Quantitative and Financial Modelling (QFM)

Extreme Values Distributions

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Introduction

INTRODUCTION

Extreme Value

- **Extremes:** natural disasters, stock market crashes,...
- **Extreme value:**
  - minimum value and/or the maximum value of a function (i.e. the single smallest and/or largest number in the set);
  - synonym for an outlier

Extreme Value Distribution (EVD)

- A type of distribution that is used to model the occurrences of extreme values;
- 3 types of Extreme Value Distribution: EVD type I, II, and III.

** **EVD Type I: Gumbel Distribution**

– Only EVD with two forms: one for the minimum and one for the maximum;

– Unbounded (not restricted to a range) and is defined on the entire range of real numbers;

– Two parameters: distribution with unchanging shape

Extreme Value Distribution (EVD)

** **EVD Type II: Fréchet Distribution**
– Model Maximum values in a dataset:
phenomena like maximum rainfalls and river discharges in hydrology, gold price analysis,...
– Bounded (restricted) on the lower side;
– Distribution with three parameters: can take many shapes

Extreme Value Distribution (EVD)

** **EVD Type III: Weibull Distribution**

– Appears practical when we observe data that deal with minima values;
– It is bounded (restricted) on the upper side;
– Distribution with three parameters: can take many shapes

Generalized Extreme Value (GEV) Distribution

The pdf is:

$$f(x;\mu,\beta,\alpha) = \frac{1}{\beta}\left[1+\alpha\left(\frac{x-\mu}{\sigma}\right)\right]^{(-1/\alpha)-1} \exp\left\{-\left[1+\alpha\left(\frac{x-\mu}{\beta}\right)\right]^{-1/\alpha}\right\} \tag{1}$$

$$f(x;\mu,\beta,0) = \frac{1}{\beta}\exp\left(-\frac{x-\mu}{\beta}\right)\exp\left[-\exp\left(-\frac{x-\mu}{\beta}\right)\right] \tag{2}$$

– $\alpha \in \mathbb{R}$ is the shape . β and μ are respectively the scale and location parameters.
– $\alpha = 0$ (intermediate or light tail), $\alpha > 0$ (fat tail) and $\alpha < 0$ (thin tail), correspond respectively, to the Gumbel, Fréchet and Weibull distributions.

Domain of attraction

Table: EVDs's Domain of attraction

Gumbel	**Fréchet**	**Weibull**
Normal	Cauchy	Uniform
Exponential	Pareto	Beta
Log-normal	Chi-squared	
Gamma	Student	

GEV densities

- Gumbel: $x \in \mathbb{R}$, Fréchet: $x \geq \mu - \frac{\beta}{\alpha}$, Weibull: $x \leq \mu - \frac{\beta}{\alpha}$
- Some key statistics: *summary*(*evd_data*)

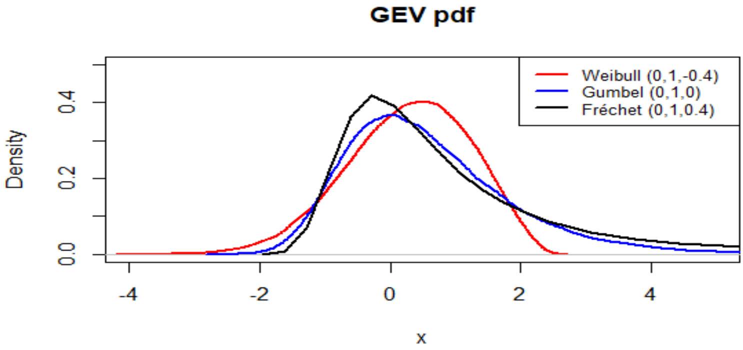

Figure: GEV densities with $\mu = 0$ and $\beta = 1$

Histograms

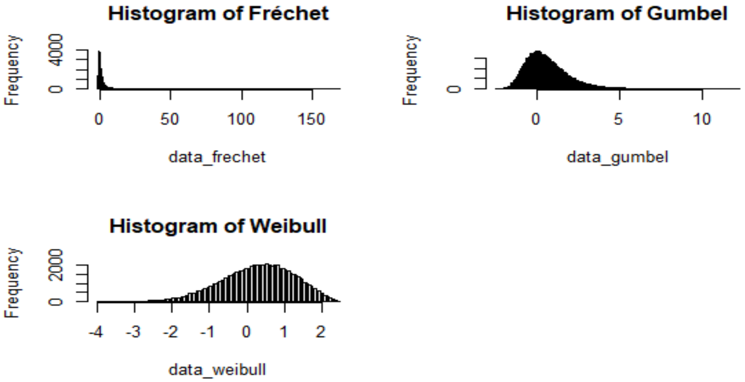

Figure: Histograms of the three EVD

Bloxplots

- Values above upper or lower whisker: can be called potential outliers.

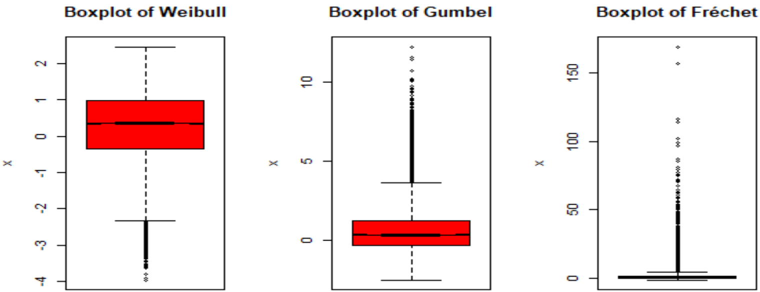

Figure: Bloxplots

Identify of the heavy tail using QQ-plot

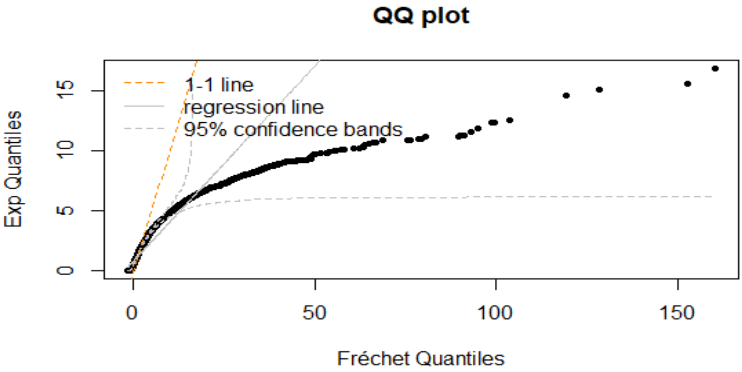

Figure: QQ-plot of Fréchet data with exponential distribution

Influence of the parameters on EVD densities

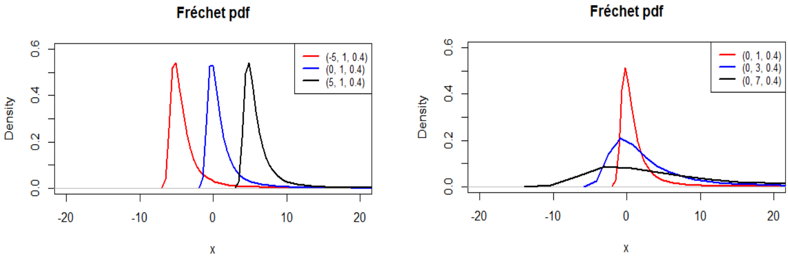

Figure: Influence of μ and β on the Fréchet pdf

– $\alpha \to \infty \Rightarrow$ the distribution is more left-bound.

The problem statement

– The last decades have been exposed to instabilities in financial markets worldwide and for that matter, Rwanda.

– Different periods of financial markets have been affected by several crises such as the Stock Market Crash (1987), the Crisis (1997-1998) and Global Financial Crisis (2007-2008).

– Then the question one will ask is what the right thing being done in finance? What was the cause for this? What went wrong?

The relevance of the problem

– Financial crisis and others rare events can have significant impacts on financial markets, including unemployment and debt, and it is important to be able to anticipate and manage these risks.

– Risk assessment is critical for informed decision-making in finance, and the results of this research could help to improve the ability of individuals and organizations to anticipate and manage financial risks.

These reasons make the problem very relevant.

Methods

-Block Maxima (BM): Block Maxima theory deals with the convergence of maxima, that is, the limits law for the maxima.

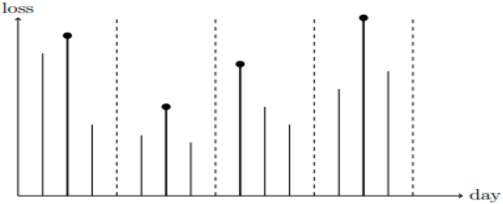

Figure: Block maxima method

GEV is used to approach the distribution of block maxima data.

Methods

-Peak Over Threshold (POT): Considering Peak over Threshold, the focus is on set of realized values exceeding a certain threshold μ. The distribution of these values is approached by the Generalized Pareto Distribution (GPD).

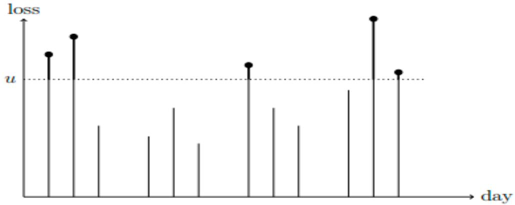

Figure: Peak Over Threshold method

-Value at Risk (VaR) and Expected Shortfall (ES): Two financial tools of risks measure.

Conclusion and Business implications

This study showed how Extreme Value Distributions and Extreme Value Theory in general, can be a useful tool to model Bank of Kigali's stock risks.

The study recommends Rwandan investors, especially risk-averse ones, who would prefer not to take up risky investments in light of the propensity of losing their investments to go on and invest.

META adjuced closing price and return

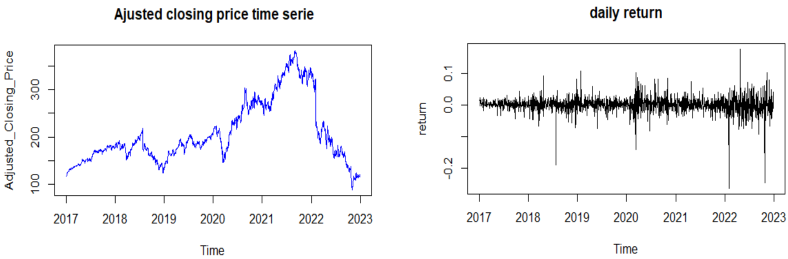

Figure: Adjuced closing prices and return

– Two tails for return: left (max losses) and right (max gains) tail

Block Maxima and GEV fitting

– 76 blocks with 20 values each

Table: Estimation by MLE

Parameters	**location(μ)**	**scale(β)**	**shape(α)**
Estimates	0.02932643	0.01824550	0.30072201
Standard error	0.002360138	0.001954631	0.094424579

– $\alpha > 0$, so the EVD type is II: Fréchet

Block Maxima and GEV fitting

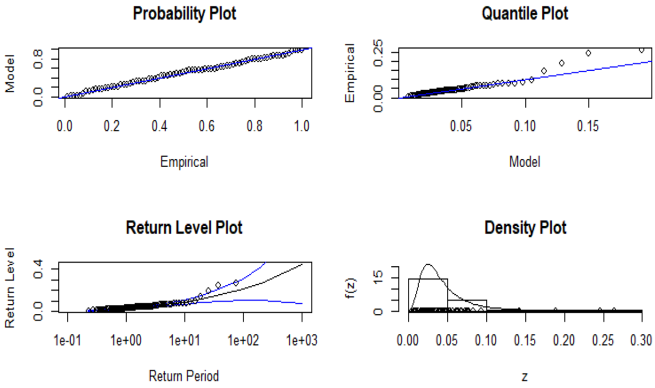

Figure: Diagnostic plot depicting GEV fitting to the block maxima data

POT, VaR and ES

– **POT method:** based on GPD

Table: Estimation by MLE

Parameters	**scale(δ)**	shape(ϵ)
Estimates	0.0110902	0.5173850
Standard error	0.002197907	0.176031128

–VaR and ES

a- BM method: EVD(μ, β, α)

$$\text{VaR}_p = \begin{cases} \mu - \frac{\beta}{\alpha}\left(1 - (-n * \ln(1-p))^{-\alpha}\right) & \text{for} \quad \alpha \neq 0 \\ \mu - \beta \ln\left(-n * \ln(1-p)\right) & \text{otherwise} \end{cases} \tag{3}$$

POT, VaR and ES

b- POT method: GPD(δ, ϵ)

$$VaR_p = s + \frac{\delta}{\epsilon}\left[\left(\frac{n}{N_s}(1-p)\right)^{-\epsilon} - 1\right] \quad (4)$$

$$ES_p = E(loss|loss > VaR_p) = \frac{VaR_p}{1-\epsilon} + \frac{\delta - \epsilon s}{1-\epsilon} \quad (5)$$

Table: VaR and ES

Confidence level	95%	97%	99%
Block Maxima VaR	0.05999799	0.04674735	0.02460466
POT VaR	0.02215508	0.02210514	0.02205674
ES	0.02714837	0.02704489	0.02694461

The next days, with a confidence level of 99%, the maximum potential loss will be 2.205674%. The corresponding loss in the situation where loss will exceed 2.205674% is 2.694461%.

Conclusion

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