Non linear preference models: why, how?

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Abstract. Literature involving preferences of artificial agents or human beings often assume their preferences are linear, that is, the preference can be represented using a complete transitive antisymmetric binary relation. Much has been written however on more complex, or more interesting, models of preferences. In this article we review some of the reasons that have been put forward to justify more complex modeling, and review some of the techniques that have been proposed to obtain models of such preferences. (Optional: we connect to various related literature about argumentation, …)

1 Introduction

Preferences of agents are usually assumed to be linear, that is complete, transitive and antisymmetric. In practice, such assumptions limit the scope of models (of uncertainty, of preferences) one can consider as legitimate, and are also falsified by observed empirical evidences. While models relaxing antisymmetry to allow indifferences between options have been around for some time *** ELECTRE ?***, models relaxing the assumptions of completeness or transitivity are more recent.

It is clear that our paper will not be complete, yet we think it gives a fair overview of why one would like to drop some of the above assumption. In particular, we will review both empirical evidence that preferences do not always follow the above assumption, thus relating them to more descriptive approaches, as well as formal models that allows one to induce intransitive or incomplete preferences.

2 Model of preference: multiple criteria and uncertainty

There are two canonical fields where the modelling of preferences is a central topic: choosing an alternative when it is evaluated according to different aspects (a.k.a. multi-criteria decision making, or MCDM), and picking an alternative whose quality depends on states of the world that are uncertainly known. Although the two frameworks are formally similar to some extent, they also present some key differences, if not formally then at least conceptually.

In MCDM, the common assumption is that the alternatives, i.e., the state of the world is known without ambiguity, and the difficulty is to

determine what are the user preferences over these different, but well-defined states. In decision making under uncertainty (DMU), the preferences of the user are usually assumed to be well-known, or at least to have been previously assessed in the form of utility functions, and the problem is to recommend an alternative given our uncertainty about the world.

We will adopt the following basic scheme for the two settings (MCDM and DMU), that we will question later on, but that are necessary to set some notations:

- In MCDM, the classical approach is to assume that there is some real-valued function $f : \times_{i=1}^{N} X^i \to \mathbb{R}$ mapping alternatives to their values, and that $a_i > a_j$ iff $f(a_i) > f(a_j)$. As all alternatives are precisely defined, the resulting order is indeed a linear one;
- In the simplest form of DMU, we usually have one space X of possible states of the world, and each act $a_i : X \to \mathbb{R}$ is modelled as a function where $a_i(x)$ is the reward or utility of performing a_i when x is the actual state of the world. When uncertainty is modelled by a probability mass p over X, we have that $a_i > a_j$ iff $\sum_x p(x)a_i(x) > \sum_x p(x)a_j(x)$, that is if the expected reward of a_i is higher than the one of a_j

In principle, such approaches can already provide order that are not antisymmetric as indifferences are possible, yet one can consider that indifferences are very unlikely in these frameworks, or at least not considered explicitly by the framework.

3 Dropping transitivity

3.1 Empirical evidences of intransitivity

* Fisher, …

3.2 Intransitivity in MCDM

3.3 Intransitivity in MDU

* Statistical preferences, strongly related to the Condorcet paradox in social choice theory

3.4 Intransivity: a default to be repaired, or a fact to be modelled?

4 Dropping completeness

4.1 Empirical evidence of incompleteness

* not clear what to put there... difficult to witness incompleteness?

4.2 Incompleteness in MCDM

* Robust MCDM a la Greco etc...
    * MCDM model that induce incomplete orders (CP-net... others?)

4.3 Incompleteness in DMU

* Stochastic dominance inducing possibly incomparability
    * Incompleteness from incompletely specified probabilities, including decision under risk

4.4 Incompleteness: absence of knowledge or knowledge of absence?