Non linear preference models: why, how?

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Abstract. Literature involving preferences of artificial agents or human beings often assume their preferences are linear, that is, the preference can be represented using a complete transitive antisymmetric binary relation. Much has been written however on more complex, or more interesting, models of preferences. In this article we review some of the reasons that have been put forward to justify more complex modeling, and review some of the techniques that have been proposed to obtain models of such preferences. (Optional: we connect to various related literature about argumentation, …)

1 Introduction

Here is a possible approach we might want to use.

- What's a model of preference? Example with weighted sum. But: forbids some possibly interesting evaluations; need to recode evaluations and hence is dependent of irrelevant alternatives; w1 > w2 unclear.
- Utility: is holistic. Forbids incomparability. Mandates transitivity.
- Models of preferences can have (at least) two interpretations: normative or descriptive. Goals and (sometimes) tools to model preferences and reasonable hypothesis that can be postulated about the shape of the preferences vary according to these two lines.
- (Reasonable) hypothesis about descriptive preferences. Why it's more difficult than can be thought naïvely.
- Reasonable hypothesis about normative preferences. Why it's more difficult than can be thought naïvely. Talk about incompleteness. Applying normative approaches: talk about prescription a la Roy…
- Focus on incompleteness rather than non-transitivity. Can be lack of information or intrinsic.
- Present usual models (Savage? MAVT?) and alternative ones (belief functions? Electre?)?
- Talk about choice under uncertainty wrt incompleteness.
- Impact of relaxations on problems of choice, ranking, classification.
- How to obtain information under incompleteness hypothesis, meaning and interpretation.
- (If space permits) Links to argumentation theory: build argumentative models that explain / justify a decision.?
- (If space permits) Path to validation of normative models. Talk about empirical social choice; reflective equilibrium. Use the resulting indeterminacy (lability). Maximizing without transitivity or completeness?

2 Model of preference

When considering a Decision Maker (DM) and a choice situation, it may be useful to try to obtain a formal model of her preferences. This model can be used to describe the way she chooses, or to recommend an option. (More on the uses of models later.) Let us see first what a simple model may look like. We start by defining a multiple-criteria choice decision situation (MCDS), one of the situations considered in this paper.

We consider a set $\mathscr{A}$ called the set of "alternatives", that represents the possible options to choose from. Alternatives are here considered as mutually exclusive and the set $\mathscr{A}$ is exhaustive: the choice situation mandates to choose exactly one alternative from the set. Let us further assume some structure on $\mathscr{A}$: assume each alternative is evaluated on multiple criteria. The set $\mathcal{G}$ supposedly represents all the criteria, or aspects, that the DM may wish to consider when choosing. For example, say the DM must choose what to plant in her garden. The set of alternatives $\mathscr{A}$ are vegetables, and the criteria $\mathcal{G}$ measure the taste, quantity, and aesthetic quality of each vegetable. Each criterion $g \in \mathcal{G}$ comes with an evaluation scale X_g, and an evaluation function $f_g : \mathscr{A} \to X_g$ that evaluates each alternative. To simplify notations, we will denote the evaluation function f_g simply by g, thus equate the criterion with its evaluation function. In our example, we could have $X_{g_1} = \{A, B, C, D\}$, a set of labels, with $g_1(a)$ representing the taste of the vegetable $a \in \mathscr{A}$ as subjectively considered by the DM (A is the best taste, D the worst), $X_{g_2} = [0, 100]$, with $g_2(a)$ representing the number of meals that the DM would enjoy if deciding to plant a, and $X_{g_3} = \{++, +, 0, -, --\}$, where $g_3(a) = 0$ indicates that the DM considers a neither beautiful nor ugly. An MCDS is one which can be described using such exhaustive sets of alternatives and criteria.

Two natural ideas come to mind when facing an MCDS, and the reader certainly has used them already in real life. The first one is to design a value function $v : \mathscr{A} \to \mathbb{R}$, which associates a value (a real number) to each alternative, such that better alternatives receive better scores, in order to reduce the problem to one of choosing an alternative that maximizes v. The second natural idea serves to define v: suffices to find a set of weights $\{w_g, g \in \mathcal{G}\}$, and a way of transforming each evaluation function $g : \mathscr{A} \to X_g$ into $g' : \mathscr{A} \to \mathbb{R}$, an evaluator that returns real numbers. Suffices then to define $v(a) = \sum_{g \in \mathcal{G}} w_g g'(a)$.