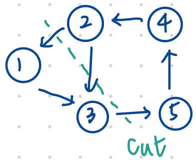
Q1

(a) Let M^* be a max matching, I^* be a max independent set.
For each of $|M^*|$ matched pairs, I^* can include at most one of the vertex

$\rightarrow |I^*| \leq |V| - |M^*| \rightarrow |I^*| + |M^*| \leq |V|$

$\because$ M^* and I^* are maximum, thus $|M| + |I| \leq |V|$ holds for $\forall$ M and I

(b) false



cut $[S, \bar{S}]$ $\rightarrow$ intersect odd numbers of edges

1, 3   2, 4, 5

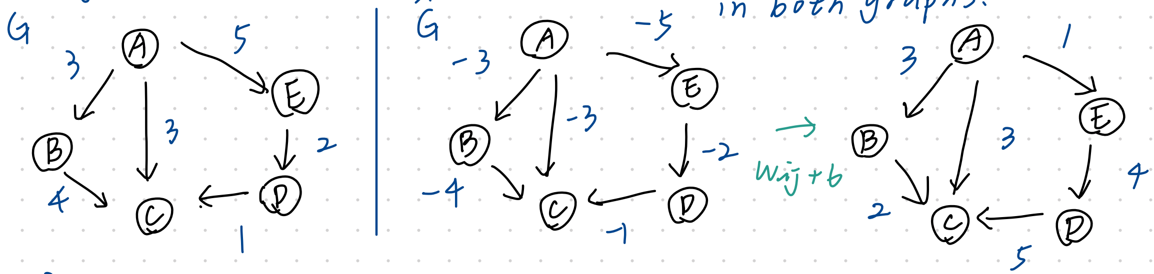
(C)

1、false

To update the weights without affecting the minimum spanning tree in $\hat{G}$, we add a big positive number C to every weight, $C = \max\{w_{ij} \in E\}$

If we update $\hat{w}_{ij} = \hat{w}_{ij} + C$ then $\hat{G}$ becomes a graph of non-negative weights.

By Dijkstra's algorithm, the shortest path from $A \rightarrow C$ is A-C in both graphs.



2、false

Multiplying edge weights by any positive constant factor preserves their relative order, as well as the relative order of any linear combination of the weigts. All path weights are linear combinations of edge weights, so the relative order of path weights is preserved.

Hence, the shortest path in G will be the shortest path in $\tilde{G}$