Sampling: Cluster sampling

```
$ echo "Data Sciences Institute"
```

Learning Outcomes

How might our study be impacted if we sample entire groups of individuals from our population based on shared characteristics? How do we effectively study a sample selected in this manner?

- Identify benefits of using cluster sampling
- Compute sample statistics for cluster samples
- Design a study using cluster sampling
- Distinguish between different types of cluster sampling, and between cluster sampling and stratified sampling

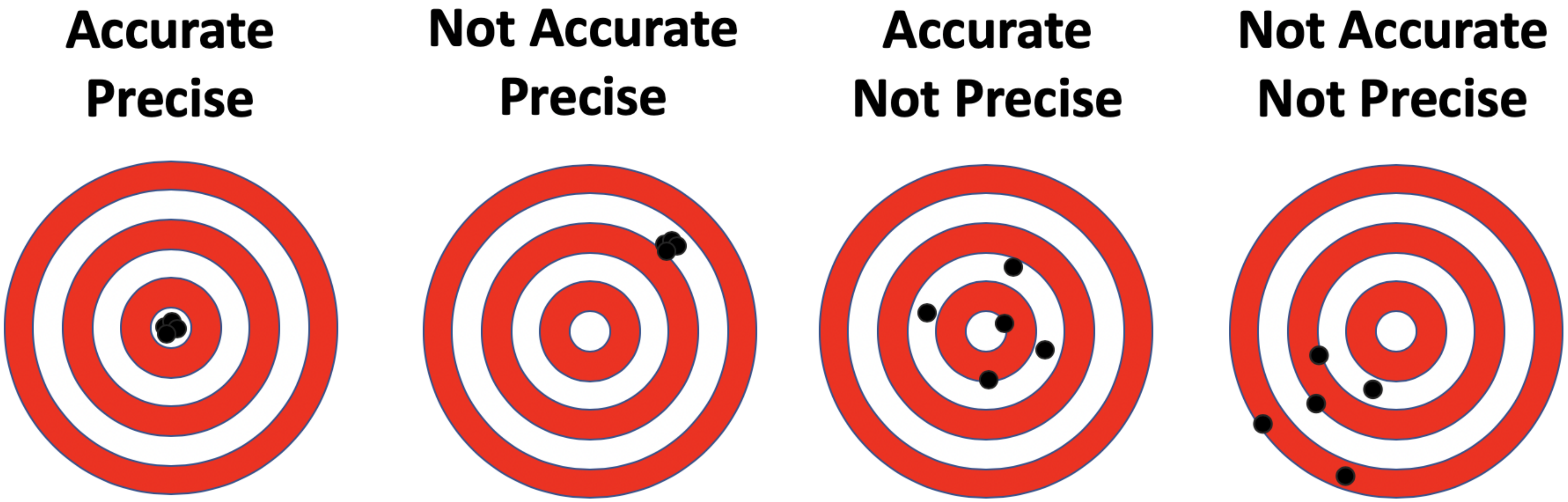
Calculating Sample Size

Recall: Choosing sample size in strata

- Proportional Allocation
  - Sample the same proportion of units from each stratum
  - Sample weights (π_{hi}) are the same for each sampled unit regardless of stratum
- Optimal Allocation
  - Variation among larger sampling units may be greater than variation among smaller sampling units, so a higher proportion of large units should be sampled
  - Useful for businesses, cities, and institutions like schools or hospitals
- Allocation for **Precision** with Strata
  - Sample to reduce the variation in stratum-level estimates, not population-level estimates
  - Useful when the goal is comparing estimates between strata

What is precision?

- **Precision** = How close our measurements are to each other
- **Accuracy** = How close our measurements are to the 'true' value



Calculating Sample Size

1. Determine the desired precision for the quantities that will be estimated from the sample.
    - What are the consequences of the study results? How much error is tolerable?
2. Find an equation relating the sample size n and your desired precision from step 1.
    - Precision should be in terms of error or variation
3. Estimate unknown quantities in the equation and solve for n.

Calculating Sample Size: Takeaways

- We can decide how precise we need our study to be, and by stating that precision in terms of error and confidence, can calculate the needed sample size
- Doing that calculation will require making some assumptions (about the distribution of our population, about the standard deviation) and justifying them

Sample size: Power

- Other than focusing on precision (when we're estimating), it is common to calculate sample size based on power (when we're trying to test a hypothesis)
- **Power**
    - How small of a difference or treatment effect do I need to be able to detect?
    - High power = High chance of my study correctly identifying a true effect (lower chance of false negatives)

Sample size considerations

- Often the initial sample size calculated will be much larger than what is realistic
  - Is the study feasible given the available budget and desired precision?
- Adjust estimates or precision expectations accordingly
  - Larger sample = smaller sampling error, but a larger sample size may increase non-sampling errors

Cluster Sampling

Cluster Sampling

1. Divide the whole population into non-overlapping subpopulations based on shared characteristics. These subpopulations are called **clusters**.
2. Randomly select a sample of clusters.
3. Survey every individual unit within each sampled cluster.

Sampling Units

- **Primary sampling units (PSUs)**
  - Groupings in the first iteration of sampling – in this case, clusters
- **Secondary sampling units (SSUs)**
  - Individuals units who are selected and/or surveyed directly
  - Also known as the observational units
- Observational units are only included in the sample if they belong to the sampled PSU (cluster)

Why use cluster sampling?

- It may be difficult, expensive, or impossible to create a sampling frame of individual (non-clustered) sampling units
  - For example, all birds in a forest or all individuals in a city at a given time
- Population may occur in natural or pre-existing clusters
  - For example, households or schools
  - For geographically widespread populations, sampling by cluster reduces the chance of extensive travel to reach a single individual

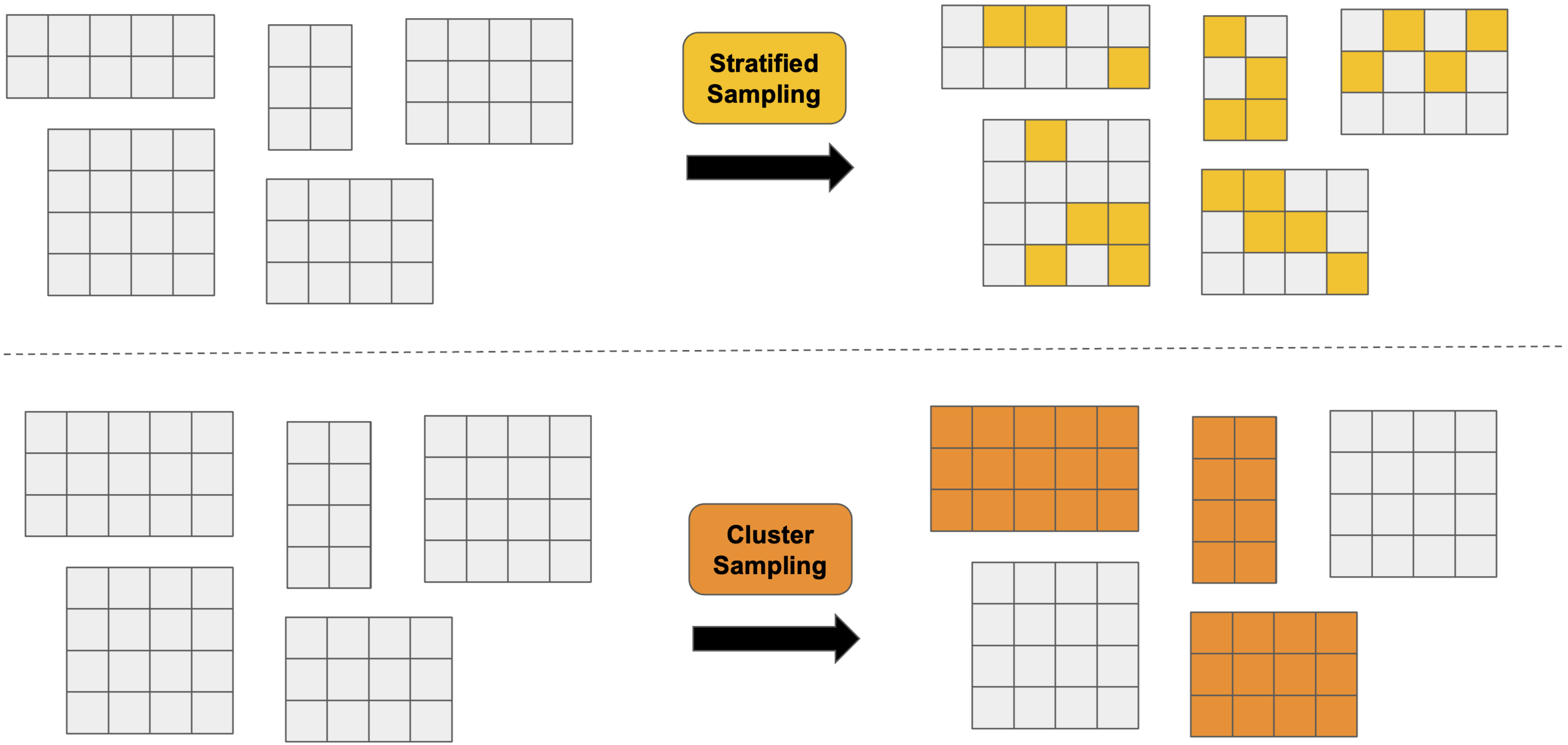
Why *not* use cluster sampling?

- Decreased precision
  - SSUs in each cluster tend to share similar characteristics
  - More difficult to generalize to population-level estimates

Cluster sampling versus stratified sampling

- Both are non-overlapping subpopulations for a given population
- Clusters are often defined for convenience, while strata may be defined to benefit particular types of analysis
- Sampling procedure is different
  - Stratified sampling: define strata → sample within each stratum → survey/observe units in the samples
  - Cluster sampling: define clusters → sample a subset of clusters → survey/observe all units in each sampled cluster

- Based on Lohr, 2019, Figure 5.1



One-Stage Cluster Sampling

One-Stage Cluster Sampling

- A random subset of PSUs (clusters) is sampled, and all SSUs within each sampled PSU are measured
- Used when the cost of measuring SSUs is small compared with the cost of sampling PSUs

Clusters of *Equal* Sizes: Notation

- Let *N* represent the total number of PSUs. Let *n* represent the number of sampled PSUs. Let t_i represent the total for all elements in PSU *i* .
- Let *M* represent the number of people in each cluster. In one stage cluster sampling with samples of equal size, $M = M_i = m_i$ for all *i* .
    - Interpretation: The number of people in each cluster (M_i) is the same for all clusters, and all units from each sampled cluster are measured (m_i)

Clusters of Equal Sizes: PSU Total

- Let y_{ij} represent the measurements from SSU (observational unit) *j* within PSU (cluster) *i* . The total measurement within PSU *i* is,

> $$t_i = \sum_{j=1}^{M} y_{ij}$$

- The total across all PSUs can be estimated with,

> $$\hat{t} = \frac{N}{n} \sum_{i=1}^{n} t_i$$

- This is a weighted sum of the total measurements from each individual cluster.

Clusters of Equal Sizes: Sample Mean

- To estimate the average per SSU (observational unit), divide the estimated total by the total number of SSUs:

> $$\hat{\bar{y}} = \frac{\hat{t}}{NM}$$

- $\hat{\bar{y}}$ is an estimator for the sample mean $\bar{y}$. Since the calculation involves the estimator for the population total, *this is not a direct calculation of the sample mean*.

Clusters of Equal Sizes: Sample Variance

- The sample variance of the PSU totals is,

> $$s_t^2 = \frac{1}{n-1}\sum_{i=1}^{N}(t_i - \frac{\hat{t}}{N})^2$$

- s_t^2 can then be used to compute the standard error of the estimated sample mean:

> $$SE(\hat{\bar{y}}) = \sqrt{\frac{1}{M}\left(1 - \frac{n}{N}\right)\frac{s_t^2}{n}}$$

Clusters of Equal Sizes: Weights

- One-stage cluster sampling with clusters of equal sizes produces a self-weighting sample, with weights,

> $$w_{ij} = \frac{N}{n}$$

- These weights can be used to estimate the sample total and mean directly from SSU measurements y_{ij} :

> $$\hat{t} = \sum_{i=1}^{N} \sum_{j=1}^{M} w_{ij} y_{ij}$$

> $$\hat{\bar{y}} = \frac{\hat{t}}{NM} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} w_{ij} y_{ij}}{\sum_{i=1}^{N} \sum_{j=1}^{M} w_{ij}}$$

Clusters of *Unequal* Sizes: Notation

- The definitions for *n*, *N*, and t_i are the same as previously.
- Let M_i represent the number of people in PSU (cluster) *i* . For clusters of unequal sizes, it is now possible that $M_i \neq M_j$ for $i \neq j$.
- One-stage sampling means that $m_i = M_i$ still (all SSUs in each cluster are sampled).

Clusters of *Unequal* Sizes: SSU Total

- The total number of SSUs in the population is defined as,

> $$M_0 = \sum_{i=1}^{N} M_i$$

- This can be computed directly when the size of every PSU is known. However, this is not always possible. M_0 can thus be estimated:

> $$\hat{M}_0 = \frac{N}{n} \sum_{i=1}^{n} M_i$$

Clusters of *Unequal* Sizes: PSU Total

- The total within each PSU can be estimated nearly the same way as for clusters of equal sizes, with the difference that M_i may be different for different clusters:

> $$t_i = \sum_{j=1}^{M_i} y_{ij}$$

- The total across all PSUs can be estimated with,

> $$\hat{t} = \frac{N}{n} \sum_{i=1}^{n} t_i$$

- This is the same as for clusters of equal sizes.

Clusters of *Unequal* Sizes: Sample Mean

- The sample mean can be calculated using the estimates for *t* and M_0 :

> $$\hat{\bar{y}} = \frac{\hat{t}}{\hat{M}_0} = \frac{\sum_{i=1}^{n} t_i}{\sum_{i=1}^{n} M_i}$$

- This can also be calculated using weights, with the same weights and calculation as for clusters of equal sizes:

> $$\hat{\bar{y}} = \frac{\hat{t}}{\hat{M}_0} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M_i} w_{ij} y_{ij}}{\sum_{i=1}^{N} \sum_{j=1}^{M_i} w_{ij}}$$

Clusters of *Unequal* Sizes: Sample Mean Variance

- The standard error for the sample mean can be estimated as follows,

> $$SE(\hat{\bar{y}}) = \sqrt{(1 - \frac{n}{N}) \frac{1}{n\bar{M}^2} \frac{\sum_{i=1}^{N} M_i^2 (\bar{y}_i - \hat{\bar{y}})^2}{n-1}}$$

- where $\bar{y}_i$ represents the sample mean within PSU *i* and $\bar{M}$ represents the mean number of SSUs in each PSU.

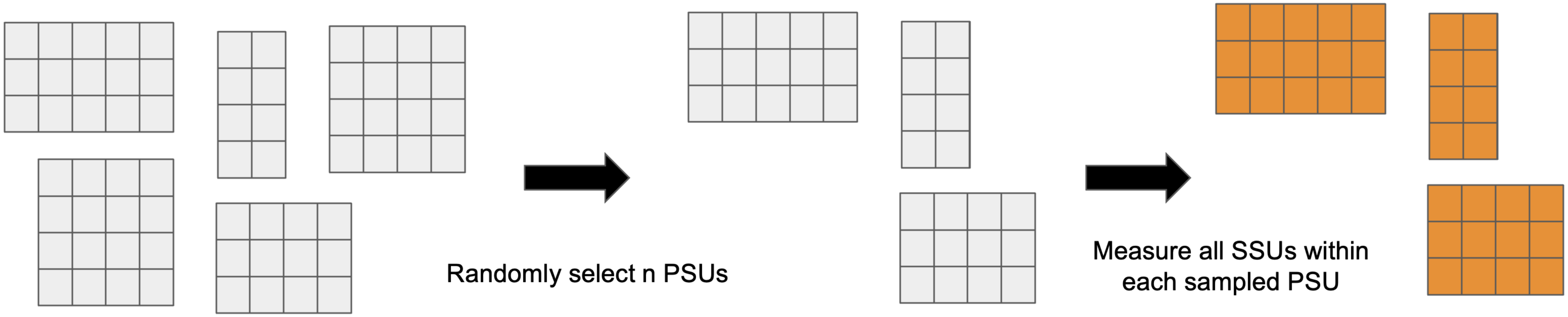
- ⚠️ **Takeaway:** If our clusters are different sizes, the ways that we estimate sample mean, error, and variance change. It matters! ⚠️

Two-Stage Cluster Sampling

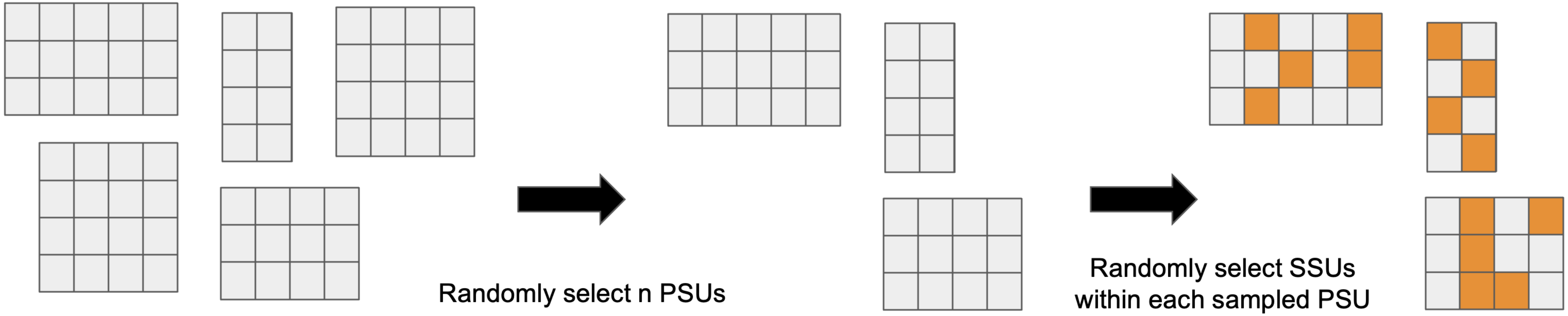
Two-Stage Cluster Sampling

- A random subset of PSUs (clusters) is selected, and then a random sample of the SSUs (observational units) within each PSU is selected for observation.
- Two stage sampling might be used if:
  - The cost of sampling SSUs is relatively high compared with the cost of sampling PSUs
  - Elements in a cluster are very similar to each other

- Based on Lohr, 2019, Figure 5.2



Two-Stage



Two-Stage Cluster Sampling: Selection Probability

- Since sampling is occurring at two different stages now, the selection probability of y_{ij} (the j^{th} SSU in PSU *i*) is a combination of the probability of PSU *i* being selected, and the probability of SSU *j* being selected within PSU *i*. Assuming an SRS is taken at both stages, we have:

> $$\pi_{ij} = P(j^{th} \text{ SSU in } i^{th} \text{ PSU selected})$$
> $$= P(i^{th} \text{ PSU selected}) \cdot P(j^{th} \text{ SSU selected} \mid i^{th} \text{ PSU selected})$$

> $$= \frac{n}{N}\frac{m_i}{M_i}$$

Two-Stage Cluster Sampling: Weights

- As always, the weight of observational unit y_{ij} is the reciprocal of its selection probability.

> $$w_{ij} = \frac{1}{\pi_{ij}} = \frac{NM_i}{nm_i}$$

- If m_i/M_i is approximately constant for all PSUs (i.e. a proportional sample from all clusters), this is considered a self-weighting sample.

Two-Stage Cluster Sampling: Population Total

- The population total can be estimated in the same was as one-stage cluster sampling.

> $$\hat{t} = \sum_{i=1}^{N} \sum_{j=1}^{M_i} w_{ij} y_{ij}$$

Two-Stage Cluster Sampling: Sample Mean

- The sample mean can be calculated using the estimates for t and M_0. This is much the same as in one-stage cluster sampling, except the totals for each PSU must now be estimated as well:

> $$\hat{\bar{y}} = \frac{\hat{t}}{\hat{M}_0} = \frac{\sum_{i=1}^{n} t_i}{\sum_{i=1}^{n} M_i}$$

- This can also be calculated using weights, with the same weights and calculation as for one-stage sampling:

> $$\hat{\bar{y}} = \frac{\hat{t}}{\hat{M}_0} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M_i} w_{ij} y_{ij}}{\sum_{i=1}^{N} \sum_{j=1}^{M_i} w_{ij}}$$

Two-Stage Cluster Sampling: Sample Variance

- With two-stage sampling, there are two types of variance: **between** PSUs, and **within** PSUs. Both calculations follow a familiar structure.
- The variance between PSUs can be calculated,

> $$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(M_i\bar{y}_i - M_i\hat{\bar{y}})^2$$

- The variance within PSU *i* can be calculated,

> $$s_i^2 = \frac{1}{m_i-1}\sum_{j=1}^{m_i}(y_{ij} - \bar{y}_i)^2$$

Two-Stage Cluster Sampling: Estimator Variance

- The estimated variance of the sample mean is also comprised of variance within and between PSUs,

> $$\hat{V}(\hat{\bar{y}}) = \frac{1}{\bar{M}^2}(1 - \frac{n}{N})\frac{s^2}{n} + \frac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2(1 - \frac{m_i}{M_i})\frac{s_i^2}{m_i}$$

- where s^2 and s_i^2 are defined as previous, and $\bar{M}$ is the average PSU size.
- ⚠️ **Takeaway**: Two-stage sampling changes our mean calculations and changes the chances for any individual to be included in our sample ⚠️

Choosing a PSU Size

- Often will come about naturally
  - There may be pre-existing groupings that can be used as clusters
  - Examples: classrooms, farms, stores
- Larger PSU size means larger variability within a PSU
- PSUs that are too large or too small may reduce cost saving benefits of cluster sampling

Choosing a Sub-Sample Size (m_i)

- Cost
  - Is measuring more SSUs marginally expensive or inexpensive?
- Accessibility
  - Do you have access to all SSUs in a given PSU? How difficult is it to measure more SSUs?
- Homogeneity
  - Are all the SSUs in a given PSU relatively similar? How much more information is gained by measuring more SSUs?
- **In general, the same considerations as an SRS apply.**

Choosing a Sample Size (n)

This process is similar to selecting sample sizes for SRS.

1. Determine precision needed.
2. Propose PSU and sub-sample sizes.
3. Calculate the variance that will be achieved.
4. Choose n to achieve desired precision.
5. Iterate until n is realistic given available resources.

Next

Errors