Question A.1

Here we document the construction of Price-to-Dividend ratio PD_t and the log dividend growth over the past year Δd_t.

Price-to-Dividend Ration PD_t

We first wish to construct the Price-to-Dividend ratio PD_t. Using H_t and V_t, we find:

$$H_t + 1 \equiv \frac{P_t + D_{t-1Y_r,t}}{P_{t-1Y_r}} \quad \text{and} \quad V_t + 1 \equiv \frac{P_t}{P_{t-1Y_r}}$$

Then, by taking the fraction between these two values:

$$\frac{H_t + 1}{V_t + 1} = \frac{\frac{P_t + D_{t-1Y_r,t}}{P_{t-1Y_r}}}{\frac{P_t}{P_{t-1Y_r}}} = \frac{P_t + D_{t-1Y_r,t}}{P_t} = 1 + \frac{D_{t-1Y_r,t}}{P_t}$$

If we now rearrange, we find:

$$\frac{H_t + 1}{V_t + 1} - 1 = \frac{D_{t-1Y_r,t}}{P_t}$$

and simplifying the expression on the left:

$$\frac{H_t + 1}{V_t + 1} - 1 = \frac{H_t + 1}{V_t + 1} - \frac{V_t + 1}{V_t + 1} = \frac{H_t - V_t}{V_t + 1} = \frac{D_{t-1Y_r,t}}{P_t}$$

and then taking the inverse, we find:

$$\frac{V_t + 1}{H_t - V_t} = \frac{P_t}{D_{t-1Y_r,t}} = PD_t$$

as required.

Log dividend growth over the past year Δd_t.

We now wish to construct the log dividend growth over the past year Δd_t using H_t and V_t. As before, we have:

$$H_t + 1 \equiv \frac{P_t + D_{t-1Y_r,t}}{P_{t-1Y_r}} \quad \text{and} \quad V_t + 1 \equiv \frac{P_t}{P_{t-1Y_r}}$$

Which implies:

$$H_t + 1 - V_t + 1 = H_t - V_t = \frac{P_t + D_{t-1Y_{r,t}}}{P_{t-1Y_r}} - \frac{P_t}{P_{t-1Y_r}} = \frac{D_{t-1Y_{r,t}}}{P_{t-1Y_r}}$$

Similarly, we have:

$$H_{t-1} - V_{t-1} = \frac{D_{t-2Y_{r,t-1Y_r}}}{P_{t-2Y_{r,t-1Y_r}}}$$

Now if we divide these two equations:

$$\frac{H_t - V_t}{H_{t-1} - V_{t-1}} = \frac{\frac{D_{t-1Y_{r,t}}}{P_{t-1Y_r}}}{\frac{D_{t-2Y_{r,t-1Y_r}}}{P_{t-2Y_{r,t-1Y_r}}}} = \frac{D_{t-1Y_{r,t}}}{D_{t-2Y_{r,t-1Y_r}}} \cdot \frac{P_{t-2Y_{r,t-1Y_r}}}{P_{t-1Y_r}}$$

Now from the definition of V_t, we have:

$$\frac{H_t - V_t}{H_{t-1} - V_{t-1}} = \frac{D_{t-1Y_{r,t}}}{D_{t-2Y_{r,t-1Y_r}}} \cdot \frac{1}{1 + V_{t-1}}$$

Which implies:

$$\frac{(H_t - V_t)(V_{t-1} + 1)}{H_{t-1} - V_{t-1}} = \frac{D_{t-1Y_{r,t}}}{D_{t-2Y_{r,t-1Y_r}}}$$

Now we can take the log of both sides:

$$\log\left(\frac{(H_t - V_t)(V_{t-1} + 1)}{H_{t-1} - V_{t-1}}\right) = \log\left(\frac{D_{t-1Y_{r,t}}}{D_{t-2Y_{r,t-1Y_r}}}\right)$$

Which by the law of logarithms, is then:

$$\log\left((H_t - V_t)(V_{t-1} + 1)\right) - \log\left(H_{t-1} - V_{t-1}\right) = \Delta d_t$$

As required