A Game-Theoretic Approach to Integration of Modules

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***Abstract*—This paper offers a new approach to integration of modules in an intelligent sensor system. Such a system requires that a set of modules—each doing a smaller portion of the overall task—be integrated within a unifying framework. From the perspective of computational systems, this problem holds a considerable interest because it is characterized by a set of coexisting mathematical objectives that need to be optimized simultaneously. In this sense, the design considerations necessitate the introduction of problem solving with multiple objectives. This paper explores these issues in the instance when each module is associated with a mathematical objective that is a function of the outputs of other modules. The integration problem is formulated and what is required of a good solution is presented. This examination interprets the decentralized mediation of conflicting subgoals as promoting a N-player game amongst the modules to be integrated and proposes a game-theoretic integration framework. We model the interaction among the modules as a noncooperative game and argue that this strategy leads to a framework in which the solutions correspond to a compromise decision. The rich mathematical literature greatly enhances our ability to examine issues of convergence, and based on this theory we present some analytical results on computation of equilibria. The application of this framework in image analysis motivates the hope that a framework such as game-theoretic integration will facilitate the development of general design principles for "modular" systems.**

***Index Terms*—Integration, fusion, game-theory, image analysis, multi-objective decision making.**

I. Introduction

INTELLIGENT systems are those which are aimed at tasks requiring sensory input and output, which are large in dimension, have a multifarious nature and require complex information processing. From the viewpoint of a robust and flexible development, their input/output behaviors can be best understood by partitioning them into modules and aggregating their modularized subsystems [28]. Consequently, a key to making an intelligent system operational is to provide an effective model of assembling its modules together and incorporating the information provided by each module into the operation of the whole system. This is referred to as the *integration problem.* This paper is concerned precisely with the integration problem.

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The problem is explored in a generic setting: the system is partitioned into modules—each of which is mathematically modelled—and integration refers to the approach used in relating the mathematical models to each other. Within this formulation, it is observed that some applications may require the multiobjective nature of the problem to be preserved. This paper is then offered in the spirit of developing the conceptual and the mathematical framework of such an approach. The solution and analysis it introduces hold a promise of generalization across the horizon of a general systematic design.

There is, of course, a large and growing literature in the design of intelligent sensor systems and careful examination of this work reveals four major—not mutually exclusive—design considerations: 1) The integration framework should enable multiobjective decision making [3], [36]; 2) a formalization of the principles underlying the integration of the framework is important if it is to offer a systematic understanding of the system behavior [37]—analytic and computational tractability; 3) as distributed and autonomous modules are desirable in many applications, the integration framework should provide a decentralized architecture [29]; 4) it should be readily applicable as more modules and capabilities are added to the system, or when a new system with a different task is designed.

Yet, it is observed that the previous approaches can be categorized roughly into two main groups. The first viewpoint focuses on realizing most of the design objectives; that is handling multiple objectives, having a decentralized architecture and being extensible. These approaches include systems based on artificial intelligence techniques and systems that are a mixture of mathematical, control and algorithmic techniques such as in [21], [26], [14], [6], [20], [34], [27], [2], [18] and in many references in [32], [5], [9]. These approaches have provided powerful solutions; however because they have not been too concerned about analytic and computational tractability, the understanding of these systems with regards to their robustness has not been complete—which in turn has narrowed their usage. A second viewpoint—in order to overcome this problem—provides a formal foundation to integration via mathematically modeling each module and then prescribing how to relate these models together. In particular, a mathematical objective is associated with each module, and then these modules are integrated by specifying how to solve all the objectives [3], [1], [33], [23], [38], [19], [31]. Combining evidence into a single measure [15], [36], [7] also falls within the category of the global objective approach. However, the integration mechanisms in these systems have been commonly based on the following two assumptions: 1) the overall system optimum can be defined as a single function of the optima of each vision module and 2) each module's objective can be solved in a decoupled sequential manner. The effect of these assumptions is to remove all conflict (in case of single objective) or cooperation (in case of stagewise single objective) between the modules and thereby exclude a very interesting characteristic of integration. Moreover, they preclude cases where each module is assumed to have access only to the outputs of other modules, so there is a need for *partial* feedback [13].

In summary, none of the papers mentioned above offers an integration framework that simultaneously meets all the four design considerations. Rather, a tradeoff exists between 1) multiobjective decision making and decentralization versus 2) analytic and computational tractability. The principal contribution of this paper is to offer an alternative systematic methodology for achieving integration in a modular system that simultaneously achieves all the design objectives and to demonstrate how to use this approach in the development of an image analysis system. The main idea behind the proposed approach is the recognition of the relevance of game-theoretic concepts to the integration problem [11], [12]. In particular, this paper provides the following contributions: 1) A formal statement of the integration problem in intelligent sensor system design, 2) An integration framework based on game-theoretic concepts, 3) The design and development of an image analysis system based on this new framework. The paper is organized as follows: A precise formulation of the integration problem is developed in Section II. Section III presents the proposed new approach. A quantitative comparison of this framework with the two former approaches is given in Section IV. Section V presents a qualitative and comparative study of the game-theoretic integration against the previous approaches in a specific vision task. The concluding section summarizes the paper briefly and assesses the larger implications of this work.

II. INTEGRATION PROBLEM

The design of a modular system requires a precise formulation of 1) a *set of modules* that perform their own tasks and support each other toward a common target, 2) the *integration problem* that arises when these modules are assembled together into a complete system, and 3) the *integration framework* that refers to a particular model chosen to solve the integration problem.

A. Modules

Let us assume N modules and $I = \{1, \cdots, N\}$ be the set of these modules. Each module $i, i = 1, \cdots, N$ is associated with: 1) A vector of output variables $\mathbf{p}_i \in P^i \subset R^{d_i}$, ($P^i$ is chosen to be closed and bounded). Each module has an initial estimate $\mathbf{p}_i^0$ of its output vector; 2) A set of input vectors $\{\mathbf{p}_j \in P^j \subset R^{d_j}: j = 1, \cdots, N, j \neq i\}$ from its neighbor modules $j \in N_i$. N_i specifies the modules with which module i needs to interact. In this paper, each input vector $\mathbf{p}_j$ is the output vector of the corresponding module j; 3) A set of sensory inputs s_k: $k = 1, \cdots, K_i$ from the external world, where K_i is the number of external inputs to the ith module; 4) A function $g_i \in C[\oplus_{j=1}^N P^j, P^i]$ that describes the dynamics of the module; 5) An objective function $F_i \in C[\oplus_{j=1}^N P^j, R]$ that is a measure of how well that module's output vector $\mathbf{p}_i$ satisfies the task of the module, given its inputs from all its surrounding modules and in general be a multimodal function as follows:

$$\begin{aligned} F_i(\mathbf{p}_1, \cdots, \mathbf{p}_{i-1}, \mathbf{p}_i, \mathbf{p}_{i+1}, \cdots, \mathbf{p}_N) \\ = f_i(\mathbf{p}_i) + \sum_j f_{ji}(\mathbf{p}_i, \mathbf{p}_j) \\ + \lambda_i g_i(\mathbf{p}_1, \cdots, \mathbf{p}_{i-1}, \mathbf{p}_i, \mathbf{p}_{i+1}, \cdots, \mathbf{p}_N) \end{aligned}$$

$\lambda_i \in R$. $f_i(\mathbf{p}_i) \in C[P^i, R]$ is the part that is a function of only its output vector $\mathbf{p}_i$, which is defined based on *a priori* modeling. $\{f_{ji}(\mathbf{p}_i, \mathbf{p}_j) \in C[P^i \times P^j, R]: j = 1, \cdots, N; j \neq i\}$ is a set of functions where each function specifies how $\mathbf{p}_i$ is coupled with the output vector $\mathbf{p}_j$ of its neighboring module j. Although in this paper, only pairwise constraints are considered, in general these could be constraints on m*-tuples* of output variables. The third component $\lambda_i g_i(\cdot)$ incorporates the module dynamics into the objective function in order to ensure that they are satisfied. The problem of each module i can then be stated qualitatively as evolving its output variables $\mathbf{p}_i$ so as to find the output vector $\mathbf{p}_i^*$ that optimizes its objective function F_i.

B. Integration of Modules

As a result of the decomposition of the overall system task into modules and the formulation of the task of each module as a single objective problem, a family of coupled and coexisting objectives is generated. The integration problem is then defined to be a set of coupled and coexisting objectives $F_i, i = 1, \cdots, N$ that need to be simultaneously optimized:

$$\textbf{Module i:} \quad \min_{\mathbf{p}_i} F_i(\mathbf{p}_1, \cdots, \mathbf{p}_i, \cdots, \mathbf{p}_N).$$

Due to coexisting nature of the objectives, there no longer exists an unambiguous system optimum toward which the optima of each module should converge. It is no longer clear what is required of optimality. In fact, one may need to consider rationality instead.

C. The Integration Framework

A solution to the integration problem requires 1) a concept of optimal/rational behavior and 2) a decision-making model for attaining optimality/rationality. As there are possibly many solutions to this problem, the *integration framework* refers to the particular solution chosen.

In order to solve the integration problem, the integration framework must first specify what constitutes optimal/rational behavior for the overall system. Optimality/rationality is defined in terms of an equilibrium point $\mathbf{p}^* \in \oplus_{k=1}^N P^k$—an ordered set of individual solutions $\mathbf{p}_i^*$ of the individual modules $i = 1, \cdots, N$ as $\mathbf{p}^{*^T} = (\mathbf{p}_1^*, \cdots, \mathbf{p}_N^{*^T})^T$. Hence, first a well-defined $\mathbf{p}^*$ must be given. Next, the integration framework must specify how to attain the equilibrium point $\mathbf{p}^*$. This is expressed in terms of a decision-making model, which is defined in terms of an appropriate iterative algorithm $A \in C\ [\oplus_{k=1}^{N} P^k, \oplus_{k=1}^{N} P^k]$. The decision-making model becomes especially important if the well-posed integration problem admits multiple solutions and if different decision-making models lead to different solutions. The appropriate iterative algorithm generates a sequence of points $\{\mathbf{p}^t = (\mathbf{p}_1^{t^T}, \cdots, \mathbf{p}_N^{t^T})^T\}_{t=0}^{\infty}$, where each point is being calculated in terms of points preceding it as $\mathbf{p}^t = A(\mathbf{p}^{t-1})$ and where $\lim_{t\to\infty} \mathbf{p}^t = \mathbf{p}^*$ [25]. This is simply an adjustment process that models the manner in which the information is to be transmitted between the modules and how to coordinate the activities of the interacting modules while each module performs its own task simultaneously with all the others.

D. Traditional Integration Frameworks

Two integration frameworks have been considered previously in the intelligent sensor system design literature: the global objective approach and the sequential approach.

The global objective approach defines the concept of optimality by proposing a solution based on the additivity assumption. The solution $\mathbf{p}^* \in \oplus_{k=1}^{N} P^k, \mathbf{p}^* = (\mathbf{p}_1^{*^T}, \cdots, \mathbf{p}_N^{*^T})^T$ is defined to be the equilibrium point of a single global objective—which is formed by combining the multiple objectives into a single global objective (see [22], [3], [17], [16]). In our work, we consider the most common method—linear combination:

$$\mathbf{p}_i^* \in \arg \min_{p_i} \sum_{k=1}^{N} \gamma_k F_k(\mathbf{p}_1^*, \cdots, \mathbf{p}_i, \cdots, \mathbf{p}_N^*).$$

It should be noted that in general nonlinear techniques can also be employed. In all cases, the integration problem is defined to be a single objective problem. Under certain conditions, the decision-making model is then simply defined to be the iterative algorithm

$$\mathbf{p}_i^t \in \arg \min_{p_i} \sum_{k=1}^{N} \gamma_k F_k(\mathbf{p}_1^{t-1}, \cdots, \mathbf{p}_i, \cdots, \mathbf{p}_N^{t-1})_{\mathbf{p}_i^{t-1}}.$$

There are three major cautions to such an approach. The first is that merging the objectives into a global objective may not be possible in the cases when 1) the objectives are not commensurable [36], or 2) the coupling constraints are not symmetric. Moreover, the effect of these assumptions is to remove all conflict among the modules. Second, although in [17] techniques for determining the weights that avoid the problems of being extremely low or high, are developed, finding the actual weights can still be problematic and computationally expensive since they are not described by a closed form solution. Finally, each module i has to optimize not only with respect to its output variable $\mathbf{p}_i$, but also with respect to the output variables of all other modules and this may not be possible in the cases where modules do not have that kind of access to each other.

The sequential approach defines the concept of optimality by proposing a solution based on the sequential separability assumption. The solution $\mathbf{p}^* \in \oplus_{k=1}^{N} P^k, \mathbf{p}^* = (\mathbf{p}_1^*, \cdots, \mathbf{p}_N^*)$ is defined to be the set of equilibria obtained from solving the objectives in a cascade or Ping-Pong manner. Thus, it assumes a hierarchy in the decision making, and permits one module to enforce its decision on the other modules. If $i = 1, \cdots, N$ corresponds to the hierarchy among the modules, then this solution is

$$\mathbf{p}_i^* \in \arg \min_{\mathbf{p}_i} F_i(\mathbf{p}_1^*, \cdots, \mathbf{p}_{i-1}^*, \mathbf{p}_i, \mathbf{p}_{i+1}^0, \cdots, \mathbf{p}_N^0).$$

It thus transforms the multiple objectives problem into a *stage-wise* single objective problem. Thus, a *sequential solution* can be computed by a sequential optimization of a set of objectives. Under certain conditions, the decision-making model is then simply defined to be the iterative algorithm:

$$\mathbf{p}_i^t \in \arg \min_{\mathbf{p}_i} F_i(\mathbf{p}_1^*, \cdots, \mathbf{p}_i^*, \mathbf{p}_i, \mathbf{p}_i^0, \cdots, \mathbf{p}_N^0)_{p_i^t}.$$

The major drawback of this approach is that it does not provide an integration mechanism that allows many modules to cooperate and compete with one another to reinforce partial information wherever possible [30].

III. Game-Theoretical Integration

The general idea of a game with N players is a familiar concept as follows: There are N modules. Each player is associated with a decision vector and a payoff function as: 1) An individual *decision vector* $\mathbf{p}_i \in P_i \subset R^{d_i}$ (P_i closed and bounded in d_i-dimensional Euclidean[1] space), which the player has the capability of manipulating directly. During the course of the game, which is a sequence of stages and moves, at each stage t, each player chooses a specific decision vector $\mathbf{p}_i^t$ for that stage of the game as its move (action) for that stage. 2) A *payoff function* $F_i(\mathbf{p}_1, \cdots, \mathbf{p}_{i-1}, \mathbf{p}_i, \mathbf{p}_{i+1}, \cdots, \mathbf{p}_N)$, $F_i \in C[\oplus_{k=1}^{N} P^k, R]$. The payoff function evaluates the performance of the player based on its decision $\mathbf{p}_i$ and the decisions $\{\mathbf{p}_j \in P^j \subset R^{d_j}\colon j = 1, \cdots, N \neq i\}$ from the players that influence the decision of player i.

Once all the players are determined, they assemble together and the N-player game starts. Each player, being a rational decision maker, wants to find the ultimate decision—a decision that optimizes its payoff—and the collection of these players constitutes a N-player game problem. As the objectives of the players are coexisting and coupled, there no longer exists an unambiguous system optimum toward which the players should converge [4]. In particular, a solution requires: 1) a precise concept of what is meant by optimality/rationality by the players and 2) decision-making models that allow for the computation of these equilibria. One of the proposed solutions is based on noncooperative games, where there exists no cooperation among the players and each player pursues its interests—which may be partly in conflict with others'—independently. For historical reasons, lack of cooperation among players refers to the fact that no coalitions can be formed among the players so that the N-player game cannot be reduced to a general-sum game [4]. This should not be confused with the cooperation that exists between the players with respect to letting each other know what their

[1]For a more general formulation, Euclidean space can be extended to a Hilbert space.

decisions are. The *game-theoretic integration framework* is a particular solution to the integration problem where the concept of rationality and the decision-making model are defined in the context of noncooperative games. Within this framework, modules are identified in a one-to-one relation with the players and will be used interchangeably.

A. Nash Solution and Parallel Decision-Making

One of the commonly known rationales for defining a noncooperative equilibrium point is the *Nash equilibrium* [41]. Let $P^1, \cdots, P^N$ be the given subsets of the Euclidean spaces $R^{d_1}, \cdots, R^{d_N}$ respectively. Let $d = \Sigma_{k=1}^N d_k$, and let $P \subset R^d$ be the Cartesian product $P = \oplus_{k=1}^N P^{d_k}$. For $i = 1, \cdots, N$, let $F_i \in C[P, R]$ be the objectives for the N modules respectively, and $\mathbf{p}_i \in P^i$ is any admissible decision for module i. Nash equilibrium point $\mathbf{p}^* = (\mathbf{p}_1^*, \cdots, \mathbf{p}_N^*)$ is the optimal/rational decision for each ith module on the assumption that all of the other modules are holding fast to their Nash decisions. For each module $i = 1, \cdots, N$, this can be expressed as:

$$\mathbf{p}_i^* \in \arg \min_{\mathbf{p}^i} F_i(\mathbf{p}_1^*, \cdots, \mathbf{p}_{i-1}^*, \mathbf{p}_i, \mathbf{p}_{i+1}^*, \cdots, \mathbf{p}_N^*).$$

If all the module's objectives require only individual optimality, the Nash equilibrium is a natural definition for rationality. Otherwise, the solution of interest needs to be redefined to be a Pareto-efficient Nash equilibrium or other types of equilibria [4], [35].

As an example, consider a simulated 2-module game. The output of module 1 is $p_1 \in P^1 \subset R$, and the output of module 2 is $p_2 \in P^2 \subset R$. The objective of module 1 is $F_1 \in C[P^1 \times P^2, R]$ and the level curves of $F_1^{-1}(c)$ are pictorially depicted in Fig. 1 (left), where the x-axis represents the decision $p_1 \in P^1 \subset R$ of module 1, the y-axis represents the decision $p_2 \in P^2 \subset R$ of module 2 and each closed contour corresponds to the set of points $(\mathbf{p}_1, \mathbf{p}_2)$ such that $F_1(\mathbf{p}_1, \mathbf{p}_2) = c$. The objective of module 2 is $F_2 \in C[P^1 \times P^2, R]$ and is similarly shown in Fig. 1 (right). For a fixed set of input decisions from the other modules, $\overline{\mathbf{p}} = (\overline{\mathbf{p}}_1, \cdots, \overline{\mathbf{p}}_{i-1}, \overline{\mathbf{p}}_{i+1}, \cdots, \overline{\mathbf{p}}_N)$, the best that module i can do is to minimize F_i constrained to be in this set. For each different $\overline{\mathbf{p}}$, a different optimal $\mathbf{p}_i$ can thus be found for module i, and the collection of all the optimal responses forms the *reaction map* $R_i \in C[\oplus_{k=1, k\neq i}^N P^k, P^i]$ [4] (p. 160). The Nash equilibrium must be in the range of all the reaction maps, and corresponds geometrically to the points that are in the intersection of the ranges of all the reaction maps. Consequently, if the intersection of all the reaction maps is not an empty set, Nash solutions exist. For our case, they are as shown in Fig. 2. The intersection of the reaction maps has only one element $(\mathbf{p}_1^*, \mathbf{p}_2^*)$ so the Nash solution is unique for this example.

Once a notion of what the modules are seeking is defined, the next issue is how can they attain it. Desirable as it may be for the modules to find their respective equilibria in one step, in practice this is rarely possible. Rather, a complete solution to the game problem must also provide a model of decision making—namely a model of how to attain the desired equilibrium point. The decision-making model is expressed

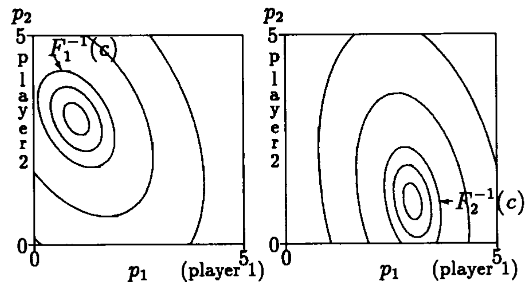

Fig. 1. 2-module example. Left: level curves $F_1^{-1}(c)$ of the objective $F_1 \in C[P^1 \times P^2, R]$ of module 1. Right: Level curves $F_2^{-2}(c)$ of the objective $F_2 \in [P^1 \times P^2, R]$ of module 2.

as an algorithm that is designed to solve the fixed point problem of computing an equilibrium point. The algorithm is defined as a mapping. Operated iteratively, the algorithm is repeatedly reapplied to the new points it generates as to produce a sequence of points [25] (p. 183). To this end, a general decision-making model [8] is considered. Let

$$\mathcal{I}_i = \{(k, t_{i_k}): \; k = 1, \cdots, N; k \neq i; \; t_{i_k} \in Z\}$$

be the *computation index* set for the ith module; i.e., at each stage t of the game, module i determines its action $\mathbf{p}_i^t$ based on the actions $\mathbf{p}_k^{t-t_{i_k}}$ of each of its neighboring module k as:

$$\arg \min_{\mathbf{p}_i} F_i(\mathbf{p}_1^{t-t_{i_1}}, \cdots \mathbf{p}_{i-1}^{t-t_{i_{i-1}}}, \mathbf{p}_i, \mathbf{p}_{i+1}^{t-t_{i_{i+1}}}, \cdots, \mathbf{p}_N^{t-t_{i_N}})_{\mathbf{p}_i^{t-1}}.$$

With a given set of computation index sets, an N-module game has the following progression: The set of modules start the game with an initial set of outputs $\{\mathbf{p}_1^0, \cdots, \mathbf{P}_N^0\}$. At each stage t of the game, each module, simultaneously with the rest of the modules, uses its objective and the information available from the other modules as determined by its index set $\mathcal{I}_i$ to make a new decision $\mathbf{p}_i^t$ and broadcasts it. Consequently, at the end of each stage, there is an associated set of actions $\mathbf{p}^t = \{\mathbf{P}_1^t, \cdots, \mathbf{p}_N^t\}$ that are the ordered set of output variables $\mathbf{p}_i^t$ of each module i. The stage of the game updated to $t+1$ and the whole iteration pattern is repeated. The game continues, until termination, which is guaranteed to occur if all the conditions for convergence are satisfied and is assumed to occur when the decision $\mathbf{p}_i^t$ of each the module i converges to its respective equilibrium point $\mathbf{p}_i^*$.

If the roles of the modules are symmetric and the modules make their decisions concurrently, one has to introduce parallel decision making. In this case, each module decides on its next action in parallel with all the other modules. In the case of two modules, the computation index sets $\mathcal{I}_1$ and $\mathcal{I}_2$ are specified as follows:

$$\mathcal{I}_1 = \{(2, 1)\} \quad \mathcal{I}_2 = \{(1, 1)\}.$$

This can be equivalently expressed by the following iteration:

$$\mathbf{p}_1^{P^t} \in \arg \min_{p_1} F_1(\mathbf{p}_1, \mathbf{p}_2^{P^{t-1}})$$
$$\mathbf{p}_2^{P^t} \in \arg \min_{p_2} F_2(\mathbf{p}_1^{P^{t-1}}, \mathbf{p}_2).$$

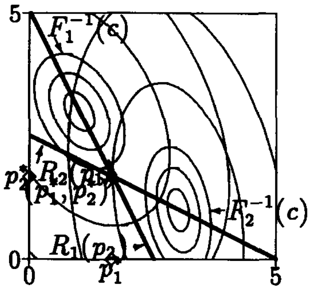

Fig. 2. Nash solution in an example 2-module game corresponds to the intersection of the reaction functions $R_1(\mathbf{p}_2)$ and $R_2(\mathbf{p}_1)$.

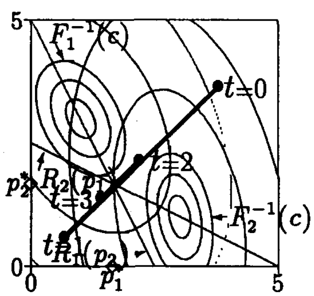

Fig. 3. Parallel decision-making in 2-module game example.

Consider again the example of Fig. 2. Assume that the initial point at $t = 0$ is $(\mathbf{p}_1^{0^T}, \mathbf{p}_2^{0^T})^T$. The sequence generated by this mode is as shown by the thick solid path in Fig. 3. For example, at stage $t = 1$, module 1 adjusts $\mathbf{p}_1$ at the same time as module 2 is doing the same for $\mathbf{p}_2$. Thus, the game proceeds to the point associated with $t = 1$ shown in the figure. This adjustment process is repeated until the Nash solution is attained.

B. Existence and Stability of Nash Equilibria

The applicability of the game-theoretic integration framework is dependent on the existence and stability of Nash equilibria. By restricting the class of objective functions, it has been possible to obtain sufficiency conditions [24]—namely conditions that ensure that a given integration problem has stable Nash equilibria. In this work, the objective functions $F_i \in C[\oplus_{k=1}^N P^k, R] \forall i = 1, \cdots, N$ satisfy the following: 1) F_i is bounded below in $\mathbf{p}_i \in \mathbf{P}^i$, 2) F_i is continuously second-order differentiable in $\mathbf{p}_i \in P^i$, 3) $\exists$ a closed neighborhood $U^i \subseteq P^i$ s.t. F_i is strongly convex in U^i. If closely examined, it will be seen that these conditions ensure that the reaction maps of each player is a singleton (namely a reaction function) as $R_i \in C^1\ [\oplus_{k=1, k\neq i}^N P^k, P^i]$, which implies that each player has one and only one decision response given other players' decisions. The reader is referred to [10] for the sufficiency conditions for existence and stability of Nash games under parallel decision-making under these assumptions. A close examination reveals that these conditions ensure that reaction functions are contractions whose limit points are Nash equilibria. With this conditions in effect and letting $\mathbf{p}^{P^*} = (\mathbf{p}_1^{P^{*T}}, \cdots, \mathbf{p}_N^{P^{*T}})^T$ refer to the Nash solution, the parallel decision-making model converges

$$\lim_{t\to\infty} \left(\mathbf{p}_1^{P^t}, \cdots, \mathbf{p}_N^{P^t}\right) = \left(\mathbf{p}_1^{P^*}, \cdots, \mathbf{p}_N^{P^*}\right)$$

and its limit point is a Nash equilibrium

$$\mathbf{p}_i^{P^*} \in \arg\min_{p_i} F_i\left(\mathbf{p}_1^{P^*}, \cdots, \mathbf{p}_{i-1}^{P^*}, \mathbf{p}_i, \mathbf{p}_{i+1}^{P^*}, \cdots, \mathbf{p}_N^{P^*}\right).$$

Let it be remarked that the theory claims simply that the stable Nash equilibrium in the closest neighborhood will be found and does not put any qualitative characterization.

IV. Quantitative Comparison

The quantitative comparison is comprised of two parts: First, the game-theoretic integration is compared with the global objective approach. Next, it is compared with the sequential approach. It should be pointed out that the analysis is only for the restricted class of game problems of Section III. Let $S_P = \{(\mathbf{p}_1^{P^*}, \cdots, \mathbf{p}_N^{P^*})\}$ be the set of parallel Nash solutions. Under the above conditions, the necessary and sufficient conditions for a Nash equilibrium $(\mathbf{p}_1^*, \cdots, \mathbf{p}_N^*) \in \oplus_{j=1}^N U^i$ are:

$$\nabla_{p_i} F_i(\mathbf{p}_1^{P^*}, \cdots, \mathbf{p}_{i-1}^{P^*}, \mathbf{p}_i, \mathbf{p}_{i+1}^{P^*}, \cdots, \mathbf{p}_N^{P^*}) = 0. \quad (1)$$

A. Game-Theoretic Model versus Global Objective Approach

This section compares the parallel game-theoretic integration model with the global objective approach. In the global objective approach the objectives are merged together into a single objective. The global objective then is optimized with respect to all the output vectors of all the modules in order to determine the optimal output vectors

$$\mathbf{p}_i^{G^*} \in \arg\min_{p_i} \sum_{j=1}^{N} F_j(\mathbf{p}_1^{G^*}, \cdots, \mathbf{p}_i, \cdots, \mathbf{p}_N^{G^*}),\ i=1, \cdots, N.$$

Now, let $S_G = \{(\mathbf{p}_1^{G^*}, \cdots, \mathbf{p}_N^{G^*})\}$ be the set of solutions to this optimization problem. Thus, in order to compare the Nash solutions and the Pareto-optimal solutions quantitatively, S^P and S^G should be compared. The following two propositions verify that in general $S^P \not\subset S^G$. The reader is referred to [10] for the proofs and a more detailed discussion.

Proposition 1: If $S^P \subset S^G$ then for all $i = 1, \cdots, N$

$$\sum_{\substack{j=1\\ j\neq i}}^{N} \nabla_{p_i} F_j(\mathbf{p}_1^{P^*}, \cdots, \mathbf{p}_{i-1}^{P^*}, \mathbf{p}_i, \mathbf{p}_{i+1}^{P^*}, \cdots, \mathbf{P}_N^{P^*}) = 0. \quad (2)$$

□

Proposition 2: $S^G \subset S^P$ iff $\forall_i = 1, \cdots, N$

$$\nabla_{p_i} F_i(\mathbf{p}_1^{G^*}, \cdots, \mathbf{p}_{i-1}^{G^*}, \mathbf{P}_i, \mathbf{p}_{i+1}^{G^*}, \cdots, \mathbf{p}_N^{G^*}) = 0. \quad □$$

B. Game-Theoretic Model versus Sequential Approach

Let the sequence associated with the sequential model be $\{(\mathbf{p}_1^{S^t}, \cdots, \mathbf{p}_N^{S^t})\}_{t=0}^{\infty}$. Its limit point, if it exists, is denoted by $(\mathbf{p}_1^{S^*}, \cdots, \mathbf{p}_N^{S^*})$. Similarly, let the sequence associated with the parallel model be $\{(\mathbf{p}_1^{P^t}, \cdots, \mathbf{p}_N^{P^t})\}_{t=0}^{\infty}$. Its limit point, if it exists, is the parallel Nash solution $(\mathbf{p}_1^{P^*}, \cdots, \mathbf{p}_N^{P^*})$.

In the example introduced earlier on, there is one possible limit point for both of the sequences, and hence both the sequential approach and the parallel game-theoretic approach converge to the same solution. However, in general, a game problem may admit multiple Nash equilibrium solutions. The following proposition establishes that the two approaches will in general lead to different solutions. The reader is referred to [10] for the proof and a more detailed discussion.

Proposition 3: Let $\forall i \in N, F_i \in C[\oplus_{k=1}^{N} P^i, R]$ be continuously second-order differentiable for $\mathbf{p}_i \in P^i$, be locally strongly convex and bounded below in $\mathbf{p}_i \in U^i \subset P^i$. Furthermore, let $\{(\mathbf{p}_1^{S^t}, \cdots, \mathbf{p}_N^{S^t})\}_{t=0}^{\infty}$, and $\{(\mathbf{p}_1^{P^t}, \cdots, \mathbf{p}_N^{P^t})\}_{t=0}^{\infty}$, be the two sequences generated by the sequential approach and parallel game-theoretic model, starting from the same initial conditions. Let the limit point of each sequence be denoted by $(\mathbf{p}_1^{S^*}, \cdots, \mathbf{p}_N^{S^*})$ and $(\mathbf{p}_1^{P^*}, \cdots, \mathbf{p}_N^{P^*})$ respectively. Then, there exist instances of the problem, for which

$$(\mathbf{p}_1^{S^*}, \cdots, \mathbf{p}_N^{S^*}) \neq (\mathbf{p}_1^{P^*}, \cdots, \mathbf{p}_N^{P^*}) \qquad \square$$

V. Qualitative Comparison

A comparative experimental study was performed in the context of *three different* image analysis systems, all of which have the same task and consist of identical vision modules but differ as:

System I: The integration framework is based on global objective approach.

System II: The integration framework is based on sequential approach.

System III: The third system is based on parallel game-theoretic integration framework.

A. Modules

The task of Module 1 is the inference of edge-like structures and the task of module 2 is the inference of 2-D contours (see [39] and [40], respectively, for the basis of these ideas). The two inference processes are formulated within the framework of two models, which specify the output, the input, and the objectives of each module respectively as $\mathbf{p}_1 \in P^1, F_1 \in C[P^1 \times P^2, R]$ and $\mathbf{p}_2 \in P^2, F_2 \in C[P^1 \times P^2, R]$. The output of module 1—locally curvilinear edgelike structures—are denoted by $\mathbf{p}_1$ and is defined to be the ordered set obtained via concatenating the edge vectors associated with all the points in the discrete edge vector field. The output of module 2–closed contours—are denoted by $\mathbf{p}_2$ and are parameters associated with the contours.

The objective of module 1 is to infer the local curvilinear edgelike structures $\mathbf{p}_1$ in the given image. A mathematical objective $F_1 \in C^1[P^1 \times P^2, R]$ is formulated to express a model that has two components: 1) Edge reinforcement & noise removal and 2) Consistency of edges with contours. Corresponding to the two components of the model, F_1 has two components as:

$$F_1(\mathbf{p}_1, \mathbf{p}_2) = f_1(\mathbf{p}_1 + f_{21}(\mathbf{p}_2, \mathbf{p}_1)$$

$f_1 \in C^1[P^1, R]$ mathematically expresses the first component of the model and $f_{21} \in C^1[P^2 \times P^1, R]$ mathematically expresses the second component of the model. The goal is to find the edges $\mathbf{p}_1^*$ that optimize this objective F_1. The objective of module 2 is to infer the 2-D contours $\mathbf{p}_2$ in the given image. A mathematical objective $F_2 \in C[P^1 \times P^2, R]$ is formulated based on a model that has two primary components: 1) a flexible (deformable) model of 2-D contours and 2) consistency of 2-D contours with the edges. Corresponding to the two components of the model, F_2 has two components as:

$$F_2(\mathbf{p}_1, \mathbf{p}_2) = f_2(\mathbf{p}_2) + f_{12}(\mathbf{p}_1, \mathbf{p}_2)$$

$f_2 \in C[P^2, R]$ measures the goodness of the match of relationships between the actual set of objects to those of the reference model, and $f_{12} \in C[P^1 \times P^2, R]$ measures the goodness of the match between the actual objects and the outputs of modules 1—namely the edges. The goal is to find the contours $\mathbf{p}_2^*$ that optimize the objective F_2.

It should be noted that the models used by the modules constitute extensions of the approaches used previously to address the image analysis problems in question. For brevity reasons, we are not able to present these models here and the reader is referred to [10], [39], [40]. We would like to emphasize that there are many ways of developing these models, and this paper has chosen certain models either due to better experimental performance or ease of implementation. Finally, let us also point out that in the experiments, in order for the three systems to have the identical integration problem and to keep the integration problem computationally realizable for System 1 (with the global objective integration framework), the objective F_1 of module 1 has only the term f_1.

B. Three Image Analysis Systems

In System I, the integration framework is based on global objective approach and is architecturally depicted in Fig. 4. The concept of optimality is defined by the solution $(\mathbf{p}_1^*, \mathbf{p}_2^*)$, which optimizes this single objective as

$$(\mathbf{p}_1^*, \mathbf{p}_2^*) = \arg \min_{\mathbf{p}_1, \mathbf{p}_2} (\gamma_1 F_1(\mathbf{p}_1, \mathbf{p}_2) + \gamma_2 F_2(\mathbf{p}_1, \mathbf{p}_2)).$$

The decision-making model is described as follows: Given $\mathbf{p}_1^0, \mathbf{p}_2^0$:

$$\mathbf{p}_1^t \in \arg \min_{p_1} \left(\gamma_1 F_1(\mathbf{p}_1, \mathbf{p}_2^{t-1}) + \gamma_2 F_2(\mathbf{p}_1, \mathbf{p}_2^{t-1})\right)$$
$$\mathbf{p}_2^t \in \arg \min_{p_2} \left(\gamma_1 F_1(\mathbf{p}_1^{t-1}, \mathbf{p}_2) + \gamma_2 F_2(\mathbf{p}_1^{t-1}, \mathbf{p}_2)\right).$$

Let it be noted that the minimization is local and involves an inaccurate search. The inaccurate search is based on gradient-descent with one-step update. A range of step-sizes has been used and it has been determined empirically that within this range, the performance of the system is the same.

In System II, the integration framework is based on the sequential approach as shown in Fig. 5. The concept of

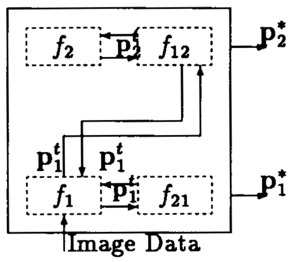

Fig. 4. Global objective integration.

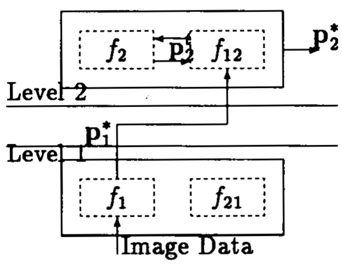

Fig. 5. Sequential integration.

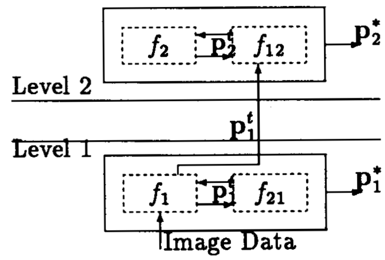

Fig. 6. Game-theoretic integration.

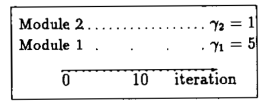

Fig. 7. Scheduling in parallel gradient descent.

optimality is defined by the solution $(\mathbf{p}_1^*, \mathbf{p}_2^*)$, which optimizes the objectives sequentially as

$$\mathbf{p}_1^* \in \arg \min_{p_1} F_1(\mathbf{p}_1, \mathbf{p}_2^*) \rightarrow \mathbf{p}_2^* \in \arg \min_{p_2} F_2(\mathbf{p}_1^*, \mathbf{p}_2).$$

The decision-making is described as follows:

$$\mathbf{p}_1^t \in \arg \min_{p_1} F_1(\mathbf{p}_1, \mathbf{p}_2^{t-1})$$
$$\mathbf{p}_2^t \in \arg \min_{p_2} F_2(\mathbf{p}_1^t, \mathbf{p}_2).$$

Let it be noted that each minimization is local and again involves inaccurate search. Each inaccurate search is based on gradient descent with one-step update. A range of step-sizes—empirically determined—has been used by each module.

In System III, the integration framework is based on the parallel game-theoretic approach as shown in Fig. 6. The concept of optimality is defined by the Nash solution $(\mathbf{p}_1^*, \mathbf{p}_2^*)$, which optimizes the 2-module game problem:

$$\mathbf{p}_1^* \in \arg \min_{p_1} F_1(\mathbf{p}_1, \mathbf{p}_2^*)$$
$$\mathbf{p}_2^* \in \arg \min_{p_2} F_2(\mathbf{p}_1^*, \mathbf{p}_2).$$

The parallel decision-making model is described as follows:

$$\mathbf{p}_1^t \in \arg \min_{p_1} F_1(\mathbf{p}_1, \mathbf{p}_2^{t-1}) \,\|\, \mathbf{p}_2^t \in \arg \min_{p_2} F_2(\mathbf{p}_1^{t-1}, \mathbf{p}_2).$$

Let it be noted that each minimization occurs in parallel with the other minimization and hence each module has access to the same initial conditions. As the parallel decision-making model cannot be exactly implemented, we resort to an inaccurate search algorithm. A parallel gradient descent algorithm has been developed for this purpose. The scheduling was similar to that shown in Fig. 7 where γ_1 and γ_2 are the iteration frequencies for each module. The parallel gradient descent increases the number of stages in the game and the reader is referred to [10] for a detailed exposition. The analysis shows that step sizes and the time spent at each stage of the game are critical considerations and gives an analytic tool for their determination.

C. Experiments

All the three systems were implemented in Pascal and C languages on DEC 3200 workstation running VMS operating system. Their performance was evaluated and compared by testing them under a variety of gray level images ranging from synthetic images to vidicon images to medical images. The vidicon images were taken in our laboratory. The medical images were supplied by Department of Diagnostic Radiology of Yale University. In order to evaluate the performance of the systems, a good measure of performance or error is necessary. Such a measure can be defined for synthetic images based on measuring the difference between the ideal performance and the observed performance. However, for real images, evaluation methods better than qualitative evaluation (visual inspection) seem necessary, but at the moment such methods for evaluating the performance of the system in isolation do not exist. Hence, the performance of the three systems in real images is compared based on qualitative analysis.

1) Synthetic Image Example: The image shown in top left of Fig. 8 is a simple synthetic 15×46 image. The decision space of module 1 $P^1 \subset R^{2(15\times46)}$. The initial edges $\mathbf{p}_1^0$ are obtained by applying a Sobel operator to the original image and are shown in Fig. 8 (top center-left). The displayed edges are both length and intensity coded based on their magnitudes. Darker and longer vectors represent stronger edges. This standard is observed when displaying edges of all the experiments. The decision space of module 2 is $P^2 \subset R^{2(4+2)}$. The ideal values for the contour parameters are known exactly and are represented as $\mathbf{p}_{2_j}^M j = 1, 2$. Different initial points $\mathbf{p}_2^0$ can thus be generated by adding a randomly generated perturbation vector $\Delta \mathbf{p}_{2_j}$ to each $\mathbf{p}_2^M$. The correlation error—a measure of the non-overlap of the region inside the contour of the model contour parameters $\mathbf{p}_2^M$ and the region inside the final output variable $\mathbf{p}_2^*$ can be used to assess the goodness of the results.

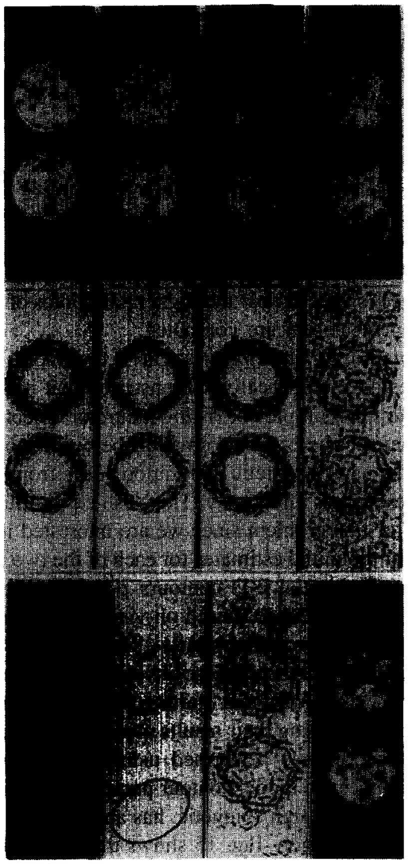

Fig. 8. Experiment 1—Synthetic Image. *Top row: Initial Conditions.* Left: Image, Center-left: Initial vector field $\mathbf{p}_1^0$. Center-right: Initial 2-D contours due to $\mathbf{p}_2^0$. Right: Image and initial 2-D contours. *Middle row: Edges* $\mathbf{p}_1$ *and 2-D Contours* $\mathbf{p}_2$. Left: $\mathbf{p}_1^0$ and $\mathbf{p}_2^0$. Center-left: System I results for $\mathbf{p}_1^*$ and$\mathbf{p}_2^*$. Center-right: System II results for $\mathbf{p}_1^*$ and $\mathbf{p}_2^*$. Right: Game-theoretic system results for $\mathbf{p}_1^*$ and $\mathbf{p}_2^*$. *Bottom row: Image and 2-D contours.* $\mathbf{p}_2$ Left: Image and $\mathbf{p}_2^0$. Center left: Image and system I $\mathbf{p}_2^*$. Center right: Image and system II $\mathbf{p}_2^*$. Right: Image and game-theoretic system $\mathbf{p}_2^*$.

If $\mathcal{R}^M \subset R^2$ denotes the set whose boundary is represented by $\mathbf{p}_2^M, \mathcal{R}^* \subset R^2$ denotes the set whose boundary is represented by $\mathbf{p}_2^*$, and $m \in C[R^2, [0,1]]$ a mapping that gives the area of a given set in R^2, the correlation error CE is

$$\mathrm{CE} = 1 - \frac{m(\mathcal{R}^M \cap \mathcal{R}^*)}{m(\mathcal{R}^M)}.$$

One sample experiment is shown in Fig. 8. The final edges $\mathbf{p}_1^*$ and 2-D contours $\mathbf{p}_2^*$ from System I, System II and System III are shown overlaid in Fig. 8 (middle center-left, center-right, right). The bottom row in Fig. 8 shows the initial 2-D contours and the final contours of the three systems overlaid on the image. System I, in which all the objectives are merged together into a single objective ($\gamma_1 = 0.4, \gamma_2 = 0.6$) fails to correctly delineate the contours. The effects of assigning the appropriate γ's were also studied experimentally. It was found that tuning γ's correctly is both timewise expensive and dependent on the exact forms of the functionals used, and hence difficult to tune perfectly. It appears that top-down control from the shape parameters wrongly bias the edges $\mathbf{p}_1^*$. System II performs better than the previous system, but it still fails to correctly delineate the two objects. System III produces a good result.

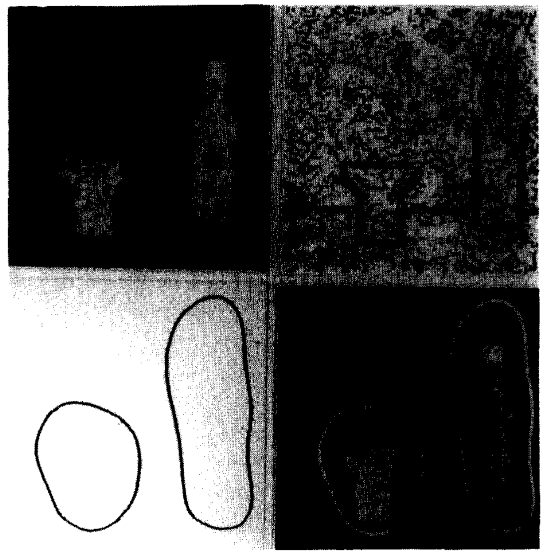

Fig. 9. Experiment 2, bottle-cup image. Upper left: image. Upper right: initial edge vector field. Lower left: Initial 2-D shapes. Lower right: Initial shapes on image.

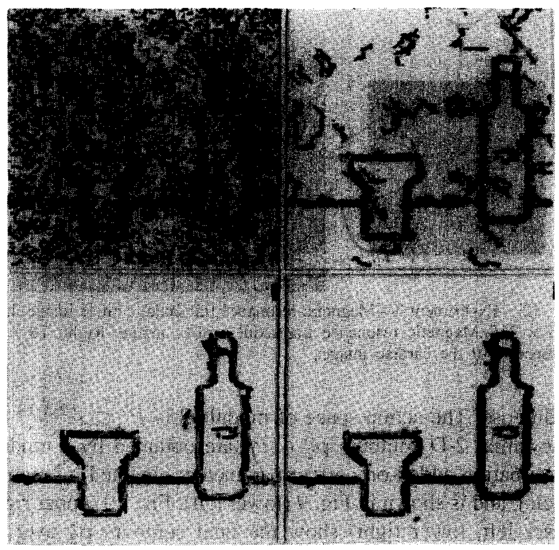

Fig. 10. Experiment 2 (cont.)—Edges and 2-D contours $\mathbf{p}_2$. Upper left: Initial edges $\mathbf{p}_1^0$ and initial 2-D contours $\mathbf{p}_2^0$. Upper right: System I results $\mathbf{p}_1^*$ and $\mathbf{p}_2^*$. Lower left: System II results $\mathbf{p}_1^*$ and $\mathbf{p}_2^*$. Lower right: Game-theoretic system results $\mathbf{p}_1^*$ and $\mathbf{p}_2^*$.

2) Camera Image Example: Another example is a noisy image of a bottle and cup scene shown in Fig. 9, acquired using a vidicon camera and then altered by adding Gaussian white noise. In this case the action space of module 1 $P^1 \subset R^{2(64\times 64)}$. The initial edge vector field as specified by $\mathbf{p}_1^0 \in P^1$ and computed using the Sobel edge operator, is very noisy

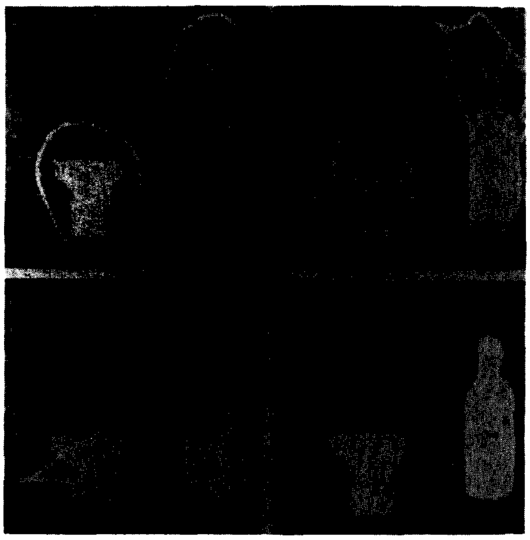

Fig. 11. Experiment 2 (cont.)—Image and 2-D contours $\mathbf{p}_2$. Upper left: Image and initial 2-D contours $\mathbf{p}_2^0$. Upper right: Image and system I 2-D contours $\mathbf{p}_2^0$. Lower left: Image and system II 2-D contours $\mathbf{p}_2^*$. Lower right: Image and game-theoretic system 2-D contours $\mathbf{p}_2^*$.

Fig. 12. Experiment 3—Magnetic resonance transaxial cardiac image. Left: 256 × 256 Magnetic resonance transaxial cardiac image. Right: 78 × 64 Subsection of the cardiac image.

and dense. The action space of module 2 is $P^2 \subset R^{2(4\cdot 10+2)}$. The initial 2-D contours $\mathbf{p}_2^0 \in P^2$ are obtained by a random perturbation added on correct parameters obtained by manual tracing and is shown in Fig. 9 (lower left). Fig. 10 (upper right, lower left, lower right) show the final contours $\mathbf{p}_2^*$ overlaid on the final edges $\mathbf{p}_1^*$ of the three systems respectively. Fig. 11 (upper left) shows the initial contours $\mathbf{p}_2^0$ overlaid on the image. Fig. 11 (upper right, lower left, lower right) show the final contours $\mathbf{p}_2^*$ of the three systems overlaid on the image respectively.

3) MRI Image Example: An MRI image example is a 78 × 64 image of the left and right ventricle in Fig. 12 (right) from a 256 × 256 magnetic resonance transaxial cardiac image of Fig. 12 (left). In this case the action space of module 1 is $P^1 \subset R^{2(78\times 64)}$. The action space of module 2 is $P^2 \subset R^{2(4\cdot 10+2)}$. The initial 2-D contours $\mathbf{p}_2^0 \in P^2$ are obtained by a random perturbation added to the correct parameters obtained by manual tracing of the ventricles and is shown in Fig. 13 (lower left). An anatomical atlas of the heart is used in constructing the relational coupling between the two ventricles. The final contours from the three systems overlaid on the image are shown consecutively in Figs. 14 (bottom upper-right, bottom lower left, bottom lower right). Both System I and System II fail completely at their tasks. In System III, the performance is better although it is observed that the trabeculations in the left endocardial surface of the left ventricle are not picked up as accurately as the ones in the right endocardial surface of the same ventricle. This can be explained by computational considerations. Our experience has shown that higher harmonics are adjusted locally and the convergence is very slow if the higher harmonics of the elliptic Fourier representation are computed.

4) Discussion of Experiments: Our experiments with several images have empirically demonstrated that it is important to adhere to the multiple coexisting nature of objectives and to have exchange of partial information between modules. The power of the game-theoretic integration arises from addressing each of these issues effectively. The Nash equilibrium solution is natural for this problem since we are interested in solutions that are near the initial estimates for each of the two parameters $\mathbf{p}_1$ and $\mathbf{p}_2$. For initial 2-D contours $\mathbf{p}_2^0$ that were relatively similar to the model case (based on visual inspection), all the three systems performed similarly. The effect of the initial output variables $\mathbf{p}_2^0$ can thus be investigated by running the same problem from different starting points with all the three systems and comparing their results using the correlation error. The investigation was performed using the synthetic image (see Fig. 8) since the model contour parameters $\mathbf{p}_2^M$ are known and the image, although synthetic, has a reasonable amount of noise and complication. Fig. 15 shows the comparative sensitivity of the three systems to different initial input variables. The x-axis represents the norm of the perturbation vector $\Delta\mathbf{p}_2$ and the y-axis represent the correlation error. It is observed that for perturbations of relatively lower scale, the game-theoretic system outdoes the rest of the systems. For progressively increased amounts of perturbation, the performance of all the three systems deteriorate as shown. Experiments showed that with *large* amounts of perturbation (where largeness was dependent on the image and the objects), none of the systems would succeed in their performance. Although it is desirable to define the limits of the tolerable perturbation, it has been found that it is extremely difficult to do so for a general case since it depends on the form of the functionals used as objectives.

The multimodal nature of F_2 affects the system behavior since the limit point will then depend on initial point p_1. This is the intuitive reason as to why the results of System II differ from those of System III. Let us recall that p_1 represents a discrete edge vector field and p_2 represents a parameterization of the boundaries of 2-D objects. The initial p_1^0 corresponds to a vector field that is relatively dense, noisy, and ambiguous. The objective of module 1 is designed to improve the interpretability of the edge vector field by 1) making the vector field relatively sparse and 2) minimizing the noise. The objective of module 2 is designed to improve the initial estimate of shapes (obtained from *a priori* model of 2-D boundaries) based on the correlation measure and the geometrical constraints between the objects. The system can proceed to find the actual 2-D shape parameters according

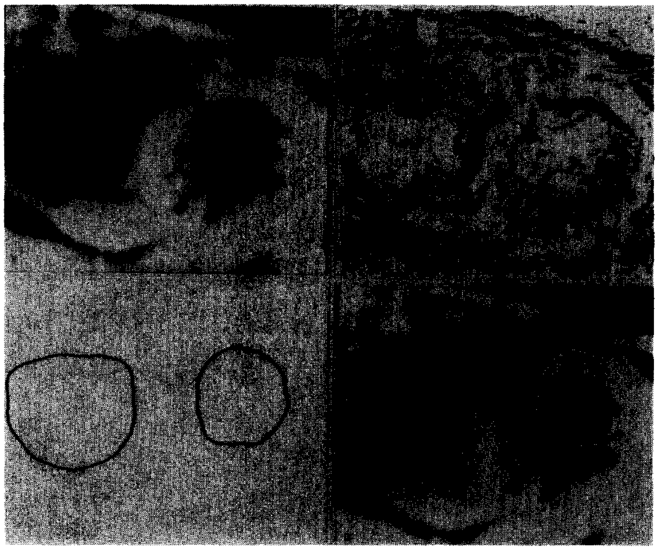

Fig. 13. Experiment 3 (cont.)—Magnetic resonance transaxial cardiac image. Upper left: Image. Upper right: Initial edges $\mathbf{p}_1$. Lower left: Initial shapes $\mathbf{p}_2^0$. Lower right: Initial shapes on image.

to three different scenarios. In the first scenario, module 2 operates directly on the raw edge vector field (i.e., $p_1 = p_1^0$). In this case, there will be no need for module 1. However, the noise and ambiguity present in the edge vector field can be very confusing, thus system performance turns out to be poor when there is noise. The next scenario is sequential processing. Module 1 improves the edge vector field as best as it can (i.e., finds $p_1 = p_1^0$) and communicates this to module 2, which then operates on this information. This is System II. However, this approach yields poor results if the initial shape estimates (i.e., p_2^0) are not very close to their actual shapes (as observed by a human observer). Intuitively, this is to be expected. Easier interpretability implies that the final vector field should be sparse, but a sparse vector field may not provide the sufficient cues to module 2 for it to find the correct shape parameters. The third scenario (System III) overcomes both the noise and sparsity problem. Since both the noise and denseness are gradually removed, module 2 can be guided into finding the correct 2-D shapes without being confused by the noise as to degrade its performance. Therefore, if the initial edge vector field is good and sparse and the initial contours are very poor, both System II and System III will fail to find the contours accurately. Hence the results *do not suggest* that one needs to start with a poor estimate of the edge vector field although at first glance this might be the conclusion. Rather, the experiments suggest that a sparse vector field (such as represented by $\mathbf{p}_1^*$) may not provide the sufficient cues to module 2 for it to find the correct shape parameters. The fact that the edge vector field is initially dense and that it becomes gradually sparser and cleaner seems to be key to the better performance of game-theoretic approach.

Another interesting question is: why talk about levels when the game-theoretic formalism has nothing to do with the visual hierarchy? It is noted that this framework treats the modules, whether at the same level or at different levels, homogeneously. Thus, the first reason is related to the fact that systems in which the integration problem arises are in general hierarchical and modular. The concept of *levels* is discussed to point out that this framework is applicable both in systems having an hierarchical modular structure and those having a flat modular structure. From a complementary perspective, although in an integration framework based on Nash games, the modules at different levels are treated similarly to those at the same level, this is not necessarily the case in game-theoretic integration frameworks based on other types of solutions. In particular, some solutions enable domination among the modules and are of great interest.

It should be remarked that the discussion of the experiments did not include discussion of convergence times of the three systems. One of the reasons was that in the results shown where there are qualitative differences in the performance, it is not meaningful to compare convergence characteristics. However, it should be added that some experimentation in the cases where the three system all performed relatively well, the convergence time of the parallel game-theoretic framework was in most cases better than the other two systems, simply because the computation is simpler (as compared to System I) or there is a gain by not making module 2 wait for the convergence of module 1 (as compared to System II). Finally, in this task, the output of module 1—the edges $\mathbf{p}_1$—are relevant only with respect to a good segmentation. However, it should be pointed out that in some applications, such as stereo matching, the output of module 1—the edges $\mathbf{p}_1$—might be a useful output and of direct interest.

VI. Conclusion

This paper has provided a systematic formulation of the integration problem in the design of intelligent sensor systems. The goal was to develop an integration framework that 1)

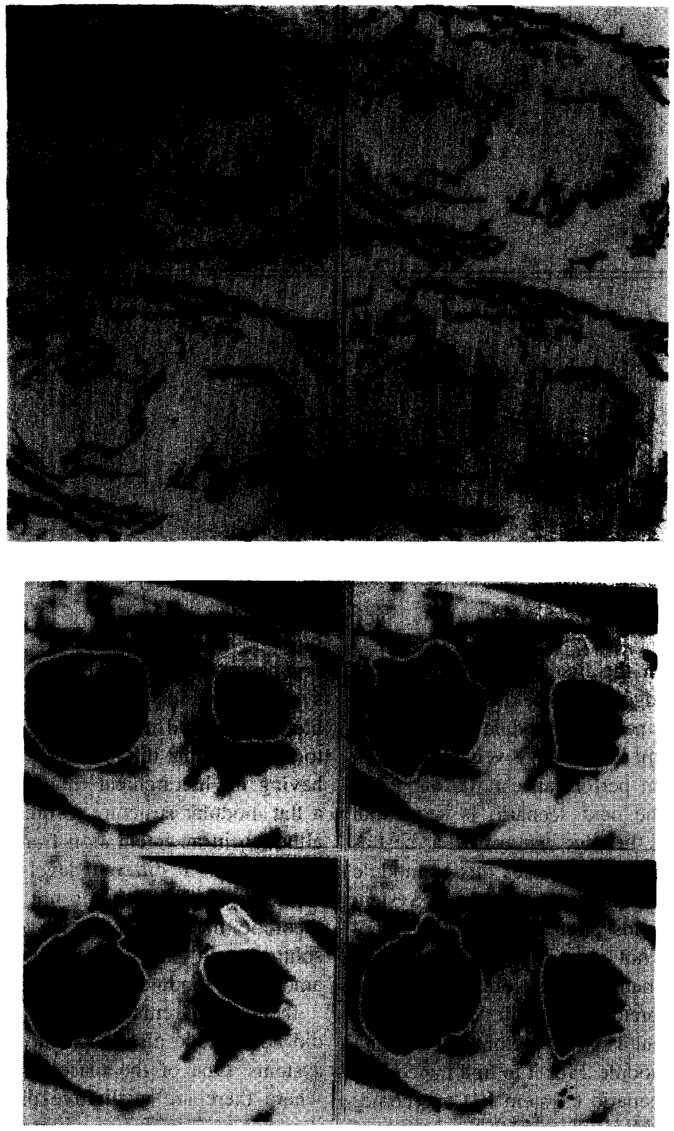

Fig. 14. Experiment 3 (cont.)—*Top 2 rows: Edges* $\mathbf{p}_1$ *and 2-D contours* $\mathbf{p}_2$. Upper left: Initial edges $\mathbf{p}_1^0$ and initial 2-D contours $\mathbf{p}_2^0$. Upper right: System I results $\mathbf{p}_1^*$ and $\mathbf{p}_2^*$. Lower left: System II results $\mathbf{p}_1^*$ and $\mathbf{p}_2^*$. Lower right: Game-theoretic system results $\mathbf{p}_1^*$ and $\mathbf{p}_2^*$; *Bottom 2 rows: Image and 2-D contours* $\mathbf{p}_2$. Upper left: Image and initial 2-D contours $\mathbf{p}_2^0$. Upper right: System I. Lower left: System II. Lower right: Game-theoretic system.

preserves the multiple and coexisting nature of the objectives of the modules, 2) has decentralized decision making, 3) is analytically and computationally tractable, and 4) is general in nature, and to apply this framework in designing an image analysis system. It is the authors' belief that this approach has opened up the possibility of new research directions by demonstrating the relevance of game-theoretic concepts in the design of intelligent sensor systems. Although it is certainly recognized that based on a single application it might be too premature to make such a claim, it is also strongly felt that the generality of the formulation lends itself very naturally to other design problems in intelligent sensor systems. It is hoped that this paper has presented the beginnings of a framework that can be used in such applications. By focusing on the problem of preserving the multiple nature of the objectives, the paper has brought a fairly unquestioned assumption back into question. The Nash equilibrium—a solution concept in nonzero-sum games under a noncooperative setting—is proposed to be a competing and in certain problems a more natural type of solution to the multiple objectives problem than the global objective solution. Although the distinctiveness of the two types of solutions is proved theoretically for certain classes of problems, the question of "Why are the Nash solutions better for certain problems?" has not been completely addressed. Perhaps, the philosophical question is destined to remain unresolved. Nevertheless, it is worthwhile to use every possible other tool to think about this question. Intuition offers the perspective that its betterness in certain problems is due

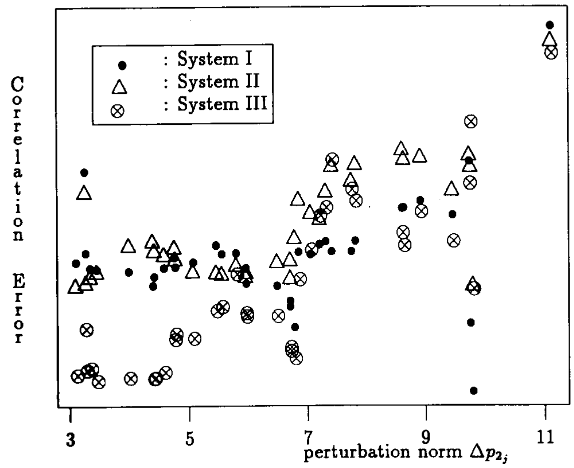

Fig. 15. Quantitative comparison of three systems.

to its nature of being a *compromise* solution, at which the objectives of all the players are satisfied to roughly the same extent. The Prisoners' Dilemma problem is a good example of this. Moreover, it seems that using any available partial information might provide important cues to the modules about how they should be updating their decisions. It is believed that an evaluation of betterness can potentially be assessed after constructing many systems based on the different approaches and comparing their performance. Initially, there remains a lot to be done in both the theory and application areas. For starters, consideration of other types of equilibria and applications in different domains are being investigated.

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