

A COMPLETE REPORT ON THE KALMAN FILTERS

UNDERTAKEN BY

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DEDICATION

This report is dedicated to The Almighty God because of the strength He gave us to complete this study despite all we had to do as final-year students. He gave us grace.

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Our Gratitude Goes to God for his help during this study, To our dear lecturer, Dr Engr Somefun for teaching us with a passion and always looking out for our best. To all members of Group C that worked tirelessly to put all this together.

ABSTRACT

This report contains a detailed explanation of Kalman Filters; it's history, operational principle, types, advantages, disadvantages, and economic impact.

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CHAPTER ONE

1.0 WHAT IS A KALMAN FILTER?

Kalman filtering, which is also known as linear quadratic estimation (LQE), is an algorithm that provides estimates of some unknown variables by using a series of measurements observed over time. The estimates of the unknown variables using the series of observed measurements tend to be more accurate than those based on a single measurement alone.

Simply put, The Kalman Filter has inputs and outputs. The inputs are noisy and sometimes inaccurate measurements. The outputs are less noisy and sometimes more accurate estimates. The output estimates can be system state parameters that were not measured or observed. Again, the Kalman Filter estimates system parameters that are not observed or measured.

In short, you can think of the Kalman Filter as an algorithm that can estimate observable and unobservable parameters with great accuracy in real-time by making use of a series of observed measurements

This superpower of the Kalman Filter is its ability to estimate system parameters that can not be measured or observed with accuracy.

The Kalman Filter is a model which uses a set of mathematical equations to provide an efficient relatively simple form requiring small computational means to evaluate the state of a process, in a way that minimizes the mean of the squared error. It is very powerful as it supports estimations of all states with respect to time; past, present, and future even when the precise nature of the modeled system is not known.

The Kalman filter provides optimal estimates of variables of interests when they cannot be measured directly using linear models with additive Gaussian noises.

1.1 HISTORY OF KALMAN FILTERS

1.1.1 BEFORE THE KALMAN FILTER

Dr. Schmidt from the ARC (Ames Research Center) and other researchers who were working on midcourse navigation and guidance for the circumlunar expedition from 1959 were exposed to its known publication by Dr. Kalman in 1960. Although the nonlinear systems utilized to model the issues these researchers were working with were utilized, the filter was linear. The proposed filter might be modified and employed not only as a remedy for their problems but also to lessen computing calculating problems in IBM 704 machines, which attracted these researchers to Kalman's suggestion. Even though it had been utilized in beamrider and homing missile guidance and navigation at the time, the Weiner filter was unable to resolve these issues

because it made estimations that greatly limited system observation or damaged inherent precision.

1.1.2 INTRODUCTION OF THE KALMAN FILTER

The Kalman filter was pioneered by Rudolf Emil Kalman in 1960, originally designed and developed to solve the navigation problem in the Apollo Project. The filter was named after him.

In 1960, Kalman published his famous paper describing a recursive solution to the discrete-data linear filtering problem. Although Thorvald Nicolai Thiele and Peter Swerling developed a similar algorithm earlier. Richard S. Bucy of the Johns Hopkins Applied Physics Laboratory contributed to the theory, causing it to be known sometimes as Kalman–Bucy filtering. Stanley F. Schmidt is generally credited with developing the first implementation of a Kalman filter. He realized that the filter could be divided into two distinct parts, with one part for time periods between sensor outputs and another part for incorporating measurements. It was during a visit by Kalman to the NASA Ames Research Center that Schmidt saw the applicability of Kalman's ideas to the nonlinear problem of trajectory estimation for the Apollo program resulting in its incorporation in the Apollo navigation computer.

1.2 PURPOSE OF KALMAN FILTERS

One of the biggest challenges of tracking and control systems is providing an accurate and precise estimation of the hidden states in the presence of uncertainty. Hence, the goal of the filter is to produce evolving optimal estimates of a modelled process from noisy measurements of the process.

It is a powerful technology for estimating the states of a dynamic system, finding many wide applications in the areas of data integration, navigation, tracking, pattern recognition and control systems.

In GPS receivers, the measurement uncertainty depends on many external factors, such as thermal noise, atmospheric effects, slight changes in satellite positions, receiver clock precision, and many more. The Kalman Filter is one of the most important and common estimation algorithms. The Kalman Filter produces estimates of hidden variables based on inaccurate and uncertain measurements. Also, the Kalman Filter predicts the future system state based on past estimations. Today the Kalman filter is used in target tracking (Radar), location and navigation systems, control systems, computer graphics, and much more.

Furthermore, Kalman filtering is a concept much applied in time series analysis used for topics such as signal processing and econometrics. Kalman filtering is also one of the main topics of robotic motion planning and control and can be used for trajectory optimization. Kalman filtering also works for modelling the central nervous system's control of movement. Due to the time delay between issuing motor commands and receiving sensory feedback, the use of

Kalman filters provides a realistic model for making estimates of the current state of a motor system and issuing updated command.

CHAPTER TWO

2.0 OPERATIONAL PRINCIPLES AND APPLICATIONS

The operational principles and applications of the Kalman Filters are discussed extensively in this chapter.

2.1 OPERATIONAL PRINCIPLES OF KALMAN FILTERS

Kalman filter is an iterative mathematical process that uses a set of equations and consecutive data inputs to quickly estimate the true value, position, velocity e.t.c of the object being measured when the measured values contain errors or uncertainty.

They are used to estimate states based on linear dynamical systems in state space format. The process model defines the evolution of the state from time $k - 1$ to time k as:

$$\mathbf{x}_k = F\mathbf{x}_{k-1} + B\mathbf{u}_{k-1} + \mathbf{w}_{k-1} \text{ ----- (2.1)}$$

where F is the state transition matrix applied to the previous state vector \mathbf{x}_{k-1} , B is the control-input matrix applied to the control vector \mathbf{u}_{k-1} , and \mathbf{w}_{k-1} is the process noise vector that is assumed to be zero-mean Gaussian with the covariance Q , i.e., $\mathbf{w}_{k-1} \sim \mathcal{N}(0, Q)$.

The process model is paired with the measurement model that describes the relationship between the state and the measurement at the current time step k as:

$$\mathbf{z}_k = H\mathbf{x}_k + \mathbf{v}_k \text{ ----- (2.2)}$$

where \mathbf{z}_k is the measurement vector, H is the measurement matrix, and \mathbf{v}_k is the measurement noise vector that is assumed to be zero-mean Gaussian with the covariance R , i.e., $\mathbf{v}_k \sim \mathcal{N}(0, R)$. Note that sometimes the term ‘measurement’ is called ‘observation’ in different literature.

The role of the Kalman filter is to provide estimate of \mathbf{x}_k at time k , given the initial estimate of \mathbf{x}_0 , the series of measurement, $\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_k$, and the information of the system described by F, B, H, Q , and R . Note that subscripts to these matrices are omitted here by assuming they are invariant over time as in most applications. Although the covariance matrices are supposed to reflect the statistics of the noises, the true statistics of the noises is not known or not Gaussian in many practical applications. Therefore, Q and R are usually used as tuning parameters that the user can adjust to get desired performance.

Kalman filtering uses a system's dynamic model (e.g., physical laws of motion), known control inputs to that system, and multiple sequential measurements (such as from sensors) to form an estimate of the system's varying quantities (its state) that is better than the estimate obtained by using only one measurement alone. As such, it is a common sensor fusion and data fusion algorithm. Noisy sensor data, approximations in the equations that describe the system evolution, and external factors that are not accounted for, all limit how well it is possible to determine the system's state. The Kalman filter produces an estimate of the state of the system

as an average of the system's predicted state and of the new measurement using a weighted average. The purpose of the weights is that values with better (i.e., smaller) estimated uncertainty are "trusted" more. The weights are calculated from the covariance, a measure of the estimated uncertainty of the prediction of the system's state. The result of the weighted average is a new state estimate that lies between the predicted and measured state, and has a better estimated uncertainty than either alone. This process is repeated at every time step, with the new estimate and its covariance informing the prediction used in the following iteration. This in turn implies that the Kalman filter only requires the last iteration scenario to determine the state of the system rather than making references to the previous system scenarios.

A BASIC BLOCK DIAGRAM DISPLAYING KALMAN FILTER CALCULATIONS

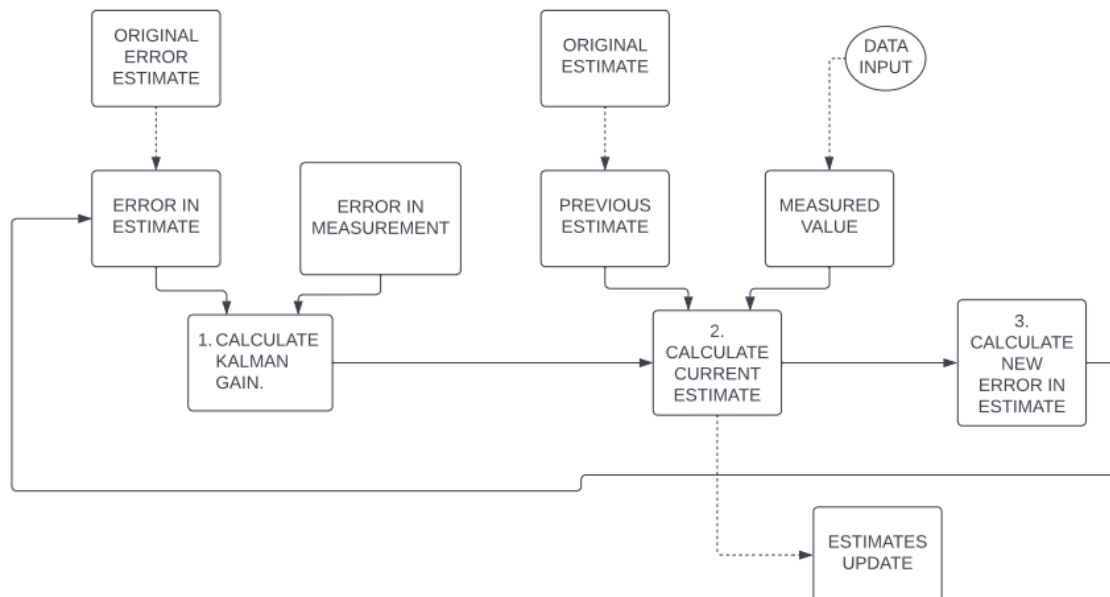


Figure 1 A Block Diagram Displaying Kalman Filter Algorithm

Basic calculations expected in Kalman filters from the above block diagram are:

$$\text{The Kalman Gain, } K_G = \frac{\text{Error in Estimate, } E_{est}}{\text{Error in Estimate, } E_{est} + \text{Error in Measurement, } E_{mea}} \dots \dots \dots (2.3)$$

Current Estimate, $EST_t = \text{Previous Estimate, } EST_{t-1} + \text{Kalman Gain, } K_G (\text{Measured Value, MEA} - \text{Previous Estimate, } EST_{t-1})$

$$\text{Error in Estimate, } E_{est} = [1 - \text{Kalman Gain, } K_G] \text{Error in Previous Estimate, } E_{EST_{t-1}}$$

2.2 KALMAN FILTER ALGORITHM

Kalman filter algorithm consists of two stages which are the predication stage and the correction stage.

i. The Prediction Stage:

Predicted state estimate: $\hat{\mathbf{x}}_k = F\hat{\mathbf{x}}_{k-1} + B\mathbf{u}_k + \mathbf{w}_{k-1} \dots \dots \dots (2.4)$

Predicted error covariance: $P_k = FP_{k-1}F^T + Q_k \dots \dots \dots (2.5)$

ii. The Correction Stage:

Measurement residual: $\tilde{\mathbf{y}}_k = \mathbf{z}_k - H\hat{\mathbf{x}}_k \dots \dots \dots (2.6)$

Kalman gain: $K_k = P_k H^T (R + H P_k H^T)^{-1} \dots \dots \dots (2.7)$

Updated state estimate: $\hat{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_k + K_k (\tilde{\mathbf{y}}_k - H \hat{\mathbf{x}}_k) \dots \dots \dots (2.8)$

Updated error covariance: $P_{k+1} = (I - HK_k) P_k \dots \dots \dots (2.9)$

In the above equations, the hat operator, $\hat{\cdot}$, means the estimate of a variable. That is, $\hat{\mathbf{x}}$ is the estimate of variable \mathbf{x} .

For the Predicted state, \mathbf{P} is called the state error covariance. Note that the covariance of a random variable \mathbf{x} is defined as $\text{cov}(\mathbf{x}) = \mathbb{E}[(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T]$ where \mathbb{E} denotes the expected (mean) value of its argument. One can observe that the error covariance becomes larger at the prediction stage due to the summation with Q , which means the filter is more uncertain of the state estimate after the prediction step.

In the Correction stage, $\tilde{\mathbf{y}}_k$ is called the measurement residual. It is the difference between the true measurement, \mathbf{z}_k , and the estimated measurement, $H\hat{\mathbf{x}}_k$. The filter estimates the current measurement by multiplying the predicted state by the measurement matrix. The residual, $\tilde{\mathbf{y}}_k$, is later then multiplied by the Kalman gain, K_k , to provide the correction, $K_k \tilde{\mathbf{y}}_k$, to the predicted estimate $\hat{\mathbf{x}}_k$. After it obtains the updated state estimate, the Kalman filter calculates the updated error covariance, P_k , which will be used in the next time step. Note that the updated error covariance is smaller than the predicted error covariance, which means the filter is more certain of the state estimate after the measurement is utilized in the update stage.

We need an initialization stage to implement the Kalman filter. As initial values, we need the initial guess of state estimate, $\hat{\mathbf{x}}_0$, and the initial guess of the error covariance matrix, P_0 . Together with Q and R , $\hat{\mathbf{x}}_0$ and P_0 play an important role to obtain desired performance. There is a rule of thumb called “initial ignorance”, which means that the user should choose a large P_0 for quicker convergence. Finally, one can obtain implement a Kalman filter by implementing the prediction and update stages for each time step, $k = 1, 2, 3, \dots$, after the initialization of estimates. Note that Kalman filters are derived based on the assumption that the process and measurement models are linear, i.e., they can be expressed with the matrices

F , B , and H , and the process and measurement noise are additive Gaussian. Hence, a Kalman filter provides optimal estimate only if the assumptions are satisfied.

2.3 CALCULATIONS ON KALMAN FILTERS

There are basically 3 examples which show the application of Kalman filters to give better insight to their functionality. They include:

- i. A Rising Fluid
- ii. A Falling Object
- iii. Moving vehicle

Model equations.

$$X_k = AX_{k-1} + Bu_k + w_k$$

$$Y_k = CX_k + Q_k$$

Where,

X = State Matrix

U = Control Variable Matrix (input matrix)

W = Noise in the Process

t = Time for 1 cycle

Y = Observation (Output)

Q = Measurement Noise

IN A RISING FLUID

$$A = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{2} \Delta T^2 \\ \Delta T \end{bmatrix} \quad U = 0 \quad X_{k-1} = \begin{bmatrix} P_x \\ V_x \end{bmatrix}$$

IN A FALLING OBJECT

$$A = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{2} \Delta T^2 \\ \Delta T \end{bmatrix} \quad U = g = -9.81 \quad X_{k-1} = \begin{bmatrix} P_x \\ V_x \end{bmatrix}$$

IN A MOVING VEHICLE

$$A = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{2} \Delta T^2 \\ \Delta T \end{bmatrix} \quad U = a \quad X_{k-1} = \begin{bmatrix} P_x \\ V_x \end{bmatrix}$$

EXAMPLE 1

Given the data, Noise densities: $\mathbf{v}_k \sim \mathcal{N}(0, 0.05)$, $\mathbf{w}_k \sim \mathcal{N}(0, 0.1)$. Determine the vehicle position directly using a GPS.

$$\mathbf{x}_k \sim \mathcal{N} \left(\begin{bmatrix} 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 0.01 & 0 \\ 0 & 1 \end{bmatrix} \right); \Delta T = 0.5s; u_0 = -2 \text{ m/s}^2; Y_1 = 2.2\text{m}; H = [1 \ 0]$$

Solution

Prediction Stage:

$$\hat{\mathbf{x}}_k = F \hat{\mathbf{x}}_{k-1} + B \mathbf{u}_k + \mathbf{w}_k$$

$$P_k = F P_{k-1} F^T + Q_{k-1}$$

From question:

$$\mathbf{x}_{k-1} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}; \text{ Since it is a vehicle, therefore } F = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{2} \Delta T^2 \\ \Delta T \end{bmatrix} = \begin{bmatrix} \frac{1}{2} * 0.5^2 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 0.125 \\ 0.5 \end{bmatrix}; \quad u_0 = -2 \text{ m/s}^2; \quad P_{k-1} = \begin{bmatrix} 0.01 & 0 \\ 0 & 1 \end{bmatrix}$$

$$F^T = \begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix}; \quad \text{From } \mathbf{w}_k, Q_{k-1} = 0.1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

Then,

$$\hat{\mathbf{x}}_k = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} + \begin{bmatrix} 0.125 \\ 0.5 \end{bmatrix} (-2) = \begin{bmatrix} 2.25 \\ 4 \end{bmatrix}$$

$$P_k = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.01 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.36 & 0.5 \\ 0.5 & 1.1 \end{bmatrix}$$

Correction Stage:

$$K_k = P_k H^T (R + H P_k H^T)^{-1}$$

$$\hat{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_k + K_k (\tilde{\mathbf{y}}_k - H \hat{\mathbf{x}}_k)$$

$$P_{k+1} = (I - H K_k) P_k$$

$$H^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \text{ From } v_k, R_{k-1} = 0.05; \tilde{\mathbf{y}}_k = 2.2$$

Then,

$$K_k = \begin{bmatrix} 0.36 & 0.5 \\ 0.5 & 1.1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} (0.05 + [1 \ 0] \begin{bmatrix} 0.36 & 0.5 \\ 0.5 & 1.1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix})^{-1}$$

$$= \begin{bmatrix} 0.88 \\ 1.22 \end{bmatrix}$$

$$\hat{\mathbf{x}}_{k+1} = \begin{bmatrix} 2.25 \\ 4 \end{bmatrix} + \begin{bmatrix} 0.88 \\ 1.22 \end{bmatrix} (2.2 - [1 \ 0] \begin{bmatrix} 2.25 \\ 4 \end{bmatrix}) = \begin{bmatrix} 2.206 \\ 3.939 \end{bmatrix}$$

$$\mathbf{P}_{k+1} = (1 - [1 \ 0] \begin{bmatrix} 0.88 \\ 1.22 \end{bmatrix}) \begin{bmatrix} 0.36 & 0.5 \\ 0.5 & 1.1 \end{bmatrix} = \begin{bmatrix} 0.04 & 0.06 \\ 0.06 & 0.13 \end{bmatrix}$$

***NOTE:** Kalman Filter is an iterative process so if we continue to calculate the predictive and correction stages, we attain a point of relative stability which will help determine our final variable output.

The best linear (consistent) unbiased estimator is the Kalman filter, which first forecasts the mean and covariance of the current state estimate at a given time before correcting based on those forecasts. It works well for non-linear systems even though linear problems are where it excels. The Extended Kalman Filter was developed because the bulk of systems in our daily lives are non-linear, which introduces new variables that cannot be factored in by just the average Kalman filter.

2.4 PRACTICAL APPLICATION OF KALMAN FILTERS

You can use a Kalman filter in any place where you have uncertain information about some dynamic system, and you can make an educated guess about what the system is going to do next.

The filter can be used to do the following:

1. Computer Vision Object Tracking – Use the measured position of an object to more accurately estimate the position and velocity of that object.
2. Body Weight Estimate on Digital Scale – Use the measured pressure on a surface to estimate the weight of an object on that surface.
3. Guidance, Navigation, and Control – Use Inertial Measurement Unit (IMU) sensors to estimate an object's location, velocity, and acceleration; and use those estimates to control the object's next moves. This is particularly used in vehicles such as aircraft, spacecraft and ships
4. Robotic trajectory modification and motion planning.
5. Kalman filtering is a concept much applied in time series analysis used for topics such as signal processing and econometrics.
6. Radar sensors for position awareness used in driverless vehicles' advanced driver assistance systems (ADAS).

2.5 MATLAB CODE FOR SOME PRACTICAL APPLICATION OF THE KALMAN FILTER

2.5.1 PREDICTING THE TRAJECTORY OF AN OBJECT WITH KALMAN FILTER

We will see a practical approach on how to use the **Kalman filter** to track and predict the trajectory of an object.

At first, I will show simple examples by drawing dots on the screen and having the trajectory predicted, and then we will see how to predict the trajectory of a ball.

Kalman filter is an algorithm that takes measurements over time and creates a prediction of the next measurements. This is used in many fields such as sensors, GPS, to predict the position in case of signal loss for a few seconds and this is what we will also see in computer vision.

We will see the simplest possible implementation with **OpenCV (Computer vision)**

STEP 1: Kalman filter with dots on solid blue background

First of all import **kalmanfilter.py** (algorithm sourced out online) and the **OpenCV** library

```
from kalmanfilter import KalmanFilter
```

```
import cv2
```

Now initialize Kalman filter and we try to insert values to get a prediction

```
# Kalman Filter
```

```
kf = KalmanFilter()
```

```
predicted = kf.predict(50,50)
```

```
print(predict)
```

As you can see from the image below, if we print the result, we will get (0,0) because the Kalman filter analysis function needs more values to make a prediction.

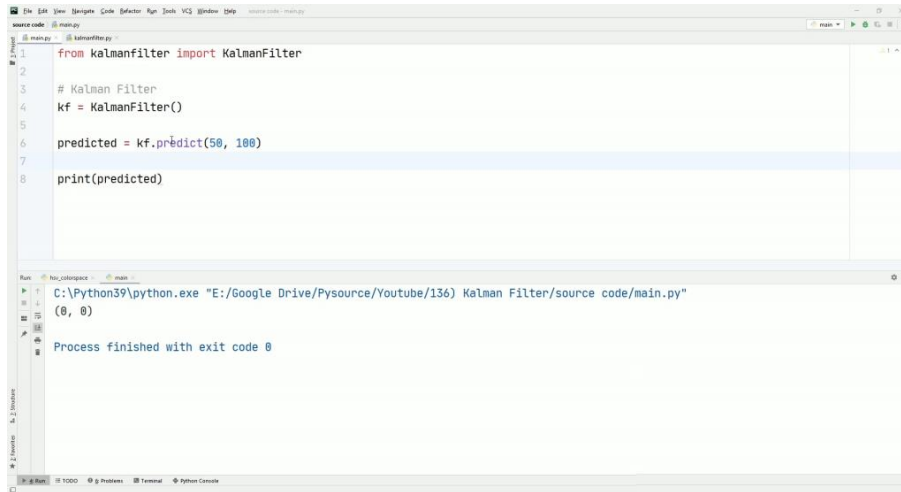


Figure 2 Code Snippet for predicting the trajectory of an object with kalman filter

STEP 2: More precision with more values

By inserting more position points, you will get a more and more precise prediction. Here is an example of code to insert more points that will make our Kalman filter better.

```

# Kalman Filter

kf = KalmanFilter()

predicted = kf.predict(50,100)
predicted = kf.predict(100,100)
predicted = kf.predict(150,100)
predicted = kf.predict(200,100)
print(predict) # result (238,114)

```

STEP 3: Simulate the movement of a ball

To verify and put into practice the theory we have seen in the previous paragraph, we simulate the trajectory of a ball that goes from right to left. We can do everything with OpenCV by specifying a series of points where the ball will be.

At the same time we pass all points to the function **kf.predict(x, y)**

```

from kalmanfilter import KalmanFilter

import cv2

# Kalman Filter

kf = KalmanFilter()

img = cv2.imread("blue_background.webp")

ball_positions = [(50, 100), (100, 100), (150, 100), (200, 100), (250, 100), (300, 100), (350, 100), (400, 100), (450, 100)]

```

```

for pt in ball2_positions:
    cv2.circle(img, pt, 15, (0, 20, 220), -1)
    predicted = kf.predict(pt[0], pt[1])
    cv2.circle(img, predicted, 15, (20, 220, 0), 4)
cv2.imshow("Img", img)
cv2.waitKey(0)

```

By executing the code, you can see from the image the solid red balls that go from left to right and the green circles that make the prediction of the next position.

As you can see at the alignment the first green circle is on the point with coordinate (0,0) because it does not have enough values to do the processing. Instead, the last green circle indicates, in a plausible way, the next possible position of the red ball.

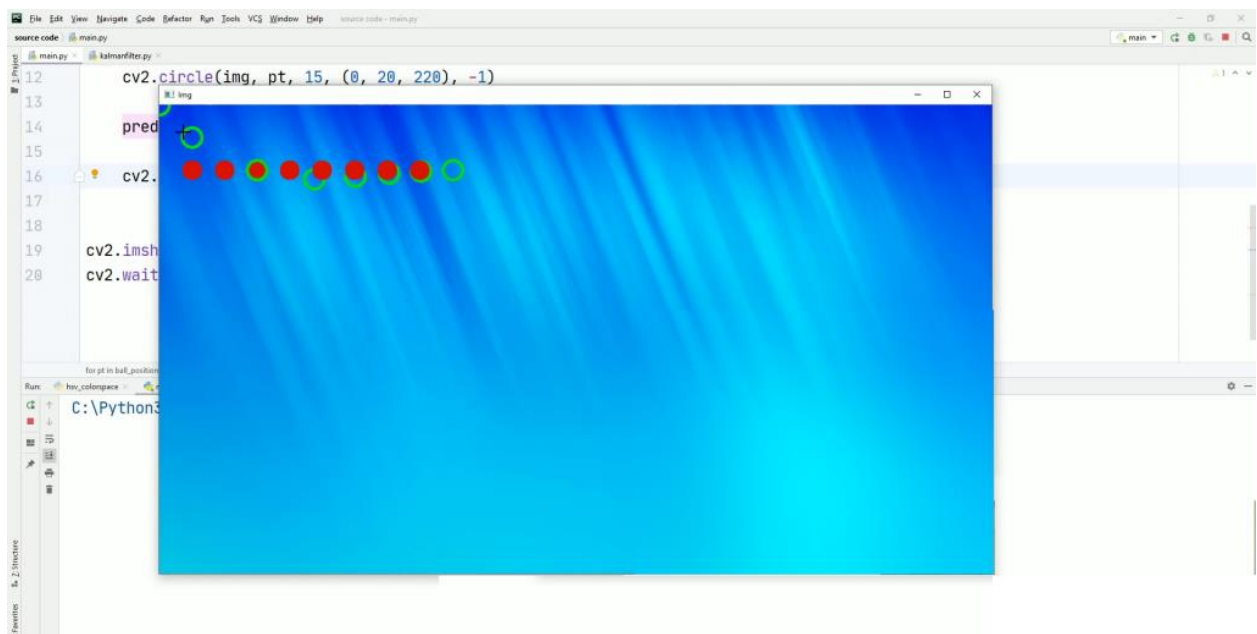


Figure 3 Code Output 1

Trying to add more points for prediction with this code

```

for i in range(10):
    predicted = kf.predict(predicted[0], predicted[1])
    cv2.circle(img, predicted, 15, (20, 220, 0), 4)
    print(predicted)

```

you can see how the Kalman filter manages to make a correct prediction. In the image below you can see other green circles that assume the position of the red ball

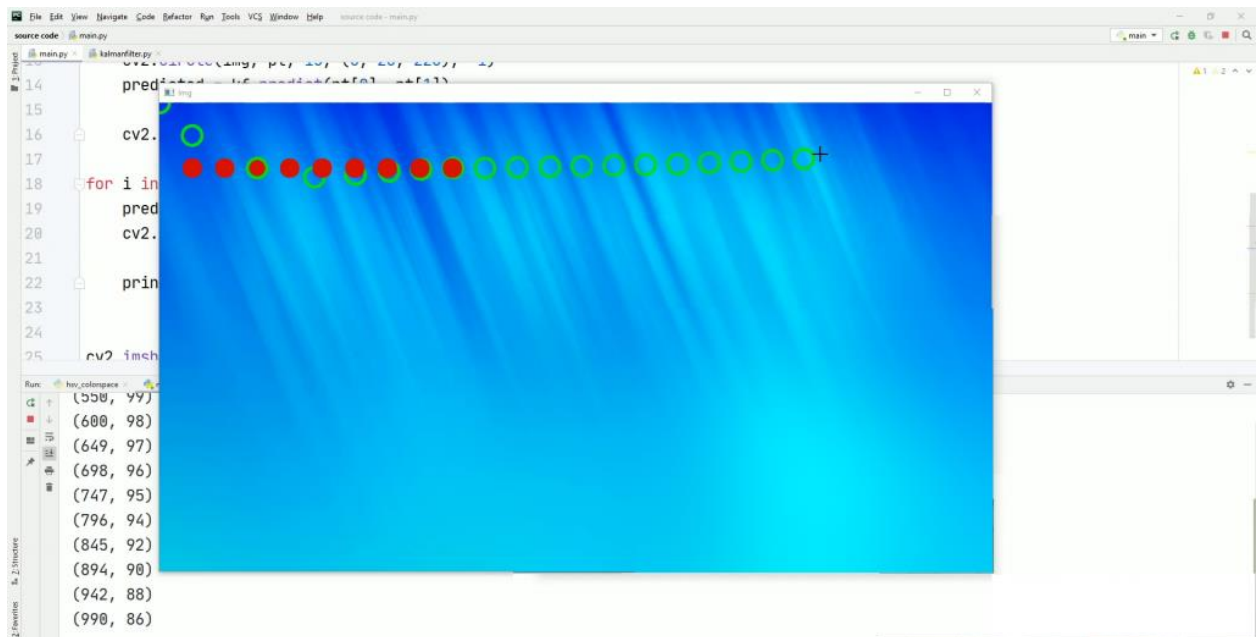


Figure 4 Code Output 2

The same procedure is valid if I simulate the launch of a ball whose trajectory draws an arc. Again Kalman filter improves from time to time as new points are awarded. The one in the image below is the result

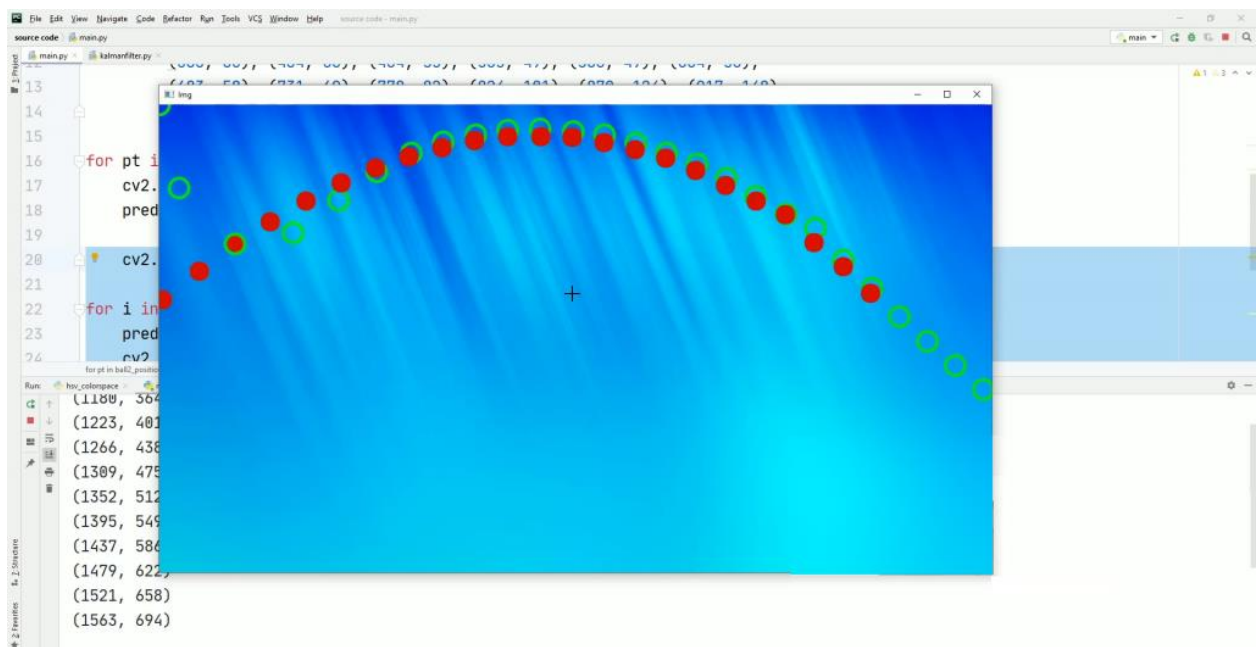


Figure 5 Code Output 3

2.5.2 USE KALMAN FILTER TO PREDICT THE TRAJECTORY OF REAL OBJECT

STEP 1: Detect the object

The first step consists of object detection, in this case of an orange, identified with the colour recognition method

We import everything necessary and proceed with obtaining the frames from the video through a loop

```
import cv2

from orange_detector import OrangeDetector
from kalmanfilter import KalmanFilter

cap = cv2.VideoCapture("orange.mp4")

# Load detector
od = OrangeDetector()

# Load Kalman filter to predict the trajectory
kf = KalmanFilter()

while True:
    ret, frame = cap.read()
    if ret is False:
        break
```

Now We use the functions for our orange detector and we get the coordinates of 2 points. However, these correspond to the top-left point and the bottom right point of the object. With a small mathematical operation, we obtain the coordinates of the center.

```
orange_bbox = od.detect(frame)
x, y, x2, y2 = orange_bbox
cx = int((x + x2) / 2)
cy = int((y + y2) / 2)
```

Now we have what we need, we just need to add the Kalman filter function to predict the future position of the object.

```
predicted = kf.predict(cx, cy)

#cv2.rectangle(frame, (x, y), (x2, y2), (255, 0, 0), 4)
cv2.circle(frame, (cx, cy), 20, (0, 0, 255), 4)
```

```
cv2.circle(frame, (predicted[0], predicted[1]), 20, (255, 0, 0), 4)
```

Showing a small red circle for the current position and a blue circle for prediction through Kalman filter, this is the result extrapolated from a single frame

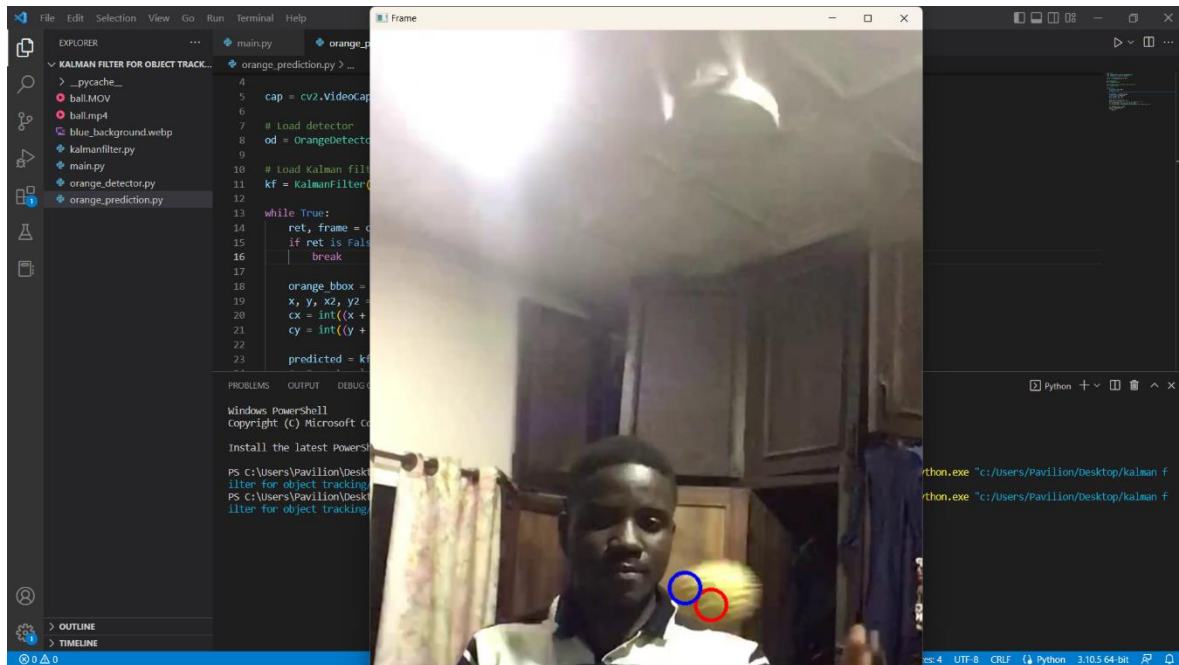


Figure 6 Code output 4

OTHER APPLICATIONS

In the 1960s, the Kalman filter was applied to navigation for the Apollo Project, which required estimates of the trajectories of manned spacecraft going to the Moon and back. With the lives of the astronauts at stake, it was essential that the Kalman filter be proven effective and reliable before it could be used

Kalman filter is also used in more complex tracking algorithms such as **SORT** and **Deep SORT**, **YOLO v5** or **7** and this allows its use in real scenarios

CHAPTER THREE

3.0 TYPES OF KALMAN FILTERS

There are 3 various types of Kalman filters, and they are as follows:

- i. Linear Kalman Filter
- ii. Extended Kalman Filter
- iii. Unscented Kalman Filter

Looking into the various filters:

3.1 LINEAR KALMAN FILTER

The linear Kalman filter is simply the most basic form of the Kalman filter also known as linear quadratic estimation (LQE), it is an algorithm that uses a series of measurements observed over time, including statistical noise and other inaccuracies, and produces estimates of unknown variables that tend to be more accurate than those based on a single measurement alone, by estimating a joint probability distribution over the variables for each timeframe.

The linear Kalman filter (trackingKF) is an optimal, recursive algorithm for estimating the state of an object if the estimation system is linear and Gaussian. An estimation system is linear if both the motion model and measurement model are linear. The filter works by recursively predicting the object state using the motion model and correcting the state using measurements.

The Kalman Filter is an algorithm to develop estimations of the true and conscious values, first by predicting a value, then estimating the uncertainty of the above value, and encountering a weighted average of both the predicted and estimated values. Most weight is assigned to the value with the least uncertainty. The outcome acquired by this method provides estimates nearer to true values.

The following figure shows in the detail the description of a linear Kalman filter's block diagram

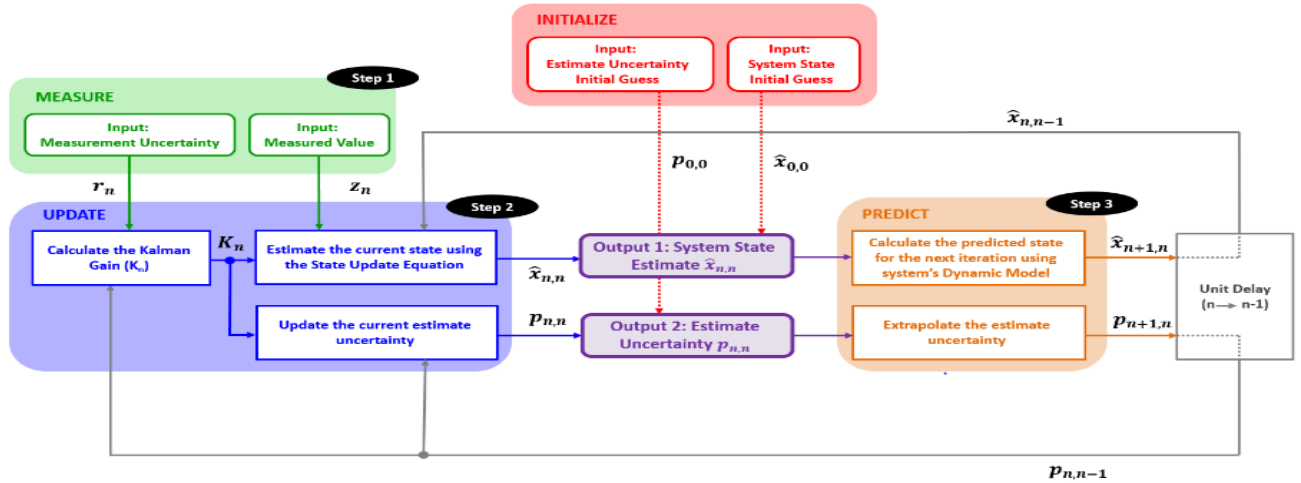


Figure 7 Diagram showing the algorithm of a linear kalman filter

3.1.1 APPLICATION OF LINEAR KALMAN FILTER

A common application is for guidance, navigation, and control of vehicles, particularly aircraft, spacecraft and ships positioned dynamically.

Kalman filtering is a concept much applied in time series analysis used for topics such as signal processing and econometrics.

3.2 EXTENDED KALMAN FILTER

The real systems that are the motivation for all the equations like Kalman filters are handled by non-linear functions. The evolved equations of filters that are primarily designed for linear functions are needed. Also, it is stated that in the estimation approach, the Extended Kalman filter (EKF) is the nonlinear arrangement of the Kalman filter.

This non-linear filter linearizes the current mean and covariance. At one moment, the EKF might have been regarded as the standard in the idea of nonlinear state estimation, navigation systems and GPS.

The Extended Kalman Filter (EKF) has become a standard technique used in several nonlinear estimations and machine learning applications. These include estimating the state of a nonlinear dynamic system, estimating parameters for nonlinear system identification (e.g., learning the weights of a neural network), and dual estimation (e.g., the Expectation Maximization (EM) algorithm) where both states and parameters are estimated simultaneously.

3.2.1 FORMULATION OF THE EXTENDED KALMAN FILTER

The basic framework for EKF involves the estimation of the state of a discrete-time nonlinear dynamic system,

$$\mathbf{X}_k = \mathbf{F}(\mathbf{X}_{k-1}, \mathbf{U}_{k-1}, \mathbf{W}_{k-1}) \dots\dots\dots(3.1)$$

$$\mathbf{Z}_k = \mathbf{H}(\mathbf{X}_k) + \mathbf{V}_k \dots \dots \dots (3.2)$$

Where \mathbf{w}_k and \mathbf{v}_k are the processes and observation noises that are both believed to be zero-mean multivariate Gaussian noise with covariance \mathbf{Q}_k and \mathbf{R}_k respectively. The functions f and h utilize the previous estimate and support in calculating the predicted state and also the predicted state is utilized to estimate the predicted measurement. Yet, f and h cannot be utilized for the covariance directly. So, a matrix of partial derivatives (the Jacobian) calculation is needed. At each time step with the use of current predicted states, the Jacobian is computed. These matrices are utilized in the KF equations. This method linearizes the nonlinear function of the present estimate.

Predicted State,

$$\mathbf{X}_{k|k-1} = f(\mathbf{X}_{k|k-1}, \mathbf{u}_{k-1}) \dots \dots \dots (3.3)$$

Predicted estimate covariance,

$$\mathbf{P}_{k|k-1} = \mathbf{F}_{k-1} \mathbf{P}_{k-1|k-1} \mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1} \dots \dots \dots (3.4)$$

Innovation (or residual) covariance,

$$\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k \dots \dots \dots (3.5)$$

Optimal Kalman gain,

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{S}_k^{-1} \dots \dots \dots (3.6)$$

Updated state estimate,

$$\mathbf{X}_{k|k} = \mathbf{X}_{k|k-1} + \mathbf{K}_k \mathbf{y}_k \dots \dots \dots (3.7)$$

Updated estimate covariance,

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} \dots \dots \dots (3.8)$$

Where the state transition and observation matrices are determined to be the following Jacobians (partial derivatives),

$$\begin{aligned} \mathbf{F}_{k-1} &= \left. \frac{\partial f}{\partial \mathbf{x}} \right|_{\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_{k-1}} \\ \mathbf{H}_k &= \left. \frac{\partial h}{\partial \mathbf{x}} \right|_{\hat{\mathbf{x}}_{k|k-1}} \dots \dots \dots (3.9) \end{aligned}$$

The EKF works on the principle that a linearized transformation of means and covariances is approximately equal to the true nonlinear transformation. However,

this approximation could be unsatisfactory in practice.

3.2.2 APPLICATIONS OF THE EXTENDED KILMAN FILTER

- i. Application of the Kalman filter in dynamic X-ray tomography. With this method, the X-ray image is reconstructed through a low-dimensional pool of parameters. This has been shown to optimize the computational speed of the overall process.
- ii. Extended Kalman Filters have extensively contributed to the optimization of robotic movements, tracking, and localization using GPS and IMU Sensors

3.2.3 ADVANTAGES OF EXTENDED KALMAN FILTERS OVER THE LINEAR KILMAN FILTERS

- i. The extended Kalman filter solves the nonlinear estimation problem by linearizing state and/or measurement equations and applying the standard Kalman filter formulas to the resulting linear estimation problem.
- ii. The Extended Kalman Filter has often a quicker convergence time which leads to a shorter time until a valid state is observed.

3.2.4 DISADVANTAGES OF EXTENDED KALMAN FILTERS

- i. Computing partial derivatives are very stressful and exhausting
- ii. Drifts when linearization is a bad approximation
- iii. It cannot handle multi-modal (multi-hypothesis) distribution.

3.3 UNSCENTED KALMAN FILTER

The UKF addresses the approximation issues of the EKF. The state distribution is again represented by a Gaussian random variable but is now specified using a minimal set of carefully chosen sample points. These sample points completely capture the true mean and covariance of the Gaussian random variable, and when propagated through the true non-linear system, capture the posterior mean and covariance accurately to the 3rd order (Taylor series expansion) for any nonlinearity.

3.3.1 FORMULATION OF THE UNSCENTED KALMAN FILTER

Since no information about the covariance is available, the additive form of the process and measurement equations

$$\mathbf{X}_k = \mathbf{f}_d(\mathbf{x}_{k-1}, k-1) + \mathbf{q}_{k-1} \dots\dots\dots(3.10)$$

$$\mathbf{Y}_k = \mathbf{h}_d(\mathbf{x}_k, k) + \mathbf{r}_k, \dots\dots\dots(3.11)$$

where $\mathbf{x}_k \in \mathbf{R}^m$ is the state, $\mathbf{y}_k \in \mathbf{R}^m$ is the measurement, $\mathbf{q}_{k-1} \in \mathbf{R}^m$ is a Gaussian process noise $\mathbf{q}_{k-1} \sim N(\mathbf{0}, \mathbf{Q}_{k-1})$, and $\mathbf{r}_k \in \mathbf{R}^m$ is a Gaussian measurement noise $\mathbf{r}_k \sim N(\mathbf{0}, \mathbf{R}_k)$. The mean and covariance of the initial state \mathbf{x}_o are \mathbf{m}_o and \mathbf{p}_o , respectively.

Prediction state: Compute the predicted state mean \mathbf{m}_k^- and the predicted covariance \mathbf{P}_k^- as

$$\begin{aligned} [\mathbf{m}_k^-, \tilde{\mathbf{P}}_k] &= \text{UT}(\mathbf{f}_d, \mathbf{m}_{k-1}, \mathbf{P}_{k-1}) \\ \mathbf{P}_k^- &= \tilde{\mathbf{P}}_k + \mathbf{Q}_{k-1}. \end{aligned} \dots\dots\dots(3.12)$$

Update state: Compute the predicted mean μ_k and covariance of the measurement \mathbf{S}_k , and the cross-covariance of the state and measurement \mathbf{C}_k :

$$\begin{aligned} [\mu_k, \tilde{\mathbf{S}}_k, \mathbf{C}_k] &= \text{UT}(\mathbf{h}_d, \mathbf{m}_k^-, \mathbf{P}_k^-) \\ \mathbf{S}_k &= \tilde{\mathbf{S}}_k + \mathbf{R}_k. \end{aligned} \dots\dots\dots(3.13)$$

Then compute the filter gain \mathbf{K}_k , the state mean \mathbf{m}_k and the covariance \mathbf{P}_k , conditional to the measurement \mathbf{y}_k :

$$\begin{aligned} \mathbf{K}_k &= \mathbf{C}_k \mathbf{S}_k^{-1} \\ \mathbf{m}_k &= \mathbf{m}_k^- + \mathbf{K}_k [\mathbf{y}_k - \mu_k] \\ \mathbf{P}_k &= \mathbf{P}_k^- - \mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T. \end{aligned} \dots\dots\dots(3.14)$$

The filtering is started from the initial mean \mathbf{m}_o and covariance \mathbf{p}_o .

3.3.2 APPLICATIONS OF THE UNSCENTED KALMAN FILTER

- i. The UKF was originally designed for the state-estimation problem and has been applied in nonlinear control applications requiring full-state feedback.
- ii. The UKF is used to minimize measuring and modelling errors for geometric and thermal errors of machine tools.
- iii. UKF has largely replaced the EKF in many nonlinear filtering and control applications, including for underwater, ground and air navigation

3.3.3 ADVANTAGES OF THE UNSCENTED KALMAN FILTER OVER THE EXTENDED KALMAN FILTER

- The UKF consistently achieves a better level of accuracy than the EKF at a comparable level of complexity.
- it uses the true nonlinear models to estimate the results as accurately as possible.
- Theoretically, captures higher-order moments of distribution than linearization.
- No closed-form derivatives or expectations are needed.

3.3.4 DISADVANTAGES OF UNSCENTED KALMAN FILTER

- Not a truly global approximation, based on a small set of trial points.
- Does not work well with nearly singular covariances, i.e., with nearly deterministic systems.
- Requires more computations than EKF
- Can only be applied to models driven by Gaussian noises.

The table below shows the mathematical comparison of the **Linear**, **Extended** and **Unscented Kalman Filter**.

LINEAR KALMAN FILTER	EXTENDED KALMAN FILTER	UNSCENTED KALMAN FILTER
$\mathbf{x}_k = \mathbf{F}\hat{\mathbf{x}}_{k-1} + \mathbf{B}\mathbf{u}_{k-1}$ $\mathbf{P}_k = \mathbf{F}\hat{\mathbf{P}}_{k-1}\mathbf{F}^T + \mathbf{Q}$	$\mathbf{F} = \frac{\partial f(\mathbf{x}, \mathbf{u})}{\partial \mathbf{x}}$ $\mathbf{x}_k = f(\hat{\mathbf{x}}_{k-1}, \mathbf{u})$ $\mathbf{P}_k = \mathbf{F}\hat{\mathbf{P}}_{k-1}\mathbf{F}^T + \mathbf{Q}$	$\mathbf{y}_\sigma = f(\chi)$ $\mathbf{x}_k = \sum \mathbf{W}_m \mathbf{y}_\sigma$ $\mathbf{P}_k = \sum w^c (\mathbf{y}_\sigma - \mathbf{x}_k)(\mathbf{y}_\sigma - \mathbf{x}_k)^T + \mathbf{Q}$
$\mathbf{Y} = \mathbf{Z} - \mathbf{H}\mathbf{x}$ $\mathbf{S} = \mathbf{H}\mathbf{P}_k\mathbf{H}^T + \mathbf{R}$ $\mathbf{K}_k = \mathbf{P}_k\mathbf{H}^T\mathbf{S}^{-1}$ $\hat{\mathbf{x}}_k = \mathbf{x}_k + \mathbf{K}_k\mathbf{Y}$ $\hat{\mathbf{P}}_k = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}_k$	$\mathbf{H} = \frac{\partial h(\mathbf{x}_k)}{\partial \mathbf{x}}$ $\mathbf{Y} = \mathbf{Z} - h(\mathbf{x})$ $\mathbf{S} = \mathbf{H}\mathbf{P}_k\mathbf{H}^T + \mathbf{R}$ $\mathbf{K}_k = \mathbf{P}_k\mathbf{H}^T\mathbf{S}^{-1}$ $\hat{\mathbf{x}}_k = \mathbf{x}_k + \mathbf{K}_k\mathbf{Y}$ $\hat{\mathbf{P}}_k = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}_k$	$\mathbf{Y} = \mathbf{Z} - \mu_z$ $\mathbf{P}_{\mathbf{Z}_k} = \sum w^c (\mathbf{Z} - \mu_{z_k})(\mathbf{Z} - \mu_{z_k})^T + \mathbf{R}$ $\mathbf{K} = [\sum w_i^c (\mathbf{y}_\sigma - \mathbf{x})(\mathbf{Z} - \mu_z)^T] \mathbf{P}_{\mathbf{Z}}^{-1}$ $\hat{\mathbf{x}}_k = \mathbf{x}_k + \mathbf{K}_k\mathbf{Y}$ $\hat{\mathbf{P}}_k = \mathbf{P}_k - \mathbf{K}\mathbf{P}_{\mathbf{Z}_k}\mathbf{K}^T$

3.4 ALTERNATIVES TO KALMAN FILTER

Kalman filters belong to a general group of filters called Bayes Filters. Bayes Filtering is the general term used to discuss the method of using a predict/update cycle to estimate the state of a dynamical system from sensor measurements. As mentioned, two types of Bayes Filters are Kalman filters and particle filters. The problem with Kalman filters is that they represent the state of the system using only single Gaussians. As you can see in the diagram excerpted from the demo just showed, sometimes it is necessary to have multimodal hypotheses about where the robot might be. If you can only choose one of the two possibilities (the most likely one), and you choose incorrectly, then it is extremely difficult to recover from your mistake. Particle filters, on the other hand, can keep track of as many hypotheses as there are particles, so if new information shows up that causes you to shift your best hypothesis completely, it is easy to do. Particle filters, or sequential Monte Carlo methods, are a set of Monte Carlo algorithms used to solve filtering problems arising in signal processing and Bayesian statistical inference. The filtering problem consists of estimating the internal states in dynamical systems when partial observations are made and random perturbations are present in the sensors as well as in the dynamical system. The objective is to compute the posterior distributions of the states of a Markov process, given the noisy and partial observations. Particle filters update their prediction in an approximate (statistical) manner. The samples from the distribution are represented by a set of particles; each particle has a likelihood weight assigned to it that represents the probability of that particle being sampled from the probability density function.

3.4.1 COMPARISON BETWEEN PARTICLE AND KALMAN FILTER

1. Kalman filter can be used for linear or linearized processes and measurement system, the particle filter can be used for nonlinear systems.
2. Also, the uncertainty of Kalman filter is restricted to Gaussian distribution, while the particle filter can deal with non-Gaussian noise distribution.
3. In cases where abrupt sensor noise is rarely observed, both filters work fairly well. However, when sensor noise exhibits jerky error, Kalman filter results in location estimation with hopping while particle filter still produces robust localization.

3.5 ADVANTAGES OF KALMAN FILTERS

1. Kalman filters are ideal for systems which are continuously changing. They have the advantage that they are light on memory (they don't need to keep any history other than the previous state), and they are very fast, making them well-suited for real-time problems and embedded systems.
2. Kalman Filter is very robust to measurement noise and does not depend on good initial conditions (after finite time the filter will converge to the correct system state) and by

calculating the Kalman gain and estimating the state-covariance a measure for the confidence in the estimate is available.

3. It can be fed multiple measurements of different natures at any time.
4. The filter computes the (sever) gains by itself. And it does that in the best way possible by choosing the gains that minimize the root mean square error over time.
5. The filtered signal will have no lag because the filter will anticipate its variations even before the next measurements are available
6. Good results in practice due to optimality and structure.
7. Convenient form for online real time processing.
8. Easy to formulate and implement given a basic understanding.
9. Measurement equations need not be inverted

The algorithm of the Kalman filter has several advantages. This is a statistical technique that adequately describes the random structure of experimental measurements. This filter can consider quantities that are partially or completely neglected in other techniques (such as the variance of the initial estimate of the state and the variance of the model error). It provides information about the quality of the estimation by providing, in addition to the best estimate, the variance of the estimation error. The Kalman filter is well suited to the online digital processing. Its recursive structure allows its real-time execution without storing observations or past estimates.

3.6 DISADVANTAGES OF KALMAN FILTER

1. In rapid change situation, sometimes Kalman filter has a slow reaction speed.
2. Kalman filter does not include model uncertainty, hence the model has to be known.
3. It assumes that the state belief is Gaussian distributed.

3.7 INNOVATIONS MADE BY THE USE OF THE KALMAN FILTER

1. A demonstration of the adaptability of Kalman original theory to nonlinear problems
2. The development of EKF, which reduces the effects of the problems arising in nonlinear systems after conducting a linearization over the best real state estimation
3. The decomposition and reformulation of the original Kalman algorithm in separate time-update and measurement-update portions so measurements could be processed at any arbitrary time interval
4. Showing the potential of the Kalman filter through its application in a full digital simulation that solved a nonlinear orientation and navigation problem that occurs on a spaceship
5. The dissemination of results from the simulation conducted at the MIT Instrument Laboratory for their possible incorporation into the control and navigation system of the Apollo spacecraft
6. The dissemination of information about the Ames Kalman filter to many scientific and aero spatial units through presentations and formal work

The EKF extension is achieved using Taylor's approximation, through which a nonlinear system can be linearized employing estimation techniques. This algorithm is a very powerful tool for positioning: it can accept different types of data, solve many parameters, and produce reliable and accurate results [4]. Both the KF and the EKF are used in robotics, where they are applied in trajectory tracking, position estimation for manipulator robots, SLAM (Simultaneous Localization and Mapping), and object detection, among others, depending on the linearity or nonlinearity of the model. The flexibility of this algorithm has enabled the integration of information from different types of sensors and techniques such as odometry, GNSS (Global Navigation Satellite System), laser and ultrasonic sensors, and recently, artificial vision, making it possible to answer the fundamental questions of autonomous navigation: Where am I? Where am I going? And how do I reach my destination?

The Kalman filter is better than other algorithms used for estimation due to the small room it needs for storage and its wide variety of uses. However, the impact on the environment surrounding it, errors from measuring equipment, and incorrect parameter selection usually cause system errors in real applications. Researchers have developed different variants or modifications in the last years. These modifications are aimed at solving the problems the algorithm presents due to the increase in the complexity of the equipment in which is applied, and owing to the accuracy and efficiency that manufacturing, medicine elaboration, and navigation, among other processes, need nowadays. An analysis of the systems in which the Kalman filter or its modifications have been employed is proposed in this work, emphasizing that the main advantages of the KF and its variants are its simplicity and capability to provide accurate estimations and prediction results

CHAPTER FOUR

4.0 THE APPLICATIONS

The Kalman filter has various applications in various sectors including but not limited to Engineering. These applications are discussed in this chapter.

4.1 APPLICATION OF KALMAN FILTERING TO NIGERIAN POWER GRID SYSTEM

The Kalman filter can be applied to the Nigerian power system in the following ways:

4.1.1 GRID SYNCHRONIZATION

The world is constantly evolving and changing one of these changes happens to be the consistent and pressing need to move the bulk of our energy generation towards renewable energy sources and one of the major ways more developed and some developing countries are undertaking this feat is by systematic replacement, that is to say decommissioning old Non-Renewable Energy power generating plants and replacing them with “Greener ” plants such as Solar Farms, Wind farms and possibly Nuclear or geothermal plants.

The problem with this system is with matching the frequency, Phase, and amplitude of the several newer plants to the tune of the older plants on the power grid although this problem can be solved using semiconductor switches and a phase locked loop which has its limits in frequency change and noise sensitivity to solve this a Kalman filter is employed.

The Kalman filter is used after generating an estimate of the phase values and measuring phase values such as amplitude, frequency, phase on the line the Kalman filter, the values are compared, and the filter generates the optimal estimate which is used by the system to make or adjust the output from the inverter.

4.1.2 PROTECTION AND DIGITAL RELAYING

Other key application of Kalman filter in power systems is to detect fault conditions and to control protection devices, a task normally done by digital relays. Based on information coming from voltages and currents, decisions must be taken to detect and protect the power system from more severe faults and maintain its stable operation (Girgis & Brown, 1981).

Of course, this information is in the transient condition and depending on the kind, frequency of occurrence and location of the fault, the effects produced in the 60 Hz components (and other frequencies), are of very particular type, allowing gathering valuable information to control a protective device or to plan a repair as soon as possible.

4.1.3 ANALYSIS AND CONTROL OF ELECTRICAL MACHINERY

Considering the digital control or analysis of induction motors, the KF has been applied in some different ways, e.g., to estimate the rotor time constant in PWM motor drives (Zai et al., 1992); to estimate the airgap flux to implement a direct flux control strategy (Pietrzak-David et al., 1992) or also to identify the rotor resistance to propose adaptive vector control schemes (Wade et al., 1997). In addition, KF has been also applied in order to reduce or avoid the use of additional sensors in the motor controlling or monitoring, considering the so called sensorless applications (Bolognani et al., 2001). It is interesting to mention that these applications use the KF as part of an automatic closed loop control system, that is, in the same way as its first applications.

4.1.4 LOAD FORECASTING AND MODAL ESTIMATION

One of the first applications of the KF in the power system area had been to forecast the total load demanded by a multi-node system (Abu-El-Magd et al., 1981; Park et al., 1991). Temporal load data, collected by the various agents in the power system administration, are used to predict load conditions, and the KF is frequently a fundamental part of the algorithm. Normally, the collected data are hourly based, and the prediction algorithm must yield short-term results, that would be useful in scheduling the actual system to supply the daily demand, and medium and long-term results, what would be useful in, for example, expansion planning and annual maintenance scheduling. Load forecasting has been gaining more importance if the electricity market becomes deregulated and the power sources become more and more distributed in the interconnected grid (Song et al., 2006).

In this kind of application, there is not much difference from the forecasting of economic data, as presented in (Clements & Hendry, 1998). There is a temporal series of power consumption and several periodicities and trends can be detected. The most evident is the daily periodicity on weekdays, which assumes the peak value around the beginning of the night. On Sundays and holydays, however, the pattern consumption tends to be less correlated. This pattern also reveals a weekly and monthly periodicity, and a yearly periodicity can obviously be assumed. Trends are always present in this kind of data, and the most important is the rising consumption that can be observed in the series. It is also important to mention the parameter dependence variations coming from the climate (mostly

temperature). In tropical countries, the load is expected to be higher in summer, by the use of air conditioning systems.

In a similar way, other interesting application is the modal estimation of the power system. Based on power systems measurements under normal conditions and defining a stochastic model relating different disturbance inputs (e.g., load changes), the KF is adjusted to estimate the outputs produced by the disturbances. Then, by monitoring the difference between the measured output and the estimated output, one can recognize if there is any change in the model parameters (Wiltshire et al., 2007).

4.2 OTHER APPLICATIONS OF KALMAN FILTERS

There are various applications for the Kalman filter and some of them will be discussed below.

4.2.1 KALMAN FILTER ON POWER ELECTRONICS AND POWER SYSTEMS – STATE OF THE ART

A concise description of different applications of the KF in power electronics and power systems areas is summarized in next sections. Even if some of them have been proposed more than 20 years ago, especially those based on off-line processing, the applications are quite limited if compared with other digital techniques applied in such areas. Nevertheless, considering the ever-increasing capacity of digital processor's technology, this line has been broken down and new KF applications have emerged, including on-line approaches. Most of them are based on the identification of the fundamental 60Hz Kalman Filter on Power Electronics and Power Systems Applications 401 components of phase voltages and/or currents (amplitude, phase, and frequency) or even based on the fundamental positive sequence components.

4.2.2 POWER CONDITIONERS CONTROL AND SYNCHRONIZATION

The requirement of synchronization of several electronic devices (such as active rectifiers, active power filters, uninterruptible power suppliers, dynamic voltage restorers, distributed generators, etc.) has been motivating the development of different algorithms to detect the amplitude, frequency, and phase angle of the power grid fundamental voltage (Padua et al., 2007; Moreno et al., 2007). Such required information can be provided by the KF output (Padua et al., 2007a; Cardoso, et al., 2007; Huang et al., 2008), as it will be demonstrated in the following.

In the matter of power conditioning, several closed-loop control schemes have been applied in order to control the voltages and currents waveforms, frequency, and amplitudes of an electrical load or point of common coupling (PCC) (Peng, 2001). Many control laws can be used in order to guarantee the voltage/current to track the references, and to compensate

for disturbances, running from classically inspired techniques (Marafão et al., 2008) to those including a KF in the control loop (Moreno et al., 2004; Kwan et al., 2005; Rosendo et al., 2007).

An active rectifier, e.g., should drain a sinusoidal current from the supply system, which should be in-phase with the fundamental component of the grid voltage, even if this one is distorted. This will ensure a high-power factor for the resulting active rectifier. In case of three-phase devices, it is also desired to ensure equal phase currents, it means that the three phase rectifier will act as resistive balanced load. In the matter of distributed generation, the control of different power sources (AC or DC) have been carried out by means of electronic power converters and usually, it depends on some synchronized signal, in order to ensure that the generated voltages have the same frequency and phase angle of the main power grid (Padua et al., 2007). Again, in this case, the required information could be achieved by means of the KF

4.3 PROGRESS REPORT FOR COUNTRIES THAT USE KALMAN FILTERS IN VARIOUS SECTORS

1. Lesotho

This study provides an empirical analysis of the time-varying price and income elasticities of electricity demand in Lesotho (a country in Southern Africa) for the period of 1995-2012 using the Kalman Filter approach. The results reveal that economic growth has been one of the main drivers of electricity consumption in Lesotho while electricity prices are found to play a less significant role since, they are monopoly-driven and relatively low when compared to international standards.

2. Korea

Electric power consumption data can become more beneficial if it is presented to the occupants of the buildings along with prediction of power consumption. This will help the residents change their power consumption behaviour, and thus have a positive impact on the electricity utilization and generation, distribution network and electricity grid operation. We present power consumption and prediction using Kalman filter in Korean homes. We have analysed the power consumption data based on daily, weekly, monthly, and yearly consumption. The aggregated data based on daily, weekly, monthly, and yearly consumption are considered. Then we can predict the maximum and average power consumption for each of the daily, weekly, monthly, and yearly power consumption and these aids the residents in purchasing an apartment relative to the consumption of power in which they deem fit and affordable

CHAPTER FIVE

5.0 CONCLUSION

This chapter provides a summary of the preceding chapter's work. It covers the different problems experienced during our project execution. It also makes recommendations to help the project's future development.

The aim of the project was to give a detailed explanation of the Kalman filter. A brief introduction was provided, along with the history and purpose of invention. The succeeding chapter was dedicated to the resolution of the different types of Kalman filters and their corresponding advantages and disadvantages. Following that, a complete description of the operational principle of the algorithm was provided along with necessary applications and codes. The report also featured the various limitations of the Kalman filter as well as its impact on the economy.

5.1 ACHIEVEMENTS

With the implementation of this project, the following objectives were accomplished;

1. A successful explanation of the workings of the Kalman filter algorithm showing relevant codes and videos.

5.2 CHALLENGES AND LIMITATIONS

In the process of developing this thesis, the following challenges were encountered:

1. Difficulty finding relevant materials that explain the concept in simple terms.
2. Difficulty breaking down complex Kalman filter codes.

5.3 CONTRIBUTORS AND THEIR CONTRIBUTIONS

Olutunde Oluwatimileyin Josiah and Omosebi David Oluwatobi

- What is a Kalman Filter
- History of Invention
- Purpose for Invention (What lead to the Invention?)

Eguwe Oghenemaero Christopher and Olaleye Adeola Paul

- The Various Types of Kalman Filters with their various applications (Uses) and a little on History on them
- How they Vary from each other

- Advantages and Disadvantages of each type

Olonisakin David Akolade, Osondu Ronald Chinedu, Egbanubi Oluwadamilare Victor, Ekere Moses Mfonido-Abasi, Olayeni Richard Olasubomi (08163059687)

- The Kalman Filter algorithm
- The mathematics behind the algorithm
- The MATLAB Code to Implement the Kalman Filter on something
- Various applications of Kalman Filters and various algorithms for them

Olotu Kesioluwa Oghenekome, Enebeli Edmond Igwebuike, Campbell Oluwabiye Oluwasanu, Oni Oluwatomiwa Boluwatife

- The advantages and disadvantages of the Kalman filter when compared to other types of filters
- Limitations of the Kalman filter
- The Impact of Kalman Filters on Economies
- What innovations and inventions have occurred due to the use of Kalman Filters
- How can the Kalman Filters be applied to the Nigerian Power system. what is its significance there
- Alternatives to Kalman Filters
- Limitations the Nigerian Power System faces without the use of Kalman Filters
- Countries that use the Kalman Filters in Their Power systems and their significant advancements/Progress when compared to each other and to Nigeria

Bosah John Chigoziem

- The Conclusion

Babalola Paul Toluwalase and Omonjade Godsfavour Osamudia

- Presentation Slides

Elele Ezinne and Olanipekun Temiloluwa Ajibade

- Report Compilation

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