The title

Olivier Cailloux

12^{th} October, 2017

1. Overview

Looking for possibilities (weak acceptance). Those propositions that are in the reflexive preferences in a large sense: there is no strong enough reason to reject those propositions, though their contrary may hold as well.

- All alternatives $\mathscr{A}$.
- Topic $T^* = \left\{ p_a, a \in \mathscr{A} \right\} \cup \left\{ p_{\neg a}, a \in \mathscr{A} \right\}$. Denoted simply a and $\neg a$. We define $\neg p$, with $p = p_a$, as equal to $p_{\neg a}$ and $\neg p$, with $p = p_{\neg a}$, as equal to p_a.
- All possible arguments: S^*, the set of all strings. It can be chosen differently, but must be non empty as it contains at least $\emptyset$, the empty argument.
- $s' \ntriangleright^p_\forall s$: absence, all the time, of strong rejection attack; s' does not render s invalid; it is sure that s' has no impact on s, even assuming that s' would in turn resist all counter-arguments to it. This is an hypothesized relation, unobservable, used to define the deliberated preferences of i (it is clear we can't observe it, thus, as we can't observe the DP). With s supporting p, s' claims that $\neg p$ is a certainty, thus s' supports that p is not weakly accepted, thus s, if rebutted, can't be used even to say that p is a possibility. Thus we have only attacks between contradictory propositions. Suffices that the attack occurs at least once over the considered time frame and unstability factors (such as submitting i to other counter-arguments) to negate $s' \ntriangleright^p_\forall s$. Here we do not condition on s' surviving: s' is declared incorrect, with no necessity of pursuing the debate and no hope of reinstatement. Example: s' has already been taken into account and countered in s; or s' does not talk about s at all; or is not understood by i. Note that when $s' \ntriangleright^p_\forall s$, further attacks to s'

have no chance to change that fact (assuming some properties over the way i reason). The negation of this should read: s' may attack s in at least some cases. For example, $\neg(s' \ntriangleright^{p}_{\forall} s)$ (s' may attack s) in case i suspects that s' is invalid (because of some counter-argument s_2 to s' that i has in mind) but wants to leave the door open to reinstatement of s'.

- Define $\neg(s' \ntriangleright^{p}_{\forall} s)$ iff $s' \rhd^{p}_{\exists} s$.
- Define $s \rhd^{\neg p,\text{sure}}_{\exists} s'$ iff s may attack s', where s has a weak claim (p is possible) and s' has a strong claim ($\neg p$ is sure). Similarly, $s \ntriangleright^{\neg p,\text{sure}}_{\forall} s'$ is a constant absence of attack.
- Require (as an axiom) that $s' \rhd^{p}_{\exists} s \vee s \rhd^{\neg p,\text{sure}}_{\exists} s'$. Equivalently: $\neg(s \ntriangleright^{\neg p,\text{sure}}_{\forall} s' \wedge s' \ntriangleright^{p}_{\forall} s)$; $\neg(s' \rhd^{p}_{\forall} s \wedge s' \ntriangleright^{p}_{\forall} s)$; $s' \rhd^{p}_{\forall} s \Rightarrow s' \rhd^{p}_{\exists} s$.

It seems that knowledge of the poss rels is insufficient to know the sure rel: assume $\neg(s' \rhd^{p}_{\exists} s)$ and $s \rhd^{\neg p}_{\exists} s'$. Thus, 1. s is sufficient for p possible, s' not sufficient for $\neg p$ sure; and 2. s may be sufficient for p sure, s' insuff for p. But is s sufficient for p sure? Thus, $s' \rhd^{p,\text{sure}}_{\exists} s$? Equivalently: $s \ntriangleright^{\neg p}_{\exists} s'$? To know this we need to know whether $\neg p \in T_i$. Or can we use $\neg(s' \rhd^{p}_{\exists} s) \Rightarrow s' \ntriangleright^{p}_{\exists} s$?

Previously:

- $\neg(s' \rhd^{p}_{\forall} s) \Leftrightarrow s' \ntriangleright^{p}_{\exists} s \Leftrightarrow s \rhd^{\neg p,\text{sure}}_{\exists} s' \Leftrightarrow \neg(s \ntriangleright^{\neg p,\text{sure}}_{\forall} s')$.
- Require (as an axiom) that $s' \rhd^{p}_{\exists} s \vee s' \ntriangleright^{p}_{\exists} s$. This yields $\neg(s' \rhd^{p}_{\forall} s \wedge s' \ntriangleright^{p}_{\forall} s)$.

Define $T_i \subseteq T^*$ as the set of propositions that are weakly accepted.

Definition 1 (Weak acceptance)**.** *Define a situation* $(\mathscr{A}, S^*, \{\rhd^{p}_{\exists}\})$. *A proposition* $p \in T^*$ *is weakly accepted,* $p \in T_i$*, iff* $\exists s \in S^* \mid \forall s' \in S^* : s' \ntriangleright^{p}_{\forall} s$.

It follows that $\neg p \notin T_i$ iff $\forall s : \exists s' \mid s' \rhd^{\neg p}_{\exists} s$.

Definition 2 (Strong rejection)**.** *Define a situation* $(\mathscr{A}, S^*, \{\rhd^{p}_{\exists}\})$. *A proposition* $\neg p \in T^*$ *is strongly rejected iff* $\forall s \in S^*, \exists s' \in S^* \mid s \ntriangleright^{p,sure}_{\forall} s'$.

Thus, strong rejection mandates s' such that $s' \rhd^{\neg p}_{\forall} s$.

If $\neg p$ is strongly rejected, then it is not weakly accepted ($\neg p \notin T_i$) as $s' \rhd^{\neg p}_{\forall} s \Rightarrow s' \rhd^{\neg p}_{\exists} s$.

Definition 3 (Clear-cut)**.** *A situation is clear-cut iff* $\neg p \notin T_i \Rightarrow \neg p$ *is strongly rejected.*

First, we want that if situation is CC, 1) exists a valid model, 2) any valid model has the right T_i. A model is valid if: a) can correctly get reinstatement; b) i is justifiably unstable; c) finite arguments; d) model is convincing.

Second, we want that given a situation, if there exists a valid model, then the situation is CC.

Third, perhaps we get as a bonus that there exists a valid model ⇔ the situation is CC3. CC1: ¬p not in wa ⇔ ¬p s rej. CC2: ¬p s rej ⇒ p w acc. CC3: ¬p s rej ⇒ p is sure.

2. Models

$\rhd_\eta$ a (possibly infinite!) DAG over (s, p) pairs, $+$ defined over arguments in $\rhd_\eta$: $(s_1, p) + (s_3, p) = (s', p)$ for some $(s', p) \in \rhd_\eta$. Define $D' \subseteq S_\gamma$ as a subset of arguments (those supporting propositions). Define $S_\gamma \subseteq S^*$ as the set of arguments used in $\rhd_\eta$. Assume it can be represented by $\geq$, a weak order defined on the basis of $\rhd_\eta$, defining at most k equivalence classes, so that our recursive definitions make sense. (TODO make this correct and precise.)

$\rhd_\eta(s_2)$: arguments that s_2 attacks, $s_1 \in \rhd_\eta(s_2) \Leftrightarrow s_2 \rhd_\eta s_1$.

Given $s_1 \in D'$, define $s_2 \succ_\exists s_1$ iff $s_2 \rhd_\exists s_1$.

Given $s_1 \in D'$, define $s_2 \not\succ_\exists s_1$ iff $s_1 \rhd_\exists^{\text{sure}} s_2$.

Define $s_1 \succ_\exists^{\text{sure}} s_2 \Leftrightarrow s_2 \not\succ_\exists s_1$.

s_1 root in $\rhd_\eta \Rightarrow s_1 \in D'$. $s_1 \in D' \Rightarrow s_3 + s_1 \in D'$.

Given $s_3 \in S_\gamma, s_2 \in S_\gamma, s_2 \notin D'$: $s_3 \succ_\exists s_2$ iff $\exists s_1 \in \rhd_\eta(s_2) \mid s_2 \succ_\exists s_1 \wedge s_2 \not\succ_\exists s_3 + s_1$.

Given $s_3 \in S_\gamma, s_2 \in S_\gamma, s_2 \notin D'$: $s_3 \not\succ_\exists s_2$ iff $\exists s_1 \in \rhd_\eta(s_2) \mid s_2 \succ_\exists s_1 \wedge s_2 \succ_\exists s_3 + s_1$.

Check: For $s_4, s_3, s_3 \notin D'$: $s_4 \succ_\exists s_3$ iff $\exists s_2 \mid s_3 \succ_\exists s_2 \wedge s_3 \not\succ_\exists s_4 + s_2$. Thus, $\exists s_2 \mid s_3 \succ_\exists s_2 \wedge \exists s_1 \mid (s_4 + s_2) \succ_\exists s_1 \wedge (s_4 + s_2) \succ_\exists s_3 + s_1$.

Given $s_3 \in S_\gamma, s_2 \in S_\gamma$, with $\exists s_1 \in \rhd_\eta(s_2) \mid s_2 \succ_\exists s_1$, we have: $s_3 \succ_\exists s_2 \vee s_3 \not\succ_\exists s_2$.

Define $\succ_\forall = \neg\not\succ_\exists$. Hence, $\neg(s \succ_\exists^{\text{sure}} s') \Leftrightarrow s \not\succ_\forall^{\text{sure}} s' \Leftrightarrow \neg(s' \not\succ_\exists s) \Leftrightarrow s' \succ_\forall s$.

Define $\not\succ_\forall = \neg\succ_\exists$.

Hence, given $s_3 \in S_\gamma, s_2 \in S_\gamma, s_2 \notin D'$: $s_3 \succ_\forall s_2$ iff $\forall s_1 \in \rhd_\eta(s_2) \cap \succ_\exists(s_2)$: $s_2 \not\succ_\forall s_3 + s_1$.

3. Conditions

Definition 4 (Reinstatement)**.** *Given a model $\mu = (\rhd_\eta, D')$ and a decision situation $(\mathscr{A}, S^*, \rhd_\exists)$, given $s_1 \in D', s_2 \rhd_\eta s_1, s_2 \succ_\exists s_1, s_3 \succ_\exists s_2, \rhd_\eta^{-1}(s_3) = \varnothing$: $\rhd_\eta(s_1) \subseteq \rhd_\eta(s_1 + s_3) \wedge \rhd_\eta^{-1}(s_1 + s_3) \subseteq \rhd_\eta^{-1}(s_1) \setminus \rhd_\eta(s_3)$.*

Definition 5 (Observable validity)**.** *Given as above, the model is observably valid iff* $\forall s_2 \rhd_\eta s : \neg(s_2 \succ_\exists s) \vee \exists s_3 \rhd_\eta s_2 \mid s_3 \succ_\exists s_2$.

Definition 6 (Justifiable unstability)**.** *Given as above…*

A. Todo

Road map.

- P1: p_a is w-a or $p_{\neg a}$ is w-a
- Define p_a is strongly accepted; $p_{\neg a}$ is strongly rejected, so that they are equivalent.
- P2: p_a is w-a or $p_{\neg a}$ is strongly accepted.

Other todos.

- If i does not consider s as supporting p, it also works: if p is not weakly acceptable by default, then any s' is considered by i as a better argument than s in favor of certain $\neg p$, and so on. In fact, whether $\emptyset \rhd_\exists^p \emptyset$ determines whether p is weakly supported by default.
- I should define $s'(\Box\rhd_\exists^p)s$ as an observable: "Assuming s' would survive, do you consider s' as leading to certainty of $\neg p$, even when considering s?". It distinguishes our knowledge and the truth: $s'(\Box\rhd_\exists^p)s \Rightarrow s' \rhd_\exists^p s$, thus, implies $\neg(s' \not\rhd_\forall^p s)$. But out of $\neg(s'(\Box\rhd_\exists^p)s)$, nothing.
- Partition (objectively) S^* (or $S^* \times T^*$) into arguments in favor of p, sure, $\neg p$, sure, and similarly for possible. Use only one rel $\rhd_\exists$, defined on contradictory arguments only, instead of $\rhd_\exists^{p,\text{sure}}$ and others. Define $s' \rhd_\exists^p s$ equals no when $\neg(s' \rightsquigarrow \neg p, \text{sure})$, equals $\rhd_\exists$ for adequate arguments, and equals yes when $\neg(s \rightsquigarrow p, \text{possible})$ and $s' \rightsquigarrow \neg p$, sure, with probably some complications needed for the argument $\emptyset$ (and related default attitude towards p).

Questions: Q1. Relationship with $s \rhd_\exists^p s$?

We want to exclude: s supports p perhaps, attacked by $s2$ (supporting $\neg p$ sure), but then $s2$ is attacked by s. Exclude $s' \rhd_\exists^p s$ and $s \rhd_\exists^{\neg p,\text{sure}} s'$. Require to assume that this situation implies another argument s_3 "attacking" s', thus, such that $s_3 + s$ is no more attacked by s_2.

B. To think

Propositions weakly self-supported $T \subseteq T^*$: weakly accepted if no arg is given. Examples: m = "eat miam"; $\neg b$ = "beurk is to exclude"; or, in a problem where there's no particularly good aliments, both a = "eat this" and $\neg a$.

When given (s, p), i may say: s does not survive; or: assuming s survives, then s supports p, or, assuming s survives, then s does not support p anyway.

When given s' against s, i may say: s' does not survive, or: assuming s' survives, then s' supports $\neg p$, …

Given $(s_2, p), (s_1, \neg p) \in D$, define $\neg(s_2 \succ_{\exists\neg p}^{\text{neg}} s_1)$ iff for some $(s, p) \in D$, where $s_1 \rhd_\exists^p s$: $s_1 \rhd_\exists^p s + s_2$. Equivalently: $s_2 \succ_{\exists\neg p}^{\text{neg}} s_1$ iff for all s, where $s_1 \rhd_\exists^p s$: $\neg(s_1 \rhd_\exists^p s + s_2)$. (This does not seem right: if given s_3 attacking s_2, and not given s_4 which would convincingly rebut s_3, then temporarily it may hold again that $s_1 \rhd_\exists^p s + s2$ (in the sense that $s_1 + s_3 \rhd_\exists^p s + s_2$).)

$s_2 \succ_{\exists\neg p} s_1$ can perhaps be queried directly by asking (in the context of some $s_1 \rhd_\exists^p s$): "assume s_2 survives, then does s_2 counter s_1?" (In the sense that s_2 is sufficiently convincing that p holds perhaps, to cancel the argument s_1 according to which $\neg p$ surely holds.)

C. Certainties

Looking for certainties. Those propositions that are in the reflexive preferences in a demanding sense: there is a strong enough reason to prefer it than its contrary.

- $s' \succ_\exists s$: weak attack; s' renders s invalid (can't be used to say that t holds for sure) (assuming s' survives)

- Propositions strongly self-supported: strongly accepted if no arg is given. Examples: m = "eat miam"; $\neg b$ = "beurk is to exclude". We might have neither c nor $\neg c$ in that set.

Definition 7 (Sure acceptance)**.** *Define a situation* $(\mathscr{A}, S^*, \{\rhd_\exists^p\})$. *A proposition* $p \in T^*$ *is accepted as sure iff* $\exists s' \in S^* \mid \forall s \in S^* : s \not\rhd_\forall^{p,sure} s'$.

Assume we use rather: if p is not sure, then $\neg p$ is weakly accepted (by def). Then we have never problems of inconsistency! But we could be in a situation where p is not accepted as sure but nobody can tell why because it is fundamentally unstable (sometimes p being accepted, sometimes not).

D. Example

s2 argues in favor of p against s1: s ”le monde n’est pas fiable”. s1 ”le monde est fiable, bhl l’a dit”. s2 ”bhl est un clown, il s’est planté sur l’Irak”. s3 ”il avait raison sur l’Irak : l’Irak a des ADM”. s4 ”l’Irak n’a pas d’ADM, Bush l’a reconnu”. Does s4 attack s3? ”bhl est un clown, il s’est planté sur l’irak” + ”l’irak n’a pas d’ADM, Bush l’a reconnu” VS ”il avait raison sur l’Irak : l’Irak a des ADM” !

Measure problem?