Introduction to Electric Circuits. Techniques of Circuit Analysis

Pablo M. Olmos (olmos@tsc.uc3m.es)
Emilio Parrado (emipar@tsc.uc3m.es)

March 3, 2018

uc3m

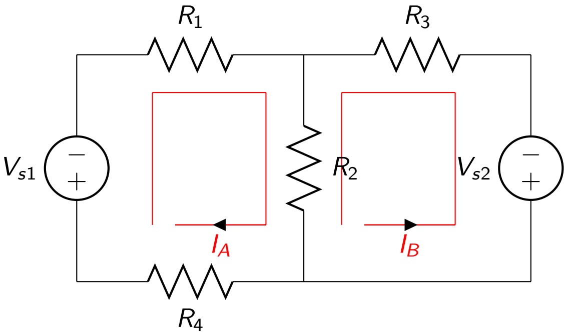
Techniques for Systematic Circuits analysis

- Using Kirchhoff's voltage and current laws we derive a set of linear equations where the voltages/currents of interests are the unknowns. Then, we make use of linear algebra to solve the system of equations.
- We present two methods: the loop current method and the node voltage method.

Loop current method

Step 1

Identify minimal loops and assign arbitrary loop currents. For loops with voltage sources we will assume the conventional direction (from the negative terminal to the positive terminal) and for loops with current sources, the direction of the source itself.



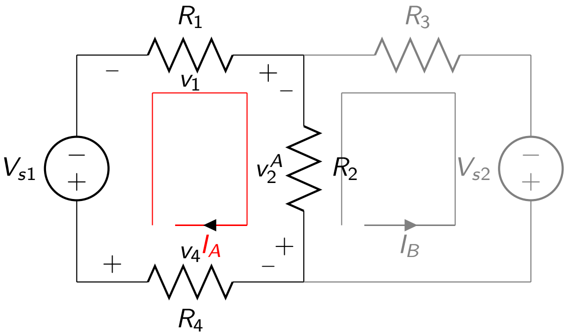
The number of loop-current assignments must be sufficient to include current through all components in the circuit.

Loop current method

Step 2

Indicate the voltage drop polarities in each loop based on the assigned current directions.

Loop I_A:

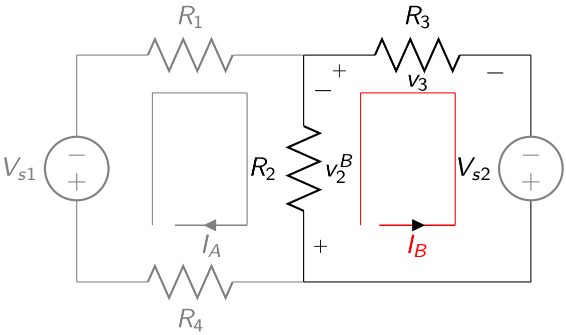


Loop current method

Step 2

Indicate the voltage drop polarities in each loop based on the assigned current directions.

Loop I_B:



Loop current method

Step 3

Apply Kirchhoff's voltage law around each closed loop. This results in one equation for each loop.

Loop I_A:

$$\begin{aligned} V_{s1} &= v_1 + v_2^A + v_2^B + v_4 \\ &= R_1 I_A + R_2(I_A + I_B) + I_A R_4 = I_A(R_1 + R_2 + R_4) + R_2 I_B. \end{aligned}$$

Loop I_B:

$$\begin{aligned} V_{s2} &= v_2^B + v_2^A + v_3 \\ &= R_2(I_B + I_A) + R_3 I_2 = I_B(R_2 + R_3) + R_2 I_A. \end{aligned}$$

Loop current method

Step 4

Solve the system of equations.

$$\left\{ \begin{array}{c} V_{s1} = I_A(R_1 + R_2 + R_4) + R_2 I_B \\ V_{s2} = I_B(R_2 + R_3) + R_2 I_A \end{array} \right.$$

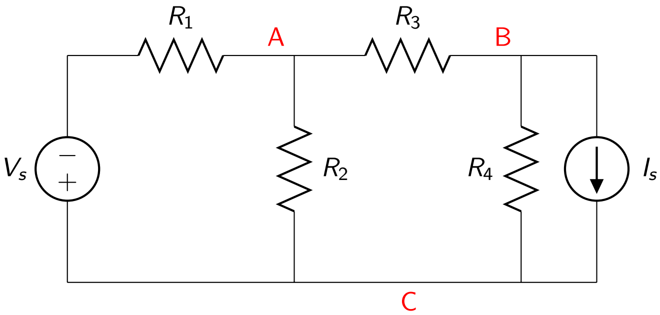
$$\begin{bmatrix} R_1 + R_2 + R_4 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} V_{s1} \\ V_{s2} \end{bmatrix}.$$

For $R_1 = R_2 = R_3 = R_4 = 20\Omega$ and $V_{s1} = 10$ v and $V_{s2} = -5$ v, we get $I_A = 0.25$ A and $I_B = -0.25$ A (check!). What does a negative value for I_B mean?

Node voltage method

Step 1

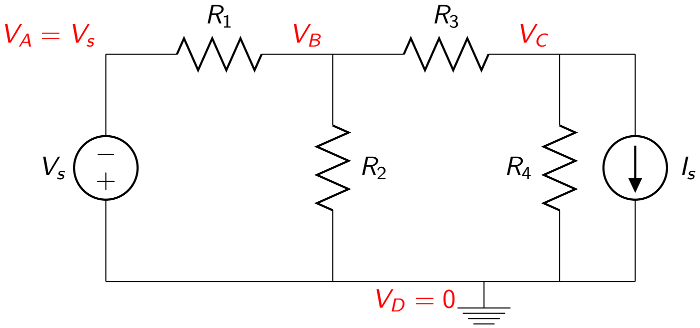
Identify essential nodes in the circuit.



Node voltage method

Step 2

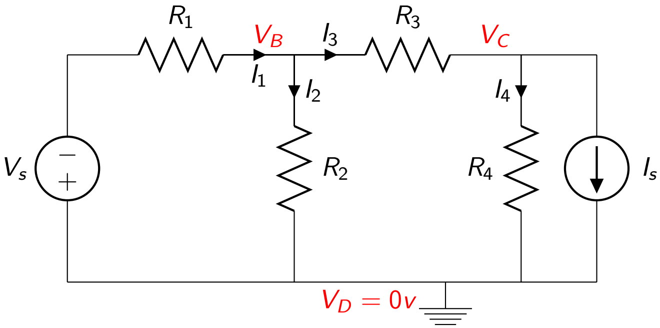
Select one node as a reference. All voltages will be relative to the reference node, for which we assume 0 v.



Node voltage method

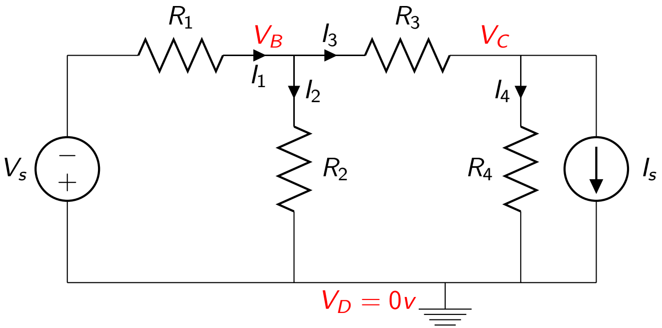
Step 3

Assigns current directions at each node where the voltage is unknown, except at the reference node. The directions are arbitrary.



Step 4

Apply Kirchhoff's current law to each node and express the equations in terms of voltages at the different nodes.



Node B

$$I_1 = I_2 + I_3 \qquad I_1 = \frac{V_s - V_B}{R_1} \qquad I_2 = \frac{V_B}{R_2} \qquad I_3 = \frac{V_B - V_C}{R_3}$$

$$\frac{V_s - V_B}{R_1} = \frac{V_B}{R_2} + \frac{V_B - V_C}{R_3}, \quad V_s R_2 R_3 = V_B(R_2 R_3 + R_1 R_3 + R_1 R_2) - V_C R_1 R_2$$

Node C

$$I_3 = I_4 + I_s \qquad I_3 = \frac{V_B - V_C}{R_3} \qquad I_4 = \frac{V_C}{R_4}$$

$$\Rightarrow \frac{V_B - V_C}{R_3} = \frac{V_C}{R_4} + I_s \Rightarrow I_s R_3 R_4 = V_B R_4 - V_C (R_3 + R_4)$$

Step 5

Solve the system of equations to compute the node voltages. Then, node currents can be computed.

$$\left\{ \begin{array}{c} V_s R_2 R_3 = V_B (R_2 R_3 + R_1 R_3 + R_1 R_2) - V_C R_1 R_2 \\ I_s R_3 R_4 = V_B R_4 - V_C (R_3 + R_4) \end{array} \right.$$

$$\begin{bmatrix} R_2 R_3 + R_1 R_3 + R_1 R_2 & -R_1 R_2 \\ R_4 & -(R_3 + R_4) \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} V_s R_2 R_3 \\ I_s R_3 R_4 \end{bmatrix}.$$