Multicriteria Decision Aid

A short introduction

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https://github.com/oliviercailloux/introduction-mcda

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Outline

Goal of MCDA

Desired:

- decide in agreement with your own preferences
- in a systematic way

Preferences: intuitive knowledge of what's right for you

One goal of Multicriteria Decision Aid (MCDA)

Model preferences

MCDA: what for? (Philosophically)

A Delphic maxim (Temple of Apollo)

"Know thyself"

MCDA: what for? (More practically)

- Make sure you take the right decision (for you)
- Discuss your intuitions
- Model expertise
- Understand intuitive notions (such as justice)
- Delegate decision making

Why model preferences?

There are three things extremely hard: Steel, a Diamond, and to know one's self.

B. Franklin [2004, p. 179]

- We hardly know our own preferences
- You can't have everything
- Which trade-offs will you agree with?

An example

- Let's choose what to plant in our garden
- Each year the performances change
- We want a systematic decision procedure

	quantity	taste	supports pollinators	resists to cold
Tomatoes	7	A	A	−−
Corn	1.5	B	D	−−
Cabbage	7.5	D	B	++
Potatoes	2.5	C	C	+
...				

Context

- Criteria $\mathcal{J}$, scales $X_j, \forall j \in \mathcal{J}$
- Action $a \in \prod_{j \in \mathcal{J}} X_j$ is a vector of performances
- Action $\mathcal{A} = \prod_{j \in \mathcal{J}} X_j$
- Available actions this year $A \subseteq \mathcal{A}$
- Winners this year $B \subseteq A$

Goal

Obtain f which maps any $A \subseteq \mathcal{A}$ to some $B \subseteq A$

	quantity	taste	sup. pollinators	res. to cold
Tomatoes	7	A	A	−−
Corn	1.5	B	D	−−
Cabbage	7.5	D	B	++
Potatoes	2.5	C	C	+

Informational basis to determine f

- We do *not* search for the absolute best f
- f models the *preference*
- Of a decision maker
- Her subjectivity is to be integrated in f

What does “preferred” mean?

Let’s determine your “preferred” university

- Help a researcher choose her university?
- Help a student choose his university?
- Help government spread funding?

Preference in MCDA

- A decision problem
- A decision maker
- Preference typically defined in terms of *desired action*

Descriptive or prescriptive perspective

MCDA typically adopts a weak prescriptive perspective

Descriptive perspective

The model describes the "usual" behavior of the subjects

- Example: which drink does the subject buy?
- Predictive model

Prescriptive perspective

The model recommends actions coherent with the values of the Decision Maker (DM)

- Example: you might want to consider this drink
- Possibly talk about hypothetical decisions
- Different validation

MCDA methods

- f: a preference model (here, a strategy of choice)
- f represents the subjectivity of the DM
- f maps $A \subseteq \mathscr{A}$ to $B \subseteq A$
- $\mathcal{F}$ the set of possible functions
- How do we determine $f \in \mathcal{F}$?

MCDA method

- Defines a class of functions $F \subseteq \mathcal{F}$
- Together with a class of *preferential parameters* Ω
- Bijection maps $\omega \in \Omega$ to $f \in F$

The Weighted sum method

- Class of functions $F_{\text{weighted sum}}$: those that sum performances and compare the resulting scores
- Preference model ω: a set of weights
- $\omega = \{w_j \in \mathbb{R}, \text{for each criterion } j\}$
- f_ω compares the weighted sums

Weights: $\omega = \langle 0.3, 0.3, 0.2, 0.2 \rangle$

	quantity	taste	supports pollinators	resists to cold	$\xrightarrow{f_\omega}$
Tomatoes	7	10	7	0	?
Corn	1.5	5	1	0	
Cabbage	7.5	2	5	10	
Potatoes	2.5	3	3	5	

The Weighted sum method

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Weights: $\omega = \langle 0.3, 0.3, 0.2, 0.2 \rangle$

	quantity	taste	supports pollinators	resists to cold	f_ω
Tomatoes	7	10	7	0	$\longrightarrow$ 6.5
Corn	1.5	5	1	0	$\longrightarrow$ 2.15
Cabbage	7.5	2	5	10	$\longrightarrow$ 5.85
Potatoes	2.5	3	3	5	$\longrightarrow$ 3.25

A problem with the weighted sum

The f you want may not be in $F_{\text{weighted sum}}$

- You prefer St 1 to the other two?
- Score(St 1) $>$ score(St 2) requires $w_{\text{course 1}} > w_{\text{course 2}}$
- Score(St 1) $>$ score(St 3) requires $w_{\text{course 2}} > w_{\text{course 1}}$

	course 1	course 2	f_ω
St 1	14	14	$\longrightarrow$
St 2	8	20	$\longrightarrow$
St 3	20	8	$\longrightarrow$

Goal of preference elicitation

- Assume we chose the method
- Method determines class of functions F and class of parameters Ω
- How shall we determine parameters $\omega \in \Omega$?
- The DM does not know the answer
- Her usual behavior does not determine the answer
- BUT we assume she is available to answer questions

Elicitation

Elicit (*Oxford English Dictionary*, excerpt)

1. To draw forth (what is latent or potential) into sensible existence.
2. to extract, draw out (information) from a person by interrogation
3. To draw forth, evoke (a response, manifestation, etc.) from a person.

The edge of one [fissure] which elicited other sentiments than those of admiration.

J. Tyndall, Glaciers of Alps, i. §25. 188

Preference elicitation

- Ask questions to the DM
- Questions must be: understandable
- Interpretable rigorously
- Informative
- Hopefully questions that the DM can answer confidently

Goal: obtain a "satisfactory" f

- Elicitation can be: by parameters, by examples, or a mix, possibly using axiomatization

Elicitation by parameters

- Assume a method has three parameters with identifiable "roles"
- Explain the effects of the parameters on f
- Ask the DM to fix the parameters
- Also permits to check whether the DM accepts the method (accepts F)
- Possibly: show the effects of f on samples

Elicitation by examples

- Use a set of examples to constrain f
- The DM declares that f should satisfy $f(\{a, b\}) = \{a\}$
- Sometimes: use historic examples
- Ideally these should be examples the DM knows how to treat
- Detect whether some $f \in F$ represents all examples

Axiomatic elicitation

- A method can be axiomatized
- Sometimes means that we know exactly which questions to ask, in which sequence, to determine f
- And the conditions under which f exists coherent with those answers
- May be compatible with other two approaches

Some topics of study

- (Axiomatic) study of classes of functions
- Elicitation procedures
- Extend to group decision making
- Extend to uncertainty on performances
- Cases study

Open questions

Is the model f we obtain necessary?

- Several reasonable models may be possible
- What's the part of arbitrariness in the model?

Will the DM possibly accept (somehow) unreasonable models?

- What does unreasonable mean?

How do we compare models?

- How do we validate?
- What does f model precisely?

Elicitation: are we asking questions the DM can answer?

Thank you for your attention!

References

B. Franklin. *Poor Richard's Almanack*. Barnes & Noble Publishing, 2004. ISBN 9780760762011.

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