CS441 Artificial Intelligence

Programming Assignment 1 Name: Zoey Lee

Due: 11/07/17

0.1 Part I: Problem-Solving as Search

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 \begin{aligned} & \text{init1} = [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] \\ & \text{init2} = [[2, 8, 3], [1, 6, 4], ['b', 7, 5]] \\ & \text{init3} = [[1, 2, 3], [7, 8, 4], ['b', 6, 5]] \\ & \text{init4} = [[1, 'b', 3], [8, 2, 4], [7, 6, 5]] \\ & \text{init5} = [[1, 2, 3], [8, 6, 4], [7, 'b', 5]] \\ & \text{goal} = [[1, 2, 3], [8, 'b', 4], [7, 6, 5]] \\ \end{aligned}
```

0.1.1 Best-first Search:

Heuristic 1 (number of misplaced tiles):

```
>> Best-first Search - number of misplaced tiles
[[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], ['b', 1, 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 'b', 3], [1, 8, 4], [7, 6, 5]] -> [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] -> [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]] Goal!
>> Best-first Search - number of misplaced tiles
[[2, 8, 3], [1, 6, 4], ['b', 7, 5]] -> [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], ['b', 1, 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 8, 4], [7, 6, 5]] -> [['b', 2, 3], [1, 8, 4], [7, 6, 5]] -> [[1, 2, 3], [b', 8, 4], [7, 6, 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
Goal!
>> Best-first Search - number of misplaced tiles
[[1, 2, 3], [7, 8, 4], ['b', 6, 5]] -> [[1, 2, 3], [7, 8, 4], [6, 'b', 5]] -> [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] ->
[[1, 2, 3], [8, 'b', 4], [7, 6, 5]] Goal!
>> Best-first Search - number of misplaced tiles
[[1, 'b', 3], [8, 2, 4], [7, 6, 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]] Goal!
>> Best-first Search - number of misplaced tiles
[[1, 2, 3], [8, 6, 4], [7, 'b', 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]] Goal!
>>>>Average steps : 5
    Heuristic 2 (Manhattan):
>> Best-first Search - Manhattan
[[2, 8, 3], [1, 6, 4], ['b', 7, 5]] -> [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] ->
[[1, 2, 3], [8, 'b', 4], [7, 6, 5]] Goal!
>> Best-first Search - Manhattan
[[2, 8, 3], [1, 6, 4], ['b', 7, 5]] -> [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] ->
[[2, 'b', 3], [1, 8, 4], [7, 6, 5]] -> [['b', 2, 3], [1, 8, 4], [7, 6, 5]] -> [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]] Goal!
>> Best-first Search - Manhattan
[[1, 2, 3], [7, 8, 4], ['b', 6, 5]] -> [[1, 2, 3], [7, 8, 4], [6, 'b', 5]] -> [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] ->
[[1, 2, 3], [8, 'b', 4], [7, 6, 5]] Goal!
>> Best-first Search - Manhattan
[['b', 1, 3], [8, 2, 4], [7, 6, 5]] -> [[1, 'b', 3], [8, 2, 4], [7, 6, 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
>> Best-first Search - Manhattan
[[1, 2, 3], [8, 6, 4], ['b', 7, 5]] -> [[1, 2, 3], [8, 6, 4], [7, 'b', 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
Goal!
```

>>>>Average steps : 4

Heuristic 3 (My heuristic):

```
>> Best-first Search - Zoey's heuristic
[[2, 8, 3], [1, 6, 4], ['b', 7, 5]] -> [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 'b', 3], [1, 8, 4], [7, 6, 5]] -> [[2, 3, 'b'], [1, 8, 4], [7, 6, 5]] -> [[2, 3, 4], [1, 8, 'b'], [7, 6, 5]] ->
 [[2, 3, 'b'], [1, 8, 4], [7, 6, 5]] -> [[2, 3, 4], [1, 8, 'b'], [7, 6, 5]] -> [[2, 'b', 3], [1, 8, 4], [7, 6, 5]] -> [['b', 3, 4], [2, 1, 8], [7, 6, 5]] -> [[2, 3, 4], ['b', 1, 8], [7, 6, 5]] -> [[3, 'b', 4], [2, 1, 8], [7, 6, 5]] -> [[3, 4, 'b'], [2, 1, 8], [7, 6, 5]] -> [[3, 1, 4], ['b', 2, 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [2, 'b
[[3, 'b', 4], [2, 1, 8], [7, 6, 5]] -> [[3, 'b', 4], [2, 1, 8], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 4], [2, 8, 'b'], [7, 6, 5]] -> [[3, 1, 4], [2, 8, 'b'], [7, 6, 5]] -> [[3, 1, 4], [2, 8, 4], [7, 6, 5]] -> [[b', 3, 4], [2, 1, 8], [7, 6, 5]] -> [[b', 2, 3], [1, 8, 4], [7, 6, 5]] -> [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] ->
[[1, 2, 3], [8, 'b', 4], [7, 6, 5]] Goal!
>> Best-first Search - Zoey's heuristic
[[2, 8, 3], [1, 6, 4], ['b', 7, 5]] -> [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 'b', 3], [1, 8, 4], [7, 6, 5]] -> [[2, 3, 'b'], [1, 8, 4], [7, 6, 5]] -> [[2, 3, 4], [1, 8, 'b'], [7, 6, 5]] ->
 \left[\left[2,\ 3,\ 4\right],\left[1,\ 'b',\ 8\right],\left[7,\ 6,\ 5\right]\right] \rightarrow \left[\left[2,\ 3,\ 4\right],\left['b',\ 1,\ 8\right],\left[7,\ 6,\ 5\right]\right] \rightarrow \left[\left[2,\ 3,\ 4\right],\left[1,\ 'b',\ 8\right],\left[7,\ 6,\ 5\right]\right] \rightarrow \left[1,\ 'b',\ 8\right]
[[2, 3, 4], [1, b, 6], [7, 6, 5]] -> [[2, 3, 4], [1, 8, 'b'], [7, 6, 5]] -> [[2, 5, 4], [1, b, 6], [7, 6, 5]] -> [[2, 3, 4], [1, 8, 4], [7, 6, 5]] -> [[3, 4, 4], [2, 1, 8], [7, 6, 5]] -> [[2, 3, 4], [2, 1, 8], [7, 6, 5]] -> [[3, 4, 'b'], [2, 1, 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 'b', 8], [7, 6, 5]] -> [[3, 1, 4], [2, 1, 8], [7, 6, 5]] -> [[3, 1, 4], [2, 1, 8], [7, 6, 5]] -> [[3, 1, 4], [2, 1, 8], [7, 6, 5]] -> [[3, 1, 4], [2, 1, 8], [7, 6, 5]] -> [[3, 1, 4], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 4], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 4], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 4], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 4], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 4], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 4], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 4], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [7, 6, 5]] -> [[3, 1, 'b'], [2, 8, 4], [2, 1, 8], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2, 8, 4], [2,
[['b', 3, 4], [2, 1, 8], [7, 6, 5]] \rightarrow [['b', 2, 3], [1, 8, 4], [7, 6, 5]] \rightarrow [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] \rightarrow [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] \rightarrow [[1, 2, 3], ['b', 8, 4], [7, 6, 5]]
[[1, 2, 3], [8, 'b', 4], [7, 6, 5]] Goal!
>> Best-first Search - Zoey's heuristic
[[1, 2, 3], [7, 8, 4], ['b', 6, 5]] -> [[1, 2, 3], [7, 8, 4], [6, 'b', 5]] -> [[1, 2, 3], [7, 'b', 4], [6, 8, 5]] ->
[[1, 2, 3], [7, 4, 'b'], [6, 8, 5]] -> [[1, 2, 3], [7, 'b', 4], [6, 8, 5]] -> [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]] Goal!
>> Best-first Search - Zoey's heuristic
[['b', 1, 3], [8, 2, 4], [7, 6, 5]] -> [[1, 'b', 3], [8, 2, 4], [7, 6, 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
Goal!
>> Best-first Search - Zoey's heuristic
[[1, 2, 3], [8, 6, 4], ['b', 7, 5]] -> [[1, 2, 3], [8, 6, 4], [7, 'b', 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
Goal!
```

>>>>Average steps: 15

0.1.2 A* Search:

>>>>Average steps: 17

Heuristic 1 (number of misplaced tiles):

```
>> A* Search - number of misplaced tiles
 [[2, 8, 3], [1, 6, 4], ['b', 7, 5]] -> [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] ->
[[2, 8, 3], ['b', 6, 4], [1, 7, 5]] -> [[2, 8, 3], ['b', 1, 4], [7, 6, 5]] -> [[2, 'b', 3], [1, 8, 4], [7, 6, 5]] -> [[2, 8, 3], [1, 6, 4], ['b', 7, 5]] -> [[2, 8, 3], [1, 6, 4], ['b', 7, 5]] -> [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 6, 4], [7, 6, 5]] -> [[2, 8, 3], [2, 1, 4], [7, 6, 5]] -> [[2, 8, 3], [2, 1, 4], [7, 6, 5]] -> [[b', 8, 3], [2, 1, 4], [7, 6, 5]] -> [[b', 8, 3], [2, 1, 4], [7, 6, 5]] -> [[b', 2, 3], [1, 8, 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -> [[2, 8, 3], [2, 1, 4], [2, 5]] -
[['b', 8, 3], [2, 6, 4], [1, 7, 5]] -> [[2, 8, 3], [1, 6, 4], ['b', 7, 5]] -> [[2, 8, 3], [7, 1, 4], ['b', 6, 5]] -> [[2, 3, 'b'], [1, 8, 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 6, 'b'], [7, 5, 4]] -> [[2, 8, 'b'], [1, 4, 3], [7, 6, 5]] -> [[2, 8, 3], [1, 4, 5], [7, 6, 'b']] -> [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] -> [[8, 'b', 3], [2, 1, 4], [7, 6, 5]] -> [[2, 8, 3], ['b', 1, 4], [7, 6, 5]] ->
 [[2, 8, 3], [6, 4, 'b'], [1, 7, 5]] -> [[2, 'b', 3], [6, 8, 4], [1, 7, 5]] -> [[8, 'b', 3], [2, 6, 4], [1, 7, 5]] -> [[2, 8, 3], ['b', 6, 4], [1, 7, 5]] -> [[2, 8, 3], ['b', 6, 4], [1, 7, 5]] -> [[2, 'b', 3], [1, 8, 4], [7, 6, 5]] -> [[2, 8, 3], [7, 1, 4], [6, 'b', 5]] -> [[2, 8, 3], ['b', 1, 4], [7, 6, 5]] -> [[2, 3, 4], [1, 8, 'b'], [7, 6, 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
 Goal!
 >> A* Search - number of misplaced tiles
 [[2, 8, 3], [1, 6, 4], ['b', 7, 5]] -> [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] ->
 [[2, 8, 3], ['b', 6, 4], [1, 7, 5]] -> [[2, 8, 3], ['b', 1, 4], [7, 6, 5]] -> [[2, 'b', 3], [1, 8, 4], [7, 6, 5]] ->
[[2, 8, 3], [1, 6, 4], ['b', 7, 5]] -> [[2, 8, 3], [1, 6, 4], [7, 5, 'b']] -> [[2, 8, 3], [1, 4, 'b'], [7, 6, 5]] -> [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 6, 4], [7, 6, 5]] -> [[2, 8, 3], [2, 1, 4], [7, 6, 5]] -> [['b', 8, 3], [2, 1, 4], [7, 6, 5]] -> [['b', 8, 3], [2, 1, 4], [7, 6, 5]] -> [['b', 8, 3], [2, 1, 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5], [2, 5
[['b', 8, 3], [2, 6, 4], [1, 7, 5]] -> [[2, 8, 3], [1, 6, 4], ['b', 7, 5]] -> [[2, 8, 3], [7, 1, 4], ['b', 6, 5]] -> [[2, 3, 'b'], [1, 8, 4], [7, 6, 5]] -> [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 6, 'b'], [7, 5, 4]] -> [[2, 8, 3], [1, 6, 'b'], [7, 5, 4]] -> [[2, 8, 3], [1, 4, 5], [7, 6, 'b']] ->
 [[2, 8, 3], ['b', 6, 4], [1, 7, 5]] -> [[2, 8, 3], [6, 7, 4], [1, 'b', 5]] -> [[2, 'b', 3], [1, 8, 4], [7, 6, 5]] ->
 [[2, 8, 3], [6, 4, 'b'], [1, 7, 5]] -> [[2, 'b', 3], [6, 8, 4], [1, 7, 5]] -> [[8, 'b', 3], [2, 6, 4], [1, 7, 5]] -> [[2, 8, 3], ['b', 6, 4], [1, 7, 5]] -> [[2, 'b', 3], [1, 8, 4], [7, 6, 5]] -> [[2, 8, 3], [7, 1, 4], [6, 'b', 5]] ->
 [[2, 8, 3], ['b', 1, 4], [7, 6, 5]] -> [[2, 3, 4], [1, 8, 'b'], [7, 6, 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
Goal!
>> A* Search - number of misplaced tiles
 [[1, 2, 3], [7, 8, 4], ['b', 6, 5]] -> [[1, 2, 3], [7, 8, 4], [6, 'b', 5]] -> [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] ->
 [[1, 2, 3], [8, 'b', 4], [7, 6, 5]] Goal!
 >> A* Search - number of misplaced tiles
 [['b', 1, 3], [8, 2, 4], [7, 6, 5]] -> [[1, 'b', 3], [8, 2, 4], [7, 6, 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
 >> A* Search - number of misplaced tiles
 [[1, 2, 3], [8, 6, 4], ['b', 7, 5]] -> [[1, 2, 3], [8, 6, 4], [7, 'b', 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
 Goal!
```

Heuristic 2 (Manhattan):

```
>> A* Search - Manhattan
[[2, 8, 3], [1, 6, 4], ['b', 7, 5]] -> [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 'b', 3], [1, 8, 4], [7, 6, 5]] -> [['b', 2, 3], [1, 8, 4], [7, 6, 5]] -> [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] ->
[[1, 2, 3], [8, 'b', 4], [7, 6, 5]] Goal!
>> A* Search - Manhattan
[[2, 8, 3], [1, 6, 4], ['b', 7, 5]] \rightarrow [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] \rightarrow [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] \rightarrow [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] \rightarrow [[2, 8, 3], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4], [1, 5, 4],
[[1, 2, 3], [8, 'b', 4], [7, 6, 5]] Goal!
>> A* Search - Manhattan
[[1, 2, 3], [7, 8, 4], ['b', 6, 5]] -> [[1, 2, 3], [7, 8, 4], [6, 'b', 5]] -> [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] ->
[[1, 2, 3], [8, 'b', 4], [7, 6, 5]] Goal!
>> A* Search - Manhattan
[['b', 1, 3], [8, 2, 4], [7, 6, 5]] -> [[1, 'b', 3], [8, 2, 4], [7, 6, 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
Goal!
>> A* Search - Manhattan
[[1, 2, 3], [8, 6, 4], ['b', 7, 5]] -> [[1, 2, 3], [8, 6, 4], [7, 'b', 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
Goal!
>>>>Average steps: 4
Heuristic 3 (My heuristic):
>> A* Search - Zoey's heuristic
[[2, 8, 3], [1, 6, 4], ['b', 7, 5]] -> [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 'b', 3], [1, 8, 4], [7, 6, 5]] -> [[2, 3, 4], [1, 8, 'b'], [7, 6, 5]] -> [[2, 3, 4], [1, 'b', 8], [7, 6, 5]] -> [[2, 3, 4], [1, 'b', 8], [7, 6, 5]] -> [[2, 3, 4], [1, 'b', 8], [7, 6, 5]] ->
[[2, 3, 'b'], [1, 8, 4], [7, 6, 5]] -> [[2, 3, 4], [1, 8, 'b'], [7, 6, 5]] -> [[2, 'b', 3], [1, 8, 4], [7, 6, 5]] ->
[['b', 2, 3], [1, 8, 4], [7, 6, 5]] -> [[2, 8, 3], ['b', 1, 4], [7, 6, 5]] -> [[b', 3, 4], [2, 1, 8], [7, 6, 5]] -> [[2, 'b', 4], [1, 3, 8], [7, 6, 5]] -> [[2, 3, 4], [b', 1, 8], [7, 6, 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
Goal!
>> A* Search - Zoey's heuristic
[[2, 8, 3], [1, 6, 4], ['b', 7, 5]] -> [[2, 8, 3], [1, 6, 4], [7, 'b', 5]] -> [[2, 8, 3], [1, 'b', 4], [7, 6, 5]] -> [[2, 'b', 3], [1, 8, 4], [7, 6, 5]] -> [[2, 3, 'b'], [1, 8, 4], [7, 6, 5]] -> [[2, 3, 4], [1, 8, 'b'], [7, 6, 5]] ->
[[2, 3, 4], [1, 'b', 8], [7, 6, 5]] -> [[2, 3, 4], ['b', 1, 8], [7, 6, 5]] -> [[2, 3, 4], [1, 'b', 8], [7, 6, 5]] ->
[[2, 3, 4], [1, 8, 4], [7, 6, 5]] -> [[2, 3, 4], [1, 8, 'b'], [7, 6, 5]] -> [[2, 'b', 3], [1, 8, 4], [7, 6, 5]] -> [['b', 2, 3], [1, 8, 4], [7, 6, 5]] -> [[2, 8, 3], ['b', 1, 4], [7, 6, 5]] -> [[2, 1, 8], [7, 6, 5]] -> [[2, 'b', 4], [1, 3, 8], [7, 6, 5]] -> [[2, 3, 4], [1, 'b', 8], [7, 6, 5]] ->
[[2, 8, 3], [1, 'b', 4], [7, 6, 5]] \rightarrow [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] \rightarrow [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
Goal!
>> A* Search - Zoey's heuristic
[[1, 2, 3], [7, 8, 4], ['b', 6, 5]] -> [[1, 2, 3], [7, 8, 4], [6, 'b', 5]] -> [[1, 2, 3], [7, 'b', 4], [6, 8, 5]] ->
[[1, 2, 3], [7, 4, 'b'], [6, 8, 5]] -> [[1, 2, 3], ['b', 8, 4], [7, 6, 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
Goal!
>> A* Search - Zoey's heuristic
[['b', 1, 3], [8, 2, 4], [7, 6, 5]] -> [[1, 'b', 3], [8, 2, 4], [7, 6, 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
Goal!
>> A* Search - Zoey's heuristic
[[1, 2, 3], [8, 6, 4], ['b', 7, 5]] -> [[1, 2, 3], [8, 6, 4], [7, 'b', 5]] -> [[1, 2, 3], [8, 'b', 4], [7, 6, 5]]
Goal!
>>>>Average steps: 10
```

I used three heuristics to solve 8 puzzle in two different algorithms, Best-First Search(BFS) and A* Search. First heuristic was by counting the number of misplaced tiles. When I used this heuristic with BFS, I got 5 as the average steps of 5 trials and I got 17 when I used it with A*.

Then, the next heuristic was Manhattan, which I add the distance from tiles in each state to tiles in goal state. And, as the average steps, I got 4 steps both times when I used this heuristic with BFS and when I used it with A*. From this result, I can observe that Manhattan is slightly more efficient than number of misplaced tiles.

Lastly, I devised a new heuristic, which doesn't seem as bad as I thought it would be. My approach was to number each state, (e.g.(0,0) = 0, (0,1) = 1, ..., (2,2) = 8)), then multiply it by its data if it's not 'b', (e.g.if(1,2) = 4 - - > 5 * 4), and finally add them all together. So, I got two sums, one for the goal state and the other for the current state, and subtracted it. This heuristic didn't lead me to the goal state. When I used this heuristic with BFS, I got 15 steps. And, when I used it with A*, I got 10 steps.

EXTRA: Produce the same data for the 15-puzzle

1. BFS

1)number of misplaced tiles: 13 steps

2)Manhattan: 13 steps 3)my heuristic: 21 steps

2. A*

1)number of misplaced tiles: 22 steps

2)Manhattan: 13 steps3)my heuristic: 17 steps

0.2 Part II: The game of Nim

I wrote a game of Nim playing program using minimax search with alpha-beta pruning of the complete game tree. I adapted the pseudo-code provided in textbook. I realized that depending on which player starts playing first was the winning point in this algorithm. It was interesting to see how it looks for the best way by moving forward and backward in really a short amount of time.

This program is a game playing between to computers basically, but I let users to enter the number of piles, configuration, and preference of which player going first. I limited the number of pile to be between 1 and 5. The configuration basically tells us that the first element is to store all eliminated data, second element is to let us stop the game, and the last element tells us how many full stacks are there. Every turn, the program displays the current pile and which player thinks about which step to win the game. After the game terminates, you will be able to see which player won the game.