The title

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1. Overview

Looking for possibilities (weak acceptance). Those propositions that are in the reflexive preferences in a large sense: there is no strong enough reason to reject those propositions, though their contrary may hold as well.

- All alternatives $\mathscr{A}$.
- Topic $T^* = \{ p_a, a \in \mathscr{A} \} \cup \{ p_{\neg a}, a \in \mathscr{A} \}$. Denoted simply a and $\neg a$. We define $\neg p$, with $p = p_a$, as equal to $p_{\neg a}$ and $\neg p$, with $p = p_{\neg a}$, as equal to p_a.
- All possible arguments: S^*, the set of all strings.
- $s' \rhd^{\mathrm{d}}_{\exists p} s$: strong rejection attack; s' renders s invalid (with s supporting p, s' claims that $\neg p$ is a certainty, thus s can't be used even to say that p is a possibility, thus s' supports that p is not weakly accepted). Thus we have only attacks between contradictory propositions. Suffices that the attack occurs at least once over the considered time frame and unstability factors (such as submitting i to other counter-arguments). Here we do not condition on s' surviving: s' is declared incorrect, with no necessity of pursuing the debate and no hope of reinstatement. Example: s' has already been taken into account and countered in s. (TODO exploit the fact that we probably only need the negation, $\neg(s' \rhd^{\mathrm{d}}_{\exists p} s)$, which is pretty clear as further attacks to s' have no impact.)
- Define $\leadsto$, decisive support, as $s \leadsto p$ iff i declares that s is a definitive argument in that weakly supports p: there are no s' that changes her position (once she has heard about s), in the sense of weak acceptancy, thus, no s' attacking s in the sense of strong rejection attack.

Definition 1 (Clear-cut)**.** *Define a situation* $(T^*, S^*, \rightsquigarrow)$*. It is clear-cut iff* $\forall a \in \mathscr{A} : \rightsquigarrow^{-1}(\{p_a, p_{\neg a}\}) \neq \emptyset$*.*

We want to prove that, under suitable assumptions (justifiable unstability, and so on): situation is clear cut iff a model (with no cycles) exist.

2. Models

D a DAG over (s, p) pairs.

Given $(s_2, p), (s_1, \neg p) \in D$, define $s_2 \succ_{\exists\neg p} s_1$ iff $\forall s \mid (s, p) \in D : [\neg(s_1 \rhd^{\mathrm{d}}_{\exists p} s) \vee \neg(s_1 \rhd^{\mathrm{d}}_{\exists p} (s + s_2)) \vee s_2$ is itself attacked]. Can perhaps be queried directly by asking (in the context of some $s_1 \rhd^{\mathrm{d}}_{\exists p} s$): "assume s_2 survives, then does s_2 counter s_1?" (In the sense that s_2 is sufficiently convincing that p holds perhaps, to cancel the argument s_1 according to which $\neg p$ surely holds.)

Given $(s_3, \neg p), (s_2, p) \in D$, define $s_3 \rhd_{\exists p} s_2$ iff $\forall (s_1, \neg p) \in D : [\neg(s_2 \succ^{\mathrm{dec}}_{\exists\neg p} s_1) \vee \neg(s_2 \succ^{\mathrm{dec}}_{\exists\neg p} (s_1 + s_3)) \vee s_3$ is itself attacked].

Define $\neg(s_2 \succ^{\mathrm{dec}}_{\exists\neg p} s_1) = \neg(s_2 \succ_{\exists\neg p} s_1)$. Then, $(s_3, \neg p) \rhd_{\exists} (s_2, p)$ iff $\forall (s_1, \neg p) : [(s_1 \rhd^{\mathrm{d}}_{\exists p} s_2) \vee ((s_1 + s_3) \rhd^{\mathrm{d}}_{\exists p} s_2) \vee s_3$ is itself attacked].

2.1. Perhaps equivalent and better

Let D be a DAG over (s, p) pairs, such that $\forall s : (s, p) \notin D \vee (s, \neg p) \notin D$, thus, a given argument is never used both with p and with $\neg p$.

Define $\rhd^D_p$ as follows.

- $s_2 \rhd^D_p s_1$ iff $(s_2 \rhd^{\mathrm{d}}_{\exists p} s_1)$ when $(s_1, p), (s_2, \neg p) \in D$,
- $s_2 \rhd^D_p s_1$ iff $\neg(s_1 \rhd^{\mathrm{d}}_{\exists p} s_2)$ when $(s_2, p), (s_1, \neg p) \in D$,
- $s_2 \rhd^D_p s_1$ not defined otherwise.

Thus, $s_2 \rhd^D_{\neg p} s_1$ iff $\neg(s_1 \rhd^{\mathrm{d}}_{\exists\neg p} s_2)$ when $(s_1, p), (s_2, \neg p) \in D$.

Given $(s_3, \neg p), (s_2, p) \in D$, define $s_3 \rhd_{\exists p} s_2$ iff $\forall (s_1, \neg p) \in D : [\neg(s_2 \rhd^D_p s_1) \vee \neg(s_2 \rhd^D_p (s_1 + s_3)) \vee s_3$ is itself attacked].

Given $(s_3, p), (s_2, \neg p) \in D$, define $s_3 \succ_{\exists\neg p} s_2$ iff $\forall (s_1, p) \in D : [\neg(s_2 \rhd^{\mathrm{d}}_{\exists p} s_1) \vee \neg(s_2 \rhd^{\mathrm{d}}_{\exists p} (s_1 + s_3)) \vee s_3$ is itself attacked].

2.2. Newer try

$s' ntepd s$ (for not triangle exists p d) iff $\neg(s' tepd s)$, iff it is sure that s' has no impact on s, even assuming that s' would in turn resist all counter-arguments to it.

$s2naenps1$ iff it is sure that $s2$ has no sufficient impact on $s1$, even assuming that $s2$ survives, more precisely, iff for some s, where $\neg(s1ntepds)$: $\neg(s1ntepd(s+s2))$.

3. Models - try

Additionally.

- Propositions weakly self-supported $T \subseteq T^*$: weakly accepted if no arg is given. Examples: m = "eat miam"; $\neg b$ = "beurk is to exclude"; or, in a problem where there's no particularly good aliments, both a = "eat this" and $\neg a$.
- $s' \succ_{\exists p} s$: weak attack; s' renders s invalid because too strong (with s supporting p, s' claims that $\neg p$ is a possibility, thus s can't be used to say that p is a certainty, thus s' supports that $\neg p$ is weakly accepted). Assuming s' survives. This is a relation on the arguments that defend p times the arguments that defend $\neg p$, union the converse, union on all propositions. Thus we have only attacks between contradictory propositions.

When given (s, p), i may say: s does not survive; or: assuming s survives, then s supports p, or, assuming s survives, then s does not support p anyway.

When given s' against s, i may say: s' does not survive, or: assuming s' survives, then s' supports $\neg p$, …

We might use the following hyp. (We can dispense of it?) (TODO what does that mean?)

Definition 2 (Completeness of T)**.** *At least one of a and $\neg a$ is in T, for each alternative.*

If none, we should assume the DM means that both are (which can be done if falsifying against observed choices).

I think we need a primitive definition of (s, p) attacked by $s1$.

Then we can define: s_{cc} attacks s_c iff $[(p + s_{cc}, p)$ not attacked by s_c, or s_{cc} not decisive].

Except this is maybe not well defined in case of circularities! (But this should not be a problem as we will not need this definition in that case.)

Definition 3. *Given*

A. Certainties

Looking for certainties. Those propositions that are in the reflexive preferences in a demanding sense: there is a strong enough reason to prefer it than its contrary.

- $s' \succ_\exists s$: weak attack; s' renders s invalid (can't be used to say that t holds for sure) (assuming s' survives)

- Propositions strongly self-supported: strongly accepted if no arg is given. Examples: m = "eat miam"; $\neg b$ = "beurk is to exclude". We might have neither c nor $\neg c$ in that set.