

# Unit Step, Impulse

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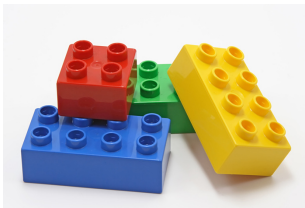
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## Today

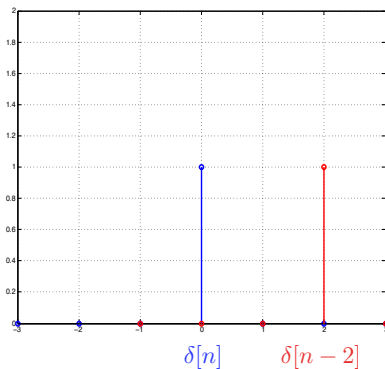
We describe a set of basic signal models whose properties are extremely important to analyze and design complex signal processing systems.



## Unit Impulse Sequence (Kronecker delta)

Probably the simplest sequence we can imagine:

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \quad \delta[n - n_0] = \begin{cases} 1 & n = n_0 \\ 0 & n \neq n_0 \end{cases}$$



## Unit Impulse Sequence. Properties

It is an even signal:  $\delta[n] = \delta[-n]$ .

Is  $\delta[n - n_0]$  an even signal?

$$x[n]\delta[n - n_0] = \begin{cases} x[n_0] & n = n_0 \\ 0 & n \neq n_0 \end{cases}$$

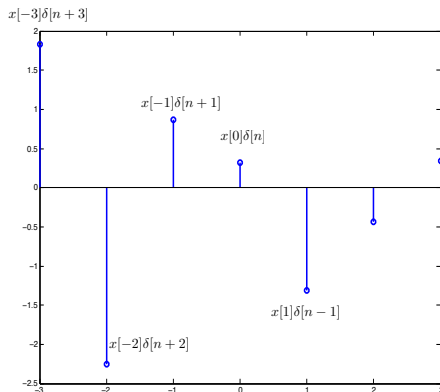
$$\sum_{n=-\infty}^{\infty} \delta[n] = 1$$

$\delta[n]$  is a signal of finite energy.

## Unit Impulse Sequence. Properties

Any signal can be decomposed as a sum of unit impulse sequences:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$



## Unit Step Sequence

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

### From $\delta[n]$ to $u[n]$

As we have seen, we can decompose it as a sum of unit impulse sequences:

$$u[n] = \sum_{k=-\infty}^{\infty} u[k] \delta[n - k] = \sum_0^{\infty} \delta[n - k]$$

$$u[n - n_0] = \sum_{k=-\infty}^{\infty} u[k - n_0] \delta[n - k] = \sum_{n_0}^{\infty} \delta[n - k]$$

## Unit Step Sequence

### From $u[n]$ to $\delta[n]$

We can also obtain  $\delta[n]$  from  $u[n]$ :

$$\begin{aligned}\delta[n] &= u[n] - u[n-1] \\ \delta[n-n_0] &= u[n-n_0] - u[n-n_0-1]\end{aligned}$$



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## Unit Step signal

Similar to the discrete case:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

### Continuous approximation

Let

$$u_L(t) = \begin{cases} 0 & t < 0 \\ t/L & 0 < t < L \\ 1 & t \geq L \end{cases}$$

then

$$\lim_{L \rightarrow 0} u_L(t) = u(t)$$

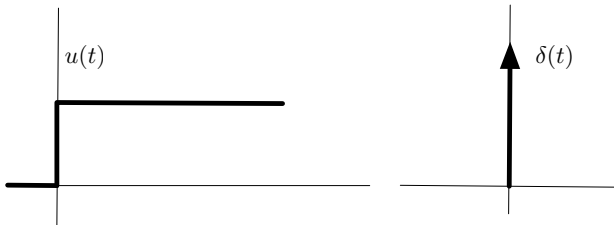
## Continuous-time impulse (Dirac Delta function)

The continuous-time impulse function  $\delta(t)$  is related to the unit step by the equation

$$u(t) = \int_{-\infty}^t \delta(\tau) d\tau$$

and this suggests that

$$\delta(t) = \frac{\partial u(t)}{\partial t} = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$$



# Continuous-time impulse (Dirac Delta function)

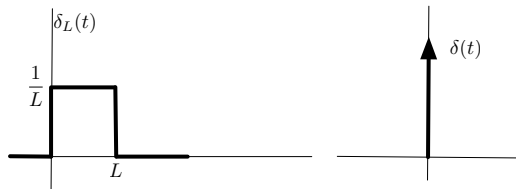
## Continuous approximation

Let

$$\delta_L(t) = \frac{\partial u_L(t)}{\partial t} \begin{cases} L^{-1} & 0 < t < L \\ 0 & \text{otherwise} \end{cases}$$

then

$$\delta(t) = \lim_{L \rightarrow 0} \delta_L(t)$$



For any  $L$  value,

$$\int_{-\infty}^{\infty} \delta_L(t) dt = L \frac{1}{L} = 1 \Rightarrow \int_{-\infty}^{\infty} \delta(t) dt = 1.$$

The signal  $\delta(t)$  has unit area.

$$\int_{-\infty}^{\infty} \delta(t - t_0) dt = 1$$

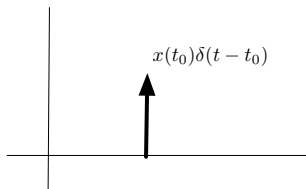
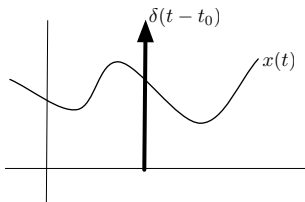
## Continuous-time Impulse. Properties.

Multiplication by a constant  $\rightarrow$  Area multiplication

$$\int_{-\infty}^{\infty} k\delta(t)dt = k \int_{-\infty}^{\infty} \delta(t)dt = k$$

$$x(t)\delta(t - t_0)$$

$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$$



## Continuous-time Impulse. Properties (II).

Any signal  $x(t)$  can be decomposed as a linear combination of an infinite number of impulses:

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t - \tau)d\tau$$

### Unit step signal

$$u(t) = \int_{-\infty}^{\infty} u(\tau)\delta(t - \tau)d\tau = \int_0^{\infty} \delta(t - \tau)d\tau$$

## Relationship with the rectangular pulse

