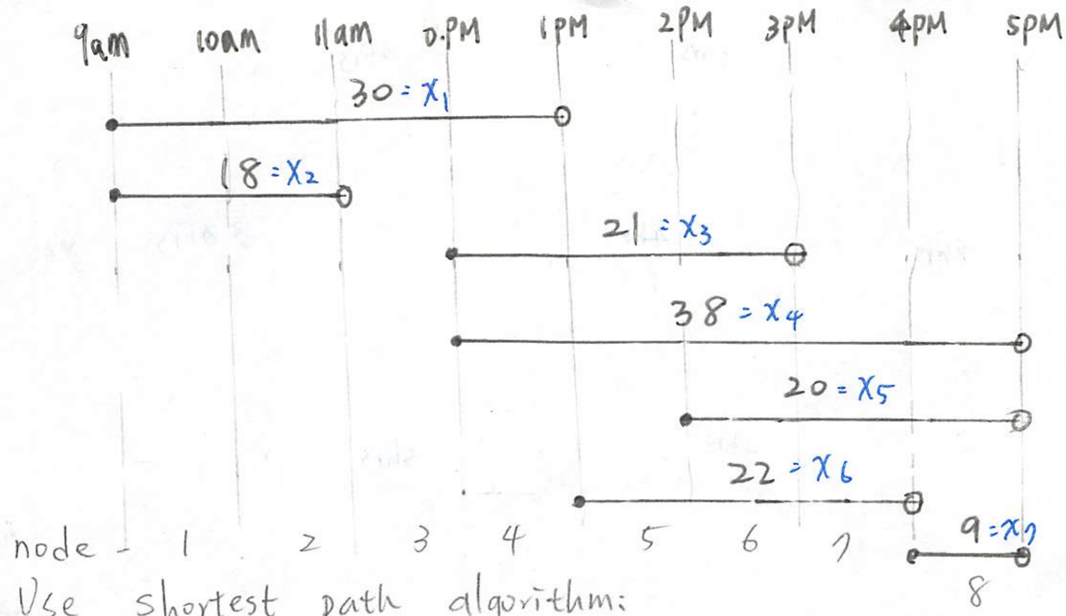
1.



Use shortest path algorithm:

define $x_1 \sim x_7$ and the objective is:

$$\min \quad 30x_1 + 18x_2 + 21x_3 + 38x_4 + 20x_5 + 22x_6 + 9x_7.$$

also, at each node (time period) must satisfy $\sum_{j:(i,j)\in E} x_{ij} \geq 1$

$\Rightarrow$ node

1:	$x_1 + x_2$	≥ 1
2:	$x_1 + x_2$	≥ 1
3:	x_1	≥ 1
4:	$x_1 + x_3 + x_4$	≥ 1
5:	$x_3 + x_4 + x_6$	≥ 1
6:	$x_3 + x_4 + x_5 + x_6$	≥ 1
7:	$x_4 + x_5 + x_6$	≥ 1
8:	$x_4 + x_5 + x_7$	≥ 1

$\Rightarrow$ Use gurobi to solve the shortest path problem:

$\Rightarrow$
- 9am - 1pm = $x_1 = 1$
- 9am - 11am = $x_2 = 0$
- noon - 3pm = $x_3 = 0$
- noon - 5pm = $x_4 = 0$
- 2pm - 5pm = $x_5 = 0$
- 1pm - 4pm = $x_6 = 1$
- 4pm - 5pm = $x_7 = 1$

and total cost = $30\times1 + 22\times1 + 9\times1 = 61$

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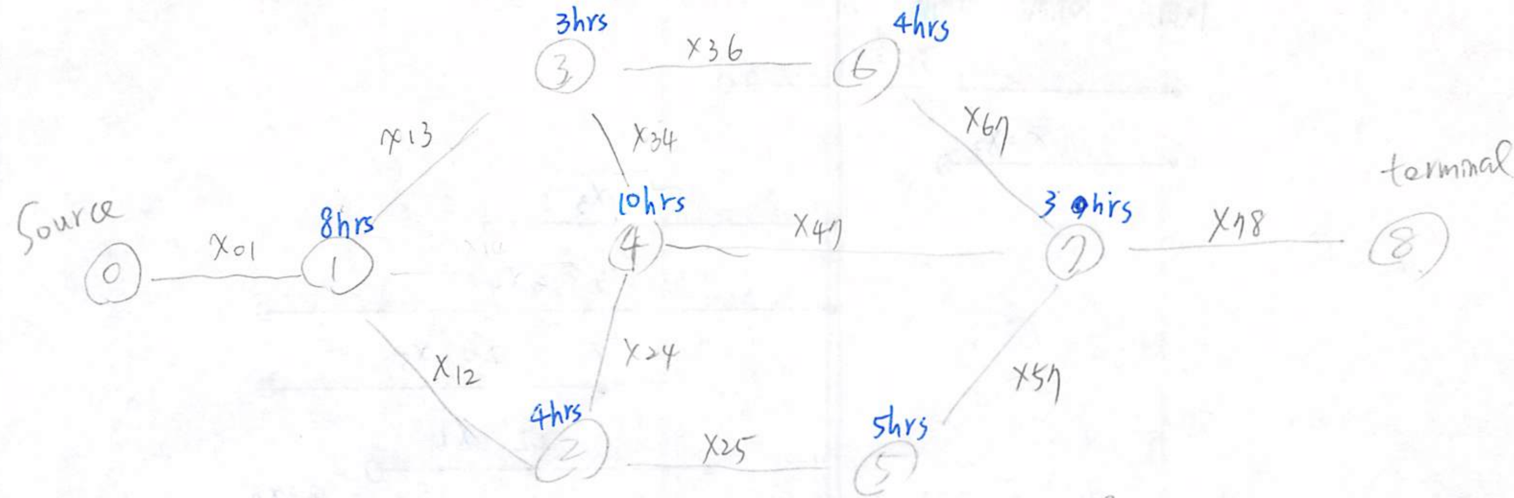
1. b



defin $x_{ij} \in \{0,1\}$ and $i \in \{0,8\}$, $j \in \{0,8\}$

$$\sum_{j:(i,j)\in E} x_{ij} - \sum_{j:(j,i)\in E} x_{ji} = \begin{cases} 1 & \text{for } i = \text{source} = 0 \\ -1 & \text{for } i = \text{terminal} = 8 \\ 0 & \forall i \in V \setminus \{s,t\} \end{cases}$$

x_{34} x_{36} x_{47} x_{57} x_{67} x_{78}.

node			dual
0 (source)	x_{01}	$= 1$	y_0
1	$-x_{01} + x_{12} + x_{13}$	$= 0$	y_1
2	$-x_{12} + x_{24} + x_{25}$	$= 0$	y_2
3	$-x_{13} + x_{34} + x_{36}$	$= 0$	y_3
4	$-x_{24} - x_{34} + x_{47}$	$= 0$	y_4
5	$-x_{25}$ x_{57}	$= 0$	y_5
6	$-x_{36}$ x_{67}	$= 0$	y_6
7	$-x_{47} - x_{57} - x_{67}$ x_{78}	$= 0$	y_7
8 (terminal)	$-x_{78}$	$= -1$	y_8

and $x_{01}, x_{12}, x_{13}, x_{14}, x_{24}, x_{25}, x_{34}, x_{36}, x_{47}, x_{57}, x_{67}, x_{78} \geq 0$

find minimum $\sum_{(i,j)\in E} c_{ij} \times \sum_{j:(i,j)\in E} x_{ij}$

=

Σ minimum $\quad \Sigma_{(i,j)\in E} \quad c_{ij} \sum_{j:(i,j)\in E} x_{ij}$

$= 8 \times x_{01} + 4 \times x_{12} + 3 \times x_{13} + 10 \times (x_{24} + x_{34}) + 5 \times x_{25} + 4 \times x_{36}$

node 1 | node 2 | node 3 | node 4 | node 5 | node 6

$+ (x_{47} + x_{57} + x_{67}) \times 3 + 0 \times x_{78}$

node 7 | node 8

$= 8x_{01} + 4x_{12} + 3x_{13} + 10x_{24} + 5x_{25} + 10x_{34} + 4x_{36} + 3x_{47} + 3x_{57}$
$3x_{67} + 0 \times x_{78}$

$\Downarrow$

change to dual

$\rightarrow$ find max $b^T w$

$= y_0 - y_8$

s.t.

x_{01}: $y_0 - y_1 \leq 8$

x_{12}: $y_1 - y_2 \leq 4$

x_{13}: $y_1 - y_3 \leq 3$

x_{14}: $y_1 - y_4 \leq 10$

x_{24}: $y_2 - y_4 \leq 10$

x_{25}: $y_2 - y_5 \leq 5$

x_{34}: $+y_3 - y_4 \leq 10$

x_{36}: $y_3 - y_6 \leq 4$

x_{47}: $-y_7 \leq 7$

x_{57}: $y_5 - y_7 \leq 7$

x_{67}: $y_6 - y_7 \leq 7$

x_{78}: $y_7 - y_8 \leq 0$

I using the gurobi to solve the $y_0 \sim y_8$

$\Rightarrow y_0 = 18 \quad y_8 = 0$

$\therefore$ the minimum cost is 18

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