

$\{P \equiv 20 \leq N < 50 \wedge 0 \leq i, j, k < N \wedge i \neq j \wedge j \neq k \wedge i \neq k\}$

fun torneo(Valores[0..N] de real, i: ent, j: ent, k: ent) dev <pos_padre: ent, pos_madre: ent>

si Valores[i] < Valores[j] entonces

$\{P \wedge \text{Valores}[i] < \text{Valores}[j]\}$

si Valores[j] < Valores[k] entonces

$\{P \wedge \text{Valores}[i] < \text{Valores}[j] < \text{Valores}[k]\}$

<pos_padre, pos_madre> := <k, j>

$\{Q\}$

si no

$\{P \wedge \text{Valores}[i] < \text{Valores}[j] \wedge \text{Valores}[j] \geq \text{Valores}[k]\}$

si Valores[i] < Valores[k] entonces

$\{P \wedge \text{Valores}[i] < \text{Valores}[j] \wedge \text{Valores}[j] \geq \text{Valores}[k] \wedge$

$\wedge \text{Valores}[i] < \text{Valores}[k]\}$

<pos_padre, pos_madre> := <j, k>

$\{Q\}$

si no

$\{P \wedge \text{Valores}[i] < \text{Valores}[j] \wedge \text{Valores}[j] \geq \text{Valores}[k] \wedge$

$\wedge \text{Valores}[i] \geq \text{Valores}[k]\}$

<pos_padre, pos_madre> := <j, i>

$\{Q\}$

fsi

fsi

si no

$\{P \wedge \text{Valores}[i] \geq \text{Valores}[j]\}$

si Valores[j] >= Valores[k] entonces

$\{P \wedge \text{Valores}[i] \geq \text{Valores}[j] \geq \text{Valores}[k]\}$

<pos_padre, pos_madre> := <i, j>

$\{Q\}$

si no

$\{P \wedge \text{Valores}[i] \geq \text{Valores}[j] \wedge \text{Valores}[j] < \text{Valores}[k]\}$

si $\text{Valores}[i] < \text{Valores}[k]$ entonces

$\{P \wedge \text{Valores}[i] \geq \text{Valores}[j] \wedge \text{Valores}[j] < \text{Valores}[k] \wedge$
 $\wedge \text{Valores}[i] < \text{Valores}[k]\}$

$\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle k, i \rangle$

$\{Q\}$

si no

$\{P \wedge \text{Valores}[i] \geq \text{Valores}[j] \wedge \text{Valores}[j] < \text{Valores}[k] \wedge$
 $\wedge \text{Valores}[i] \geq \text{Valores}[k]\}$

$\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle i, k \rangle$

$\{Q\}$

fsi

fsi

fsi

ffun

$\{Q \equiv (\exists p, q, r : p, q, r \in \{i, j, k\} \wedge p \neq q \wedge q \neq r \wedge p \neq r : \text{Valores}[p] \geq \text{Valores}[q] \geq \text{Valores}[r] \wedge$
 $\wedge \text{pos_padre} = p \wedge \text{pos_madre} = q)\}$

Verificación formal de la función 'torneo'

(1) $P \Rightarrow \text{def}(\text{Valores}[i] < \text{Valores}[j])$

$\text{def}(\text{Valores}[i] < \text{Valores}[j]) \Leftrightarrow 0 \leq i, j < N$

$P \equiv 20 \leq N < 50 \wedge 0 \leq i, j, k < N \wedge i \neq j \wedge j \neq k \wedge i \neq k \Rightarrow 0 \leq i, j < N$

(2) $P \wedge (\text{Valores}[i] < \text{Valores}[j]) \Rightarrow \text{def}(\text{Valores}[j] < \text{Valores}[k])$

$\text{def}(\text{Valores}[j] < \text{Valores}[k]) \Leftrightarrow 0 \leq j, k < N$

$P \wedge \text{Valores}[i] < \text{Valores}[j] \equiv 20 \leq N < 50 \wedge 0 \leq i, j, k < N \wedge i \neq j \wedge j \neq k \wedge i \neq k \wedge \text{Valores}[i] < \text{Valores}[j] \Rightarrow 0 \leq j, k < N$

(3) $\{P \wedge \text{Valores}[i] < \text{Valores}[j] < \text{Valores}[k]\} \langle \text{pos_padre}, \text{pos_madre} \rangle := \langle k, j \rangle \{Q\}$

Calculamos $\text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle k, j \rangle, Q)$:

$\text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle k, j \rangle, Q) \Leftrightarrow \text{def}(k) \wedge \text{def}(j) \wedge Q_{\text{pos_padre}, \text{pos_madre}}^{k, j} \Leftrightarrow$

$\Leftrightarrow \text{cierto} \wedge \text{cierto} \wedge (\exists p, q, r : p, q, r \in \{i, j, k\} \wedge p \neq q \wedge q \neq r \wedge p \neq r : \text{Valores}[p] \geq \text{Valores}[q] \geq \text{Valores}[r] \wedge k = p \wedge j = q) \Leftrightarrow$

$\Leftrightarrow (\exists r : r \in \{i, j, k\} \wedge k \neq j \wedge j \neq r \wedge k \neq r : \text{Valores}[k] \geq \text{Valores}[j] \geq \text{Valores}[r])$

Por último, demostramos que $(P \wedge \text{Valores}[i] < \text{Valores}[j] < \text{Valores}[k]) \Rightarrow$

$\Rightarrow \text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle k, j \rangle, Q)$:

$P \wedge \text{Valores}[i] < \text{Valores}[j] < \text{Valores}[k] \Rightarrow$

$\Rightarrow 0 \leq i, j, k < N \wedge i \neq j \wedge j \neq k \wedge i \neq k \wedge \text{Valores}[i] < \text{Valores}[j] < \text{Valores}[k] \Rightarrow$

$\Rightarrow \text{Tomando } r = i, \text{ podemos afirmar que } (\exists r : r \in \{i, j, k\} \wedge k \neq j \wedge j \neq r \wedge k \neq r : \text{Valores}[k] \geq \text{Valores}[j] \geq \text{Valores}[r])$

(4) $P \wedge (\text{Valores}[i] < \text{Valores}[j]) \wedge (\text{Valores}[j] \geq \text{Valores}[k]) \Rightarrow \text{def}(\text{Valores}[i] < \text{Valores}[k])$

$\text{def}(\text{Valores}[i] < \text{Valores}[k]) \Leftrightarrow 0 \leq i, k < N$

$P \wedge \text{Valores}[i] < \text{Valores}[j] \wedge \text{Valores}[j] \geq \text{Valores}[k] \equiv 20 \leq N < 50 \wedge 0 \leq i, j, k < N \wedge i \neq j \wedge j \neq k \wedge i \neq k \wedge \text{Valores}[i] < \text{Valores}[j] \wedge \text{Valores}[j] \geq \text{Valores}[k] \Rightarrow 0 \leq i, k < N$

(5) $\{P \wedge \text{Valores}[i] < \text{Valores}[j] \wedge \text{Valores}[j] \geq \text{Valores}[k] \wedge \text{Valores}[i] < \text{Valores}[k]\} \langle \text{pos_padre}, \text{pos_madre} \rangle := \langle j, k \rangle \{Q\}$

Calculamos $\text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle j, k \rangle, Q)$:

$\text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle j, k \rangle, Q) \Leftrightarrow \text{def}(j) \wedge \text{def}(k) \wedge Q_{\text{pos_padre}, \text{pos_madre}}^{j, k} \Leftrightarrow$

$$\Leftrightarrow \text{cierto} \wedge \text{cierto} \wedge (\exists p, q, r : p, q, r \in \{i, j, k\} \wedge p \neq q \wedge q \neq r \wedge p \neq r : \text{Valores}[p] \geq \text{Valores}[q] \geq \text{Valores}[r] \wedge j = p \wedge k = q) \Leftrightarrow$$

$$\Leftrightarrow (\exists r : r \in \{i, j, k\} \wedge j \neq k \wedge k \neq r \wedge j \neq r : \text{Valores}[j] \geq \text{Valores}[k] \geq \text{Valores}[r])$$

Por último, demostramos que $(P \wedge \text{Valores}[i] < \text{Valores}[j] \wedge \text{Valores}[j] \geq \text{Valores}[k] \wedge \text{Valores}[i] < \text{Valores}[k]) \Rightarrow \text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle j, k \rangle, Q)$:

$$P \wedge \text{Valores}[i] < \text{Valores}[j] \wedge \text{Valores}[j] \geq \text{Valores}[k] \wedge \text{Valores}[i] < \text{Valores}[k] \Rightarrow$$

$$\Rightarrow 0 \leq i, j, k < N \wedge i \neq j \wedge j \neq k \wedge i \neq k \wedge \text{Valores}[i] < \text{Valores}[j] \wedge \text{Valores}[j] \geq \text{Valores}[k] \wedge \text{Valores}[i] < \text{Valores}[k] \Rightarrow$$

$$\Rightarrow \text{Tomando } r = i, \text{ podemos afirmar que } (\exists r : r \in \{i, j, k\} \wedge j \neq k \wedge k \neq r \wedge j \neq r : \text{Valores}[j] \geq \text{Valores}[k] \geq \text{Valores}[r])$$

(6) $\{P \wedge \text{Valores}[i] < \text{Valores}[j] \wedge \text{Valores}[j] \geq \text{Valores}[k] \wedge \text{Valores}[i] \geq \text{Valores}[k]\} \langle \text{pos_padre}, \text{pos_madre} \rangle := \langle j, i \rangle \{Q\}$

Calculamos $\text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle j, i \rangle, Q)$:

$$\text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle j, i \rangle, Q) \Leftrightarrow \text{def}(j) \wedge \text{def}(i) \wedge Q_{\text{pos_padre}, \text{pos_madre}}^{j, i} \Leftrightarrow$$

$$\Leftrightarrow \text{cierto} \wedge \text{cierto} \wedge (\exists p, q, r : p, q, r \in \{i, j, k\} \wedge p \neq q \wedge q \neq r \wedge p \neq r : \text{Valores}[p] \geq \text{Valores}[q] \geq \text{Valores}[r] \wedge j = p \wedge i = q) \Leftrightarrow$$

$$\Leftrightarrow (\exists r : r \in \{i, j, k\} \wedge j \neq i \wedge i \neq r \wedge j \neq r : \text{Valores}[j] \geq \text{Valores}[i] \geq \text{Valores}[r])$$

Por último, demostramos que $(P \wedge \text{Valores}[i] < \text{Valores}[j] \wedge \text{Valores}[j] \geq \text{Valores}[k] \wedge \text{Valores}[i] \geq \text{Valores}[k]) \Rightarrow \text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle j, i \rangle, Q)$:

$$P \wedge \text{Valores}[i] < \text{Valores}[j] \wedge \text{Valores}[j] \geq \text{Valores}[k] \wedge \text{Valores}[i] \geq \text{Valores}[k] \Rightarrow$$

$$\Rightarrow 0 \leq i, j, k < N \wedge i \neq j \wedge j \neq k \wedge i \neq k \wedge \text{Valores}[i] < \text{Valores}[j] \wedge \text{Valores}[j] \geq \text{Valores}[k] \wedge \text{Valores}[i] \geq \text{Valores}[k] \Rightarrow$$

$$\Rightarrow \text{Tomando } r = k, \text{ podemos afirmar que } (\exists r : r \in \{i, j, k\} \wedge j \neq i \wedge i \neq r \wedge j \neq r : \text{Valores}[j] \geq \text{Valores}[i] \geq \text{Valores}[r])$$

(7) $P \wedge (\text{Valores}[i] \geq \text{Valores}[j]) \Rightarrow \text{def}(\text{Valores}[j] \geq \text{Valores}[k])$

$$\text{def}(\text{Valores}[j] \geq \text{Valores}[k]) \Leftrightarrow 0 \leq j, k < N$$

$$P \wedge \text{Valores}[i] \geq \text{Valores}[j] \equiv 20 \leq N < 50 \wedge 0 \leq i, j, k < N \wedge i \neq j \wedge j \neq k \wedge i \neq k \wedge \text{Valores}[i] \geq \text{Valores}[j] \Rightarrow 0 \leq j, k < N$$

(8) $\{P \wedge \text{Valores}[i] \geq \text{Valores}[j] \geq \text{Valores}[k]\} \langle \text{pos_padre}, \text{pos_madre} \rangle := \langle i, j \rangle \{Q\}$

Calculamos $\text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle i, j \rangle, Q)$:

$$\text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle i, j \rangle, Q) \Leftrightarrow \text{def}(i) \wedge \text{def}(j) \wedge Q_{\text{pos_padre}, \text{pos_madre}}^{i,j} \Leftrightarrow$$

$$\Leftrightarrow \text{cierto} \wedge \text{cierto} \wedge (\exists p, q, r : p, q, r \in \{i, j, k\} \wedge p \neq q \wedge q \neq r \wedge p \neq r : \text{Valores}[p] \geq \text{Valores}[q] \geq \text{Valores}[r] \wedge i = p \wedge j = q) \Leftrightarrow$$

$$\Leftrightarrow (\exists r : r \in \{i, j, k\} \wedge i \neq j \wedge j \neq r \wedge i \neq r : \text{Valores}[i] \geq \text{Valores}[j] \geq \text{Valores}[r])$$

Por último, demostramos que $(P \wedge \text{Valores}[i] \geq \text{Valores}[j] \geq \text{Valores}[k]) \Rightarrow$

$\Rightarrow \text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle i, j \rangle, Q)$:

$$P \wedge \text{Valores}[i] \geq \text{Valores}[j] \geq \text{Valores}[k] \Rightarrow$$

$$\Rightarrow 0 \leq i, j, k < N \wedge i \neq j \wedge j \neq k \wedge i \neq k \wedge \text{Valores}[i] \geq \text{Valores}[j] \geq \text{Valores}[k] \Rightarrow$$

$$\Rightarrow \text{Tomando } r = k, \text{ podemos afirmar que } (\exists r : r \in \{i, j, k\} \wedge i \neq j \wedge j \neq r \wedge i \neq r : \text{Valores}[i] \geq \text{Valores}[j] \geq \text{Valores}[r])$$

(9) $P \wedge (\text{Valores}[i] \geq \text{Valores}[j]) \wedge (\text{Valores}[j] < \text{Valores}[k]) \Rightarrow \text{def}(\text{Valores}[i] < \text{Valores}[k])$

$$\text{def}(\text{Valores}[i] < \text{Valores}[k]) \Leftrightarrow 0 \leq i, k < N$$

$$P \wedge \text{Valores}[i] \geq \text{Valores}[j] \wedge \text{Valores}[j] < \text{Valores}[k] \equiv 20 \leq N < 50 \wedge 0 \leq i, j, k < N \wedge i \neq j \wedge j \neq k \wedge i \neq k \wedge \text{Valores}[i] \geq \text{Valores}[j] \wedge \text{Valores}[j] < \text{Valores}[k] \Rightarrow 0 \leq i, k < N$$

(10) $\{P \wedge \text{Valores}[i] \geq \text{Valores}[j] \wedge \text{Valores}[j] < \text{Valores}[k] \wedge \text{Valores}[i] < \text{Valores}[k]\} \langle \text{pos_padre}, \text{pos_madre} \rangle := \langle k, i \rangle \{Q\}$

Calculamos $\text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle k, i \rangle, Q)$:

$$\text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle k, i \rangle, Q) \Leftrightarrow \text{def}(k) \wedge \text{def}(i) \wedge Q_{\text{pos_padre}, \text{pos_madre}}^{k,i} \Leftrightarrow$$

$$\Leftrightarrow \text{cierto} \wedge \text{cierto} \wedge (\exists p, q, r : p, q, r \in \{i, j, k\} \wedge p \neq q \wedge q \neq r \wedge p \neq r : \text{Valores}[p] \geq \text{Valores}[q] \geq \text{Valores}[r] \wedge k = p \wedge i = q) \Leftrightarrow$$

$$\Leftrightarrow (\exists r : r \in \{i, j, k\} \wedge k \neq i \wedge i \neq r \wedge k \neq r : \text{Valores}[k] \geq \text{Valores}[i] \geq \text{Valores}[r])$$

Por último, demostramos que $(P \wedge \text{Valores}[i] \geq \text{Valores}[j] \wedge \text{Valores}[j] < \text{Valores}[k] \wedge \text{Valores}[i] < \text{Valores}[k]) \Rightarrow \text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle k, i \rangle, Q)$:

$$P \wedge \text{Valores}[i] \geq \text{Valores}[j] \wedge \text{Valores}[j] < \text{Valores}[k] \wedge \text{Valores}[i] < \text{Valores}[k] \Rightarrow$$

$$\Rightarrow 0 \leq i, j, k < N \wedge i \neq j \wedge j \neq k \wedge i \neq k \wedge \text{Valores}[i] \geq \text{Valores}[j] \wedge \text{Valores}[j] < \text{Valores}[k] \wedge \text{Valores}[i] < \text{Valores}[k] \Rightarrow$$

$$\Rightarrow \text{Tomando } r = j, \text{ podemos afirmar que } (\exists r : r \in \{i, j, k\} \wedge k \neq i \wedge i \neq r \wedge k \neq r : \text{Valores}[k] \geq \text{Valores}[i] \geq \text{Valores}[r])$$

(11) $\{P \wedge \text{Valores}[i] \geq \text{Valores}[j] \wedge \text{Valores}[j] < \text{Valores}[k] \wedge \text{Valores}[i] \geq \text{Valores}[k]\}$
 $\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle i, k \rangle \{Q\}$

Calculamos $\text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle i, k \rangle, Q)$:

$$\text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle i, k \rangle, Q) \Leftrightarrow \text{def}(i) \wedge \text{def}(k) \wedge Q_{\text{pos_padre}, \text{pos_madre}}^{i, k} \Leftrightarrow$$

$$\Leftrightarrow \text{cierto} \wedge \text{cierto} \wedge (\exists p, q, r : p, q, r \in \{i, j, k\} \wedge p \neq q \wedge q \neq r \wedge p \neq r : \text{Valores}[p] \geq \text{Valores}[q] \geq \text{Valores}[r] \wedge i = p \wedge k = q) \Leftrightarrow$$

$$\Leftrightarrow (\exists r : r \in \{i, j, k\} \wedge i \neq k \wedge k \neq r \wedge i \neq r : \text{Valores}[i] \geq \text{Valores}[k] \geq \text{Valores}[r])$$

Por último, demostramos que $(P \wedge \text{Valores}[i] \geq \text{Valores}[j] \wedge \text{Valores}[j] < \text{Valores}[k] \wedge \text{Valores}[i] \geq \text{Valores}[k]) \Rightarrow \text{pmd}(\langle \text{pos_padre}, \text{pos_madre} \rangle := \langle i, k \rangle, Q)$:

$$P \wedge \text{Valores}[i] \geq \text{Valores}[j] \wedge \text{Valores}[j] < \text{Valores}[k] \wedge \text{Valores}[i] \geq \text{Valores}[k] \Rightarrow$$

$$\Rightarrow 0 \leq i, j, k < N \wedge i \neq j \wedge j \neq k \wedge i \neq k \wedge \text{Valores}[i] \geq \text{Valores}[j] \wedge \text{Valores}[j] < \text{Valores}[k] \wedge \text{Valores}[i] \geq \text{Valores}[k] \Rightarrow$$

$$\Rightarrow \text{Tomando } r = j, \text{ podemos afirmar que } (\exists r : r \in \{i, j, k\} \wedge i \neq k \wedge k \neq r \wedge i \neq r : \text{Valores}[i] \geq$$

$$\geq \text{Valores}[k] \geq \text{Valores}[r])$$