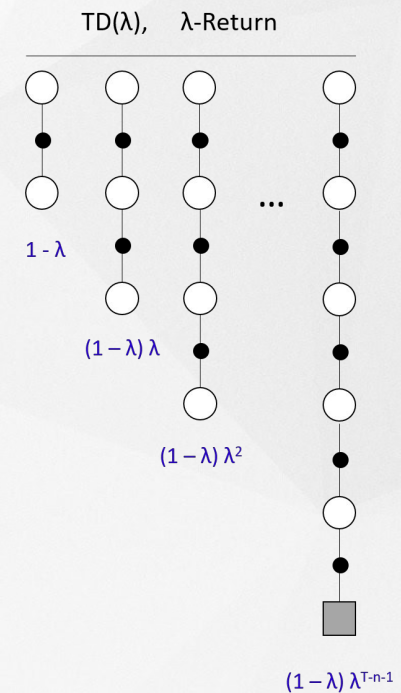


## Chapter 06. 스스로 전략을 짜는 강화학습 ( Reinforcement Learning )

# Temporal Difference



# Monte Carlo Method

## Monte Carlo ES (Exploring Starts), for estimating $\pi \approx \pi_*$

Initialize:

$\pi(s) \in \mathcal{A}(s)$  (arbitrarily), for all  $s \in \mathcal{S}$

$Q(s, a) \in \mathbb{R}$  (arbitrarily), for all  $s \in \mathcal{S}, a \in \mathcal{A}(s)$

$Returns(s, a) \leftarrow$  empty list, for all  $s \in \mathcal{S}, a \in \mathcal{A}(s)$

Loop forever (for each episode):

Choose  $S_0 \in \mathcal{S}, A_0 \in \mathcal{A}(S_0)$  randomly such that all pairs have probability  $> 0$

Generate an episode from  $S_0, A_0$ , following  $\pi$ :  $S_0, A_0, R_1, \dots, S_{T-1}, A_{T-1}, R_T$

$G \leftarrow 0$

Loop for each step of episode,  $t = T-1, T-2, \dots, 0$ :

$G \leftarrow \gamma G + R_{t+1}$

Unless the pair  $S_t, A_t$  appears in  $S_0, A_0, S_1, A_1, \dots, S_{t-1}, A_{t-1}$ :

Append  $G$  to  $Returns(S_t, A_t)$

$Q(S_t, A_t) \leftarrow \text{average}(Returns(S_t, A_t))$

$\pi(S_t) \leftarrow \operatorname{argmax}_a Q(S_t, a)$

# Temporal Difference

에피소드 마다 가 아니라 매 타임스텝 마다 가치함수를 업데이트

$$\text{MC : } V(S_t) \leftarrow V(S_t) + \alpha(G_t - V(S_t))$$

$$\text{TD(0) : } V(S_t) \leftarrow V(S_t) + \alpha(R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$

Input : the policy  $\pi$  to be evaluated

Initialize  $V(s)$  arbitrarily (e.g.,  $V(s) = 0, \forall s \in S^+$ )

Repeat (for each episode):

Initialize  $S$

Repeat (for each step of episode):

$A \leftarrow$  action given by  $\pi$  for  $S$

Take action  $A$ ; observe reward,  $R$ , and next state,  $S'$

$V(S) \leftarrow V(S) + \alpha[R + \gamma V(S') - V(S)]$

$S \leftarrow S'$

until  $S$  is terminal

# Bellman Equation

The value function can be decomposed into two parts:

- immediate reward  $R_{t+1}$
- discounted value of successor state  $\gamma v(S_{t+1})$

$$\begin{aligned} v(s) &= \mathbb{E} [G_t \mid S_t = s] \\ &= \mathbb{E} [R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots \mid S_t = s] \\ &= \mathbb{E} [R_{t+1} + \gamma (R_{t+2} + \gamma R_{t+3} + \dots) \mid S_t = s] \\ &= \mathbb{E} [R_{t+1} + \gamma G_{t+1} \mid S_t = s] \\ &= \mathbb{E} [R_{t+1} + \gamma v(S_{t+1}) \mid S_t = s] \end{aligned}$$



# Temporal Difference

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- Return  $G_t = R_{t+1} + \gamma R_{t+2} + \dots + \gamma^{T-1} R_T$  is *unbiased* estimate of  $v_\pi(S_t)$
- True TD target  $R_{t+1} + \gamma v_\pi(S_{t+1})$  is *unbiased* estimate of  $v_\pi(S_t)$
- TD target  $R_{t+1} + \gamma V(S_{t+1})$  is *biased* estimate of  $v_\pi(S_t)$
- TD target is much lower variance than the return:
  - Return depends on *many* random actions, transitions, rewards
  - TD target depends on *one* random action, transition, reward

- MC = unbiased, high variance
- TD = biased, small variance

# TD vs MC

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## Driving Home Example

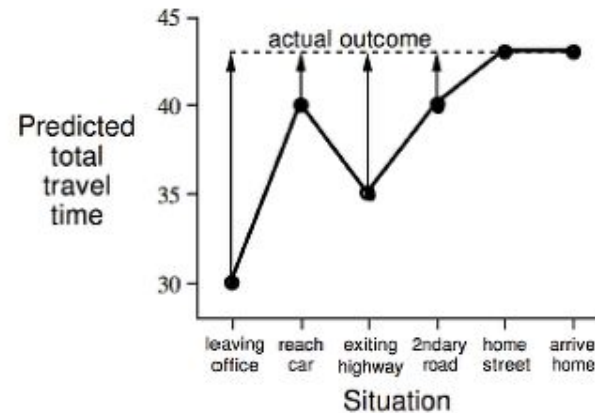
State	Elapsed Time (minutes)	Predicted Time to Go	Predicted Total Time
leaving office	0	30	30
reach car, raining	5	35	40
exit highway	20	15	35
behind truck	30	10	40
home street	40	3	43
arrive home	43	0	43

# TD vs MC

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## Driving Home Example: MC vs. TD

Changes recommended by Monte Carlo methods ( $\alpha=1$ )



Changes recommended by TD methods ( $\alpha=1$ )

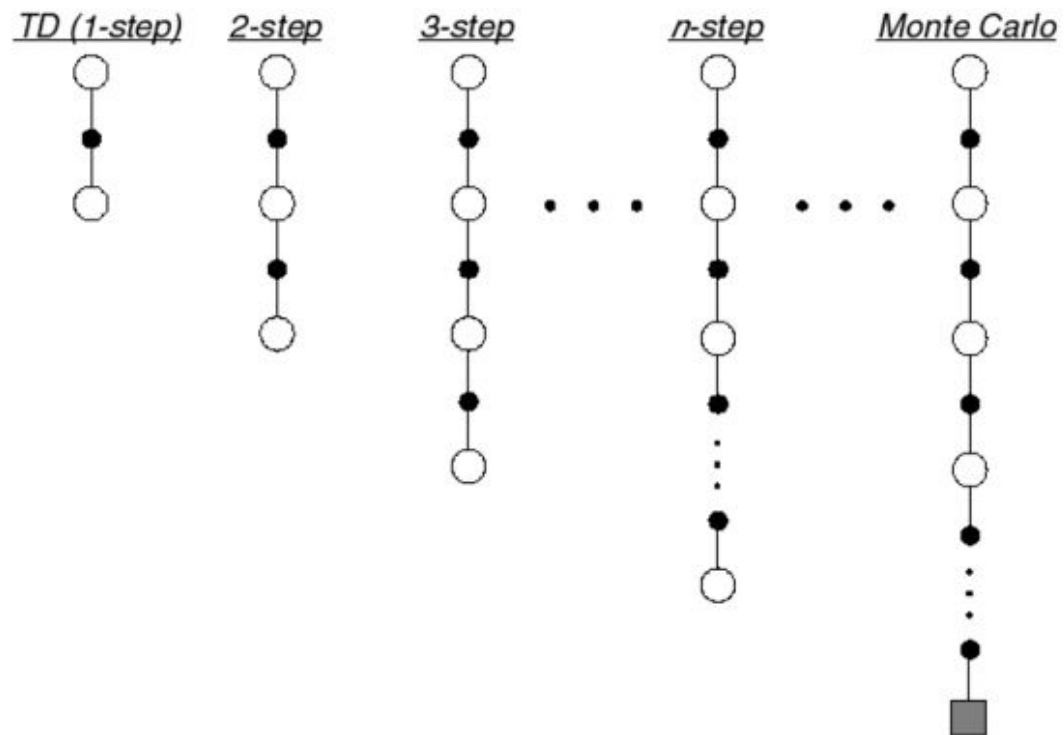


# N- Step TD

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## $n$ -Step Prediction

- Let TD target look  $n$  steps into the future





# N- Step TD

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## $n$ -Step Return

- Consider the following  $n$ -step returns for  $n = 1, 2, \infty$ :

$$n = 1 \quad (TD) \quad G_t^{(1)} = R_{t+1} + \gamma V(S_{t+1})$$

$$n = 2 \quad G_t^{(2)} = R_{t+1} + \gamma R_{t+2} + \gamma^2 V(S_{t+2})$$

$$\vdots$$
$$\vdots$$

$$n = \infty \quad (MC) \quad G_t^{(\infty)} = R_{t+1} + \gamma R_{t+2} + \dots + \gamma^{T-1} R_T$$

- Define the  $n$ -step return

$$G_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \dots + \gamma^{n-1} R_{t+n} + \gamma^n V(S_{t+n})$$

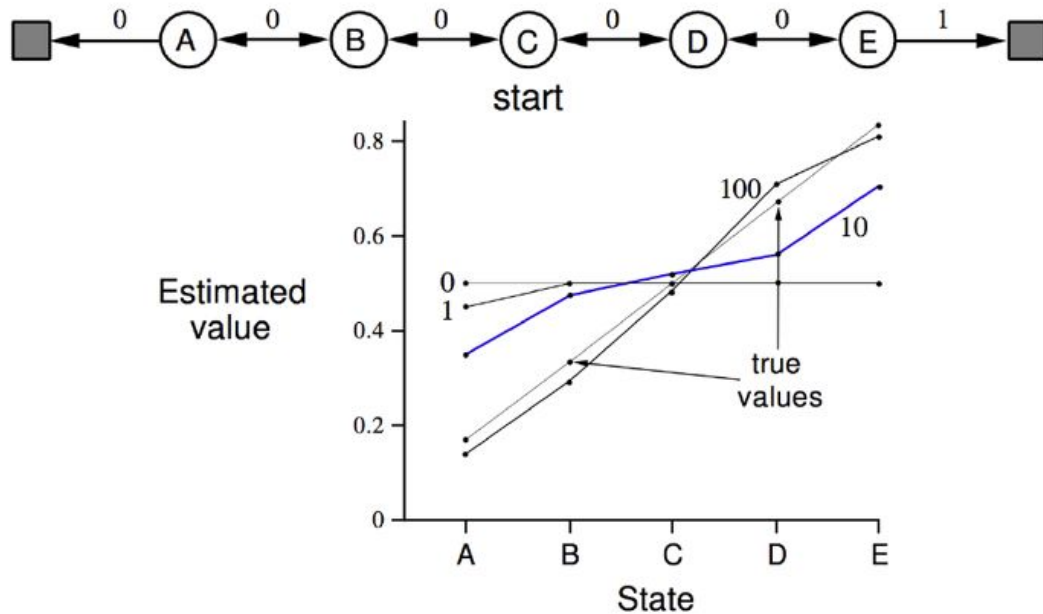
- $n$ -step temporal-difference learning

$$V(S_t) \leftarrow V(S_t) + \alpha (G_t^{(n)} - V(S_t))$$

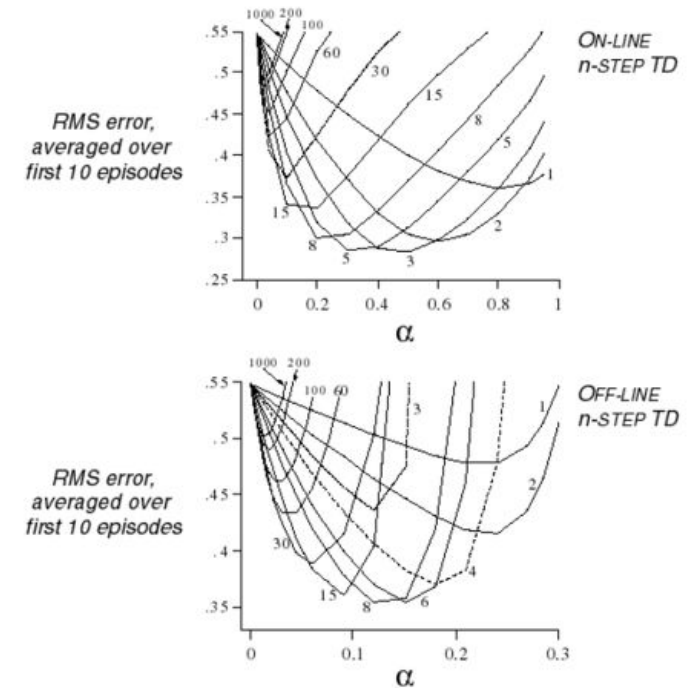
# TD( $\lambda$ )

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## Random Walk Example



## Large Random Walk Example



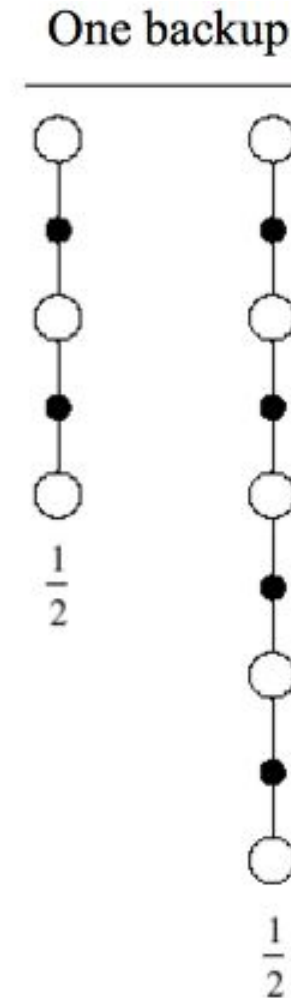
# Forward view TD( $\lambda$ )

<http://www0.cs.ucl.ac.uk/staff/d.silver/web/Home.html>

- We can average  $n$ -step returns over different  $n$
- e.g. average the 2-step and 4-step returns

$$\frac{1}{2}G^{(2)} + \frac{1}{2}G^{(4)}$$

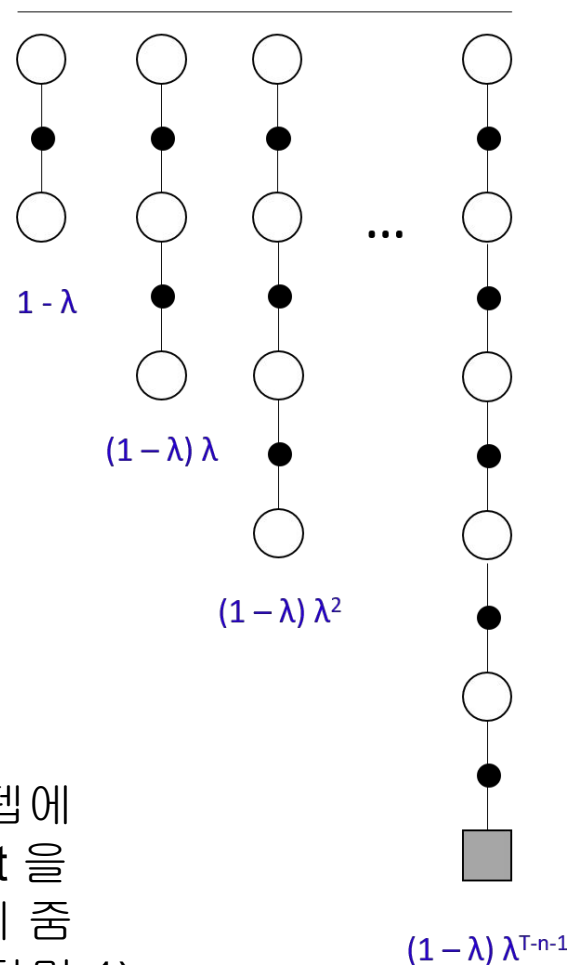
- Combines information from two different time-steps
- Can we efficiently combine information from all time-steps?



# Forward view TD( $\lambda$ )

<http://www0.cs.ucl.ac.uk/staff/d.silver/web/Home.html>

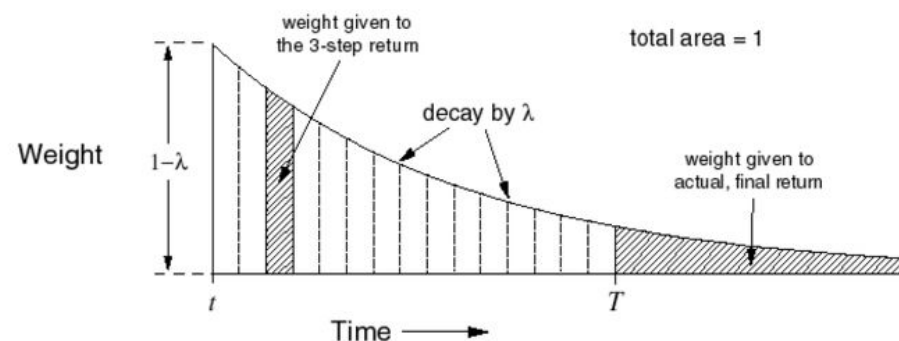
TD( $\lambda$ ),  $\lambda$ -Return



각 스텝에  
weight 을  
다르게 줌  
(다더하면 1)

$$V(S_t) \leftarrow V(S_t) + \alpha(G_t^\lambda - V(S_t))$$

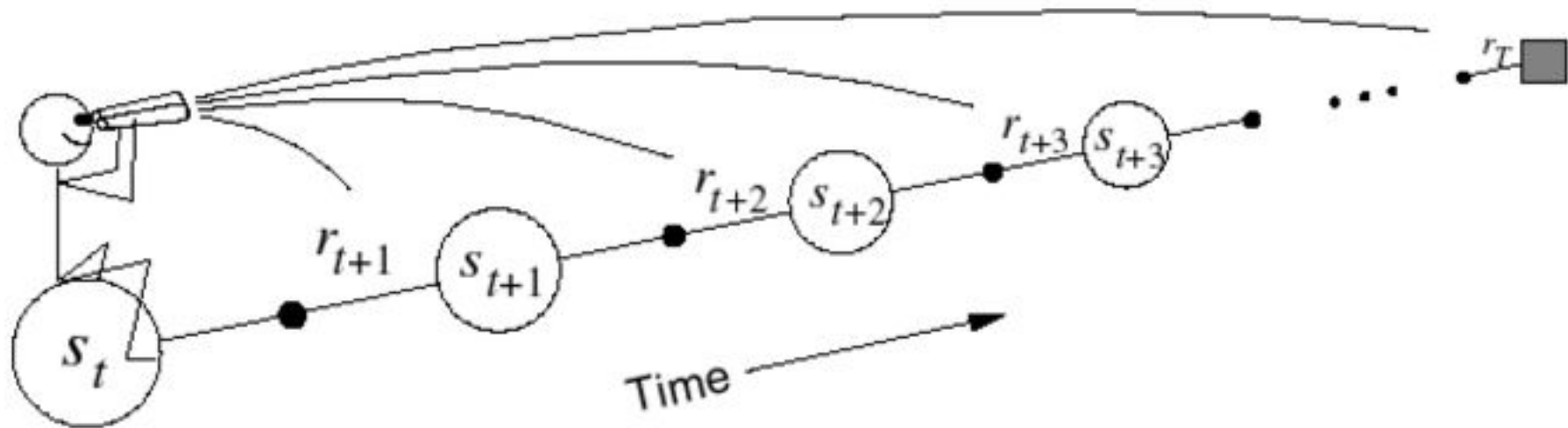
when  $\lambda$ -return,  $G_t^\lambda = (1 - \lambda) \sum_{n=1}^{\infty} \lambda^{n-1} G_t^{(n)}$



$$G_t^\lambda = (1 - \lambda) \sum_{n=1}^{\infty} \lambda^{n-1} G_t^{(n)}$$

# Forward view TD( $\lambda$ )

<http://www0.cs.ucl.ac.uk/staff/d.silver/web/Home.html>



- Update value function towards the  $\lambda$ -return
- Forward-view looks into the future to compute  $G_t^\lambda$
- Like MC, can only be computed from complete episodes



## Backward view TD ( $\lambda$ )

<http://www0.cs.ucl.ac.uk/staff/d.silver/web/Home.html>

- Forward view provides theory
- Backward view provides mechanism
- Update online, every step, from incomplete sequences

## Backward view TD ( $\lambda$ )

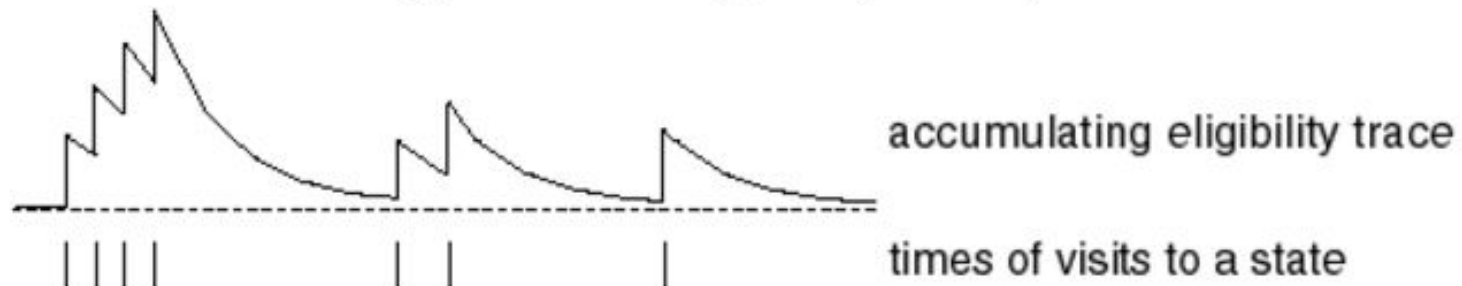
<http://www0.cs.ucl.ac.uk/staff/d.silver/web/Home.html>



- Credit assignment problem: did bell or light cause shock?
- **Frequency heuristic**: assign credit to most frequent states
- **Recency heuristic**: assign credit to most recent states
- *Eligibility traces* combine both heuristics

$$E_0(s) = 0$$

$$E_t(s) = \gamma\lambda E_{t-1}(s) + \mathbf{1}(S_t = s)$$

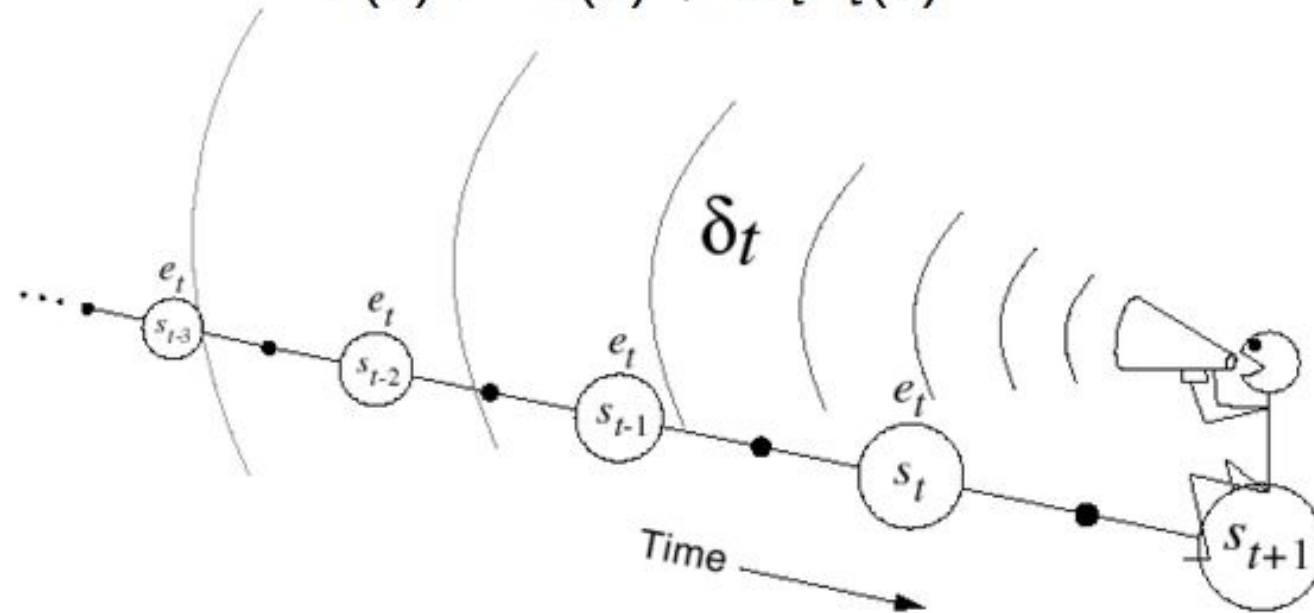


## Backward view TD ( $\lambda$ )

- Keep an eligibility trace for every state  $s$
- Update value  $V(s)$  for every state  $s$
- In proportion to TD-error  $\delta_t$  and eligibility trace  $E_t(s)$

$$\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$$

$$V(s) \leftarrow V(s) + \alpha \delta_t E_t(s)$$



## Backward view TD ( $\lambda$ )

<http://www0.cs.ucl.ac.uk/staff/d.silver/web/Home.html>

- When  $\lambda = 1$ , credit is deferred until end of episode
- Consider episodic environments with offline updates
- Over the course of an episode, total update for TD(1) is the same as total update for MC

### Theorem

*The sum of offline updates is identical for forward-view and backward-view TD( $\lambda$ )*

$$\sum_{t=1}^T \alpha \delta_t E_t(s) = \sum_{t=1}^T \alpha \left( G_t^\lambda - V(S_t) \right) \mathbf{1}(S_t = s)$$

## For/Backward view TD ( $\lambda$ )

forward-view TD( $\lambda$ )는 MC와 TD의 장점을 모두 취하기 위해 n-step을 쓰려했으나, n에 따라 각기 다른 장점이 있기에 MC의 update식에 있는 target(return)을  $\lambda$ -return으로 사용하여 각 n-step의 장점을 모두 취하는 것에 의미를 두었다면, backward-view TD( $\lambda$ )는 TD의 update식에서 eligibility trace라는 방법을 이용해서 새롭게 weight를 주는 것에 의미를 두었다고 생각할 수 있습니다. 개인적으로는 forward-view TD( $\lambda$ )는 MC의 high variance 문제를 episode의 수를 끝날 때까지가 아닌 특정 n으로 줄여서 해결하려했더니  $\alpha$ 와 n에 따라 optimal한 정도가 다르기 때문에 이를 아우르기 위한 방법, 그리고 backward-view TD( $\lambda$ )는 TD의 high bias문제를 그 동안 지나왔던 state에 heuristic으로 기준을 주어 바로 다음 step뿐만 아니라 이 전의 event도 영향을 주게끔 해결하려는 시도로 이해했습니다.



• *Thank you*