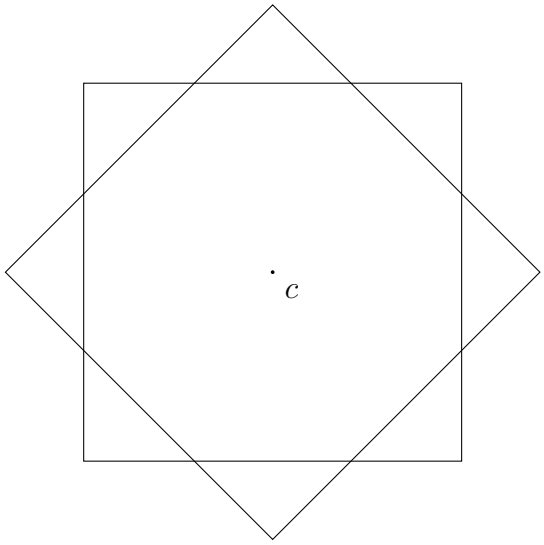
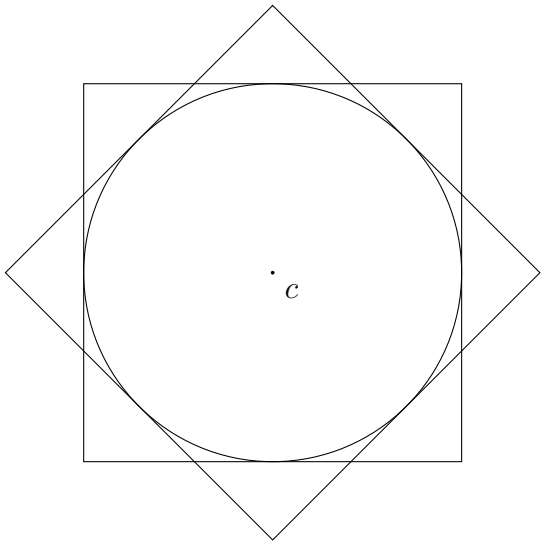
Problem 1

Two squares of side length 1 have a common centre c. Show that the area of their intersection is at least $\frac{3}{4}$



Solution

We gain insight by observing the two squares are identical, except one is rotated 45 degrees. This means we can inscribe a circle inside one square, and the same circle will perfectly inscribe the second square, like so:



We conclude that the area of the intersection must be at least the area of the circle.

We know the radius of the circle is $\frac{1}{2}$ and so the area is $\pi \cdot (\frac{1}{2})^2 = \frac{\pi}{4}$

We then have $\frac{\pi}{4} > \frac{3}{4}$, completing the proof.