Example 6.1 **Determining the Voltage, Given the Current, at the Terminals of an Inductor**

The independent current source in the circuit shown in Fig. 6.2 generates zero current for $t < 0$ and a pulse $10te^{-5t}$A, for $t > 0$.

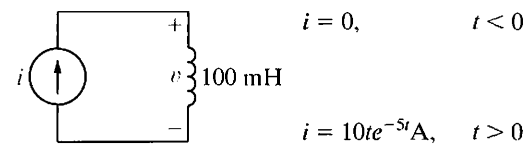

Figure 6.2 ▲ The circuit for Example 6.1.

a) Sketch the current waveform.
b) At what instant of time is the current maximum?
c) Express the voltage across the terminals of the 100 mH inductor as a function of time.
d) Sketch the voltage waveform.
e) Are the voltage and the current at a maximum at the same time?
f) At what instant of time does the voltage change polarity?
g) Is there ever an instantaneous change in voltage across the inductor? If so, at what time?

Solution

a) Figure 6.3 shows the current waveform.
b) $di/dt = 10(-5te^{-5t} + e^{-5t}) = 10e^{-5t}(1 - 5t)$ A/s; $di/dt = 0$ when $t = \frac{1}{5}$ s. (See Fig. 6.3.)
c) $v = Ldi/dt = (0.1)10e^{-5t}(1 - 5t) = e^{-5t}(1 - 5t)$ V, $t > 0$; $v = 0, t < 0$.
d) Figure 6.4 shows the voltage waveform.
e) No; the voltage is proportional to di/dt, not i.
f) At 0.2 s, which corresponds to the moment when di/dt is passing through zero and changing sign.
g) Yes, at $t = 0$. Note that the voltage can change instantaneously across the terminals of an inductor.

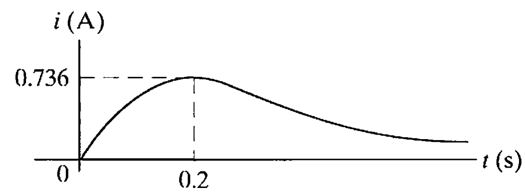

Figure 6.3 ▲ The current waveform for Example 6.1.

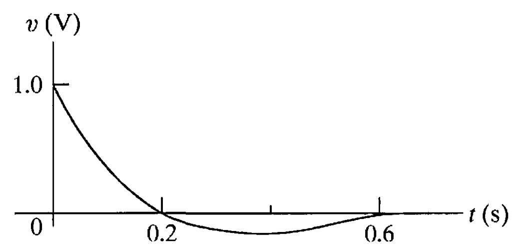

Figure 6.4 ▲ The voltage waveform for Example 6.1.

Current in an Inductor in Terms of the Voltage Across the Inductor

Equation 6.1 expresses the voltage across the terminals of an inductor as a function of the current in the inductor. Also desirable is the ability to express the current as a function of the voltage. To find i as a function of v, we start by multiplying both sides of Eq. 6.1 by a differential time dt:

$$v\,dt = L\left(\frac{di}{dt}\right)dt. \tag{6.2}$$

Multiplying the rate at which i varies with t by a differential change in time generates a differential change in i, so we write Eq. 6.2 as

$$v\,dt = L\,di. \tag{6.3}$$

As before, we use different symbols of integration to avoid confusion with the limits placed on the integrals. In Eq. 6.12, the energy is in joules, inductance is in henrys, and current is in amperes. To illustrate the application of Eqs. 6.7 and 6.12, we return to Examples 6.1 and 6.2 by means of Example 6.3.

Example 6.3 Determining the Current, Voltage, Power, and Energy for an Inductor

a) For Example 6.1, plot i, v, p, and w versus time. Line up the plots vertically to allow easy assessment of each variable's behavior.

b) In what time interval is energy being stored in the inductor?

c) In what time interval is energy being extracted from the inductor?

d) What is the maximum energy stored in the inductor?

e) Evaluate the integrals

$$\int_0^{0.2} p\,dt \quad \text{and} \quad \int_{0.2}^{\infty} p\,dt,$$

and comment on their significance.

f) Repeat (a)–(c) for Example 6.2.

g) In Example 6.2, why is there a sustained current in the inductor as the voltage approaches zero?

Solution

a) The plots of i, v, p, and w follow directly from the expressions for i and v obtained in Example 6.1 and are shown in Fig. 6.8. In particular, $p = vi$, and $w = (\frac{1}{2})Li^2$.

b) An increasing energy curve indicates that energy is being stored. Thus energy is being stored in the time interval 0 to 0.2 s. Note that this corresponds to the interval when $p > 0$.

c) A decreasing energy curve indicates that energy is being extracted. Thus energy is being extracted in the time interval 0.2 s to ∞. Note that this corresponds to the interval when $p < 0$.

d) From Eq. 6.12 we see that energy is at a maximum when current is at a maximum; glancing at the graphs confirms this. From Example 6.1, maximum current = 0.736 A. Therefore, w_{max} = 27.07 mJ.

e) From Example 6.1,

$$i = 10te^{-5t}\text{ A} \quad \text{and} \quad v = e^{-5t}(1 - 5t)\text{ V}.$$

Therefore,

$$p = vi = 10te^{-10t} - 50t^2e^{-10t}\text{ W}.$$

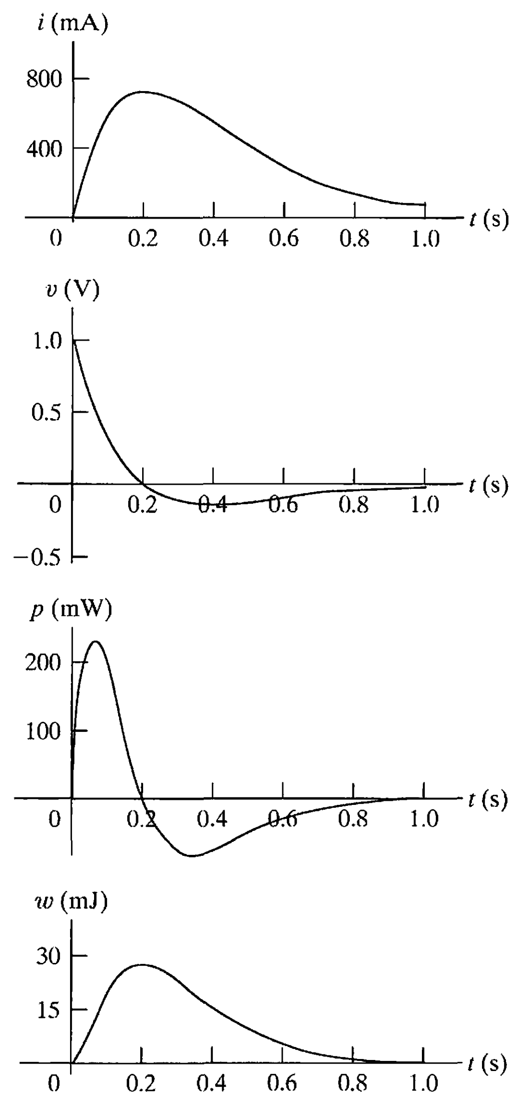

Figure 6.8 ▲ The variables i, v, p, and w versus t for Example 6.1.