Introduction to Electric Circuits. Techniques of Circuit Analysis (II)

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Even though the node-voltage method and the loop-current methods are powerful techniques for solving circuits, we are still interested in methods that can be used to simplify circuits.

- Today: we provide new tools to simplify/transform a given circuit.
- Recall: we already know how to reduce a given circuit by merging resistors that are in parallel or in series.

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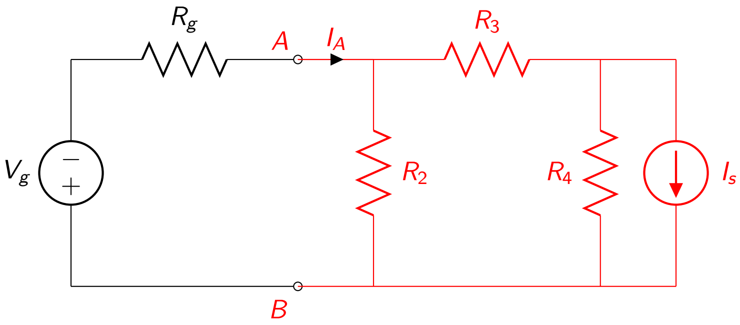
Equivalent voltage/current sources

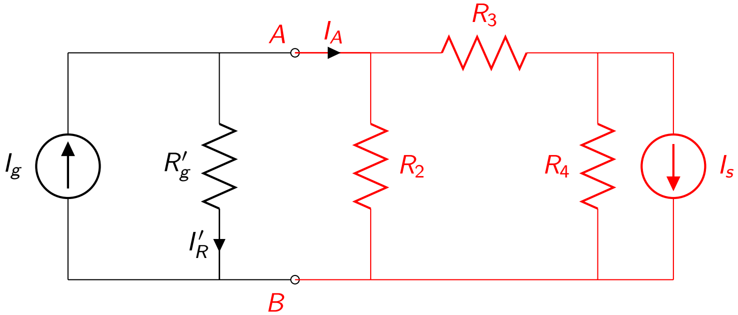
The two following configurations are equivalent **for any circuit we connect at the right to nodes** A **and** B. Namely, none of the voltages/currents in the red circuit at the right of A-B will change.

Configuration 1



Equivalent voltage/current sources

Configuration 2



where $I_g = \frac{V_g}{R_g}$ and $R'_g = R_g$.

Proof:

- The red circuit does not see any difference between both configurations as long as V_A-V_B and I_A are the same for both scenarios.
- KVL in the left-most loop in configuration 1:

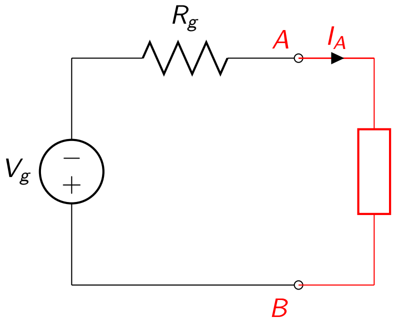
$$0 = -V_g + I_A R_g + V_A - V_B \Rightarrow V_g = I_A R_g + V_A - V_B;$$

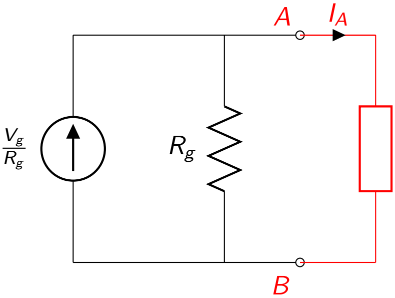
- KCL in node A in configuration 2:

$$I_g = I'_R + I_A \Rightarrow I_g = \frac{V_A - V_B}{R'_g} + I_A$$
$$\Rightarrow R'_g I_g = (V_A - V_B) + I_A R'_g$$

- If we compare the two expressions in blue: $V_g = I_g R'_g$ and $R_g = R'_g$.

We have not assumed anything about the red circuit. This is true for any possible circuit that we connect to nodes A and B.





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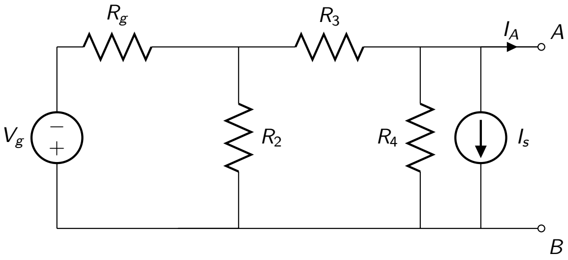
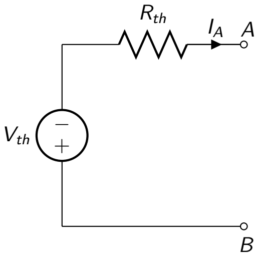
Thévenin Equivalent

- Assume a circuit composed by independent voltage/current sources, dependent voltage/current sources and resistors.
- For any two nodes A and B of the circuit, the circuit can be replaced by a voltage source V_{th} and a single resistor R_{th} in series configuration. This is called the **Thvenin equivalent**.



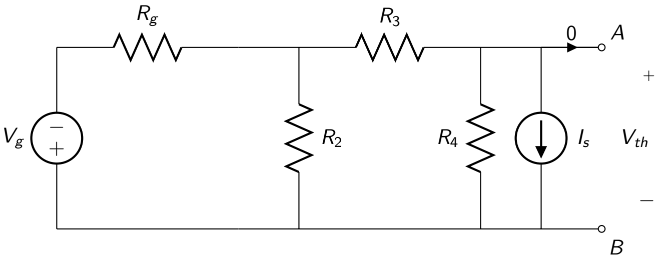


Any circuit that we connect to A and B does not see any difference. $V_A - V_B$ and I_A are exactly the same in both cases.

Thévenin equivalent. Computing V_{th} and R_{th}

V_{th}

V_{th} is the voltage drop between A and B in open circuit. Namely, we connect a resistor of $R_{open} = \infty\ \Omega$ between A and B. Thus, $I_A = 0$.

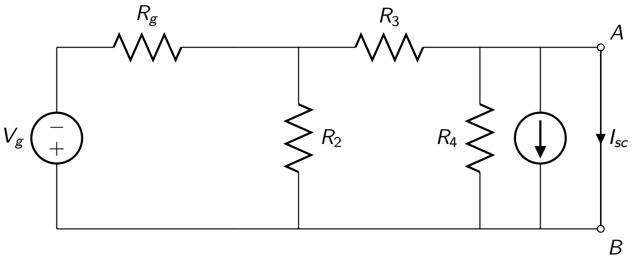


Thévenin equivalent. Computing V_{th} and R_{th}

R_{th}

We place a short circuit across nodes A and B and calculate the resulting the short-circuit current I_{sc}. The resistor R_{th} is computed as follows

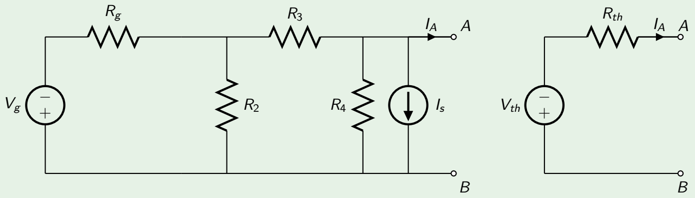
$$R_{th} = \frac{V_{th}}{I_{sc}}$$



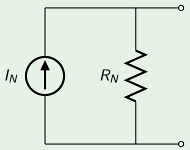
In the circuit above, what is the current across the resistor R_4?

Norton equivalent

If we compute the Thévenin equivalent circuit of a given circuit, e.g.,



Then we can transform the the voltage source V_{th} plus the resistor in series R_{th} into a current source $I_N = V_{th}/R_{th}$ in parallel to a resistor $R_N = R_{th}$. This configuration is called the **Norton equivalent** of the circuit with respect nodes A and B.



$$I_N = \frac{V_{th}}{R_{th}} \text{ (Short circuit current)}$$

$$R_N = R_{th}$$

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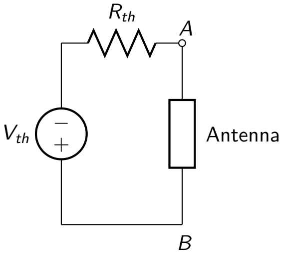
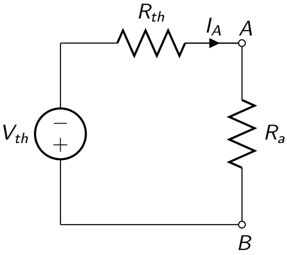
- Circuit analysis plays an important role in the analysis of systems designed to transfer power from a source to a load.
- In Communication and instrumentation systems are designed to transmit information via electro signals. The power available at the transmitter or the detector is typically limited.
- Thus, transmitting as much of this power as possible to the receiver, or load, is critical.

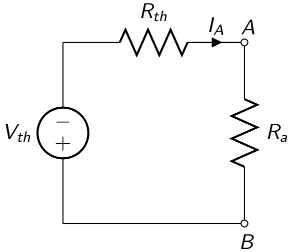
Maximum power transfer theorem

Mobile phone:
Battery+Circuitry+Antenna.

Assume we compute the Thévenin equivalent of the first two and we model the antenna by a a resistor R_a.







The power provided by V_{th} is $P = -V_{th}^2/(R_{th} + R_a)$ Watt

The power dissipated in R_{th} is

$$P_{R_{th}} = \frac{V_{th}^2}{(R_{th} + R_a)^2} R_{th} \text{ Watt}$$

The power dissipated in R_a is

$$P_{R_a} = \frac{V_{th}^2}{(R_{th} + R_a)^2} R_a \text{ Watt}$$

Our goal is to design the antenna resistivity R_a so that we maximize the power dissipated in R_a:

$$\frac{\partial P_{R_a}}{\partial R_a} = \frac{V_{th}^2}{(R_{th}+R_a)^2} - 2R_a \frac{V_{th}^2}{(R_{th}+R_a)^3} = 0$$
$$\Rightarrow (R_{th}+R_a) - 2R_a = 0 \Rightarrow R_{th} = R_a$$

The power is maximized at the antenna if we design it to have a resistivity equal to R_{th}, the Thévenin resistor for the rest of the circuit.