

Below is the formulation for the load balancing problem. We define some  $L$  such that this is the maximum tolerated difference between the highest and lowest demand faced

Sets

$C$  set of Customers  
 $F$  set of Facilities

Parameters

$c_{ij}$   $(i, j) \in C \times F$  cost of assigning customer  $i$  to facility  $j$   
 $k_f$   $f \in F$  cost of opening facility  $f$   
 $d_c$   $c \in C$  demand of customer  $c$   
 $l_f$   $f \in F$  capacity of facility  $f$   
 $L$  Maximal allowable difference in demand

Variables

$x_{ij} \in \{0, 1\}$   $(i, j) \in C \times F$  Assigning customer  $i$  to facility  $j$   
 $b_f \in \{0, 1\}$   $f \in F$  Opening facility  $f$   
 $u_{max} \geq 0$  Demand of the busiest facility  
 $u_{min} \geq 0$  Demand of the least busy facility

Objective

$$\min \sum_{(i,j) \in C \times F} c_{ij}x_{ij} + \sum_{f \in F} k_f b_f \quad (1)$$

Constraints

$$\sum_{j \in F} x_{ij} = 1 \quad \forall i \in C \quad (2)$$

$$x_{ij} \leq b_j \quad \forall (i, j) \in C \times F \quad (3)$$

$$\sum_{i \in C} d_i x_{ij} \leq l_j b_j \quad \forall j \in F \quad (4)$$

$$\sum_{i \in C} d_i x_{ij} \geq u_{min} \quad \forall j \in F \quad (5)$$

$$\sum_{i \in C} d_i x_{ij} \leq u_{max} \quad \forall j \in F \quad (6)$$

$$u_{max} - u_{min} \leq L \quad (7)$$

From the Capacitated Model, we have added three new constraints and two new decision variables. Our new decision variables include a positive limit to the demand faced by any facility and a positive lower limit. Constraints (5) and (6) then ensure that any demand faced by any facility  $f$  is bounded above and below. Finally, constraint (7) ensures that the difference between these limits are no more than some pre-defined limit  $L$