International Economics II: course notes *

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1.1 Set up

- $U(\bar{c}) = \sum_{1 \leq i \leq n} u(c_i)$, with $\bar{c} = \{c_i\}_{1 \leq i \leq n}$
- $u(0) = 0, u'(0) > 0, u''(0) < 0$
- Elasticity of demand of good i: $\epsilon_i = \frac{\partial c_i}{\partial p_i} \frac{p_i}{c_i}$, we assume $\epsilon_i < -1$
- Define $\sigma(c_i) = -\epsilon_i$, thus $1 < \sigma(c_i)$
- Increasing Return to Scale (IRS): workload required for producing q_i units is $l_i = f + q_i/\varphi$; f the fixed cost, φ the productivity

*Course given by Gabriel Smagghue

1.2 Consumption

- From Lagrangien for utility maximization subject to revenue constraint, we obtain $u'(c_i) = \lambda p_i$
- $d\lambda p_i + \lambda dp_i = u''(c_i)dc_i$
- We assume $\frac{d\lambda}{dp_i} = 0$ (large number of varieties), whence $\lambda = u''(c_i)\frac{dc_i}{dp_i}$
- Thus, $u'(c_i)/p_i = u''(c_i)\frac{dc_i}{dp_i}$
- We obtain $\sigma(c_i) = -\frac{u'(c_i)}{u''(c_i)c_i} > 0$

1.3 Production

- Cost of producing q_i units is wl_i; w the wage
- L identical consumers thus $q_i = Lc_i$; whence $\sigma(c_i) = -\frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i}$
- Marginal Cost is $MC = \frac{w}{\varphi}$
- Revenue of the firm on product i is $R_i = p_i q_i$ (and $R = \sum_{1\leq i\leq n} p_i q_i$)
- Marginal Revenue is $MR_i = \frac{dp_i}{dq_i}q_i + p_i = p_i(1 - \frac{1}{\sigma(c_i)})$
- To optimize profit $\pi_i = p_i q_i - wf - \frac{w}{\varphi}q_i$, set $MR_i = MC$
- We obtain the Profit max condition (PP): $\frac{p_i}{w} = \frac{\sigma(c_i)}{\sigma(c_i)-1}\frac{1}{\varphi}$
- (Average Cost is $AC_i = f/q_i + w/\varphi$)

1.4 Solving

- From $\pi_i = 0$, we obtain the Free entry condition (ZZ): $\frac{p_i}{w} = \frac{f}{Lc_i} + \frac{1}{\varphi}$
- $L = \sum_{1\leq i\leq n} l_i = nf + \frac{\sum_{1\leq i\leq n} q_i}{\varphi}$
- By symmetry, $c_i = c$, $q_i = q$, $p_i = p$
- $L = nf + \frac{nLc}{\varphi}$ thus $n = \frac{L}{f + \frac{Lc}{\varphi}} = \frac{1}{\frac{f}{L} + \frac{c}{\varphi}}$

1.5 In the $(c, p/w)$ space

ZZ curve:

- strictly decreasing
- higher consumption, higher production, lower average costs, lower p/w
- with c constant, bigger L, more sales, lower p/w

PP curve:

- Flat iff $\sigma'(c) = 0$
- Strictly increasing if $\sigma'(c) < 0$ ($\frac{\sigma(c_i)}{\sigma(c_i)-1} = 1 + \frac{1}{\sigma(c_i)-1}$, $\frac{1}{\sigma(c_i)-1}$ composes two decreasing functions)
- $\sigma'(c) < 0$ behaviorally reasonable: bigger consumers are richer thus less sensitive to prices
- Higher consumption, less elastic demand, higher markup, more market power, higher prices

1.6 Doubling market size

Let's double L

- ZZ curve shifts downwards
- $c_1 < c_0$
- More varieties: $n_0 = \frac{1}{\frac{f}{L} + \frac{c_0}{\varphi}} < \frac{1}{\frac{f}{2L} + \frac{c_1}{\varphi}} = n_1$
- Under assumption $\sigma'(c) < 0$, $\sigma(c_0) < \sigma(c_1)$ thus $(\frac{p}{w})_1 < (\frac{p}{w})_0$
- Under assumption $\sigma'(c) < 0$, some firms exit: $n_1 = \frac{1}{\frac{f}{2L} + \frac{c_1}{\varphi}} < \frac{1}{\frac{f}{2L} + \frac{c_0}{2\varphi}} = 2n_0$, equivalently, $c_0 < 2c_1$. Proof: let $\left(c_1, \left(\frac{p}{w}\right)_1\right)$ be the equilibrium after doubling L, thus $\left(2c_1, \left(\frac{p}{w}\right)_1\right)$ is on the ZZ curve before doubling L, and all points with $c \geq 2c_1$ are strictly below the PP curve (because its derivative is positive) and not below the ZZ curve (because its derivative is negative) so not equilibrium points.

2 CES utility

2.1 Consumer

2.1.1 Set up

- A single consumer utility is $U(\bar{c}) = (\sum_{1\leq i\leq n} c_i^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$ (a country utility is $U(L\bar{c}) = LU(\bar{c})$)
- Revenue constraint is $w = \sum p_i c_i$

2.1.2 Demand

- From the Lagrangian we obtain $\frac{\sigma}{\sigma-1}U(\bar{c})^{\frac{1}{\sigma}}\frac{\sigma-1}{\sigma}c_i^{\frac{\sigma-1}{\sigma}-1} - \lambda p_i = 0$ (because $\frac{\sigma}{\sigma-1} - 1 = \frac{\sigma}{\sigma-1}\frac{1}{\sigma}$), thus $c_i = \lambda^{-\sigma}p_i^{-\sigma}U(\bar{c})$, so that $\sum p_i c_i = w = \sum p_i^{1-\sigma}\lambda^{-\sigma}U(\bar{c})$
- Thus, $\lambda^{-\sigma}U(\bar{c}) = \frac{w}{\sum p_i^{1-\sigma}}$, then $c_i = p_i^{-\sigma}\frac{w}{\sum p_k^{1-\sigma}}$, and using $q_i = Lc_i$, we obtain $q_i = p_i^{-\sigma}\frac{wL}{\sum p_k^{1-\sigma}}$

2.1.3 Price index

- $U(\bar{c}) = (\sum p_i^{1-\sigma})^{\frac{\sigma}{\sigma-1}}\lambda^{-\sigma}U(\bar{c})$, whence $\lambda^{-\sigma} = (\sum p_i^{1-\sigma})^{\frac{\sigma}{1-\sigma}}$, and using $\frac{w}{U(\bar{c})} = \sum p_i^{1-\sigma}\lambda^{-\sigma}$, we get $\frac{w}{U(\bar{c})} = (\sum p_i^{1-\sigma})^{\frac{1}{1-\sigma}}$
- Define the ideal price index as $P = \frac{w}{U(\bar{c})} = (\sum p_i^{1-\sigma})^{\frac{1}{1-\sigma}}$: it increases in the same way that the welfare decreases (price-index elasticity of utility is minus one); it is the cost of one unit of happiness
- We can write $q_i = (\frac{p_i}{P})^{-\sigma}\frac{wL}{P}$
- We obtain elasticity of demand $\epsilon_i = \frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i} = \left(-\sigma q_i(\frac{p_i}{P})^{-1}\frac{\partial}{\partial p_i}(\frac{p_i}{P})\right)\frac{p_i}{q_i}$, and, considering P as constant with respect to p_i (which seems to hold if considering a continuous world of varieties which each have null measure), $\epsilon_i \approx -\sigma$
- Given goods $i \neq j$, elasticity of substitution $\epsilon_{ij} = \frac{\partial ln(q_i/q_j)}{\partial ln(p_j/p_i)} = -\sigma$ (using $\frac{q_i}{q_j} = (\frac{p_i}{p_j})^{-\sigma}$)
- Using symmetric prices, $P = n^{\frac{1}{1-\sigma}}p$
- We have $U_j(\bar{q}) = \frac{Lw}{p}n^{\frac{1}{\sigma-1}}$ thus $\frac{\partial \log U}{\partial \log n} = \frac{1}{\sigma-1}$
- When varieties increase, welfare increases at rate $\frac{\partial \log U}{\partial \log n} = \frac{1}{\sigma-1}$

2.2 Producer

- As above, $l_i = f + q_i/\varphi$; to optimize profit set $MR_i = \frac{\partial p_i}{\partial q_i} q_i + p_i = \frac{p_i}{\epsilon_i} + p_i = MC = \frac{w}{\varphi}$, obtain $\frac{p_i}{w} = \frac{\sigma}{\sigma-1}\frac{1}{\varphi}$
- At zero profit $\frac{\pi_i}{w} = \frac{p_i}{w} q_i - f - \frac{1}{\varphi} q_i = 0$, we obtain a constant output per variety, $q_i = \varphi f(\sigma - 1)$
- Using symmetric goods and market clearing condition $L = nf + n\frac{q}{\varphi}$, we obtain $n = \frac{L}{\sigma f}$

2.3 Solving, in autarky

- We get $P = (\frac{\sigma f}{L})^{\frac{1}{\sigma-1}} \frac{\sigma}{\sigma-1}\frac{w}{\varphi}$
- We see that increasing n, or increasing L, decreases the price index, thus increases welfare

2.4 Iceberg trading

Two countries $\{D, X\}$ with same parameters except for sizes L_j ($j \in \{D, X\}$) open up and starts trading; we write their initial variable levels (in autarky) as $v_j^{(A)}$ and resulting variable levels (under exchange) as v_j; given $j \in \{D, X\}$, let $\bar{j}$ denote the other country.

- Prices of goods produced and sold in a given country $j \in \{D, X\}$ are still $p_j = \frac{\sigma}{\sigma-1}\frac{w_j}{\varphi}$
- Iceberg type trading cost: goods sold in the other country have price $p_{j\to\bar{j}} = p_{j\to\bar{j}} = \tau p_j$, with $1 \leq \tau$
- Total production of a good produced in j becomes $q_j = q_{j\to j} + \tau q_{j\to\bar{j}}$, with $q_{j\to\bar{j}} = q_{j\to\bar{j}}$ the quantity that effectively arrives abroad
- Thus the enterprise sells q_j at price p_j, equivalently, $q_{j\to j}$ at price p_j domestically and $q_{j\to\bar{j}}$ at price τp_j abroad
- It makes profit $\pi_j = p_j q_j - w_j(f + \frac{q_j}{\varphi}) = \frac{1}{\sigma-1}\frac{w_j q_j}{\varphi} - w_j f$
- From $\pi_j = 0$ we obtain $q_j = q_j = \varphi f(\sigma - 1) = q_j^{(A)}$
- Using $L_j = n_j f + n_j \frac{q_j}{\varphi}$, we obtain $n_j = n_j = \frac{L_j}{\sigma f} = n_j^{(A)}$
- $P_j = \left[n_j {p_j}^{1-\sigma} + n_{\bar{j}} (\tau p_{\bar{j}})^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$
- With $\tau = 1$ and $n_j = n_{\bar{j}}$, we get $P_j = \frac{P_j^{(A)}}{2^{\frac{1}{\sigma-1}}}$: price index decreases
- No pro-competitive effect: σ fixed, does not contribute to lowering prices

- From above, $q_{j\to\bar{j}} = \left(\frac{p_{j\to\bar{j}}}{P_{\bar{j}}}\right)^{-\sigma} \frac{w_{\bar{j}} L_{\bar{j}}}{P_{\bar{j}}}$, thus $q_{j\to\bar{j}} = \left(\frac{\tau p_j}{P_{\bar{j}}}\right)^{-\sigma} \frac{w_{\bar{j}} L_{\bar{j}}}{P_{\bar{j}}}$

- It follows that the value $X_j = \tau p_j n_j q_{j\to\bar{j}}$ of aggregate exports (the gravity equation) depends linearly on $L_j L_{\bar{j}}$ (shouldn't account for P which depends on n which depends on L?) and on $\tau^{1-\sigma}$ (note also that $0 \le \tau^{1-\sigma} \le 1$)

- From trade balance $X_j = X_{\bar{j}}$, $\frac{p_j}{p_{\bar{j}}} = \frac{w_j}{w_{\bar{j}}}$, $\frac{n_j}{n_{\bar{j}}} = \frac{L_j}{L_{\bar{j}}}$ and $\frac{q_{j\to\bar{j}}}{q_{\bar{j}\to j}} = \frac{w_j{}^{-\sigma} w_{\bar{j}} L_{\bar{j}} P_j{}^{1-\sigma}}{w_{\bar{j}}{}^{-\sigma} w_j L_j P_{\bar{j}}{}^{1-\sigma}}$, it follows that $\frac{X_j}{X_{\bar{j}}} = 1 = \frac{w_j{}^{-\sigma} P_j{}^{1-\sigma}}{w_{\bar{j}}{}^{-\sigma} P_{\bar{j}}{}^{1-\sigma}}$, whence $\left(\frac{w_j}{w_{\bar{j}}}\right)^{\sigma} = \left(\frac{P_j}{P_{\bar{j}}}\right)^{1-\sigma}$

- Recall that largest country had smaller prices (Sl. 20, do we mean higher real wages $\frac{w_j^{(A)}}{P_j^{(A)}}$? Can we compare utilities? True even with $\tau = 1$, right?); trade balance obliges to keep wages higher in the biggest country, which lowers its exports (unless $\tau = 1$, why?)

- With $\tau = 1$, we get that $w_j = w_{\bar{j}}$

- With $1 < \tau$ (thus $t = \tau^{1-\sigma} < 1$), we can see that the largest country has higher wages: from $w^{\sigma} = \frac{Lw^{1-\sigma}+t}{1+tLw^{1-\sigma}}$, with $w = \frac{w_j}{w_{\bar{j}}}$ and $L = \frac{L_j}{L_{\bar{j}}}$, thus $w^{\sigma} + tLw - Lw^{1-\sigma} - t = 0$, plugging $w = 1$ yields $w^{\sigma} + tLw - Lw^{1-\sigma} - t = (1-t)(1-L) = (t-1)(L-1)$, whose sign is the inverse of $L-1$, and the expression is increasing in w (its derivative is positive), thus $0 < L - 1 \Leftrightarrow 1 < L \Leftrightarrow 1 < w$ (is there an easier argument?)

- With $\tau = \infty$ (thus $\tau^{1-\sigma} = 0$), we can conclude more directly: $\left(\frac{w_j}{w_{\bar{j}}}\right)^{\sigma} = \frac{n_j}{n_{\bar{j}}} \left(\frac{p_j}{p_{\bar{j}}}\right)^{1-\sigma} = \frac{L_j}{L_{\bar{j}}} \left(\frac{w_j}{w_{\bar{j}}}\right)^{1-\sigma}$, thus $\frac{w_j}{w_{\bar{j}}} = \left(\frac{L_j}{L_{\bar{j}}}\right)^{\frac{1}{2\sigma-1}}$

Mechanisms through which trade increases:

τ **finite** Trade increases through extensive margins: countries start exchanging every varieties

τ **reduces** Trade increases through intensive margins: higher export in quantity but constant number of varieties

2.5 Example

- Set $p = 1$, $\varphi = \sigma = 2$ and $f = 1/2$

- Producer: $w^{(A)} = 1$, $q = 1$

- Solving: $n_j = L_j$, $P_j^{(A)} = 1/L_j$

- Consumer: $c_j = 1/L_j$, $U(c_j) = L_j$

- Set $L_D = 1$ and $L_X = 2$
- $n_D = P_D^{(\mathrm{A})} = c_D = U(c_D) = 1$
- $n_X = U(c_X) = 2$, $P_X^{(\mathrm{A})} = c_X = \frac{1}{2}$
- $p_{j\to\bar{j}} = \tau$
- $P_D^{(\mathrm{E})} = 1/(1 + 2/\tau) = \tau/(\tau + 2)$
- $P_X^{(\mathrm{E})} = 1/(2 + 1/\tau) = \tau/(2\tau + 1)$

3 Melitz

3.1 Various notes

Problem is that more productive firms seem to export, which the model does not say; this is not solvable even with different trading costs (not sure why)

Here we implicitly normalize by setting $w = 1$.

We write φ_d the equivalent of φ_a under non-autarky so actually $\varphi_a^{(1)}$ as compared to $\varphi_a^{(0)}$

Note that $\pi(\varphi)$ is the profit of a single firm of the type φ; to get the profit of the whole type φ we'd need to consider the density g.

Note that there is no aggregate profit to redistribute because the total profit is precisely the total entry cost (zero entry condition).

3.2 Set up

- We normalize the wage to $w = 1$
- Productivity φ is drawn from a productivity space $\Phi \subseteq (0, \infty)$ under probability measure $\mu : \mathcal{B}(\Phi) \to [0, 1]$ (thus $\mu(\emptyset) = 0$, $\mu(\Phi) = 1$); we write its CDF, given $\varphi \in \Phi$, as $G(\varphi) = \mu((0, \varphi])$, corresponding to the probability that productivity is at most φ

3.3 Consumer

- The demand of a single customer of a good from firm with productivity φ is c_φ
- A single consumer utility is $U(\bar{c}) = (\int_\Phi c_\varphi^{\frac{\sigma-1}{\sigma}} d\varphi)^{\frac{\sigma}{\sigma-1}}$ (a country utility is $U(L\bar{c}) = LU(\bar{c})$)
- Revenue constraint is $1 = \sum p_i c_i$
- As above, we define $P = \frac{1}{U(\bar{c})} = (\int_\Phi p_\varphi^{1-\sigma} d\varphi)^{\frac{1}{1-\sigma}}$ and obtain $c_\varphi = \frac{1}{P^{1-\sigma}} \frac{1}{p_\varphi^\sigma}$ and $q_\varphi = \frac{L}{P^{1-\sigma}} \frac{1}{p_\varphi^\sigma}$

3.4 Producer

- Fixed costs f_d to serve domestic market and f_x to serve foreign market
- As above, we obtain $p_\varphi = \frac{\sigma}{\sigma-1}\frac{1}{\varphi}$
- Revenue of firm of type φ is $r(\varphi) = p_\varphi q_\varphi = \frac{L}{P^{1-\sigma}}\frac{1}{p_\varphi^{\sigma-1}} = A\varphi^{\sigma-1}$ with $A = \frac{L}{P^{1-\sigma}}(\frac{\sigma-1}{\sigma})^{\sigma-1}$
- Profit is $\pi(\varphi) = p_\varphi q_\varphi - \frac{q_\varphi}{\varphi} - f_d = p_\varphi q_\varphi(1-\frac{1}{p_\varphi\varphi}) - f_d = p_\varphi q_\varphi(1-\frac{\sigma-1}{\sigma}) - f_d = A\varphi^{\sigma-1}(\frac{1}{\sigma}) - f_d = B\varphi^{\sigma-1} - f_d$ with $B = \frac{A}{\sigma}$
- Zero Cutoff Profit condition: introduce the level φ_a such that $\pi(\varphi_a) = 0$
- Firms in $(0, \varphi_a)$ exit, set with measure $G(\varphi_a)$; define $\Phi^* = (\varphi_a, \infty)$ as the active firms, with measure $\mu(\Phi^*)$; define $\mu^* = \frac{\mu}{\mu(\Phi^*)}$ as the measure normalized for the active firms (thus, the conditional distribution of productivity given entry)
- Define fixed entry cost $f_e = \int_{\Phi^*}\pi d\mu$ as the expected profit of entering
- Let M_e be the mass of drawers (potential entrants), then the mass of producers (entered) is $M_a = M_e\mu(\Phi^*)$; define $\nu^* = M_a\mu^*$ as the measure $\nu^*(F)$ of firms of kind F conditional to activity
- Define aggregate revenue as $R_a = \int r(\varphi)d\nu^* = M_a\int r(\varphi)d\mu^*$
- Aggregate productivity is defined as the productivity $\bar\varphi(\varphi_a) = r^{-1}(\frac{R_a}{M_a})$ that yields average revenue, and, from $\frac{R_a}{M_a} = A\int\varphi^{\sigma-1}d\mu^*$, $\bar\varphi(\varphi_a)^{\sigma-1} = \int\varphi^{\sigma-1}d\mu^*$
- Similarly, with Π_a the aggregate profit, expected profit conditional on entry is $\frac{\Pi_a}{M_a} = \int\pi(\varphi)d\mu^* = B\int\varphi^{\sigma-1}d\mu^* - f_d = \pi(\bar\varphi(\varphi_a))$
- Expected profit (unconditionally) is $\int\pi(\varphi)d\mu = \frac{\Pi_a}{M_e} = \mu(\Phi^*)\pi(\bar\varphi(\varphi_a))$, thus $\Pi_a = M_e\mu(\Phi^*)\pi(\bar\varphi(\varphi_a)) = M_a\pi(\bar\varphi(\varphi_a)) = M_a\int\pi(\varphi)d\mu^* = \frac{M_a}{\mu(\Phi^*)}\int_{\Phi^*}\pi(\varphi)d\mu = M_e\int_{\Phi^*}\pi(\varphi)d\mu = M_e f_e$
- Price index under autarky of producers (?) is $P_a = (M_a\int_{\Phi^*}p_\varphi^{1-\sigma}d\mu^*)^{\frac{1}{1-\sigma}} = M_a^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma-1}(\int_{\Phi^*}(\frac{1}{\varphi})^{1-\sigma}d\mu^*)^{\frac{1}{1-\sigma}} = M_a^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma-1}\frac{1}{\bar\varphi(\varphi_a)}$
- We also use $B_a\varphi^{\sigma-1} = \frac{\varphi^{\sigma-1}-\varphi_a^{\sigma-1}}{\varphi_a^{\sigma-1}}$ (dunno why)
- Obtain $f_e = f_d J(\varphi_a)$ with $J(\varphi_a) = \int_{\Phi^*}\left((\frac{\varphi}{\varphi_a})^{\sigma-1} - 1\right)d\mu$, as $f_e = \int_{\Phi^*}\pi d\mu = \int_{\Phi^*}(\frac{r(\varphi)}{\sigma} - f_d)d\mu = \int_{\Phi^*}(\frac{A\varphi^{\sigma-1}}{\sigma} - f_d)d\mu$ and as $\pi(\varphi_a) = 0$, thus $\frac{A}{\sigma}\varphi_a^{\sigma-1} - f_d = 0$
- Also, $\frac{\partial f_e}{\partial\varphi_a} = -\frac{\sigma-1}{\varphi_a^{\sigma-1}}(-\varphi_a^{\sigma-1})g(\varphi_a)$

4 Gravity equation

- X_{ij} the exports from i to j
- $Y_i = \sum_j X_{ij}$ is all the exports from i to the world, including to itself, so it's the production
- $X_j = \sum_i X_{ij}$ is all the imports from the world to j, including from itself, so it's total consumption
- Here we do not assume that $Y_i = X_i$: some countries have imbalance, for reasons not explained by this approach (capital investments, which could compensate, are not considered); though we do assume market clear, so the sum of trade deficits is zero
- With some models we will assume that the quality of the good ω is $a_{ij\omega}$ thus reflects intrinsic quality of the good and its appreciation by consumers
- There is some competition that bring profit to zero, within one country, for a given variety
- Note that on slide 28 there should be no ω
- We assume a single ω per source country, so ω and i are the same thing

When total production and total consumption are equal, we can equal Y_j and X_j $X_{ij} = \frac{Y_i}{}$