ASTRO 501 — Homework 2a

Due midnight Feb 19 on Gradescope

Total: 50 points

Instructions

Show all steps and clearly state assumptions. Use Python where requested/appropriate.

Submit a single PDF on Gradescope.

Gradescope formatting (helps grading):

- Clearly label each subpart (e.g., 3(c)).
- BOX your final numerical answers (or highlight them).
- Keep narrative answers concise (bullet points are welcome).

Note: if you have trouble finding definitions of terms, please email monnier@umich.edu or post a discussion question on Canvas. You could try asking Maizey!

Code resources: A starter Jupyter notebook (with all helper functions included in the notebook) will be posted alongside this HW. You will need modify and experiment beyond what is is each cell.

Problem 1 (10 pts): 1D Fourier optics + static aberrations

Goal: Build intuition that Optical Path Difference (wavefront) slopes steer light and low-order aberrations reshape the PSF.

Format: Complete working 1D code is provided; you will modify OPD terms/parameters and interpret results.

Here we explore a 1-dimensional telescope, with pupil coordinate x in meters spanning a telescope pupil of diameter D (meters), with uniform amplitude in the pupil and a wavelength λ. Define a top-hat pupil amplitude:

$$A(x) = 1 \ \text{ for } \ |x| \leq D/2; \quad A(x) = 0 \ \text{ otherwise.}$$

Let the wavefront error be described as an optical path difference (OPD) W(x) in meters. The complex pupil field is:

$$U(x) = A(x) \exp[i \ \varphi(x)], \quad \text{with} \quad \varphi(x) = (2\pi/\lambda) \ W(x).$$

The (1D) image-plane field is proportional to the Fourier transform: $\tilde{U}(f) = F\{U(x)\}$; the PSF is proportional to $|\tilde{U}|^2$. The Fourier coordinate f has units of cycles/m, and the small-angle mapping is $\theta \approx \lambda f$ (θ in radians).

OPD models to explore using provided python code (you will vary coefficients):

- Tilt (wedge OPD): $W(x) = \alpha x$
- Defocus-like (1D cylindrical defocus): $W(x) = \beta x^2$
- Odd cubic ("coma-like" in 1D): $W(x) = \gamma x^3$

(a) (1 pts) Explain why the OPD slope dW/dx (radians) is equal to the angular shift in the image plane. ("wavefront slope steers light").

(b) (2 pts) Tilt: Using the provided code, show how increasing α shifts the PSF peak location. Measure the shift versus α and show it is linear (hand sketch of plot is fine). Compare to the steering prediction using dW/dx = α (constant).

(c) (3 pts) Defocus: Add the βx^2 term and report how the PSF width changes versus β. Define your width metric (FWHM or second moment). Make sure to explore a wide range of β, that has a very small effect to having a disastrous effect on the image quality! Include a figure.

Note that astigmatism is 2-dimensional aberration can be thought of as "defocus that is not symmetric." For example, gravity sag can create different curvatures in horizontal and vertical direction.

(d) (2 pts) Cubic: Add the coma γx^3 term and describe one clear distinctive PSF signature different than tilt and defocus. Include a figure that shows what you see.

(e) (2 pt) Consider what happens to your previous answers if you change the wavelength. Does the observing wavelength affect the amount of tilt (part a) and defocus (b) for the same OPD? Explore using the code then justify your answer.

Instructor note (context): Astigmatism can be thought of as "defocus that is not symmetric." For example, gravity sag can create different curvatures in horizontal and vertical direction.

Problem 2 (10 pts): Turbulence scalings: r0, seeing, t0, Θ0

An observing site reports seeing Θ_{seeing} = 0.80 arcsec at λ_0 = 500 nm at zenith. Assume a single effective wind speed v = 12 m/s, a turbulence-weighted height H = 5 km, and airmass X = 1.5.

Definitions (names and units; treat these as given for this class):

- Θ_{seeing} = long-exposure seeing FWHM (atmospheric image blur) [arcsec or radians]
- r0: Fried parameter (coherence diameter) [m]
- v: effective wind speed (frozen-flow) [m/s]
- t0: coherence time [s]
- H: turbulence-weighted effective height [m]
- Θ_{iso}: isoplanatic angle [arcsec or radians]
- X: airmass ($\approx$ sec z) [dimensionless]

Use standard relations derived in Lawson (2000) Ch. 5 (Quirrenbach):

- $\Theta_{seeing} \approx \lambda$ / r0 (we ignore the order-unity factor ~0.98 used in some texts)
- $r0(\lambda) = r0(\lambda\_0) * (\frac{\lambda}{\lambda_0})^{\frac{6}{5}} * X^{-\frac{3}{5}}$ (includes wavelength and airmass A dependence)
- t0 = r0 / v   (Lawson Eq. 5.33; Lawson uses tau0)
- Θ_{iso} (zenith) = 0.314 * r0 / H   (Lawson Eq. 5.37 at z = 0)
- Θ_{iso} (X) = Θ_{iso} (1) * $X^{(-8/5)}$ (from Eq. 5.37 plus the zenith-angle dependence of r0)

(a) (2 pts) Compute r0 at lambda0 = 500 nm. Report r0 in cm.

(b) (3 pts) Calculate the seeing FWHM at lambda = 1.65 μm (H band) and 10 μm . Report Θ_{seeing} in arcsec and compare to the diffraction limit (Θ_{D-L} \approx \lambda/D) for D=5m telescope. Normally image quality gets worse in the infrared due to diffraction – is that still true when considering turbulence?

(c) (2 pts) Compute t0 at 500 nm and at 1.65 μm. Convert to milliseconds and interpret (one sentence) what it implies for AO loop speed.

(d) (3 pts) What is the meaning of the isoplanatic patch? Compute Θ_{iso} at zenith and then Θ_{iso} at airmass X = 1.5 using the scaling above. Report in arcsec. Compare the isoplanatic patch at V band vs K band and consider implications, e.g., does Θ_{iso} pose an issue when you consider the typical field of view of visible and infrared cameras?

Problem 3 (10 pts): Shack-Hartmann sampling and AO loop speed (order-of-magnitude design)

This problem is meant to connect the turbulence parameters from Problem 2 to practical AO design choices. Use the following simple heuristics (treat as given for this class):

- For a Shack Hartmann wavefront sensor, you should have at least one subaperture for each coherent patch (r0) for the wavelength you want to correct.
- You need to sample fast enough so that the atmosphere doesn't change as you update the shape of the deformable mirror. That means you need to readout your camera 5-10x faster than the coherence time of the atmosphere at the wavelength you are correcting.

Consider an 8.0 m telescope at zenith and you want to do adaptive optics correction at 1.5 microns.

(a) (4 pts) Using your r0 at 500 nm, estimate number of subapertures in your sensor and thus the required number of actuators in your deformable mirror.

(b) (3 pts) Using your t0 at 500 nm, estimate a reasonable f_loop. Repeat using t0 at 1.65 μm and briefly explain why IR AO is generally easier in bandwidth.

(c) (3 pts) In 3–5 bullet points, explain how each of the following design changes would help (or not help): (i) smaller d (more subapertures), (ii) faster loop rate, (iii) observing at longer wavelength, (iv) guide star farther off-axis.

Problem 4 (10 pts): Kolmogorov phase/OPD screens → PSF, Strehl, and long-exposure halo (guided Python)

In the provided notebook, we generate a Kolmogorov optical path difference (OPD) screen $W(x,y)$ for a specified r0(550 nm), convert to phase via $\varphi = (2\pi/\lambda)\cdot W$, and compute the PSF from a pupil amplitude mask and that phase.

Use the provided code cells (do not write your own generator unless you want to experiment).

Assume a 6.5 m telescope. Use r0(@550 nm) ∈ {5 cm, 10 cm, 20 cm} as (bad, average, excellent) seeing, and scale to other wavelengths using $r0(\lambda) = r0(550\text{ nm})\cdot(\lambda/550\text{ nm})^{\wedge}(6/5)$.

For this problem you should produce only two figures (Fig. 4a and Fig. 4d) plus a small Strehl table in part (c). For the other parts, you may report results qualitatively (no additional plot required unless you want to include one).

(a) (3 pts) For observing at $\lambda = 1.5$ um, generate short-exposure PSFs for the three r0(550 nm) values using a circular pupil. Make one 2×2 panel plot showing the three cases plus the diffraction-limited PSF. Describe (2–3 sentences) how the speckle halo changes with r0.

(b) (3 pts) Long exposure (no AO): choose one of the three r0(550 nm) cases and average N = 100 independent realizations at $\lambda = 550$ nm. Measure the PSF FWHM (rough is fine) and compare to the order-of-magnitude seeing scale Theta_seeing $\approx \lambda / r0(\lambda)$. In 2–4 sentences, describe what changes between short exposure and long exposure.

(c) (2 pts) Toy AO Strehl vs wavelength: using the provided toy AO model with actuator spacing = 20 cm, measure the Strehl ratio (peak ratio relative to the diffraction-limited PSF) at $\lambda = 550$ nm, 1.65 µm, and 10 µm for your chosen r0(550 nm). Report a small table of λ versus Strehl and give one sentence explaining the trend.

(d) (2 pts) Long exposure with AO: for one wavelength of your choice (recommend 1.65 µm), average N = 100 realizations and make a single figure with three panels: (i) long-exposure PSF with no AO, (ii) long-exposure PSF with toy AO, (iii) a 1D slice through the PSF peak overlaying the no-AO and AO cases. Briefly describe what improves and what does not.

(e) (Optional, no points, for fun) Tip-tilt only: toggle the remove_tt flag (no AO) at $\lambda = 1.5$ um and describe what changes in the PSF core/halo. If you include a plot, keep it simple (one side-by-side comparison).

Problem 5 (10 pts): AO modes + Strehl from wavefront error

(a) (6 pts) AO architectures and goals (definitions + what they're for)

For each item below, write (i) a 1–2 sentence definition and (ii) a 1–2 sentence statement of what it is designed to do (typical field-of-view and performance goal; a key limitation is OK).

1. **NGAO (Natural Guide Star AO)**
2. **LGAO (Laser Guide Star AO)** (include cone effect / focus anisoplanatism in your answer)
3. **SCAO (Single-Conjugate AO)**
4. **MCAO (Multi-Conjugate AO)**
5. **GLAO (Ground-Layer AO)**
6. **Active optics** (and how it differs from adaptive optics)

Grading note: Full credit requires both "what it is" and "what it's designed to do."

Minimal prompts (to guide your thinking):

- For NGAO vs LGAO: What provides the wavefront reference? What does that imply for sky coverage and limitations?
- For SCAO vs MCAO vs GLAO: What is the field-of-view goal (narrow/high Strehl vs wider/more uniform vs wide/partial)?
- For active optics: What timescale and what aberration sources?

(b) (4 pts) Maréchal (Ruze-like) Strehl from many optics

Assume the Maréchal approximation:

$S \approx exp[- ((2\pi \, \sigma\_WFE) / \lambda)^2]$

where σ_WFE is the total RMS wavefront error(in length units). You are building a relay with 10 lenses/optics. Each optics has two air-glass surfaces, and each surface is specified as "diffraction limited": assume each surface contributes σ_surf = λ/20 RMS at the design wavelength. You can assume each surface has independent wavefront errors from each other. Compare the Strehl Ratio after 1 lens vs all 10 lenses. What would each surface wavefront error contribution be so that the total system remains "diffraction-limited"? State your assumptions and show some intermediate steps. (don't just write the final answer).