

International Economics II: course notes *

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1.1 Set up

- $U(\bar{c}) = \sum_{1 \leq i \leq n} u(c_i)$, with $\bar{c} = \{c_i\}_{1 \leq i \leq n}$
- $u(0) = 0, u'(0) > 0, u''(0) < 0$
- Elasticity of demand of good i : $\epsilon_i = \frac{\partial c_i}{\partial p_i} \frac{p_i}{c_i}$, we assume $\epsilon_i < -1$
- Define $\sigma(c_i) = -\epsilon_i$, thus $1 < \sigma(c_i)$
- Increasing Return to Scale (IRS): workload required for producing q_i units is $l_i = f + q_i/\varphi$; f the fixed cost, φ the productivity

1.2 Consumption

- From Lagrangien for utility maximization subject to revenue constraint, we obtain $u'(c_i) = \lambda p_i$
- $d\lambda p_i + \lambda dp_i = u''(c_i) dc_i$
- We assume $\frac{d\lambda}{dp_i} = 0$ (large number of varieties), whence $\lambda = u''(c_i) \frac{dc_i}{dp_i}$
- Thus, $u'(c_i)/p_i = u''(c_i) \frac{dc_i}{dp_i}$
- We obtain $\sigma(c_i) = -\frac{u'(c_i)}{u''(c_i)c_i} > 0$

*Course given by Gabriel Smagghue

1.3 Production

- Cost of producing q_i units is wl_i ; w the wage
- L identical consumers thus $q_i = Lc_i$; whence $\sigma(c_i) = -\frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i}$
- Marginal Cost is $MC = \frac{w}{\varphi}$
- Revenue of the firm on product i is $R_i = p_i q_i$ (and $R = \sum_{1 \leq i \leq n} p_i q_i$)
- Marginal Revenue is $MR_i = \frac{dp_i}{dq_i} q_i + p_i = p_i(1 - \frac{1}{\sigma(c_i)})$
- To optimize profit $\pi_i = p_i q_i - wf - \frac{w}{\varphi} q_i$, set $MR_i = MC$
- We obtain the Profit max condition (PP): $\frac{p_i}{w} = \frac{\sigma(c_i)}{\sigma(c_i)-1} \frac{1}{\varphi}$
- (Average Cost is $AC_i = f/q_i + w/\varphi$)

1.4 Solving

- From $\pi_i = 0$, we obtain the Free entry condition (ZZ): $\frac{p_i}{w} = \frac{f}{Lc_i} + \frac{1}{\varphi}$
- $L = \sum_{1 \leq i \leq n} l_i = nf + \frac{\sum_{1 \leq i \leq n} q_i}{\varphi}$
- By symmetry, $c_i = c$, $q_i = q$, $p_i = p$
- $L = nf + \frac{nLc}{\varphi}$ thus $n = \frac{L}{f + \frac{Lc}{\varphi}} = \frac{1}{\frac{f}{L} + \frac{c}{\varphi}}$

1.5 In the $(c, p/w)$ space

ZZ curve:

- strictly decreasing
- higher consumption, higher production, lower average costs, lower p/w
- with c constant, bigger L , more sales, lower p/w

PP curve:

- Flat iff $\sigma'(c) = 0$
- Strictly increasing if $\sigma'(c) < 0$ ($\frac{\sigma(c_i)}{\sigma(c_i)-1} = 1 + \frac{1}{\sigma(c_i)-1}$, $\frac{1}{\sigma(c_i)-1}$ composes two decreasing functions)
- $\sigma'(c) < 0$ behaviorally reasonable: bigger consumers are richer thus less sensitive to prices
- Higher consumption, less elastic demand, higher markup, more market power, higher prices

1.6 Doubling market size

Let's double L

- ZZ curve shifts downwards
- $c_1 < c_0$
- More varieties: $n_0 = \frac{1}{\frac{f}{L} + \frac{c_0}{\varphi}} < \frac{1}{\frac{f}{2L} + \frac{c_1}{\varphi}} = n_1$
- Under assumption $\sigma'(c) < 0$, $\sigma(c_0) < \sigma(c_1)$ thus $(\frac{p}{w})_1 < (\frac{p}{w})_0$
- Under assumption $\sigma'(c) < 0$, some firms exit: $n_1 = \frac{1}{\frac{f}{2L} + \frac{c_1}{\varphi}} < \frac{1}{\frac{f}{2L} + \frac{c_0}{2\varphi}} = 2n_0$, equivalently, $c_0 < 2c_1$. Proof: let $(c_1, (\frac{p}{w})_1)$ be the equilibrium after doubling L , thus $(2c_1, (\frac{p}{w})_1)$ is on the ZZ curve before doubling L , and all points with $c \geq 2c_1$ are strictly below the PP curve (because its derivative is positive) and not below the ZZ curve (because its derivative is negative) so not equilibrium points.

2 CES utility

2.1 Consumer

2.1.1 Set up

- A single consumer utility is $U(\bar{c}) = (\sum_{1 \leq i \leq n} c_i^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$ (a country utility is $U(L\bar{c}) = LU(\bar{c})$)
- Revenue constraint is $w = \sum p_i c_i$

2.1.2 Demand

- From the Lagrangian we obtain $\frac{\sigma}{\sigma-1} U(\bar{c})^{\frac{1}{\sigma}} \frac{\sigma-1}{\sigma} c_i^{\frac{\sigma-1}{\sigma}-1} - \lambda p_i = 0$ (because $\frac{\sigma}{\sigma-1} - 1 = \frac{\sigma}{\sigma-1} \frac{1}{\sigma}$), thus $c_i = \lambda^{-\sigma} p_i^{-\sigma} U(\bar{c})$, so that $\sum p_i c_i = w = \sum p_i^{1-\sigma} \lambda^{-\sigma} U(\bar{c})$
- Thus, $\lambda^{-\sigma} U(\bar{c}) = \frac{w}{\sum p_i^{1-\sigma}}$, then $c_i = p_i^{-\sigma} \frac{w}{\sum p_k^{1-\sigma}}$, and using $q_i = Lc_i$, we obtain $q_i = p_i^{-\sigma} \frac{wL}{\sum p_k^{1-\sigma}}$

2.1.3 Price index

- $U(\bar{c}) = (\sum p_i^{1-\sigma})^{\frac{\sigma}{\sigma-1}} \lambda^{-\sigma} U(\bar{c})$, whence $\lambda^{-\sigma} = (\sum p_i^{1-\sigma})^{\frac{\sigma}{1-\sigma}}$, and using $\frac{w}{U(\bar{c})} = \sum p_i^{1-\sigma} \lambda^{-\sigma}$, we get $\frac{w}{U(\bar{c})} = (\sum p_i^{1-\sigma})^{\frac{1}{1-\sigma}}$
- Define the ideal price index as $P = \frac{w}{U(\bar{c})} = (\sum p_i^{1-\sigma})^{\frac{1}{1-\sigma}}$: it increases in the same way that the welfare decreases (price-index elasticity of utility is minus one); it is the cost of one unit of happiness

- We can write $q_i = (\frac{p_i}{P})^{-\sigma} \frac{wL}{P}$
- We obtain elasticity of demand $\epsilon_i = \frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i} = \left(-\sigma q_i (\frac{p_i}{P})^{-1} \frac{\partial}{\partial p_i} (\frac{p_i}{P}) \right) \frac{p_i}{q_i}$, and, considering P as constant with respect to p_i (which seems to hold if considering a continuous world of varieties which each have null measure), $\epsilon_i \approx -\sigma$
- Given goods $i \neq j$, elasticity of substitution $\epsilon_{ij} = \frac{\partial \ln(q_i/q_j)}{\partial \ln(p_j/p_i)} = -\sigma$ (using $\frac{q_i}{q_j} = (\frac{p_i}{p_j})^{-\sigma}$)
- Using symmetric prices, $P = n^{\frac{1}{1-\sigma}} p$
- We have $U_j(\bar{q}) = \frac{Lw}{p} n^{\frac{1}{\sigma-1}}$ thus $\frac{\partial \log U}{\partial \log n} = \frac{1}{\sigma-1}$
- When varieties increase, welfare increases at rate $\frac{\partial \log U}{\partial \log n} = \frac{1}{\sigma-1}$

2.2 Producer

- As above, $l_i = f + q_i/\varphi$; to optimize profit set $MR_i = \frac{\partial p_i}{\partial q_i} q_i + p_i = \frac{p_i}{\epsilon_i} + p_i = MC = \frac{w}{\varphi}$, obtain $\frac{p_i}{w} = \frac{\sigma}{\sigma-1} \frac{1}{\varphi}$
- At zero profit $\frac{\pi_i}{w} = \frac{p_i}{w} q_i - f - \frac{1}{\varphi} q_i = 0$, we obtain a constant output per variety, $q_i = \varphi f(\sigma - 1)$
- Using symmetric goods and market clearing condition $L = nf + n \frac{q}{\varphi}$, we obtain $n = \frac{L}{\sigma f}$

2.3 Solving, in autarky

- We get $P = (\frac{\sigma f}{L})^{\frac{1}{\sigma-1}} \frac{\sigma}{\sigma-1} \frac{w}{\varphi}$
- We see that increasing n , or increasing L , decreases the price index, thus increases welfare

2.4 Iceberg trading

Two countries $\{D, X\}$ with same parameters except for sizes L_j ($j \in \{D, X\}$) open up and starts trading; we write their initial variable levels (in autarky) as $v_j^{(A)}$ and resulting variable levels (under exchange) as v_j ; given $j \in \{D, X\}$, let \bar{j} denote the other country.

- Prices of goods produced and sold in a given country $j \in \{D, X\}$ are still $p_j = \frac{\sigma}{\sigma-1} \frac{w_j}{\varphi}$
- Iceberg type trading cost: goods sold in the other country have price $p_{j \rightarrow \bar{j}} = p_{j \rightarrow \bar{j}} = \tau p_j$, with $1 \leq \tau$
- Total production of a good produced in j becomes $q_j = q_{j \rightarrow j} + \tau q_{j \rightarrow \bar{j}}$, with $q_{j \rightarrow \bar{j}} = q_{j \rightarrow \bar{j}}$ the quantity that effectively arrives abroad

- Thus the enterprise sells q_j at price p_j , equivalently, $q_{j \rightarrow j}$ at price p_j domestically and $q_{j \rightarrow \bar{j}}$ at price τp_j abroad
- It makes profit $\pi_j = p_j q_j - w_j(f + \frac{q_j}{\varphi}) = \frac{1}{\sigma-1} \frac{w_j q_j}{\varphi} - w_j f$
- From $\pi_j = 0$ we obtain $q_j = q_j = \varphi f(\sigma - 1) = q_j^{(A)}$
- Using $L_j = n_j f + n_j \frac{q_j}{\varphi}$, we obtain $n_j = n_j = \frac{L_j}{\sigma f} = n_j^{(A)}$
- $P_j = \left[n_j p_j^{1-\sigma} + n_{\bar{j}} (\tau p_{\bar{j}})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$
- With $\tau = 1$ and $n_j = n_{\bar{j}}$, we get $P_j = \frac{P_j^{(A)}}{2^{\frac{1}{\sigma-1}}}$: price index decreases
- No pro-competitive effect: σ fixed, does not contribute to lowering prices
- From above, $q_{j \rightarrow \bar{j}} = \left(\frac{p_{j \rightarrow \bar{j}}}{P_{\bar{j}}} \right)^{-\sigma} \frac{w_{\bar{j}} L_{\bar{j}}}{P_{\bar{j}}}$, thus $q_{j \rightarrow \bar{j}} = \left(\frac{\tau p_j}{P_{\bar{j}}} \right)^{-\sigma} \frac{w_{\bar{j}} L_{\bar{j}}}{P_{\bar{j}}}$
- It follows that the value $X_j = \tau p_j n_j q_{j \rightarrow \bar{j}}$ of aggregate exports (the gravity equation) depends linearly on $L_j L_{\bar{j}}$ (shouldn't account for P which depends on n which depends on L ?) and on $\tau^{1-\sigma}$ (note also that $0 \leq \tau^{1-\sigma} \leq 1$)
- From trade balance $X_j = X_{\bar{j}}$, $\frac{p_j}{p_{\bar{j}}} = \frac{w_j}{w_{\bar{j}}}$, $\frac{n_j}{n_{\bar{j}}} = \frac{L_j}{L_{\bar{j}}}$ and $\frac{q_{j \rightarrow \bar{j}}}{q_{\bar{j} \rightarrow j}} = \frac{w_j^{-\sigma} w_{\bar{j}} L_{\bar{j}} P_j^{1-\sigma}}{w_{\bar{j}}^{-\sigma} w_j L_j P_{\bar{j}}^{1-\sigma}}$, it follows that $\frac{X_j}{X_{\bar{j}}} = 1 = \frac{w_j^{-\sigma} P_j^{1-\sigma}}{w_{\bar{j}}^{-\sigma} P_{\bar{j}}^{1-\sigma}}$, whence $\left(\frac{w_j}{w_{\bar{j}}} \right)^{\sigma} = \left(\frac{P_j}{P_{\bar{j}}} \right)^{1-\sigma}$
- Recall that largest country had smaller prices (Sl. 20, do we mean higher real wages $\frac{w_j^{(A)}}{P_j^{(A)}}$? Can we compare utilities? True even with $\tau = 1$, right?); trade balance obliges to keep wages higher in the biggest country, which lowers its exports (unless $\tau = 1$, why?)
- With $\tau = 1$, we get that $w_j = w_{\bar{j}}$
- With $1 < \tau$ (thus $t = \tau^{1-\sigma} < 1$), we can see that the largest country has higher wages: from $w^{\sigma} = \frac{L w^{1-\sigma} + t}{1 + t L w^{1-\sigma}}$, with $w = \frac{w_j}{w_{\bar{j}}}$ and $L = \frac{L_j}{L_{\bar{j}}}$, thus $w^{\sigma} + t L w - L w^{1-\sigma} - t = 0$, plugging $w = 1$ yields $w^{\sigma} + t L w - L w^{1-\sigma} - t = (1-t)(1-L) = (t-1)(L-1)$, whose sign is the inverse of $L-1$, and the expression is increasing in w (its derivative is positive), thus $0 < L-1 \Leftrightarrow 1 < L \Leftrightarrow 1 < w$ (is there an easier argument?)
- With $\tau = \infty$ (thus $\tau^{1-\sigma} = 0$), we can conclude more directly: $\left(\frac{w_j}{w_{\bar{j}}} \right)^{\sigma} = \frac{n_j}{n_{\bar{j}}} \left(\frac{p_j}{p_{\bar{j}}} \right)^{1-\sigma} = \frac{L_j}{L_{\bar{j}}} \left(\frac{w_j}{w_{\bar{j}}} \right)^{1-\sigma}$, thus $\frac{w_j}{w_{\bar{j}}} = \left(\frac{L_j}{L_{\bar{j}}} \right)^{\frac{1}{2\sigma-1}}$

Mechanisms through which trade increases:

- τ **finite** Trade increases through extensive margins: countries start exchanging every varieties
- τ **reduces** Trade increases through intensive margins: higher export in quantity but constant number of varieties

2.5 Example

- Set $p = 1$, $\varphi = \sigma = 2$ and $f = 1/2$
- Producer: $w^{(A)} = 1$, $q = 1$
- Solving: $n_j = L_j$, $P_j^{(A)} = 1/L_j$
- Consumer: $c_j = 1/L_j$, $U(c_j) = L_j$
- Set $L_D = 1$ and $L_X = 2$
- $n_D = P_D^{(A)} = c_D = U(c_D) = 1$
- $n_X = U(c_X) = 2$, $P_X^{(A)} = c_X = \frac{1}{2}$
- $p_{j \rightarrow \bar{j}} = \tau$
- $P_D^{(E)} = 1/(1 + 2/\tau) = \tau/(\tau + 2)$
- $P_X^{(E)} = 1/(2 + 1/\tau) = \tau/(2\tau + 1)$

3 Melitz

3.1 Set up

- We normalize the wage to $w = 1$
- Productivity φ is drawn from a productivity space $\Phi \subseteq (0, \infty)$ under probability measure $\mu : \mathcal{B}(\Phi) \rightarrow [0, 1]$ (thus $\mu(\emptyset) = 0$, $\mu(\Phi) = 1$); we write its CDF, given $\varphi \in \Phi$, as $G(\varphi) = \mu((0, \varphi])$, corresponding to the probability that productivity is at most φ

3.2 Consumer

- The demand of a single customer of a good from firm with productivity φ is c_φ
- A single consumer utility is $U(\bar{c}) = (\int_\Phi c_\varphi^{\frac{\sigma-1}{\sigma}} d\varphi)^{\frac{\sigma}{\sigma-1}}$ (a country utility is $U(L\bar{c}) = LU(\bar{c})$)
- Revenue constraint is $1 = \sum p_i c_i$

- As above, we define $P = \frac{1}{U(\bar{c})} = (\int_{\Phi} p_{\varphi}^{1-\sigma} d\varphi)^{\frac{1}{1-\sigma}}$ and obtain $c_{\varphi} = \frac{1}{P^{1-\sigma}} \frac{1}{p_{\varphi}^{\sigma}}$ and $q_{\varphi} = \frac{L}{P^{1-\sigma}} \frac{1}{p_{\varphi}^{\sigma}}$

3.3 Producer

- As above, we obtain $p_{\varphi} = \frac{\sigma}{\sigma-1} \frac{1}{\varphi}$
- Revenue of (a single) firm of type φ is $r(\varphi) = p_{\varphi} q_{\varphi} = \frac{L}{P^{1-\sigma}} \frac{1}{p_{\varphi}^{\sigma-1}} = A \varphi^{\sigma-1}$ with $A = \frac{L}{P^{1-\sigma}} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}$
- Assuming still that producing q_{φ} units require labor $l_{\varphi} = \frac{q_{\varphi}}{\varphi} + f_d$, with fixed costs f_d to serve domestic market, profit is $\pi(\varphi) = r(\varphi) - l_{\varphi} = p_{\varphi} q_{\varphi} - \frac{q_{\varphi}}{\varphi} - f_d = p_{\varphi} q_{\varphi} (1 - \frac{1}{p_{\varphi} \varphi}) - f_d = p_{\varphi} q_{\varphi} (1 - \frac{\sigma-1}{\sigma}) - f_d = A \varphi^{\sigma-1} (\frac{1}{\sigma}) - f_d = B \varphi^{\sigma-1} - f_d$ with $B = \frac{A}{\sigma}$
- Zero Cutoff Profit condition: introduce the level φ_a such that $\pi(\varphi_a) = 0$
- Firms in $(0, \varphi_a)$ exit, set with measure $G(\varphi_a)$; define $\Phi^* = (\varphi_a, \infty)$ as the active firms, with measure $\mu(\Phi^*)$; define $\mu^* = \frac{\mu}{\mu(\Phi^*)}$ as the measure normalized for the active firms (thus, the conditional distribution of productivity given entry)
- Define fixed entry cost $f_e = \int_{\Phi^*} \pi d\mu$ as the expected profit of entering
- Let M_e be the mass of drawers (potential entrants), then the mass of producers (entered) is $M_a = M_e \mu(\Phi^*)$; define $\nu^* = M_a \mu^*$ as the measure $\nu^*(F)$ of firms of kind F conditional to activity
- Define aggregate revenue as $R_a = \int r(\varphi) d\nu^* = M_a \int r(\varphi) d\mu^*$
- Aggregate productivity is defined as the productivity $\bar{\varphi} = r^{-1}(\frac{R_a}{M_a})$ that yields average revenue; from $\frac{R_a}{M_a} = A \int \varphi^{\sigma-1} d\mu^*$, we obtain $\bar{\varphi}^{\sigma-1} = \int \varphi^{\sigma-1} d\mu^*$
- Similarly, with Π_a the aggregate profit, expected profit conditional on entry is $\frac{\Pi_a}{M_a} = \int \pi(\varphi) d\mu^* = B \int \varphi^{\sigma-1} d\mu^* - f_d = \pi(\bar{\varphi})$
- Expected profit (unconditionally) is $\int \pi(\varphi) d\mu = \frac{\Pi_a}{M_e} = \mu(\Phi^*) \pi(\bar{\varphi})$, thus $\Pi_a = M_e \mu(\Phi^*) \pi(\bar{\varphi}) = M_a \pi(\bar{\varphi}) = M_a \int \pi(\varphi) d\mu^* = \frac{M_a}{\mu(\Phi^*)} \int_{\Phi^*} \pi(\varphi) d\mu = M_e \int_{\Phi^*} \pi(\varphi) d\mu = M_e f_e$
- Obtain $f_e = f_d J(\varphi_a)$ with $J(\varphi_a) = \int_{\Phi^*} \left(\left(\frac{\varphi}{\varphi_a} \right)^{\sigma-1} - 1 \right) d\mu$, as $f_e = \int_{\Phi^*} \pi d\mu = \int_{\Phi^*} \left(\frac{r(\varphi)}{\sigma} - f_d \right) d\mu = \int_{\Phi^*} \left(\frac{A \varphi^{\sigma-1}}{\sigma} - f_d \right) d\mu$ and as $\pi(\varphi_a) = 0$, thus $\frac{A}{\sigma} \varphi_a^{\sigma-1} - f_d = 0$
- Also, $\frac{\partial J(\varphi_a)}{\partial \varphi_a} < 0$, as $\frac{\partial J(\varphi_a)}{\partial \varphi_a} = \int_{\Phi^*} \frac{\partial}{\partial \varphi_a} \left(\left(\frac{\varphi}{\varphi_a} \right)^{\sigma-1} - 1 \right) d\mu$ (because the boundary term $\left(\frac{\varphi_a}{\varphi_a} \right)^{\sigma-1} - 1$ vanishes), which equals $\int_{\Phi^*} -(\sigma-1) \frac{\varphi^{\sigma-1}}{\varphi_a^{\sigma-2}} d\mu$ (or without using Leibniz rule explicitly, $\frac{\partial J(\varphi_a)}{\partial \varphi_a} = \frac{\partial}{\partial \varphi_a} \left[\frac{1}{\varphi_a^{\sigma-1}} \int_{\Phi^*} \varphi^{\sigma-1} d\mu \right] - \frac{\partial \mu(\Phi^*)}{\partial \varphi_a} = \left[-\frac{\sigma-1}{\varphi_a^{\sigma-2}} \int_{\Phi^*} \varphi^{\sigma-1} d\mu + \frac{1}{\varphi_a^{\sigma-1}} (-\varphi_a^{\sigma-1}) g(\varphi_a) \right] - (-g(\varphi_a))$, with density $g = \frac{d\mu}{d\varphi}$)

- We can also write $\pi(\varphi) = r(\varphi) - l_\varphi = p_\varphi q_\varphi - \frac{q_\varphi}{\varphi} - f_d = \frac{q_\varphi}{\varphi}(p_\varphi \varphi - 1) - f_d = \frac{q_\varphi}{\varphi}(\frac{\sigma}{\sigma-1} - 1) - f_d = \frac{q_\varphi}{\varphi} \frac{1}{\sigma-1} - f_d$
- Full employment condition requires $L = \int l_\varphi d\nu^* + M_e f_e = \int (\frac{q_\varphi}{\varphi} + f_d) d\nu^* + \int \pi(\varphi) d\nu^* = \int ((\sigma-1)\pi(\varphi) + \sigma f_d + \pi(\varphi)) d\nu^* = \int (\sigma\pi(\varphi) + \sigma f_d) d\nu^* = \sigma M_a(\pi(\bar{\varphi}) + f_d)$, thus $M_a = \frac{L}{\sigma(\pi(\bar{\varphi}) + f_d)}$ (differs from slides and problem set, error?)

3.4 Open economy

- Fixed costs f_x to serve foreign market; marginal costs are $\frac{\tau}{\varphi}$; thus export prices are $p_\varphi^x = \frac{\sigma}{\sigma-1} \frac{\tau}{\varphi}$