Two-Mode Relational Similarities

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ABSTRACT

In a previous paper, Kovacs (2010) proposed a generalized relational similarity measure based on iterated correlations of entities in a network calibrated by their relational similarity to other entities. Here I show that, in the case of two-mode network data, Kovacs's approach can be simplified and generalized similarities calculated non-iteratively. The basic idea is to rely on initial similarities calculated from transforming the two-mode data into one-mode projections using the familiar duality approach due to Breiger (1974). I refer to this as two-mode relational similarities and show, using the Southern Women's data and data from Senate voting in the 112th U.S. Congress, that it yields results substantively indistinguishable from Kovacs's iterative strategy.

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1. Introduction

In a previous paper, Kovacs (2010) introduced a generalized relational similarity measure based on iterated correlations applicable to two-mode and one-mode network data. According to Kovacs, a desirable generalized similarity measure must have two desirable properties. First, (1) it should respect the *principle of equivalence*, such that it classifies actors as similar if they have similar relations to other objects and objects as similar if they have similar relations to other actors. Second, (2) the similarity measure should respect the *principle of duality* (Breiger, 1974), such that it classifies actors as similar if they have similar relations to objects *which are themselves similar*, and objects as similar if they have similar relations to actors *who are themselves similar*. Kovacs's key observation is that such a generalized relational similarity measure could be obtained by tuning the usual correlation distance measure. This adjustment is required because, while the correlation distance respects the principle of equivalence, it fails to abide by the principle of duality.

Particularly, Kovacs begins by noting that in the two-mode case, the correlation distance between any pair of actors $D(A)^{cor}_{i,j}$ be expressed as a function of their row profiles in actors $\times$ objects affiliation matrix $\mathbf{M}$ as follows:

$$D(A)^{cor}_{i,j} = \frac{(M_{i\bullet} - \bar{M}_{i\bullet})(M_{j\bullet} - \bar{M}_{j\bullet})^T}{\sqrt{(M_{i\bullet} - \bar{M}_{i\bullet})(M_{i\bullet} - \bar{M}_{i\bullet})^T (M_{j\bullet} - \bar{M}_{j\bullet})(M_{j\bullet} - \bar{M}_{j\bullet})^T}} \tag{1}$$

Where $M_{i\bullet}$ is the row profile corresponding to actor i, $M_{j\bullet}$ is the row profile corresponding to actor j, $\bar{M}_{i\bullet}$ is the row mean for actor i, and $\bar{M}_{j\bullet}$ is the row mean for actor j in the affiliation matrix. The same approach can be used to find the correlation distance between any two column objects $D(O)^{cor}_{i,j}$, by substituting their column profiles $(M_{\bullet i}, M_{\bullet j})$ and column means $(\bar{M}_{\bullet i}, \bar{M}_{\bullet j})$ into equation 1. Overall, increasingly positive values of the correlation distance indicate actor similarity, while negative values indicate actor dissimilarity, with values bounded in the $(-1, 1)$ interval.

As noted, the correlation distance classifies actors as similar if they have similar relations to other objects, but fails to incorporate the inter-object similarities. That is, actors should receive more similarity "points" if they connect to objects that are themselves similar. To this end, consider a matrix $\mathbf{S(O)}$ with cell entries $s(o)_{ij}$ capturing pairwise similarities between the objects in the two-mode network. In this case, a generalized relational similarity (GRS) measure for actors based on the correlation distance can be expressed as:

$$D(A)^{grs}_{i,j} = \frac{(M_{i\bullet} - \bar{M}_{i\bullet})\mathbf{S(O)}(M_{j\bullet} - \bar{M}_{j\bullet})^T}{\sqrt{(M_{i\bullet} - \bar{M}_{i\bullet})\mathbf{S(O)}(M_{i\bullet} - \bar{M}_{i\bullet})^T}\sqrt{(M_{j\bullet} - \bar{M}_{j\bullet})\mathbf{S(O)}(M_{j\bullet} - \bar{M}_{j\bullet})^T}} \tag{2}$$

Kovacs notes that if we have access to an analogous matrix of similarities between actors $\mathbf{S(A)}$ in the network, then we would be able to also calculate a generalized relational similarity score for objects $D(O)_{grs}$ by plugging in that matrix and the column profiles and means into equation 2, yielding:

$$D(O)^{grs}_{i,j} = \frac{(M_{\bullet i} - \bar{M}_{\bullet i})\mathbf{S(A)}(M_{\bullet i} - \bar{M}_{\bullet i})^T}{\sqrt{(M_{\bullet i} - \bar{M}_{\bullet i})\mathbf{S(A)}(M_{\bullet i} - \bar{M}_{\bullet i})^T}\sqrt{(M_{\bullet j} - \bar{M}_{\bullet j})\mathbf{S(A)}(M_{\bullet j} - \bar{M}_{\bullet j})^T}} \tag{3}$$

Typically, we only have access to the network information and no exogenous indication of pre-existing similarities between actors or objects required to compute 2 and 3. Kovacs's (ingenious) solution is to use the duality property and compute "reflective similarities" by first computing initial object similarities by plugging $D(A)^{cor}$ into 3 (equivalent to substituting the identity matrix, of dimensions $O \times O$, into the slot occupied by $\mathbf{S(O)}$ in 2), then using the resulting object similarities to estimate generalized actor similarities. This is done by substituting the $O \times O$ matrix of $D(O)^{grs}$ values obtained in the first step into the slot occupied by $\mathbf{S(O)}$ in equation 2, and substituting the $A \times A$ matrix of actor similarities obtained in this step into the slot occupied by $\mathbf{S(A)}$ in equation 3. The iterations continue until both $D(A)^{grs}$ and $D(O)^{grs}$ "freeze" according to some stopping criterion (ϵ).

2. Two-Mode Relational Similarities

The basic purpose of this comment is to show that there is a simpler alternative to the iteration approach to computing generalized relational similarities in two-mode data. This alternative respects the basic spirit of Kovacs's proposal, namely, the principles of equivalence and duality, while exploiting the duality of actors and objects in two-mode networks more directly.

The basic idea is that similarity matrices that can play the role of $\mathbf{S(A)}$ and $\mathbf{S(O)}$ can be obtained from any two-mode network without iteration. Instead, they can be computed via Breiger's (1974) well-known dual projection approach (see also Everett and Borgatti, 2013).

This procedure has two major steps. First, we obtain actor co-membership ($\mathbf{AA}$) and object overlap matrices ($\mathbf{OO}$) from the affiliation matrix ($\mathbf{M}$) following the well-known formulas:

$$\mathbf{AA} = \mathbf{MM^T} \tag{4}$$

$$\mathbf{OO} = \mathbf{M^TM} \tag{5}$$

For both $\mathbf{AA}$ and $\mathbf{OO}$, the ij^{th} entry is the number of objects shared by actors i and j and the number of actors who choose both objects i and j respectively. The diagonal entries in AA_{ij} record the number of objects each actor chooses, and the diagonal entries in OO_{ij} record the number of actors choosing each object.

Once we have these one-mode projections, it is straightforward to compute similarity measures across actors based on their co-memberships and across objects in terms of the actors they share. The basic idea is that the similarity between actors increases as the number of common objects chosen increases. In the same way, the similarity between objects increases as the number of actors who choose both objects, suitably weighted by both actors "expansiveness" and object "popularity" (the diagonal entries of $\mathbf{AA}$ and $\mathbf{OO}$, respectively).

Many options are available here, but the cosine similarity is a straightforward candidate (Goodman, 1996, 422). For actors, this is given by:

$$S(A)^{cos}_{ij} = \frac{AA_{ij}}{\sqrt{AA_{ii}AA_{jj}}} \tag{6}$$

And for objects:

$$S(O)^{cos}_{ij} = \frac{OO_{ij}}{\sqrt{OO_{ii}OO_{jj}}} \tag{7}$$

We can then compute a distance matrix containing relational similarities between actors and objects directly without iteration, which I refer to as *two-mode relationality similarity*.

For actors, this is:

$$D(A)^{tmrs}_{i,j} = \frac{(M_{i\bullet} - \bar{M}_{i\bullet})\mathbf{S(O)^{cos}}(M_{j\bullet} - \bar{M}_{j\bullet})^T}{\sqrt{(M_{i\bullet} - \bar{M}_{i\bullet})\mathbf{S(O)^{cos}}(M_{i\bullet} - \bar{M}_{i\bullet})^T}\sqrt{(M_{j\bullet} - \bar{M}_{j\bullet})\mathbf{S(O)^{cos}}(M_{j\bullet} - \bar{M}_{j\bullet})^T}} \tag{8}$$

And correspondingly, for objects, this is given by:

$$D(O)^{tmrs}_{i,j} = \frac{(M_{\bullet i} - \bar{M}_{\bullet i})\mathbf{S(A)^{cos}}(M_{\bullet i} - \bar{M}_{\bullet i})^T}{\sqrt{(M_{\bullet i} - \bar{M}_{\bullet i})\mathbf{S(A)^{cos}}(M_{\bullet i} - \bar{M}_{\bullet i})^T}\sqrt{(M_{\bullet j} - \bar{M}_{\bullet j})\mathbf{S(A)^{cos}}(M_{\bullet j} - \bar{M}_{\bullet j})^T}} \tag{9}$$

Where everything is as before. It is easy to see that equations 8 and 9 respect the principle of equivalence. Actors count as similar to the extent they connect to similar objects ($\mathbf{S(O)^{cos}}$). Conversely, objects count as similar to the extent they are chosen by similar actors ($\mathbf{S(A)^{cos}}$). Because the similarities are defined according to the dual projection method, in which the similarity of actors is based on the objects they choose, and the similarity of objects is based on the actors who choose them, the measure also respects the principle of duality.

Table 1
Mantel statistic and associated p-values for the agreement between distance matrices between actors and events in the Southern Women data computed based on generalized relational similarities and two-mode relational similarities.

Object	Corr.	p-value
Actors	0.94	0.001
Events	0.96	0.001

3. Two Empirical Illustrations

3.1. Southern Women Data

In this section, I re-analyze Davis et al.'s (1941), Southern Women Data—also analyzed by Kovacs—showing that both Kovacs's iterated approach (D^{grs}) and the more direct two-mode approach (D^{tmrs}) yield comparable results.[1] Like Kovacs, I take Doreian et al.'s generalized blockmodeling partitioning of actors and events as a reference point (2004, Table 4). An optimal "unsupervised" similarity-based partition should thus yield three clusters of actors C_1^A = {Evelyn, Laura, Theresa, Brenda, Charlotte, Frances, Eleanor, Ruth}, C_2^A = {Verne Myra, Katherine, Sylvia, Nora, Helen, Olivia, Flora}, and C_3^A = {Pearl, Dorothy} and three clusters of events $C_1^E = \{E_1, E_2, E_3, E_4, E_5\}$, $C_2^E = \{E_6, E_7, E_8, E_9\}$, $C_3^E = \{E_{10}, E_{11}, E_{12}, E_{13}, E_{14}\}$.

Figures 1 and 2 summarize the main thrust of the findings.[2] Like Kovacs, I present a multidimensional scaling (MDS) solution based on a matrix of distances, computed as $d_{ij} = \frac{(1-s_{ij})}{2}$, where d_{ij} is the distance and s_{ij} is the relevant similarity between objects i and j. I depart from Kovacs's original analysis by using a "hybrid" (hierarchical and k-means) clustering of the MDS distances to obtain data-driven groupings of the objects (Chen, Tai, Harrison and Pan, 2005), rather than just "eyeballing" the MDS plot. This also allows us to better detect more subtle differences in the quality of the MDS solutions—e.g., relative to Doreian et al. (2004)—using the generalized relational similarities and the two-mode relational similarities.

3.1.1. Agreement Between Distance Matrices

Kovacs's generalized relational similarity and the two-mode relational similarity output yield data suitable for constructing a distance matrix between objects, in this case, actors or events. As such, we may ask how closely the two ways of obtaining (dis)similarities between objects correspond. To answer this question, I compute a simple permutation-based Pearson correlation measure from biostatistics designed to estimate the agreement between two distance matrices due to Mantel (1967).[3] The results are shown in Table 1. As we can see, the two measures agree pretty closely: $r = 0.94$ for actors and $r = 0.96$ for events ($p < 0.001$).

3.1.2. Actors

Figure 1a shows the MDS two-dimensional plot of actors based on the generalized relational similarities for the Southern Women Data, closely replicating Kovacs's (2010) Figure 12b using the author's implementation of Kovacs's algorithm written as a function for the *R* statistical computing environment (see Supplementary materials). After $k = 119$ iterations, the algorithm converged (using $\epsilon = 0.001$).

As Kovacs noted from a visual inspection of the MDS plot, the distribution of points in the two-dimensional MDS space obtained using the generalized relational similarities corresponds to the generalized blockmodeling solution reported by Doreian et al. (2004). The hybrid k-means analysis with three clusters specified largely agrees with that assessment. We can distinguish two larger sets of similar actors separated from one another and the more peripheral

[1] A github repository containing replication materials and *R* functions implementing both the Kovacs generalized relational similarity and the two mode relational similarity approach can be accessed here: `https://github.com/olizardo/Two-Mode-Relational-Similarity`.

[2] All plots are built using the `ggplot2` grammar of graphics (Wickham, 2016), particularly the `ggscatter` function from the package `ggpubr` (Kassambara, 2023).

[3] I use the function `mantel` from the `vegan` Community Ecology package (Oksanen, Simpson, Blanchet, Kindt, Legendre, Minchin, O'Hara, Solymos, Stevens, Szoecs, Wagner, Barbour, Bedward, Bolker, Borcard, Carvalho, Chirico, De Caceres, Durand, Evangelista, FitzJohn, Friendly, Furneaux, Hannigan, Hill, Lahti, McGlinn, Ouellette, Ribeiro Cunha, Smith, Stier, Ter Braak and Weedon, 2022) for *R*, which relies on the implementation of the Mantel statistic developed by Legendre and Legendre (2012).

Dorothy and Pearl. Note, however, that using the generalized relational similarity approach, the k-means clustering misclassifies Pearl as belonging to the first large cluster and puts Dorothy in a singleton cluster.

As Figure 1b shows, the two-mode relational similarity approach yields substantively indistinguishable results compared to Kovacs's generalized relational similarities. The hybrid k-means clustering based on the corresponding distances reveals Doreian et al.'s three actor blocks. Notably, the k-means three-cluster solution for the two-mode relational similarities correctly assigns Pearl and Dorothy a separate cluster, thus coming closer to Doreian et al.'s generalized blockmodeling results than Kovacs's generalized relationality similarity.

3.1.3. Events

Figure 2 reports the corresponding analysis for relational similarities of the object (events). In the original paper, Kovacs (2010, 206), described these results verbally—no plot was provided—as follows:

> The generalized similarity model provides a grouping for the events as well (not shown here). This grouping differs slightly from Doreian et al.'s (2004) grouping: although the (1, 2, 3, 4, 5) and (10, 11, 12, 13, 14) clusters emerge in the generalized similarity solution as well, the picture differs for events 6, 7, 8, and 9. Event 6 here is clustered together with (1, 2, 3, 4, 5), while events 7, 8 and 9 do not fall into any group but stand separately (206).

Figure 2a reproduces Kovacs's verbal description of the results using generalized relational similarities. A three-group hybrid hierarchical k-means clustering of the distances in the multidimensional scaling space separates the closely-spaced events 1-6 and 10-14. Event 7 is assigned to a singleton cluster, and events 8 and 9 are assigned to the first and second clusters.

As Figure 2b shows, the two-mode relational similarity approach yields results that are largely consistent with but somewhat different from those obtained using the generalized relational similarities. These results are closer to the target event clusters from Doreian et al.'s blockmodel solution (2004). As with the Kovacs approach, events 1-6 and 10-14 end up closely spaced—save for event 11—and assigned to separate clusters. Events 7 and 8 fall into a separate cluster, as would be expected, given the Doreian et al. partition. Event 9, however, is grouped with the event 10-14 cluster. In all, the two-mode relational similarity approach seems more effective at avoiding singleton clusters than the generalized similarity measure.

3.2. 112th U.S. Senate Voting Network

One of the most fruitful applications of two-mode data analysis following work by Fowler (2006) has been for studying political networks (see e.g., Neal, 2014). Kovacs included such an analysis in his article using data he collected from the 109th U.S. Senate. Political networks have the advantage that the partition quality of any given approach can be compared to the intuitive "ground truth" provided exogenous knowledge of institutionalized partisan divides. In this section, I re-analyze data from one year (2011-2012) of the 112th U.S. Congress analyzed by Knoke, Diani, Hollway and Christopoulos (2021, Chap. 8) and publicly available in the *R* package `migraph` (Hollway, 2022). The two-mode network consists of 102 senators voting on 24 bills (one of the bills received unanimous support and is thus not considered in the analysis). Each cell of the affiliation matrix contains a one if a senator voted "Yea" for the bill and a zero otherwise ("Nay").[4]

3.2.1. Agreement Between Distance Matrices

As shown in Table 2, we can once again see that the two measures agree well: $r = 1.0$ for Senators and $r = 0.96$ for Bills ($p < 0.001$). The closer levels of agreement here are not surprising given that, as we will see, the political network has a simple (polarized) structure the generalized similarity approach can faithfully recover.

3.2.2. Actors

Figure 3a shows the MDS two-dimensional plot of Senators based on the generalized relational similarities (converging after 21 iterations with $\epsilon = 0.001$). As in Kovacs's analysis of the data from the 109th U.S. Senate, the Generalized Relational Similarity approach does a good job of capturing the partisan divide in the one year of data from the 112th Congress from Knoke et al. (2021). The hybrid k-means with three clusters solution based on the MDS distances separate Democrats (on the left) and Republicans (on the right), assigning the "centrist" Senator Murkowski

[4] See Knoke et al. (2021, Appendices 8.1 and 8.2) for the list of names of Senators with party affiliations and exact vote counts by political party for each bill.

Table 2
Mantel statistic and associated p-values for the agreement between distance matrices between Senators and Bills in Knoke et al. (2021) U.S. Senators data computed based on generalized relational similarities and two-mode relational similarities.

Object	Corr.	p-value
Senators	1	0.001
Bills	0.96	0.001

from Alaska to a singleton cluster. As shown in Figure 3b the MDS based on distances obtained from the two-mode relational similarities produces results substantively indistinguishable from the iterative approach. Democrats and Republicans are assigned to unambiguously distinct clusters, and Murkowski is assigned to a singleton cluster.[5]

3.2.3. Bills

Figure 4a shows the corresponding analysis for the bills. Once again, both the generalized relational similarities and the two-mode relational similarities produce largely comparable results. Both strategies separate bills that received overwhelming Democratic support (on the left) from those that received overwhelming Republican support (on the right). Both approaches also identify the one bill that received overwhelming bipartisan support (215b: Defense Authorization/Iran Sanctions). The one difference is that the two-mode relational similarity also assigns Bill 251 to the third (bipartisan) cluster. This is a bill on “Tax Rates Extensions” supported by the great majority of Democrats and Republicans but which received a few dissenting votes from both sides (three from Democrats and five from Republicans).

4. Conclusion

Kovacs (2010) made a case that a desirable metric for relational similarity should respect at least two basic desiderata, which I have referred to as the principles of equivalence and duality, proposing an approach based on iterated correlations that met these criteria. In this short note, I have shown that we can get there using a more straightforward, non-iterative approach, more directly grounded in the principle of duality via the dual projection approach to analyzing two-mode data (Everett and Borgatti, 2013).

Like Kovacs, this approach tunes the correlation distances between entities in one mode based on their connection to similar entities in the other mode, as revealed by the one-mode projections. A replication of Kovacs’s analysis of Davis et al.’s (1941) Southern Women Data and a re-analysis based on the proposed alternative metric shows that the two-mode relational similarity approach yields substantively identical results to those obtained using the generalized similarity strategy. Moreover, an original analysis of a political networks dataset based on the 112th U.S. Congress (Knoke et al., 2021) leads to the same conclusion. In both cases, the two-mode similarities produce substantively comparable results as Kovacs’s generalized relational similarities without relying on iterations and thus arbitrary convergence criteria. The measure is also simpler and more tightly linked to duality idea (Breiger, 1974).

One difference between the two approaches is that the generalized similarity approach tends to produce more closely spaced distances because similarities end up closer to their extreme values $(-1, 1)$.[6] The generalized similarity approach, in some cases, yields more distinct or tightly packed clusters than the two-mode similarities (as is the case for actors in the Southern Women data). This is a feature of the iterative strategy that Kovacs’s approach shares with such algorithms as CONCOR (Chen, 2002). It is unclear, however, whether this particular feature outweighs the disadvantage of having to rely on an iterative strategy with arbitrary stoppage criteria when a non-iterative approach that reveals the same object groupings—recoverable using standard approaches—is available. In general, non-iterative approaches should be generally preferred to iterative ones—as they are in measuring eigenvector-based centralities—for their better interpretability and their minimizing arbitrary analytic decisions (Mealy, Farmer and Teytelboym, 2019).

That said, the two-mode similarities inherit the strengths and the weaknesses of the generalized relational similarity approach. On the one hand, the two-mode similarities are preferable when comparing objects along non-independent or correlated dimensions, and can be used outside the two-mode social network analysis context proper (e.g., document

[5] Note that both approaches group Susan Collins with the Democrats, as she was then considered a “liberal republican.”
[6] Thanks to a reviewer for pointing this out.

by term or document by topic matrices in the computational analysis of text data). On the other hand, two-mode similarities, like Kovacs's generalized relational similarities, are built on the Pearson correlation distance and are thus less applicable when possible non-linearities are present in the data. Moreover, there are other approaches capable of computing similarities satisfying both the principles of equivalence and duality. Accordingly, looking at the linkages between the two-mode similarities strategy and other ways of measuring generalized similarities would be a useful pathway for future work.

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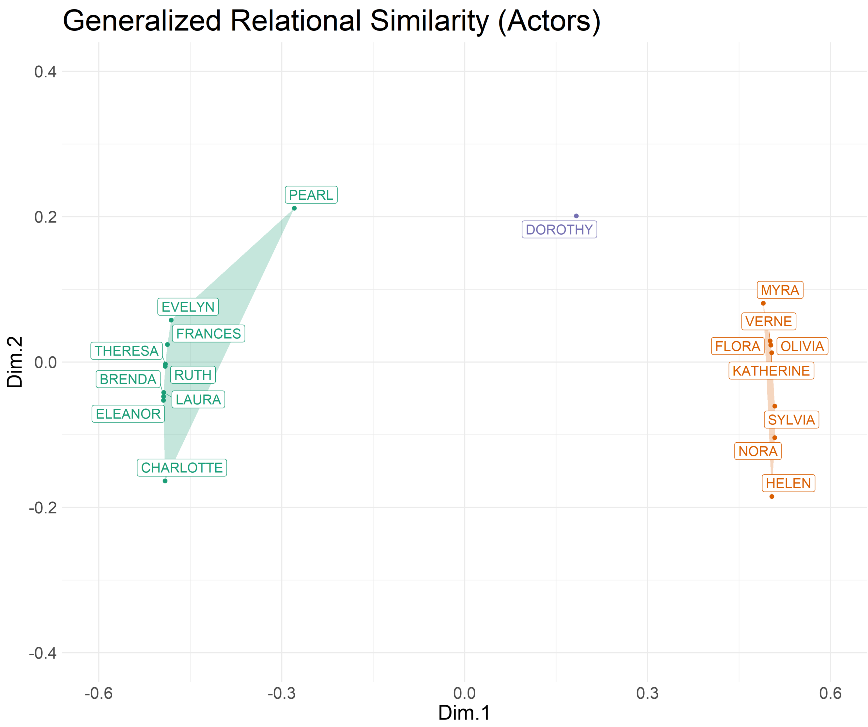

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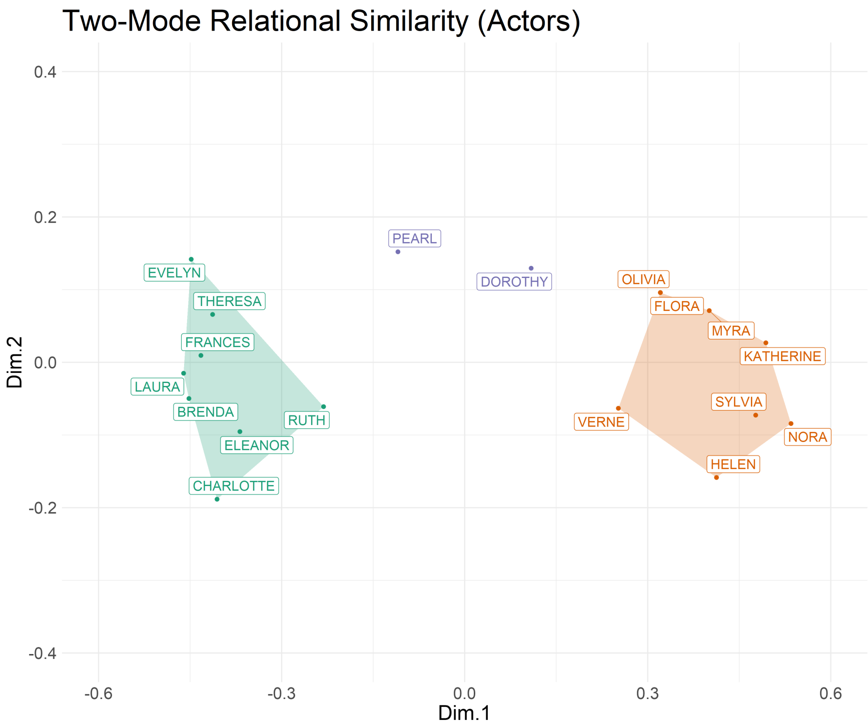
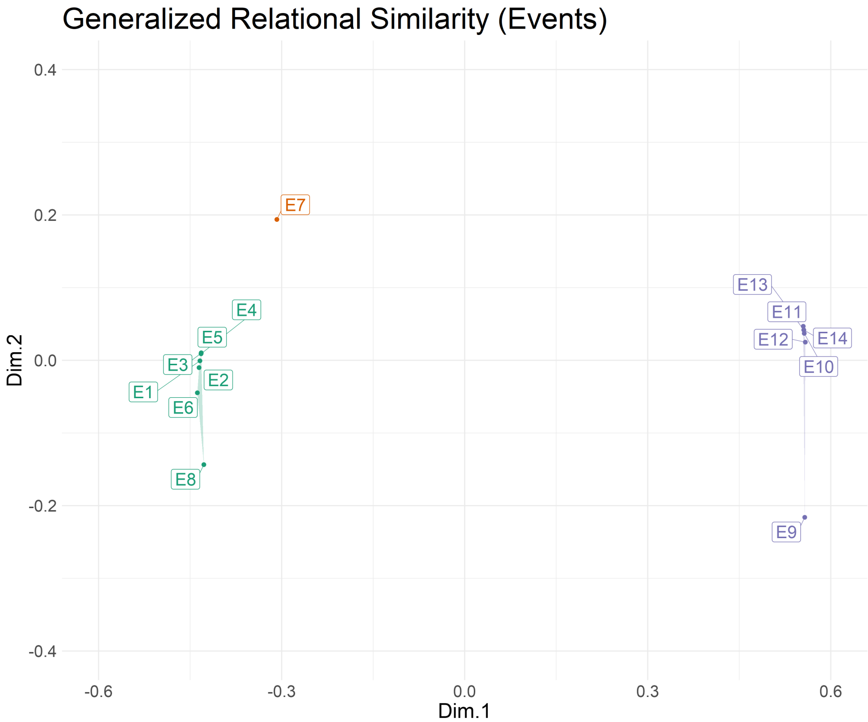


(b)

Figure 1: Two-dimensional metric multidimensional scaling plots of generalized and two-mode 0900relational similarities for actors in the Southern Women data. Colors correspond to a three-group hybrid k-means clustering of the relational similarities, with cluster centroids determined by a first-step hierarchical clustering of distances according to Ward's (1963) criterion.



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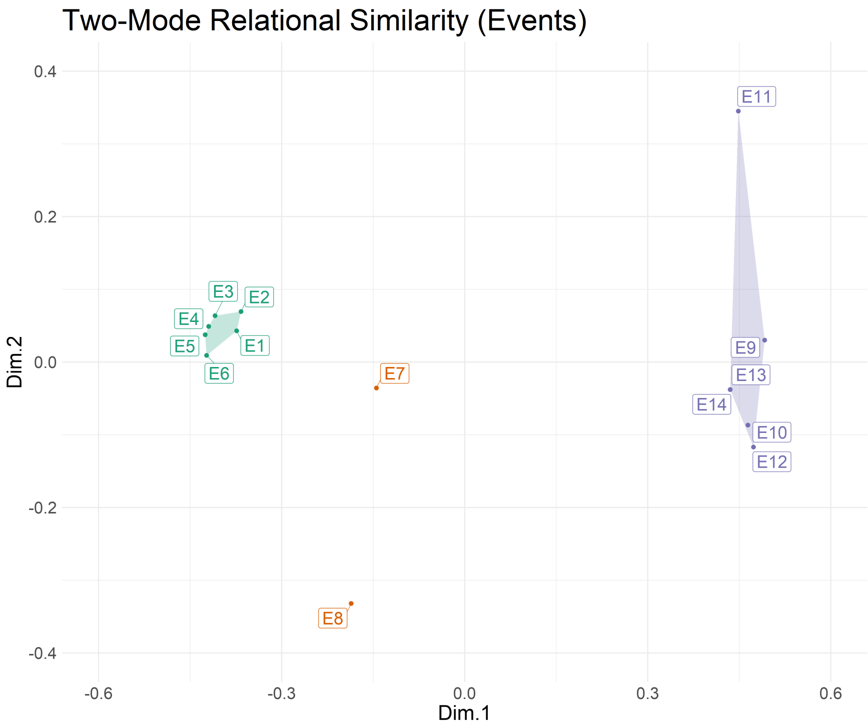
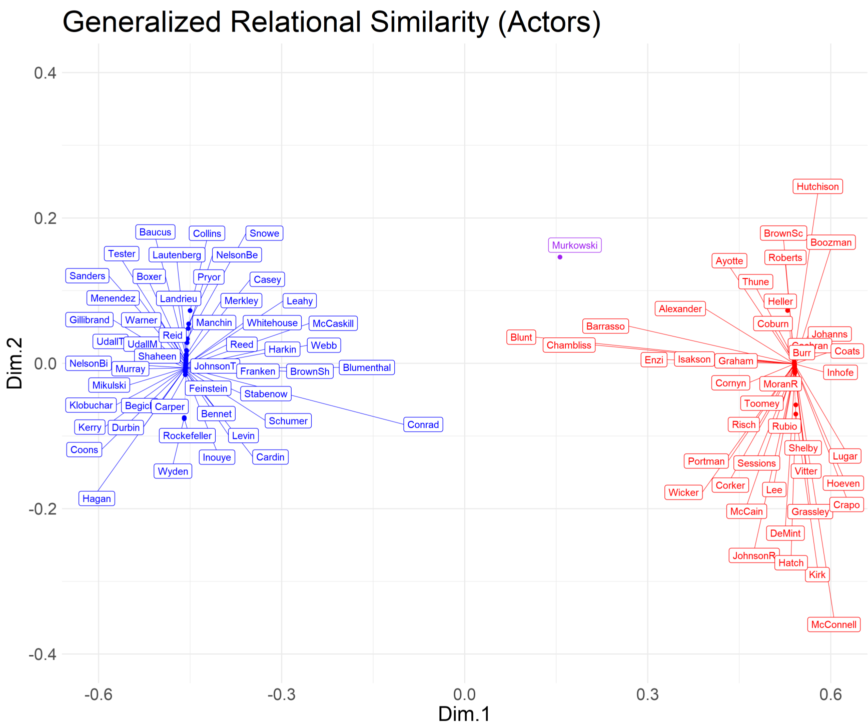


(b)

Figure 2: Two-dimensional metric multidimensional scaling plots of generalized and two-mode relational similarities for events in the Southern Women data. Colors correspond to a three-group hybrid k-means clustering of the relational similarities, with cluster centroids determined by a first-step hierarchical clustering according to Ward's (1963) criterion.



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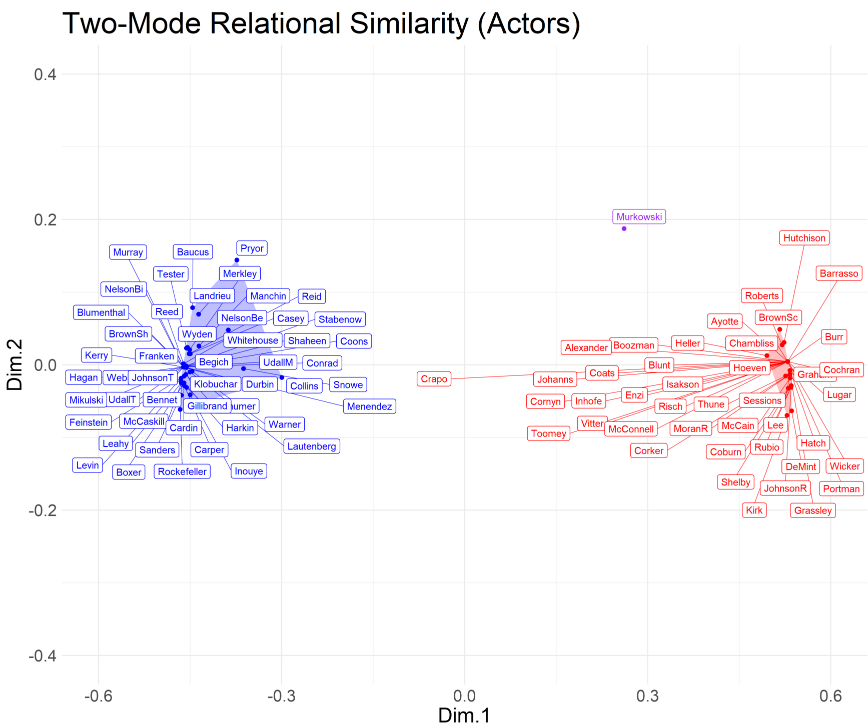
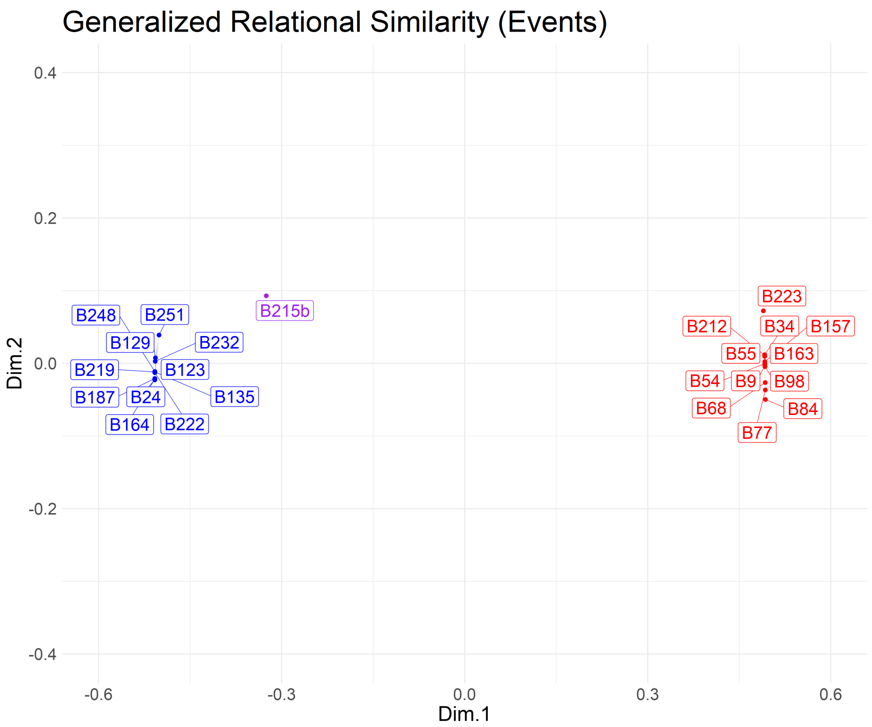


(b)

Figure 3: Two-dimensional metric multidimensional scaling plots of generalized and two-mode relational similarities for Senators in the 112th U.S. Congress (2011-2012). Colors correspond to a three-group hybrid k-means clustering of the relational similarities, with cluster centroids determined by a first-step hierarchical clustering of distances according to Ward's (1963) criterion.



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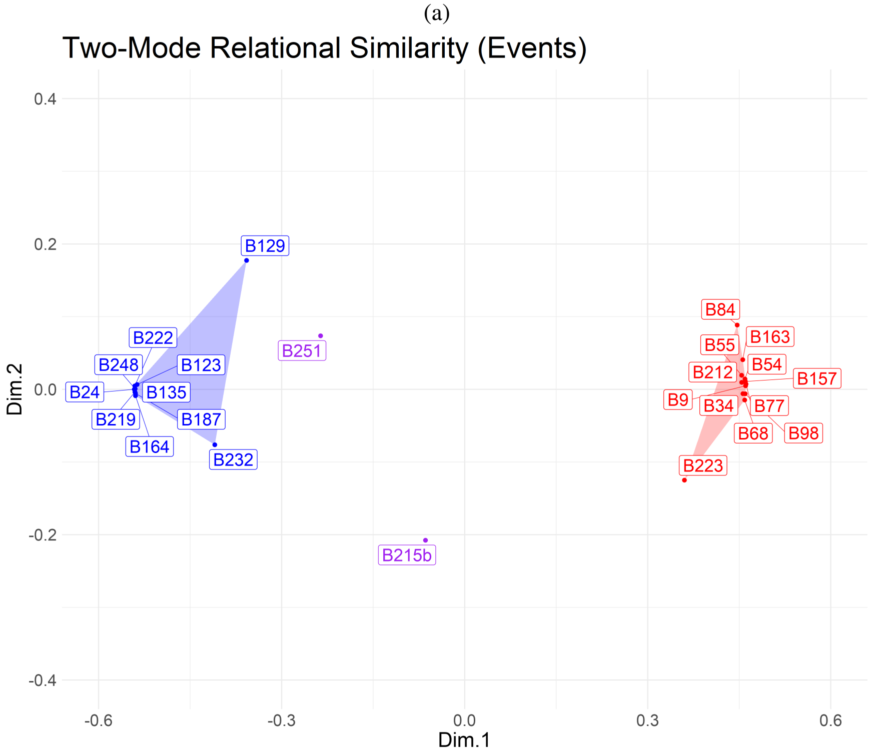

(b)

Figure 4: Two-dimensional metric multidimensional scaling plots of generalized and two-mode relational similarities for Bills voted by Senators in the 112th U.S. Congress (2011-2012). Colors correspond to a three-group hybrid k-means clustering of the relational similarities, with cluster centroids determined by a first-step hierarchical clustering according to Ward's (1963) criterion.