Introduction to Electric Circuits. Basic Magnitudes and Elements. Kirchhoff's Laws.

Pablo M. Olmos (olmos@tsc.uc3m.es)
Emilio Parrado (emipar@tsc.uc3m.es)

March 3, 2018

uc3m

- An electric circuit is composed of individual components connected by conductive wires or traces through which **electric current can flow**.
- The combination of components and wires allows various simple and complex operations to be performed: signals can be amplified, computations can be performed, and data can be moved from one place to another.

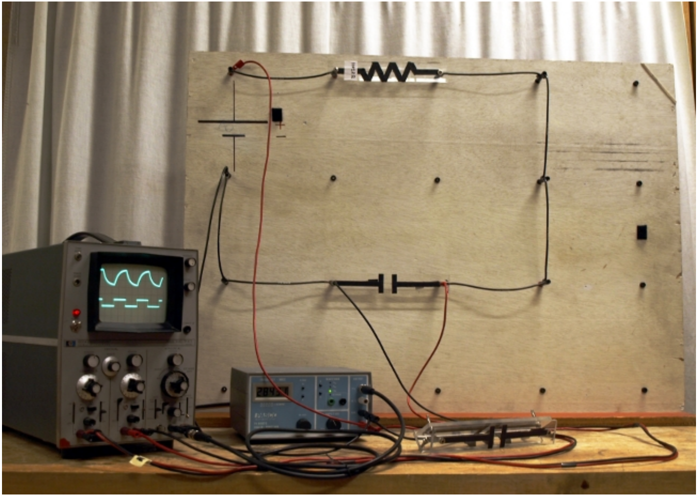
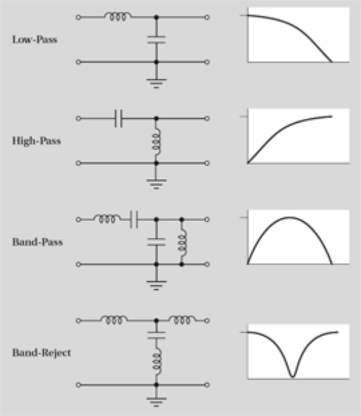


Figure: Rudimentary setting for an electric circuit containing one battery, one resistor and one capacitor. In the oscilloscope, we observe the input/output voltage signals (bottom and top signals respectively).

In this course:

- Basic analysis tools.
- Passive Electric circuits: Resistors+Capacitators+Inductions $\Rightarrow$ **filters**.



- Application of concepts on signal & systems.

We assume you have had an introductory physics course in which the electrical and magnetic phenomena were discussed.

Recommended books

- Electric circuits, James William Nilsson.
- Principles of Electric Circuits, Thomas Floyd.

Index

Circuit Theory

- Circuit theory is a special case of electromagnetic field theory, i.e., the study of static and moving electric charges.
- Applying the general electromagnetic field theory to investigate electrical circuits is cumbersome and requires the use of advanced mathematics.
- In circuit theory, we rely on three physical assumptions:

In circuit theory we ignore the propagation effects of the electrical signals throughout the circuit. Electrical effects happens instantaneously.

The net charge on every component in the system is always zero. No component can collect a net excess of charge.

There is no magnetic coupling between the components in a system.

Circuit Theory

This approach yields the following benefits:

1. Circuit theory provides **simple solutions** (of sufficient accuracy) to problems that are prohibitively complex by applying the general theory.
2. **Divide and conquer approach:** subsystems called components are first analyzed. We can use the behavior of each component to predict the behavior of bigger systems.
3. Circuits analysis introduces a **methodology for solving large networks of coupled linear equations**, which are prevalent through engineering and technology.

Index

Electrical Charge

- Property of matter: excess or deficiency of electrons. We symbolize the charge by Q.
- The electric charge exists in discrete quantities. The electron is the smallest particle that exhibits negative electrical charge.
- Coulomb is the unit of charge. $Q_e = -1.610^{-19}$ C.
- The charge of the proton is equal in magnitude to the electron charge, but with opposite sign.

Circuit theory

Electrical effects are attributed to both the separation of charges, which creates **an electrical force (voltage)**, and charges in motion, which creates an **electric fluid or current**).

Voltage

- When we separate positive and negative charges, an energy transferred to the charges.
- If it is physically possible, charges will move across the circuit until they reach a point with minimum energy.
- The voltage between two points of the circuit is the difference of electrical energy per unit charge between the two points.

The energy is never lost, just transformed.

When we separate charges we transform chemical energy (a battery) into electrical energy. When charges flow throughout the circuit, the electrical energy that they lose is transformed into a different kind of energy, e.g., heat in the case of a heater or light in the case of a lamp.

Voltage

Energy per unit charge:

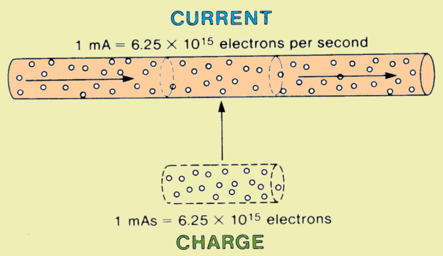
$$V = \frac{\partial W}{\partial Q} \qquad \text{volts (V)}$$

One volt is the potential difference between two points when one joule of energy is used to move one coulomb of positive charge from one point to the other.

Electrical current

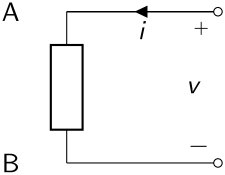
- Under normal conditions the movement of the electrons is truly random, meaning they are moving in all directions by the same amount.
- If a voltage is placed across a conductive material, it causes the free electrons to move in a single direction.
- We define the electrical current as the rate of charge flow past a given point in an electric circuit:

$$I(t) = \frac{\partial Q}{\partial t} \qquad \text{Amperes (A)}$$



One ampere is the amount of current that exists when a number of electrons having a total charge of one coulomb move trough a given cross-sectional area in one second.

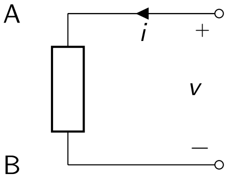
The Ideal Basic Circuit Element



Any element will be defined by a certain $v = f(i)$ function.

The voltage across the terminals $A - B$ is denoted by v. We arbitrarily assume A as the positive terminal $(+)$ and B as the negative terminal $(-)$.

- If $v > 0$, **positive charges** in A have more energy than in B.
- If $v < 0$, **positive charges** in B have more energy than in A.
- **Conventional current flow:** we consider the current as a flow of positive charges.

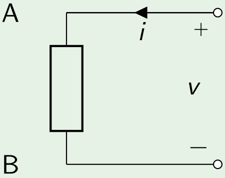


- $v = 5$v and $i = 2$A $\rightarrow$ current flows from a point of where charges have more energy (A) to a point where they have less energy (B). **The device is consuming electrical energy.**
- $v = 5$v and $i = -2A$ $\rightarrow$ current flows from a point of where charges have less energy (B) to a point where they have more energy (A). **The device is generating electrical energy.** E.g., it is a battery.

Electric Power

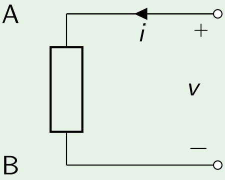
- Electric power $P(t)$ is the amount of electric energy $W(t)$ consumed/generated used per unit time:

$$P(t) = V(t)I(t) \qquad \text{Watts (W)}$$



- If $v > 0$ and $i > 0$, the charges are losing electrical energy (the device is transforming electrical energy into another kind of energy), thus $P(t) > 0$.

Electric Power

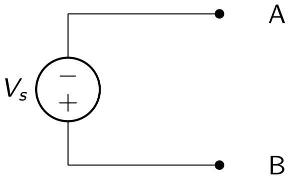


- If $v > 0$ and $i < 0$, the element is providing the charges with electrical energy, thus $P(t) < 0$.

Any device that generates power is said to be active (e.g. a voltage source) and otherwise passive (a resistor).

Index

Ideal Voltage source

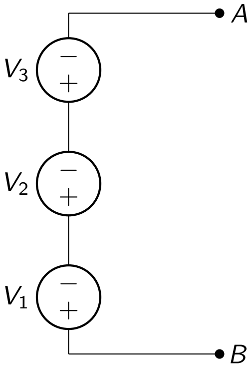


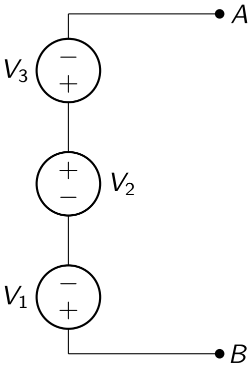
$$V_s = V_A - V_B$$

An ideal voltage source maintains a fixed voltage across its terminals V_s regardless of the current i_s in the device.

Voltage sources in series

When two or more voltage sources are in series, the total voltage is equal to the algebraic sum of the individual source voltages (take care with the polarities!).

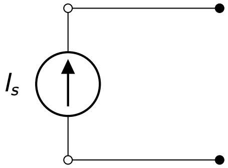




$$V_A - V_B = V_1 + V_2 + V_3$$

$$V_A - V_B = V_1 - V_2 + V_3$$

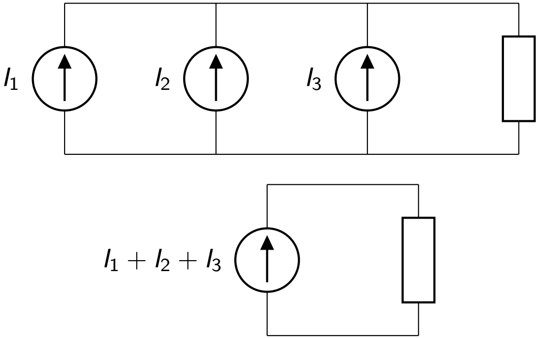
Ideal current source



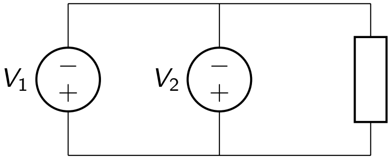
An ideal current source maintains a fixed current I_s within its terminals regardless of the voltage across them.

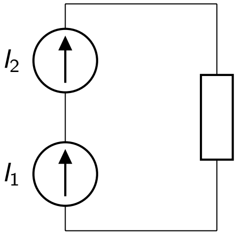
Current sources in parallel

The total current produced by current sources in parallel is equal to the algebraic sum of the individual current sources.



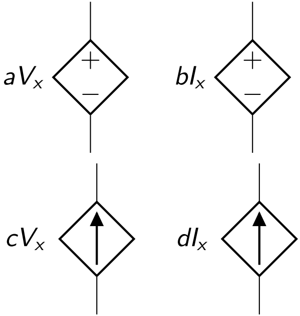
Forbidden configurations!





Dependent voltage/current sources

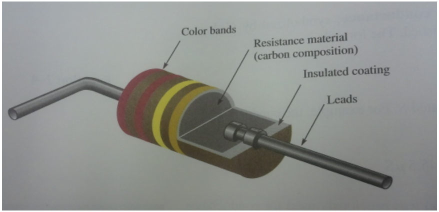
- The value of the voltage/current generated depends on another magnitude (current/voltage) in the circuit.



Where a, b, c, d are real constants and V_x and I_x are some voltage/current in the circuit.

Resistance

- When there is current through a material, the free electrons move through the material and occasionally collide with atoms.
- Collitions cause the electrons to lose some of their energy and the movement is restricted.
- The property of a material that restrict the flow of electrons is called resistance R, measured in Ohms (1Ω).



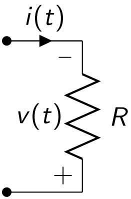
One ohm ($1\ \Omega$) of resistance exists if there is one ampere (1 A) of current in a material when one volt (1 V) is applied across the material.

A resistor is a component that is specifically designed to have a certain amount of resistance.

The Ohm's law

The current $i(t)$ going through a given resistor is directly proportional to voltage $v(t)$ and inversely proportional to the resistance R:

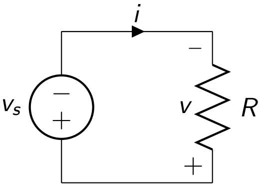
$$i(t) = \frac{v(t)}{R}$$



The device always consumes power:

$$P(t) = v(t)i(t) = \frac{v^2(t)}{R} = Ri^2(t) > 0 \text{ Watt.}$$

A very simple circuit



- The voltage sources provides electrical energy to the charges that flow across the resistor.

Resistor perspective

The voltage across terminals is v_s and the current is $i = v_s/R$ (positive, it enters through the positive terminal). Power consumed: $P = v_s^2/R$ Watt.

Voltage source perspective

The voltage across terminals is v_s and the current is $i = v_s/R$. The power consumed is $P = -v_s^2/R$ Watt (the current enters through the negative terminal of the voltage source).

Index

Kirchhoff's Laws

- A circuit is said to be solved when the voltage across and the current in every element in the circuit have been determined.
- The interconnection of elements imposes constraints on the relationships between the terminal voltages and currents.
- These constraints are referred to as Kirchhoff's Laws.

Branches, Loops and nodes

- Branch: any element of two terminals (wires do not count).
- Loop: any set of branches that form a closed loop.
- Minimal loop: any loop that does not contain another loop.
- Node: a node is any point where two or more branches are connected.
- Essential Nodes: any node where at least three branches are connected.

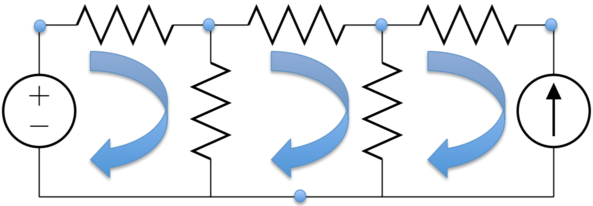
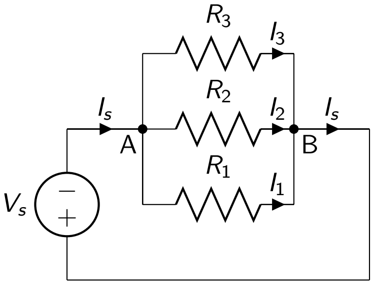


Figure: 7 devices or branches, 3 minimal loops, 5 nodes and 3 essential nodes.

Kirchhoff's current law

The sum of currents entering any node is equal to the sum of currents leaving that node.



In node A

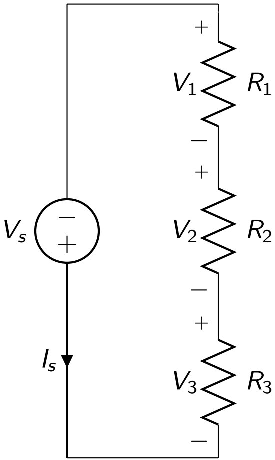
$$\underbrace{I_s}_{\text{Current in}} = \underbrace{I_1 + I_2 + I_3}_{\text{Current out}}$$

In node B

$$\underbrace{I_1 + I_2 + I_3}_{\text{Current in}} = \underbrace{I_s}_{\text{Current out}}$$

Kirchhoff's voltage law

In a single closed path, the sum of all the voltage drops must be equal to the sum of the voltage rises.

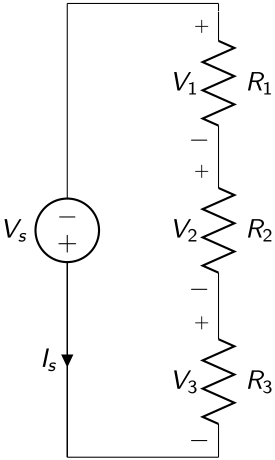


$$V_s = V_1 + V_2 + V_3$$

Kirchhoff's voltage law

Another way to express Kirchhoff's voltage law is as follows:

The algebraic sum of all the voltage drops around a single closed path in a circuit is zero.

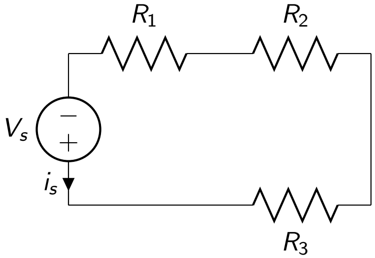


We follow the current direction and we algebraically sum the voltage drops. A voltage source represents a negative drop.

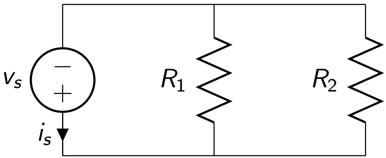
$$V_1 + V_2 + V_3 - V_s = 0$$

Index

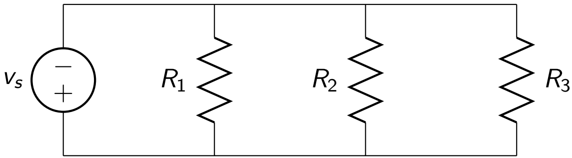
Compute i_s and the voltage drop in every resistor



Compute the current across R_2



Compute the power generated by the voltage source



Compute the voltage drop in R_2