Capacitance and Inductance

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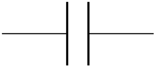
- Capacitors and Inductors are passive two-terminal electrical components used to store energy in an electric/magnetic field.
- Both are basic components used in electronics where **current and voltage change** with time, due to their ability to delay and reshape alternating currents/voltages.
- **Analog filters**: circuits containing resistors, capacitors, inductors y active sources that change with time.



Capacitor

Inductor

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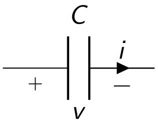
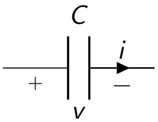
The capacitor

- Composed by two electrical conductors separated by a dielectric material.
- The application of of a voltage to the terminals creates a displace of charge within the dielectric.
- The displacement current is indistinguishable from a conduction current. We simply refer to it as the current across the capacitor.
- The current across a capacitor is proportional to the rate at which the voltage varies with time:

$$i(t) = C\frac{dv(t)}{dt} \qquad \text{(Fundamental equation of the capacitor)}$$



C is called the capacity and it is measured in Farads (F).



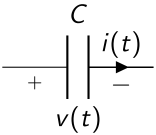
$$i(t) = C\frac{dv(t)}{dt}$$

Remarks:

1. If the voltage across the terminals is constant, the current is zero. Thus, **a capacitor behaves as an open circuit in the presence of a constant voltage**.
2. The voltage cannot change instantaneously across the capacitor. Such a change would produce infinite current, physically impossible. For any time t_0:

$$v(t_0^-) = v(t_0) = v(t_0^+)$$

The capacitor



$$i(t) = C\frac{dv(t)}{dt}$$

If we know the current across the capacitor along the time $i(t)$, we can compute the voltage $v(t)$ as follows:

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\mathrm{d}\tau = \frac{1}{C}\int_{t_0}^{t} i(\tau)\mathrm{d}\tau + v(t_0)$$

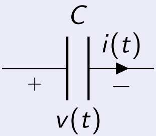
Example

The voltage pulse described by the following equations is applied to a $0.5 - \mu F$ capacitor:

$$v(t) = \begin{cases} 0 & t \leq 0 \\ 4t \ V & 0 < t \leq 1 \\ 4e^{-(t-1)} \ V & 1 < t \leq \infty \end{cases}$$

- Derive the expression for the capacitor current.
- Sketch the voltage and current as functions of time.

Power and Energy in the Capacitor



$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)\mathrm{d}\tau + v(t_0)$$

From the definition of power

$$p(t) = v(t)i(t) = Cv(t)\frac{dv(t)}{dt}$$

or

$$p(t) = v(t)i(t) = i(t)\left[\frac{1}{C}\int_{t_0}^{t} i(\tau)\mathrm{d}\tau + v(t_0)\right]$$

If $p(t) > 0$ the capacitor is storing energy. If $p(t) < 0$, it means the capacitor is delivering energy to the circuit.

Power and Energy in the Capacitor

The energy in the capacitor at any time can be computed as follows:

$$p(t) = \frac{dw(t)}{d(t)} \Rightarrow w(t) = \int_{-\infty}^{t} p(t)dt = \int_{-\infty}^{t} v(t)C\frac{dv(t)}{\cancel{dt}}\cancel{dt} = C\frac{v^2(t)}{2}$$

The energy $w(t)$ in the capacitor is always positive!

Example (cont.)

The voltage pulse described by the following equations is applied to a $0.5-\mu F$ capacitor:

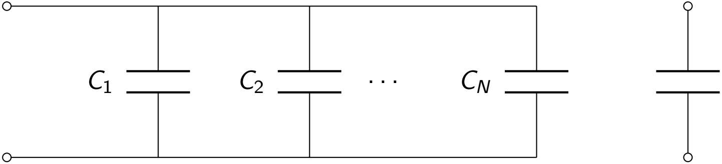
$$v(t) = \begin{cases} 0 & t \leq 0 \\ 4t\ V & 0 < t \leq 1 \\ 4e^{-(t-1)}\ V & 1 < t \leq \infty \end{cases}$$

- Derive the expression for the power $p(t)$ and the energy $w(t)$ in the capacitor.
- Specify the interval of time when energy is being stored in the capacitor.
- Specify the interval of time when energy is being delivered by the capacitor.
- Evaluate the integrals

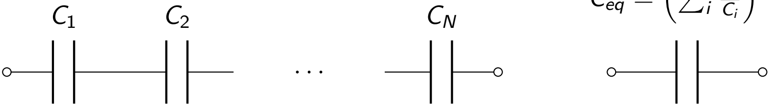
$$\int_0^1 p(t)dt \qquad \int_1^\infty p(t)dt$$

and comment their significance.

Capacitors in parallel/series



$$C_{eq} = \sum_i C_i$$



$$C_{eq} = \left(\sum_i \frac{1}{C_i} \right)^{-1}$$

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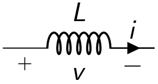
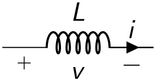
The Inductor

- Inductors are circuit elements based on phenomena associated with magnetic fields.
- The source of the magnetic field is charge in motion (current).
- If the current is varying with time, the magnetic field is varying with time, which induces a voltage in any conductor linked by the field.
- The voltage drop across the terminals of the inductor capacitor is proportional to the rate at which the current varies with time:

$$v(t) = L\frac{di(t)}{dt} \text{ (Fundamental equation of the inductor)}$$



L is called the inductance and it is measured in Henrys (H).



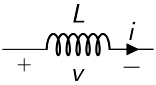
$$v(t) = L\frac{di(t)}{dt}$$

Remarks:

1. If the current across the terminals is constant, the voltage is zero. Thus, **a capacitor behaves as a short circuit in the presence of a constant current**.
2. The current cannot change instantaneously across the inductor. Such a change would produce infinite voltage drop across the terminals, physically impossible. For any time t_0:

$$i(t_0^-) = i(t_0) = i(t_0^+)$$

The inductor

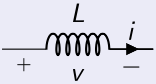


$$v(t) = L\frac{di(t)}{dt}$$

If we know the voltage in the inductor $v(t)$ along time, we can compute the current $i(t)$ as follows:

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\mathrm{d}\tau = \frac{1}{L}\int_{t_0}^{t} v(\tau)\mathrm{d}\tau + i(t_0)$$

Power and Energy in the Inductor



$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)\mathrm{d}\tau + i(t_0)$$

From the definition of power

$$p(t) = v(t)i(t) = Li(t)\frac{di(t)}{dt}$$

or

$$p(t) = v(t)i(t) = v(t)\left[\frac{1}{L}\int_{t_0}^{t} v(\tau)\mathrm{d}\tau + i(t_0)\right]$$

If $p(t) > 0$ the inductor is storing energy. If $p(t) < 0$, it means the inductor is delivering energy to the circuit.

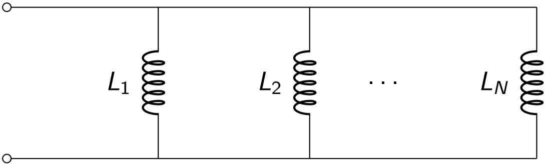
Power and Energy in the Inductor

The energy in the inductor at any time can be computed as follows:

$$p(t) = \frac{dw(t)}{d(t)} \Rightarrow w(t) = \int_{-\infty}^{t} p(t)dt = \int_{-\infty}^{t} i(t)L\frac{iv(t)}{\cancel{dt}}\cancel{dt} = L\frac{i^2(t)}{2}$$

The energy $w(t)$ in the inductor is always positive!

Inductors in parallel/series

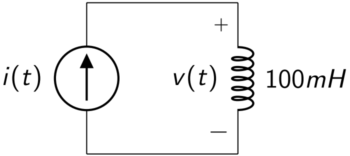


$$L_{eq} = \left(\sum_i \frac{1}{L_i}\right)^{-1}$$

$$L_{eq} = \sum_i L_i$$

Example

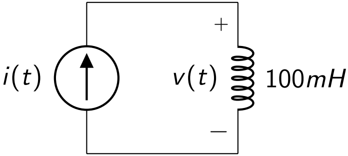
The independent current source in the circuit shown generates zero current for $t < 0$ and a pulse $i(t) = 10te^{-5t}$.



- Sketch the current waveform (computing the derivative w.r.t. helps).
- At what instant of time is the current maximum?
- Express the voltage $v(t)$ across the terminals of the 100-mH inductor as a function of time.
- Sketch the voltage waveform. At what instant does $v(t)$ change polarity?

Example (cont.)

The independent current source in the circuit shown generates zero current for $t < 0$ and a pulse $i(t) = 10te^{-5t}$.



- Plot the power $p(t)$ and energy $w(t)$ in the inductor along time.
- In what time interval is energy being stored in the inductor?
- In what time interval is energy being extracted from the inductor?
- What is the maximum energy stored in the inductor?
- Evaluate the integrals

$$\int_0^{0.2} p(t)dt \qquad \int_{0.2}^{\infty} p(t)dt$$

and comment on their significance.