Homework 2

Dan Cardall Xiaoyan Mi Tyler Olsen

October 30, 2019

Please complete the following numerical problems by hand.

Problem 1

Let $S = \$100$, $K = \$105$, $r = 8\%$, $T = 0.5$, and $\delta = 0.0$. Let $u = 1.3$, $d = 0.8$, and $n = 1$.

a. What are the premium, Δ, and B for a European call?
$\Delta = \frac{25-0}{100*(1.3-.8)} = .5$
$\beta = (e^{-.08*.5})[\frac{1.3*0-.8*25}{.5}]$ = -38.43
Premium $= .5 * 100 - 38.43 = 11.57$

b. What are the premium, Δ, and B for a European put?
Premium: 12.45
Δ: -.5
β: 62.45

Let $S = \$100$, $K = \$95$, $r = 8\%$, $T = 0.5$, and $\delta = 0.0$. Let $u = 1.3$, $d = 0.8$, and $n = 1$.

Problem 2

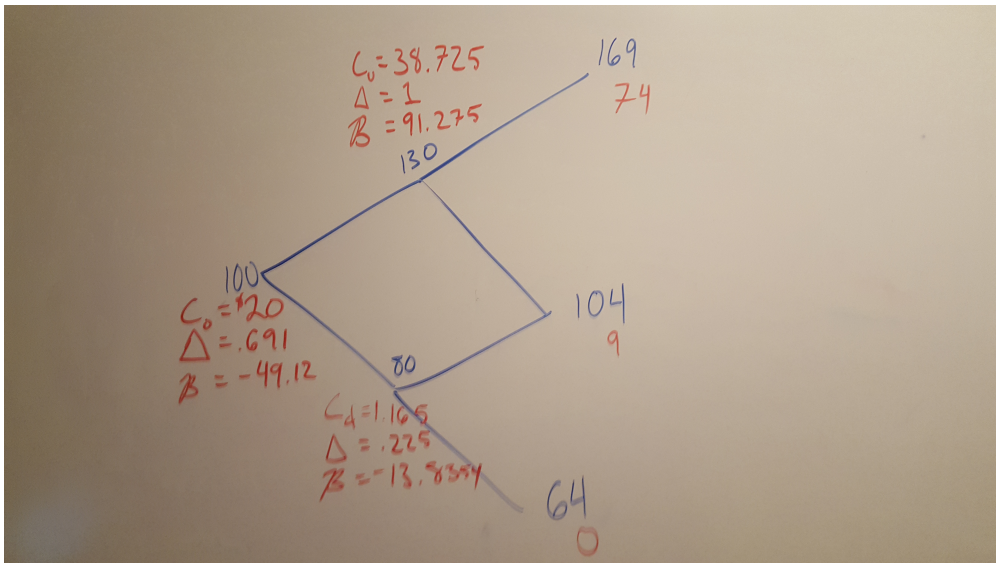
a. Verify that the price of a European put is \$7.471.
Premium: 7.471
Δ: -.3
β: 37.41
The price of a European put is indeed \$7.471.

b. Suppose you observe a call price of \$17. What is the arbitrage?
Since the quoted call price is larger than 16.196 (the call price) you will want to short the option call and long the synthetic.

c. Suppose you observe a call price of \$15.50. What is the arbitrage?
Since the quoted price is less than 16.196, you will want to buy the call, short 7/10 shares and lend 18.462 dollars.

Problem 3

Let $S = \$100$, $K = \$95$, $\sigma = 30\%$, $r = 8\%$, $T = 1$, and $\delta = 0.0$. Let $u = 1.3$, $d = 0.8$, and $n = 2$. Construct the binomial tree for a call option. At each node provide the premium, Δ, and B.



Problem 4

Repeat the option price calculation in the previous question for stock prices of \$80, \$90, \$110, \$120, and \$130, but now let $n = 3$. Keep everything else fixed. What happens to the initial option Δ as the stock price increases?

Solved using Python code:

For price \$80:
Premium: 11.0805
Δ: .4849
β: -27.716

For price \$90:
Premium: 17.19
Δ: .61
β: -37.62

For price \$110:
Premium: 29.42
Δ: .79
β: -57.42

For price \$120:
Premium: 37.35
Δ: .84
β: -63.69

For price \$130:
Premium: 46.58
Δ: .88
β: -67.35

We can see that as the price increases, so does Δ.

Problem 5

Let $S = \$100$, $K = \$95$, $r = 8\%$ (continuously compounded), $\sigma = 30\%$, $\delta = 0$, and $T = 1$ year and $n = 3$.

a. What is the premium for an American call option? Is there any early exercise?

The premium for an American call option is 18.283.

b. What is the premium for a European call option? Use the computational shortcut with the risk-neutral binomial pmf that I showed you in class. Compare the American and European premia.

The premium for an European call option is 18.283

c. What is the premium for a European put? Does put-call parity hold? (see McDonald Chapter 9). Also use the risk-neutral binomial pmf for this problem.

The premium for European put is 5.979. Yes, the put-call parity holds.

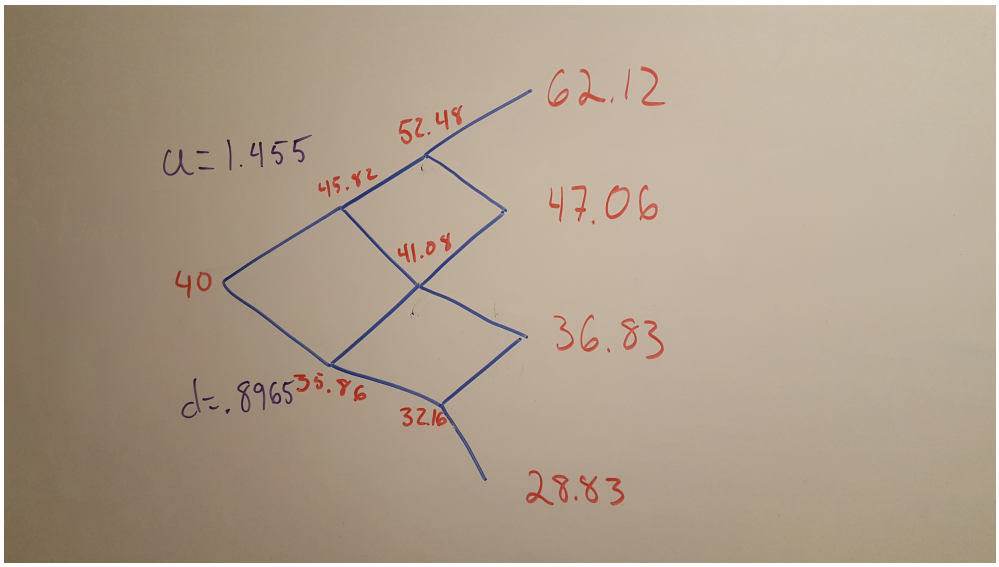
d. What is the premium of the American put? Compare with the European put. If they differ, explain why.

The premium for the American put is 5.98 which is the same as the European put.

Problem 6

Let $S = \$40$, $K = \$40$, $r = 8\%$ (continuously compounded), $\sigma = 30\%$, $\delta = 0.0$, $T = 0.5$ year, and $n = 3$.

a. Construct the binomial tree for the stock. What are u and d?



We can see that u must equal 1.455 and that d must equal .8965

b. Compute the premia of American and European calls and puts.

American Call = 4.38
American Put = 2.81
European Call = 4.38
European Put = 2.81