times \mathscr{A}$. $\succ$ denotes the strict ordering associated with $\succeq$.

Note that different definitions of $\mathscr{A}$, $\mathscr{R}$ and $\succeq$ give birth to different argumentation systems.

In the above definition, an argument is an *abstract* entity whose structure and origin are not known. Its role is only determined by its relation to other arguments via the defeat relation.

The preference order between arguments makes it possible to distinguish different types of relations between arguments:

Definition 12. Let A and B be two arguments of $\mathscr{A}$.

- B attacks A iff $B \mathscr{R} A$ and it is not the case that $A \succ B$.
- If $B \mathscr{R} A$, then A defends itself against B iff $A \succ B$.
- A set of arguments $\mathscr{S}$ defends A if there is some argument in $\mathscr{S}$ which attacks every argument B where B attacks A.

Since arguments are conflicting, it is important to define the acceptable ones (i.e. the "good" ones). Inspired by Dung's work [15], several different semantics for the notion of acceptability have been proposed in [2]. In what follows, we are interested in two kinds of extensions: *grounded extension* and *stable extensions*. These two notions are based on a coherence requirement defined as follows:

Definition 13 (*Conflict-free*). Let $\mathscr{A}$ be a set of arguments and $S \subseteq \mathscr{A}$. S is conflict-free iff there does not exist $A, B \in S$ such that $A \mathscr{R} B$ and not$(B \succ A)$.

The grounded extension is composed of arguments which are not defeated, arguments which are defeated but preferred to their defeaters and lastly arguments which are defeated but defended by acceptable arguments.

Definition 14 (*Grounded extension*). Let S be a conflict-free set of arguments, and let $\mathscr{F} : 2^{\mathscr{A}} \mapsto 2^{\mathscr{A}}$ be a function such that $\mathscr{F}(S) = \{A \mid S \text{ defends } A\}$.

The *grounded extension* of an argumentation framework $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ is

$$\underline{\mathscr{S}} = \bigcup \mathscr{F}_{i \geqslant 0}(\emptyset) = C_{\mathscr{R},\succeq} \cup \left[\bigcup \mathscr{F}_{i \geqslant 1}(C_{\mathscr{R},\succeq}) \right].$$

$C_{\mathscr{R},\succeq}$ gathers all non-defeated arguments and arguments defending themselves against all their defeaters.

Definition 15 (*Stable extension*). Let $\langle \mathscr{A}, \mathscr{R}, \succeq \rangle$ be an (AF). A conflict-free set of arguments S is a *stable extension* iff S is a fixed point of a function $\mathscr{G} : 2^{\mathscr{A}} \times 2^{\mathscr{A}}$ such that $\mathscr{G}(S) = \{A \in \mathscr{A} \mid \nexists B \in S \text{ such that } B \mathscr{R} A \text{ and not}(A \succ B)\}$.

Let $\mathscr{S}\mathscr{E} = \{S_1, \ldots, S_n\}$ be the set of stable extensions of AF.

Note that an argumentation framework has at most one grounded extension, whereas it may have several stable extensions.

5. Relating information merging with argumentation

Our aim in this section is to highlight the relationship between the two approaches to solve conflicts described in the previous sections, namely merging multiple sources information (with implicit or explicit priorities) and argumentation framework.

It has been shown in [3] that when information is modelled in propositional logic and implicit priorities are assumed, merging approaches [18–20] are recovered in standard argumentation framework. We show in this paper that a *particular* argumentation framework is needed to recover merging approaches when information is pervaded with explicit priorities [6,7,17].

In order to recover the results of the different merging operators within an argumentation framework, one needs to specify the basic argumentation framework presented in Section 4, in particular one needs to give the definitions of an argument, of the defeasibility relation between arguments, and finally of the preference relation between arguments.

There are several definitions of *argument* and *defeat* among arguments. For our purpose, we will use the definitions proposed in [16]. Indeed, these definitions will be used for capturing the results of the different merging operators defined in Section 3. However, things are different with the third parameter of an argumentation framework, namely the preference relation between arguments. We will show that a specific relation is needed for recovering each merging operator.

Let K be a propositional knowledge base. From K different arguments may be constructed. In what follows, we will denote by $\mathscr{A}(K)$ the set of all arguments that can be built from a given base K as follows.

Definition 16 (*Argument*). An *argument* is a pair $\langle H, h\rangle$ where

(1) h is a formula of the language $\mathscr{L}$,
(2) $H \subseteq K$,
(3) H is consistent,
(4) $H \vdash h$,
(5) H is minimal (no strict subset of H satisfies 1, 2, 3, 4).

H is called the *support* and h the *conclusion* of the argument.

Let Σ be a set of arguments. Supp(Σ) is a function which returns the union of the supports of all the elements of Σ.

The defeat relation which will be used throughout the paper is the following:

Definition 17 (*Attack*). Let $\langle H, h\rangle$ and $\langle H', h'\rangle$ be two arguments of $\mathscr{A}(K)$. $\langle H, h\rangle$ undercuts $\langle H', h'\rangle$ iff for some $k \in H'$, $h \equiv \neg k$. An argument is undercut if there exists at least one argument against one element of its support.

5.1. The flat case

We recall in this section how to capture the results of merging approaches described in Section 3.1, proposed in [3]. For this purpose, an argument $\langle H, h\rangle$ takes its support from $K_1 \cup \cdots \cup K_n$ i.e., $H \subseteq K_1 \cup \cdots \cup K_n$. Recall that $E = \{K_1, \ldots, K_n\}$ is the set of bases to be merged with a merging operator Δ. We say that $\langle H, h\rangle$ is constructed from E.

Then the basic idea is to associate to the support of each argument a *force*. This last corresponds to the minimal distance between the support of the argument and the different bases K_i. The following defines formally the distance between a support and a base.

Definition 18 (*Distance Support-Base*). Let $\langle H, h\rangle$ be an argument and K be a propositional base. The distance between the support H and K is computed as follows:

$$\delta(H, K) = \min\{\mathrm{dist}(\omega, \omega') \mid \omega \models H \text{ and } \omega' \models K\}.$$

Example 8. Let us consider again the bases $K_1 = \{a\}$, $K_2 = \{a \to b\}$ and $K_3 = \{\neg b\}$ given in Example 1. $H = \{a, a \to b\}$, $H' = \{\neg b\}$ are two subsets of $K_1 \cup K_2 \cup K_3$.

- $\delta(H, K_1) = \delta(H, K_2) = 0$, $\delta(H, K_3) = 1$,
- $\delta(H', K_1) = 0$, $\delta(H', K_2) = 0$, $\delta(H', K_3) = 0$.

To capture the results of the distance-based merging operator Δ, we define the *force* of a support as follows:

Definition 19. Let $E = \{K_1, \ldots, K_n\}$ and $\langle H, h\rangle$ be an argument constructed from E.

$$\mathrm{Force}(H) = \Delta(\delta(H, K_1), \ldots, \delta(H, K_n)).$$

Indeed the force of a support corresponds in some sense to the global distance. The force of a support makes it possible to define a preference relation between arguments.

Definition 20 (*Preference relation*). Let $\langle H,h\rangle$ and $\langle H',h'\rangle$ be two arguments constructed from E. $\langle H,h\rangle$ is preferred to $\langle H',h'\rangle$, denoted $\langle H,h\rangle \succ_\Delta \langle H',h'\rangle$ iff Force(H) < Force(H').

In the following, $\mathscr{A}(E)$ will denote the set of arguments constructed from E.

Proposition 1. *Let $S_1,\ldots,S_n$ be the stable extensions of the argumentation framework $\langle \mathscr{A}(E), Undercut, \succeq_\Delta\rangle$. Then,* $\mathrm{Mod}(\mathrm{Supp}(S_1)) \cup \cdots \cup \mathrm{Mod}(\mathrm{Supp}(S_n))$ *is the set of models obtained by the merging operator Δ.*

Example 9 (*continued*). Let us consider the framework $\langle \mathscr{A}(E), Undercut, \succeq_\Delta\rangle$ where: $\mathscr{A}(E) = \{A_1 = \langle\{a\},a\rangle, A_2 = \langle\{a\to b\}, a\to b\rangle, A_3 = \langle\{\neg b\},\neg b\rangle, A_4 = \langle\{a, a\to b\}, b\rangle, A_5 = \langle\{\neg b, a\to b\},\neg a\rangle, A_6 = \langle\{a,\neg b\},\neg(a\to b)\rangle\}$.

Undercut $= \{(A_4,A_3),(A_4,\ A_5),(A_4,A_6),(A_5,A_4),(A_5,A_1),(A_5,A_6),(A_6,A_5),(A_6,A_4),(A_6,A_2)\}$. Table 3 gives the distance between each argument and the bases K_1, K_2, K_3 and also the force of each argument following different merging operators.

Let us consider the $\mathscr{SUM}$ operator. Three stable extensions can be computed: $S_1 = \{A_2,A_3,A_5\}$, $S_2 = \{A_1,A_2,A_4\}$ and $S_3 = \{A_1,A_3,A_6\}$.

We have

- $\mathrm{Mod}(\mathrm{Supp}(S_1)) = \mathrm{Mod}(\{\neg b, a\to b\}) = \{\neg a,\neg b\} = \{\omega_3\}$,
- $\mathrm{Mod}(\mathrm{Supp}(S_2)) = \mathrm{Mod}(\{a, a\to b\}) = \{a,b\} = \{\omega_0\}$,
- $\mathrm{Mod}(\mathrm{Supp}(S_3)) = \mathrm{Mod}(\{a,\neg b\}) = \{a,\neg b\} = \{\omega_1\}$.

This corresponds to the result of distance-based merging where we get $\mathrm{Mod}(\mathrm{Merge}_{\mathscr{SUM}}(E)) = \{\omega_0,\omega_1,\omega_3\}$.

5.2. The prioritized case

Our aim in this section is to show that argumentation framework can also recover merging approaches when information is pervaded with explicit priorities encoded in possibilistic logic framework.

Table 3
Distance and force of the arguments

Argument	$\delta(H,K_1)$	$\delta(H,K_2)$	$\delta(H,K_3)$	$\mathrm{Force}_{\mathscr{SUM}}(H)$	$\mathrm{Force}_{\mathscr{WS}}(H)$	$\mathrm{Force}_{\mathscr{MAX}}(H)$
A_1	0	0	0	0	0	0
A_2	0	0	0	0	0	0
A_3	0	0	0	0	0	0
A_4	0	0	1	1	1	1
A_5	1	0	0	1	1	1
A_6	0	1	0	1	3	1

Let us first recall some concepts. Let $B_1, \ldots, B_n$ be different possibilistic bases. *Disj* will denote the set of all disjunctions of different size that can be formed from formulas of the n bases. *Conj* will denote the set of formulas of $B_1, \ldots, B_n$ with possibly new weights. Weights of formulas in *Disj* and *Conj* are aggregated using an operator $\otimes$. For instance, if the formula (ϕ, a) is in B_1 and (ψ, b) is in B_2, then the formula $(\phi \vee \psi, \otimes(a, b))$ will be in *Disj* and the formulas $(\phi, \otimes(a, 0))$ and $(\psi, \otimes(0, b))$ will be in *Conj*, with $\otimes(x, y)$ is for example $\max(x, y)$ or $\min(x, y)$, etc. In what follows, $\mathscr{B} = Conj \cup Disj$. In fact, it can be shown that if the aggregation operator $\otimes$ is exactly the operator $\oplus$, then the two bases $\mathscr{B}$ and $\mathscr{B}_{\oplus}$ are equivalent.

Proposition 2. *Let $B_1, \ldots, B_n$ be different possibilistic bases. If $\otimes = \oplus$, then the bases $\mathscr{B}$ and $\mathscr{B}_{\oplus}$ are equivalent.*

All the proofs are given in Appendix A.

Let us now start by defining the notion of argument. An argument has a deductive form and takes the form of an explanation. Each argument is constructed from formulas of $B_1, \ldots, B_n$ and disjunctions between formulas of these bases.

An argument in this subsection takes its support from $\mathscr{B}^*$ i.e., let $\langle H, h\rangle$ be an argument constructed from $\mathscr{B}$ then $H \subseteq \mathscr{B}^*$. Note that it is not necessary to construct the bases *Disj* and *Conj* in order to define the arguments. Fragments of these bases are constructed only when needed i.e., when building arguments.

When explicit priorities are given between the beliefs, such as certainty degrees, a preference relation between arguments may be defined such that the arguments using more certain beliefs are found stronger than arguments using less certain beliefs. The force of an argument corresponds to the *certainty degree* of the less entrenched belief involved in the argument.

Definition 21 (*Force of an argument*). Let $A = \langle H, h\rangle$ be an argument. The *force* of A, denoted by Force(A), is

$$\text{Force}(A) = \min\{a_i \mid \phi_i \in H \text{ and } (\phi_i, a_i) \in \mathscr{B}\}.$$

The following proposition shows that the force of an argument can be computed from $\mathscr{B}$ without computing explicitly the base *Disj*.

Proposition 3. *Let $B_1, \ldots, B_n$ be n possibilistic bases. Let $A = \langle H, \phi\rangle$ be an argument in $\mathscr{A}(\mathscr{B})$. It holds that*

$$Force(A) = \min\{\oplus(a_{j1}, \ldots, a_{jn}) \mid \phi_j \in H, B_i \Vdash (\phi_j, a_{ji})\}.$$

Example 10. Let us compute an argument for $\phi \vee \psi$ from $\mathscr{B}_{\oplus}$. We get $A_1 = \langle\{\phi \vee \psi\}, \phi \vee \psi\rangle$ and $A_2 = \langle\{\phi\}, s\phi \vee \psi\rangle$.

A_1 is stronger than A_2 since Force(A_1) = .96 whereas Force(A_2) = .6.
Now $B_1 \Vdash (\phi \vee \psi, .9)$ and $B_2 \Vdash (\phi \vee \psi, .6)$. Then, Force($A_1$) = min{ $\oplus(.9, .6)$} = .96.

Similarly to the flat case, the forces of an argument makes it possible to compare pairs of arguments as follows:

Definition 22 (*Preference relation*). Let A and A' be two arguments in $\mathscr{A}(\mathscr{B})$. A is *preferred* to A', denoted by $A \succ_{\oplus} A'$, iff $\text{Force}(A) > \text{Force}(A')$.

Example 11. Let us consider again the possibilistic base given in Example 6: $B = \{(\phi \vee \psi, .9), (\neg\phi, .7), (\xi \vee \psi, .6), (\neg\xi, .5)\}$. There are two arguments in favor of ψ

- $A_1 = \langle\{\phi \vee \psi, \neg\phi\}, \psi\rangle$,
- $A_2 = \langle\{\xi \vee \psi, \neg\xi\}, \psi\rangle$.

A_1 is preferred to A_2 since $\text{Force}(A_1) = .7$ whereas $\text{Force}(A_2) = .5$.

We can show easily that any plausible consequence of a given possibilistic base B is supported by an acceptable argument, if we consider only the arguments $\mathscr{A}(B)$ built from that base B.

Proposition 4. *Let B be a possibilistic base, and let $\langle\mathscr{A}(B), Undercut, \succeq_{\oplus}\rangle$ be an argumentation framework and $\underline{\mathscr{S}}$ its set of acceptable arguments.*
If (ϕ, a) is a plausible consequence of B, then $\exists A = \langle H, \phi\rangle \in \underline{\mathscr{S}}$.

Another interesting result states that any possibilistic consequence (ϕ, a) of a given possibilistic base B_i is supported by an acceptable argument A whose force is equal to a. Moreover, A is the strongest argument w.r.t $\succ$ in favor of ϕ. This means that the degree a of a possibilistic consequence ϕ corresponds to the force of the best argument in favor of ϕ.

Proposition 5. *Let B be a possibilistic base, and let $\langle\mathscr{A}(B), Undercut, \succeq_{\oplus}\rangle$ be an argumentation framework and $\underline{\mathscr{S}}$ its set of acceptable arguments.*
If (ϕ, a) is a possibilistic consequence of B, then $\exists A = \langle H, \phi\rangle \in \underline{\mathscr{S}}$ with Force(A) = a, *and $\forall A' = \langle H', \phi\rangle \in \underline{\mathscr{S}}, A \succeq_{\oplus} A'$.*

An important concept in possibilistic logic is that of *inconsistency degree* of a possibilistic base B. In what follows, we will show that the inconsistency degree can be computed from the forces of the conflicting arguments as follows:

Proposition 6. *Let B be a possibilistic base, and let $\langle\mathscr{A}(B), Undercut, \succeq_{\oplus}\rangle$ be an argumentation framework.*

$$\text{Inc}(B) = \max\{\min(\text{Force}(A_i), \text{Force}(A_j)) | A_i, A_j \in \mathscr{A}(B) \text{ and } A_i \textit{ Undercuts } A_j\}.$$

Example 12. Let us consider the base $\mathscr{B}_{\oplus}$ constructed in Example 5: $\mathscr{B}_{\oplus} = \{(\phi \vee \psi, .96), (\neg\phi \vee \neg\psi, .94), (\neg\phi, .8), (\xi \vee \neg\psi, .73), (\neg\psi, .7), (\xi \vee \phi, .64), (\phi, .6), (\xi, .1)\}$.

Table 4 summarizes the different arguments which can be constructed from $\mathscr{B}_{\oplus}$ and their force. As we mentioned before, we only focus on the best arguments (i.e., having the highest force) in favor of formulas. For example, there is an argument $A = \langle\{\phi\}, \phi \vee \psi\rangle$, with a force equal to .6, in favor of $\phi \vee \psi$ however it is not considered since there is another argument A_1 in favor of $\phi \vee \psi$ with a higher force. We have $Undercut = \{(A_6, A_3), (A_6, A_4), (A_7, A_5), (A_7, A_6), (A_6, A_7)\}$.

Table 4
The force of arguments in possibilistic logic framework

Argument	Force
$A_1 = \langle\{\phi \vee \psi\}, \phi \vee \psi\rangle$	.96
$A_2 = \langle\{\neg\phi \vee \neg\psi\}, \neg\phi \vee \neg\psi\rangle$	.94
$A_3 = \langle\{\neg\phi\}, \neg\phi\rangle$	.8
$A_4 = \langle\{\xi \vee \neg\psi, \neg\phi, \phi \vee \psi\}, \xi\rangle$	.73
$A_5 = \langle\{\neg\psi\}, \neg\psi\rangle$	.7
$A_6 = \langle\{\phi \vee \psi, \neg\psi\}, \phi\rangle$	.7
$A_7 = \langle\{\neg\phi, \phi \vee \psi\}, \psi\rangle$	.8

Then, $\max\{\min(.7,.8), \min(.7,.73), \min(.8,.7), \min(.8,.7), \min(.7,.8)\} = .7$. It can be checked that the inconsistency degree of $\mathscr{B}_\oplus$ is .7.

Indeed we have the following result:

Proposition 7. *Let B be a possibilistic base.*

(1) *A formula ϕ is a plausible consequence of B iff $\exists A = \langle H, \phi\rangle$ in $\mathscr{A}(B)$ s.t.* $\text{Force}(A) > \text{Inc}(B)$.
(2) *A formula (ϕ, a) is a possibilistic consequence of B iff $\exists A = \langle H, \phi\rangle$ in $\mathscr{A}(B)$ s.t.* $\text{Force}(A) > \text{Inc}(B)$, $\text{Force}(A) = a$ *and* $\forall A' = \langle H, \phi\rangle \in \mathscr{A}(B)$ *s.t.* $\text{Force}(A) > \text{Inc}(B)$, *we have* $\text{Force}(A) \geqslant \text{Force}(A')$.

Example 13. Let us consider the different arguments of Example 12. Only the arguments having a weight strictly greater than .7 are considered. Namely A_1, A_2, A_3, A_4 and A_7. Thus, the plausible consequences of $\mathscr{B}_\oplus$ are $\phi \vee \psi$, $\neg\phi \vee \neg\psi$, $\neg\phi$, ξ and ψ (and their consequences). The possibilistic consequences of $\mathscr{B}_\oplus$ are $(\phi \vee \psi, .96)$, $(\neg\phi \vee \neg\psi, .94)$, $(\neg\phi, .8)$, $(\xi, .73)$ and $(\psi, .8)$ (and their consequences).

The following theorem ends this section and shows that the result of merging in possibilistic logic framework is captured in argumentation framework.

Theorem 1. *Let $B_1, \ldots, B_n$ different possibilistic bases and $\oplus$ be a possibilistic merging operator. Let $\langle \mathscr{A}, Undercut, \succeq_\oplus\rangle$ be an argumentation framework constructed from $\mathscr{B}$. If $\otimes = \oplus$ then the following result holds*:

$$\mathscr{T}^* \subseteq \text{Supp}(\underline{\mathscr{S}}),$$

where $\mathscr{T}$ is given in Definition 10.

The above result shows that an argumentation framework is "stronger" than the merging operator defined in Section 3.2 in the sense that it may return more results. The reason is that possibilistic logic suffers from the so-called *drowning* problem [5]. A drowning problem means that some information that is not responsible of conflicts may be ignored. More precisely, formulas at the level and below the inconsistency degree are ignored.

Example 14. Let us consider again the bases B_1 and B_2 given in Example 5. Let $\oplus$ be the max operator. Then, $\mathscr{B}_\oplus = B_1 \cup B_2 = \{(\phi \vee \psi, .9), (\neg\phi, .8), (\neg\psi, .7), (\phi, .6), (\xi, .1)\}$.

Using the inference in possibilistic logic, plausible consequences are $\phi \vee \psi$, $\neg\phi$ and ψ while the argumentation-based inference gives $\{\phi \vee \psi, \neg\phi, \psi, \xi\}$.

6. Conclusion

We presented in this paper an argumentation-based framework for resolving conflicts between knowledge bases in a prioritized case where priorities are represented in possibilistic logic framework. The proposed approach is different from the classical way used in the literature to deal with conflicting multiple sources information.

The classical existing approaches consist of first merging individual bases into a new base from which conclusions are drawn. The new base is composed of the most prioritized consistent formulas. The drawback of this approach is that it may ignore formulas that are not responsible for the conflicts.

The argumentation-based approach proposed here builds arguments from the separate bases, evaluates them and lastly computes a set of acceptable arguments from which conclusions are drawn.

The main result of the work presented in this paper is that the argumentation framework captures the result of the merging operator defined in [6,7,17] without merging the different bases. This is of great importance since merging the bases is computationally very costly. Moreover, it is not always interesting to merge the bases as it is the case in a multi-agent system. In such a system, each agent has its own base which may conflict with the bases of the other agents.

Moreover the argumentation-based framework solves the drowning problem. Consequently, it returns more formulas than the approach which merges the bases.

The present work can also be easily extended to recover a merging approach developed in [23] to merge possibilistic bases using multiple-operators. In that work, two merging operators are used for consistent and conflicting formulas respectively. To capture this merging approach the force of an argument will be computed using two operators; an operator applied on formulas provided by consistent bases and another operator applied on formulas provided by conflicting bases.

An extension of this work would be to study the behavior of the argumentation-based approach proposed in this paper from a postulate point of view inspired from the description of possibilistic merging operators from postulate point of view given in [8]. Another extension consists in comparing the argumentation-based approach and the merging-based approach from a complexity in space and time point of view.

Appendix A

Proof of Lemma 2

Let $\Sigma = \{(\phi, \oplus(a_1, \ldots, a_n)) | \phi \in \mathscr{L} \text{ and } B_i \Vdash (\phi, a_i)\}$.

First note that (ϕ, a_i) is an existential inference of B_i means that the greatest weight with which ϕ may belong to B_i is a_i.

Now note that ϕ may be any formula D_j in $\mathscr{B}^*_\oplus$. We have $(\phi, \oplus(x_1, \ldots, x_n)) \in \mathscr{B}_\oplus$ while $(\phi, \oplus(a_1, \ldots, a_n)) \in \Sigma$. Since $\Vdash$ gives the greatest possible weight of a formula we have $a_i \geqslant x_i$ for $i = 1, \ldots, n$. Then $\oplus(a_1, \ldots, a_n) \geqslant \oplus(x_1, \ldots, x_n)$. We distinguish two cases:

Case 1: $\oplus(a_1,\ldots,a_n) = \oplus(x_1,\ldots,x_n)$. In this case $(\phi, \oplus(a_1,\ldots,a_n))$ belongs to $\mathscr{B}_\oplus$.
Case 2: $\oplus(a_1,\ldots,a_n) > \oplus(x_1,\ldots,x_n)$. This means that there exists at least a_k s.t. $a_k > x_k$. This also means that the formula in ϕ (i.e. D_j) taken from B_k can belong to B_k with the weight a_k higher than x_k. In this case we can add that formula to B_k with the weight a_k and we get a new possibilistic base equivalent to B_k following Definition 5. Indeed $(\phi, \oplus(a_1,\ldots,a_n))$ can be added to $\mathscr{B}_\oplus$ without any damage.

When ϕ is not a formula D_j we distinguish two cases:

- $\forall i = 1,\ldots,n$: $B_i \Vdash (\phi, 0)$. Then ϕ belongs to Σ with the weight $\oplus(0,\ldots,0) = 0$ so it is ignored.
- $\exists i$, $B_i \Vdash (\phi, a_i)$ with $a_i \neq 0$. This means that ϕ does not belong to B_i but is a consequence of some formulas of B_i. Following Definition 5 this formula can be added to B_i and $(\phi, \oplus(a_1,\ldots,a_n))$ can also be added to $\mathscr{B}_\oplus$ without any damage.

So each formula in Σ either belongs to $\mathscr{B}_\oplus$ or can be added without any damage and conversely. Indeed $\mathscr{B}_\oplus$ and Σ are equivalent. □

Proof of Proposition 2

Let $B_1,\ldots,B_n$ be n possibilistic bases.

Following Definition 8 we have $\mathscr{B}_\oplus = \{(D_j, \oplus(x_1,\ldots,x_n) \mid j = 1,\ldots,n\}$, where D_j are disjunctions of size j among formulas taken from different B_i's $(i = 1,\ldots,n)$ and x_i is either equal to a_i or to 0 depending respectively on whether ϕ_i belongs to D_j or not.

In order to show that $\mathscr{B}$ and $\mathscr{B}_\oplus$ are equivalent for $\otimes = \oplus$, we show that $\forall(\phi, a) \in \mathscr{B}$ we have $(\phi, a) \in \mathscr{B}_\oplus$ and conversely.

Let $(\phi_1, a_1) \in B_1,\ldots,(\phi_n, a_n) \in B_n$. Then $(\phi_1, \otimes(a_1, 0,\ldots,0)) \in Conj$, $(\phi_2, \otimes(0, a_2, 0,\ldots, 0)) \in Conj,\ldots,(\phi_n, \otimes(0,\ldots,0,a_n)) \in Conj$.

Following Definition 8, ϕ_1 belongs to $\mathscr{B}_\oplus$ with the weight $\oplus(a_1, 0,\ldots,0)$. When $\otimes = \oplus$, it also belongs to $\mathscr{B}_\oplus$. This implies as well to $(\phi_2, \otimes(0, a_2, 0,\ldots,0)),\ldots,(\phi_n, \otimes(0,\ldots,0,a_n))$. Indeed $Conj \subseteq \mathscr{B}_\oplus$.

Now $(\phi_1 \vee \cdots \vee \phi_i, \otimes(a_1,\ldots,a_i,0,\ldots,0)) \in Disj$. Following Definition 8, this formula also belongs to $\mathscr{B}_\oplus$ when $\otimes = \oplus$. Indeed $Disj \subseteq \mathscr{B}_\oplus$.

Similarly we show that each formula in $\mathscr{B}_\oplus$ belongs also to $\mathscr{B}$ when $\otimes = \oplus$. In fact D_j is either composed of one formula and thus corresponds to a formula in *Conj* or composed of more than one formula and thus corresponds to a formula in *Disj*. □

Proof of Proposition 3

The proof can be checked by noticing that the force of an argument corresponds to the minimal weight of formulas in this argument following Definition 21. Following Lemma 2, if a formula (ϕ, a) belongs to $\mathscr{B}_\oplus$ then $a = \oplus(a_1,\ldots,a_n)$ such that $B_i \Vdash (\phi, a_i)$ for $i = 1,\ldots,n$. Since $\mathscr{B} = \mathscr{B}_\oplus$ for $\otimes = \oplus$ following Proposition 2, it holds that $\text{Force}(A) = \min\{\oplus(a_{j1},\ldots, a_{jn}) | \phi_j \in H, B_i \Vdash (\phi_j, a_{ji})\}$, where H is the support of A. □

Proof of Proposition 4

Suppose that ϕ is a plausible consequence of B and let us show that $\exists A = \langle H, \phi\rangle$ in $\underline{\mathscr{L}}$.

Following Definition 6, ϕ is a plausible consequence of B iff $B_{>\mathrm{Inc}(B)} \vdash \phi$.

Let Σ be a minimal subset of $B_{>\mathrm{Inc}(B)} \vdash \phi$ s.t. $\Sigma \vdash \phi$. Then $\langle \Sigma, \phi \rangle$ is an argument in favor of ϕ. Moreover $\mathrm{Force}(\Sigma) > \mathrm{Inc}(B)$ since $\Sigma \subseteq B_{>\mathrm{Inc}(B)}$.

Notice $B_{>\mathrm{Inc}(B)}$ is consistent so each argument Σ' undercutting Σ has some or all its formulas above the inconsistency degree of B. Indeed $\mathrm{Force}(\Sigma') \leqslant \mathrm{Inc}(B)$. Then $\mathrm{Force}(\Sigma) > \mathrm{Force}(\Sigma')$ i.e. $\Sigma \succ_{\oplus} \Sigma'$. Indeed $\langle \Sigma, \phi \rangle$ is an acceptable argument i.e. $\langle \Sigma, \phi \rangle \in \underline{\mathscr{S}}$. $\square$

Proof of Proposition 5

Let (ϕ, a) be a possibilistic consequence of B.

Let us first show that there exists $A = \langle H, \phi \rangle \in \underline{\mathscr{S}}$ s.t. $\mathrm{Force}(A) = a$. Following Definition 7, (ϕ, a) is a possibilistic consequence of B implies that ϕ is a plausible consequence of B. Following Proposition 4 this means that there exists $\langle H, \phi \rangle \in \underline{\mathscr{S}}$.

Also following Definition 7, a is the maximal weight with which ϕ is inferred from B. Since the arguments are by definition minimal, there is necessarily an argument $\langle H, \phi \rangle$ in $\underline{\mathscr{S}}$ s.t. the minimal weight of formulas of H in B is a, i.e. $\mathrm{Force}(A) = a$.

Let us now show that $\forall A' = \langle H', \phi \rangle \in \underline{\mathscr{S}}$ we have $A \succeq_{\oplus} A'$. Suppose that there exists $A' = \langle H', \phi \rangle \in \underline{\mathscr{S}}$ s.t. $A \prec_{\oplus} A'$ i.e. $\mathrm{Force}(A) < \mathrm{Force}(A')$.

Let $a' = \mathrm{Force}(A')$. This means that the minimal weight of formulas of H' in B is a'. By definition of the argument, we know that H' is minimal. Indeed ϕ is a possibilistic consequence of H' with a weight equal to a'.

Since $a > \mathrm{Inc}(B)$ (following Definition 7) we have also $a' > \mathrm{Inc}(B)$. Indeed (ϕ, a') is a possibilistic consequence of B following Definition 7. However by hypothesis (ϕ, a) is also a possibilistic consequence of B and the fact that $a' > a$ contradicts item 2 of Definition 7. Indeed $a \geqslant a'$ i.e. $\mathrm{Force}(A) \geqslant \mathrm{Force}(A')$ which corresponds to $A \succeq_{\oplus} A'$. $\square$

Proof of Proposition 6

The proof can be checked by first noticing that arguments are individually consistent. Let $A_i = \langle H, \phi \rangle$ and $A_j = \langle H', \psi \rangle$ s.t. A_i undercuts A_j. This means that $\exists k \in H'$ s.t. $\phi \equiv \neg k$. This also means that $H \cup H'$ is inconsistent.

Let $\Sigma_{ij} = \{(\phi_l, a_l) : \phi_l \in H, (\phi_l, a_l) \in B\} \cup \{(\psi_{l'}, a_{l'}) : \psi_{l'} \in H', (\psi_{l'}, a_{l'}) \in B\}$.

We have $\mathrm{Inc}(\Sigma_{ij}) \geqslant \min(\mathrm{Force}(A_i), \mathrm{Force}(A_j))$. We distinguish two cases: either $\mathrm{Inc}(\Sigma_{ij}) = \min(\mathrm{Force}(A_i), \mathrm{Force}(A_j))$ or

$\mathrm{Inc}(\Sigma_{ij}) > \min(\mathrm{Force}(A_i), \mathrm{Force}(A_j))$.

Suppose that $\mathrm{Force}(A_i) \geqslant \mathrm{Force}(A_j)$.

The first case means that the formula $k \in H'$ s.t. $\phi \equiv \neg k$ has the minimal weight in H'. The second case means that this formula has not the minimal weight in H' so $\mathrm{Inc}(\Sigma_{ij}) > \mathrm{Force}(A_j)$. However this does not alter the computation of $\mathrm{Inc}(B)$ since A_i also undercuts $A_m = \langle H'', k \rangle$, where $\phi \equiv \neg k$. In this case we have $\mathrm{Inc}(\Sigma') = \min(\mathrm{Force}(A_i), \mathrm{Force}(A_m))$, where $\Sigma'^{*} = H \cup H''$. Then we have $\mathrm{Inc}(\Sigma') > \mathrm{Inc}(\Sigma_{ij})$. Now we know from Definition 4 that the inconsistency degree of B is the maximal degree in B where inconsistency is met. Indeed we have well

$$\mathrm{Inc}(B) = \max\{\min(\mathrm{Force}(A_i), \mathrm{Force}(A_j)) | A_i, A_j \in \mathscr{A}(B) \text{ and } A_i \textit{ Undercuts } A_j\}. \qquad \square$$

Proof of Proposition 7

(1)
- Suppose that ϕ is a plausible consequence of B and show that $\exists A = \langle H, \phi \rangle$ in $\mathscr{A}(B)$ s.t. Force$(A) >$ Inc(B).
  From Definition 6, ϕ is a plausible consequence of B iff $B_{>\mathrm{Inc}(B)} \vdash \phi$. Indeed there is a minimal set H in $B_{>\mathrm{Inc}(B)}$ s.t. $H \vdash \phi$. So $A = \langle H, \phi \rangle$ is an argument in favor of ϕ. Since all formulas of H are in $B_{>\mathrm{Inc}(B)}$ we have that all formulas of H in B have a weight strictly greater than Inc(B). Indeed Force$(A) >$ Inc(B).
- Suppose now that $\exists A = \langle H, \phi \rangle$ in $\mathscr{A}(B)$ s.t. Force$(A) >$ Inc(B) and let us show that ϕ is a plausible consequence of B.
  $\langle H, \phi \rangle$ is an argument in favor of ϕ means that $H \vdash \phi$. Since Force$(A) >$ Inc(B) this means that $H \subseteq B_{>\mathrm{Inc}(B)}$. Inference in propositional logic is monotonic so we have $B_{>\mathrm{Inc}(B)} \vdash \phi$. Then ϕ is a plausible consequence of B.

(2) Suppose that (ϕ, a) is a possibilistic consequence of B. From Proposition 5 we have that $\exists A = \langle H, \phi \rangle$ in $\mathscr{A}(B)$ s.t. Force$(A) = a$.
From Definition 7 we have that if (ϕ, a) is a possibilistic consequence of B then ϕ is a plausible consequence of B. Following the first item of this proposition we have Force$(A) >$ Inc(B).From Definition 7, we know that there is no $b > a$ s.t. $B_{>b} \vdash \phi$. So $\forall A' = \langle H, \phi \rangle \in \mathscr{A}(B)$ we have necessarily Force$(A) \geqslant$ Force(A').
- Suppose that $\exists A = \langle H, \phi \rangle$ in $\mathscr{A}(B)$ s.t. Force$(A) >$ Inc(B) and Force$(A) = a$. Following the first item of this proposition this means that ϕ is a plausible consequence of B i.e. $B_{>\mathrm{Inc}(B)} \vdash \phi$.
  Suppose now that $\forall A' = \langle H', \phi \rangle$ in $\mathscr{A}(B)$ s.t. Force$(A) >$ Inc(B) we have Force$(A) \geqslant$ Force(A'). This simply means that there is no $b >$ Force(A) s.t. $B_{>b} \vdash \phi$. This corresponds to the second item of Definition 7. Indeed (ϕ, a) is a possibilistic consequence of B. $\square$

Proof of Theorem 1

Suppose that $\otimes = \oplus$. Following Proposition 2 we have $\mathscr{B} = \mathscr{B}_{\oplus}$. Let us show that $\forall (\phi, a) \in \mathscr{T}$ we have $\phi \in$ Supp$(\underline{\mathscr{S}})$.

$\phi \in$ Supp$(\underline{\mathscr{S}})$ means that there exists an argument $A = \langle H, \psi \rangle$ in $\underline{\mathscr{S}}$ such that $\phi \in H$.

Notice $(\phi, a) \in \mathscr{T}$ means that ϕ is a plausible consequence of $\mathscr{B}_{\oplus}$ so it is also a plausible consequence of $\mathscr{B}$ i.e. $\mathscr{B}_{>\mathrm{Inc}(\mathscr{B})} \vdash \phi$. By definition $\mathscr{B}_{>\mathrm{Inc}(\mathscr{B})}$ is consistent. Let Σ be a minimal subset of $\mathscr{B}_{>\mathrm{Inc}(\mathscr{B})}$ s.t. $\Sigma \vdash \phi$. So $A = \langle \Sigma, \phi \rangle$ is an argument in favor of ϕ. Since $\mathscr{B}_{>\mathrm{Inc}(\mathscr{B})}$ is consistent then each argument A' undercutting $A = \langle \Sigma, \phi \rangle$ takes at least one formula from $\mathscr{B}_{\leqslant\mathrm{Inc}(\mathscr{B})}$. So Force$(A') <$ Force(A) which means that A is preferred to all its undercutting arguments. Indeed A is an acceptable argument i.e. it belongs to $\underline{\mathscr{S}}$ which implies that $\phi \in$ Supp$(\underline{\mathscr{S}})$.

References

[1] L. Amgoud, C. Cayrol, Inferring from inconsistency in preference-based argumentation frameworks, International Journal of Automated Reasoning 29 (2) (2002) 125–169.

[2] L. Amgoud, C. Cayrol, A reasoning model based on the production of acceptable arguments, Annals of Mathematics and Artificial Intelligence 34 (2002) 197–216.
[3] L. Amgoud, S.D. Parsons, An argumentation framework for merging conflicting knowledge bases, in: Proceedings of the 8th International Conference on Logics in Artificial Intelligence (JELIA'02), 2002, pp. 27–37.
[4] L. Amgoud, H. Prade, Using arguments for making decisions, in: Proceedings of the 20th Conference on Uncertainty in Artificial Intelligence, 2004, pp. 10–17.
[5] S. Benferhat, D. Dubois, C. Cayrol, J. Lang, H. Prade, Inconsistency management and prioritized syntax-based entailment, in: Proceedings of the 13th International Joint Conference on Artificial Intelligence (IJCAI'93), 1993, pp. 640–645.
[6] S. Benferhat, D. Dubois, S. Kaci, H. Prade, Possibilistic merging and distance-based fusion of propositional information, Annals of Mathematics and Artificial Intelligence 34 (1–3) (2002) 217–252.
[7] S. Benferhat, D. Dubois, H. Prade, M. Williams, A practical approach to fusing and revising prioritized belief bases, in: Proceedings of the 9th Portuguese Conference on Artificial Intelligence (EPIA'99), 1999, pp. 222–236.
[8] S. Benferhat, S. Kaci, Fusion of possibilistic knowledge bases from a postulate point of view, International Journal on Approximate Reasoning 33 (2003) 255–285.
[9] Loreto Bravo, Leopoldo E. Bertossi, Logic programs for consistently querying data integration systems, in: Proceedings of the 18th International Joint Conference on Artificial Intelligence (IJCAI'03), 2003, pp. 10–15.
[10] C.I. Chesnevar, A. Maguitman, R.P. Loui, Logical models of arguments, ACM Computing Surveys 32 (4) (2000) 337–383.
[11] L. Cholvy, A general framework for reasoning about contradictory information and some of its applications, in: Proceedings of the ECAI Workshop Conflicts Among Agents, 1998.
[12] L. Cholvy, Reasoning about merging information, Handbook of Defeasible Reasoning and Uncertainty Management Systems, vol. 3, 1998, pp. 233–263.
[13] M. Dalal, Investigations into a theory of knowledge base revision: preliminary report, in: Proceedings of the 7th National Conference on Artificial Intelligence (AAAI'88), 1988, pp. 475–479.
[14] D. Dubois, J. Lang, H. Prade, Possibilistic logic, in: D. Gabbay et al. (Eds.), Handbook of Logic in Artificial Intelligence and Logic Programming, vol. 3, Oxford University Press, 1994, pp. 439–513.
[15] P.M. Dung, On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and *n*-person games, Artificial Intelligence 77 (1995) 321–357.
[16] M. Elvang-Goransson, A. Hunter, Argumentative logics: reasoning with classically inconsistent information, Data and Knowledge Engineering 16 (1995) 125–145.
[17] S. Kaci, Connaissances et Préférences: Représentation et fusion en logique possibiliste, in: Thèse de doctorat. Université Paul Sabatier. Toulouse, 2002.
[18] S. Konieczny, R. Pino Pérez, On the logic of merging, in: Proceedings of the 6th International Conference on Principles of Knowledge Representation and Reasoning (KR'98), Trento, 1998, pp. 488–498.
[19] J. Lin, Integration of weighted knowledge bases, Artificial Intelligence 83 (1996) 363–378.
[20] J. Lin, A. Mendelzon, Merging databases under constraints, International Journal of Cooperative Information Systems 7 (1) (1998) 55–76.
[21] H. Prakken, G. Sartor, Argument-based extended logic programming with defeasible priorities, Journal of Applied Non-Classical Logics 7 (1997) 25–75.
[22] H. Prakken, G. Vreeswijk, Logics for defeasible argumentation, second ed., in: D. Gabbay (Ed.), Handbook of Philosophical Logic, 4, Kluwer Academic Publishers, 2002, p. 219.
[23] G. Qi, W. Liu, D.H. Glass, Combining individually inconsistent prioritized knowledge bases, in: Proceedings of the 10th International Workshop on Non-monotonic Reasoning, 2004, pp. 342–349.
[24] N. Rescher, R. Manor, On inference from inconsistent premises, Theory and Decision 1 (1970) 179–219.
[25] P.Z. Revescz, On the semantics of arbitration, International Journal of Algebra and Computation 7 (1997) 133–160.
[26] P.Z. Revesz, On the semantics of theory change: arbitration between old and new information, in: 12th ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Databases, 1993, pp. 71–92.
[27] P.Z. Revesz, On the semantics of arbitration, International Journal of Algebra and Computation 7 (2) (1997) 133–160.
[28] L. Zadeh, Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets and Systems 1 (1978) 3–28.