1 Dynamic System Analysis

The aircraft system dynamics is described by the following set of differential equations (which are linear and already in state space representation). State variables are α, r, θ and input variable is δ.

$$\dot{\alpha} = -0.31\alpha + 57.4r + 0.232\delta, \tag{1.1a}$$
$$\dot{r} = -0.016\alpha - 0.425r + 0.0203\delta, \tag{1.1b}$$
$$\dot{\theta} = 56.7r. \tag{1.1c}$$

The 3 transfer functions describing aircraft system dynamics:

$$G_\alpha(s) = \frac{A(s)}{\Delta(s)}, \tag{1.2a}$$
$$G_r(s) = \frac{R(s)}{\Delta(s)}, \tag{1.2b}$$
$$G_\theta(s) = \frac{\Theta(s)}{\Delta(s)}. \tag{1.2c}$$

For open-loop dynamics, we must take into account the impact of the actuator and sensor. From the question sheet, we know the actuator has transfer function

$$G_a(s) = \frac{1}{0.0145s + 1}, \tag{1.3a}$$

and the sensor has transfer function

$$G_m(s) = \frac{e^{-0.0063s}}{0.0021s + 1}. \tag{1.3b}$$

We assume that the system has zero initial conditions when determining these transfer functions. Given the transfer functions of the actuator, Equation (1.3a) and sensor, Equation (1.3b) in this system, we can find that the poles of both have negative real parts $s = -\frac{1}{0.0145}$, $s = -\frac{1}{0.0021}$ respectively; and are therefore stable. Since the open-loop system is the cascade of the actuator, aircraft dynamics, and sensor as shown in the block diagram in Figure 1. Provided the transfer function relating to the aircraft dynamics $(G_\alpha, G_r, G_\theta)$ has only poles with negative real parts, the open-loop system will be stable.

1.1 Analysis of G_α, G_r, G_θ.

From the system of differential equations the transfer functions of G_α, G_r and G_θ can be derived.

$$G_\alpha = \frac{0.232s + 1.26382}{(s + 0.31)(s + 0.425) + 0.9184} \tag{1.4a}$$
$$G_r = \frac{0.0203s + 0.002581}{(s + 0.31)(s + 0.425) + 0.9184} \tag{1.4b}$$
$$G_\theta = \frac{1.15101s + 0.1463427}{s((s + 0.31)(s + 0.425) + 0.9184)} \tag{1.4c}$$

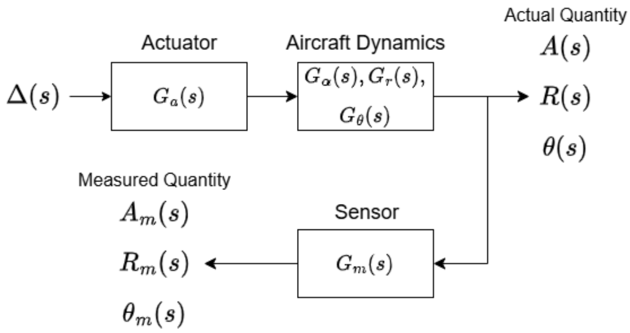
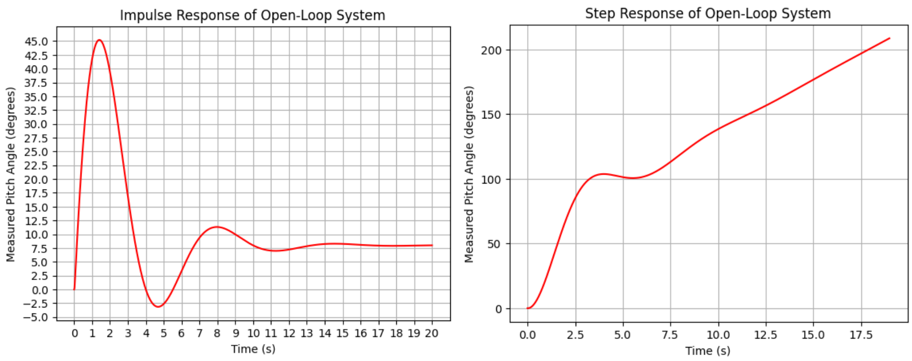


Figure 1: A block diagram of the control system.



(a) The impulse response of G_θ.

(b) The step response of G_θ.

Figure 2: Output plots of G_θ.

The poles of G_α occur when $s^2+0.735s+1.05015=0$ which can be solved using the quadratic formula to give $s=-0.3675\pm j0.9566$. The poles of G_r also occur when $s^2+0.735s+1.05015=0$ and therefore result in the same value. Since $G_\theta(s)=\frac{56.7}{s}G_r(s)$; $G_\theta(s)$ shares all of the poles of $G_r(s)$, which are all stable, but also possesses an additional pole at $s=0$. This means the poles of $G_\theta(s)$ are $s=-0.3675\pm j0.9566, s=0$. Since we have a pole with a non-negative real part, G_θ is not BIBO stable, meaning the open-loop system relating deflection angle of elevators to pitch angle is also not BIBO stable. This means a controller will be required to obtain BIBO stability of the system. Figure 2a shows that when the deflection angle of the elevators δ is a unit impulse, the pitch angle θ of the aircraft reaches a maximum of roughly 45 degrees after 1.5 seconds, before stabilising at (roughly) 7.5 degrees after around 16 seconds. Note that despite the input being a unit impulse, the pitch angle does not return to zero for the open-loop system. Figure 2b shows that as time increases, pitch angle θ is increasing unbounded. This follows from the prior discovery that $G_{\theta(\text{open-loop})}$ is not BIBO stable. The frequency response of G_θ is plotted along with the impulse, step and frequency responses of G_α and G_r in the attached Python notebook.