Chapter 2 Kinematics

2.1, 2.2, 2.3.

- Newtonian mechanics, Kinematics, dynamics.

2.1 Displacement

Position

- Position is a ~~vector~~ scalar quantity
- Is used as a reference.

Displacement.

- Displacement is a vector quantity
- $\Delta x = x_f - x_i$ (m)

Distance

- Scalar quantity
- Total length travelled
- Addition rather than the displacements subtraction.

2.2 Vectors, Scalars, and Coordinate Systems.

- Scalar is just a magnitude, Vector is a magnitude and a direction.
- In one dimension, vectors can be represented by a (+) or (-)
- A simple coordinate system for this is a 4-quadrant graph

2.3 Time, Velocity, and Speed

Time

- Used as an interval and measures change.
- SI unit → seconds (s)
- Elapsed Time = $\Delta t = t_f - t_i$

Velocity

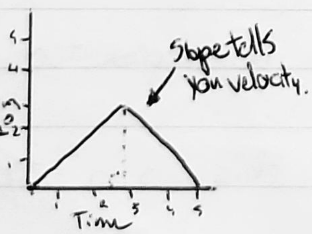
- Velocity is a vector quantity so it is how fast an object travels with a direction.
- Average Velocity = $\frac{\text{Total Displacement}}{\text{Total Time}}$ or $\vec{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$ (m/s)

Speed

- Speed is a scalar quantity so it is how fast an object travels.
- Average Speed = $\frac{\text{Distance}}{\text{Time}}$ or $v = \frac{d}{t}$ (m/s)

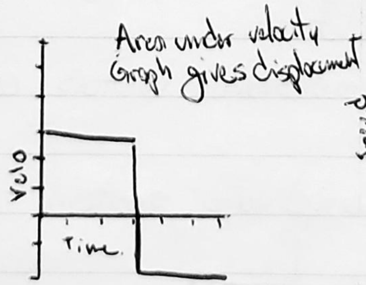
Graphs

- Position-Time



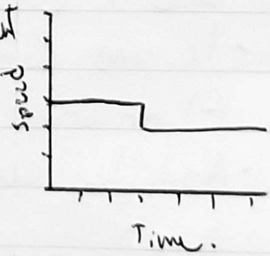
- Comparing Position to time.

- Velocity - Time



- Comparing the velocity of an object to time.

- Speed - Time



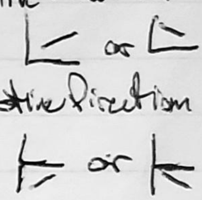
- Comparing the speed of an object to time.

2.4, 2.5

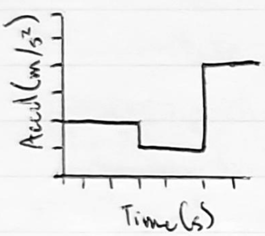
2.4 Acceleration

- Acceleration is the change in velocity in relation to time.
- Since velocity is the measurement, acceleration is a vector quantity but it can also be a scalar.
- Average Acceleration = $\frac{\text{Average Velocity}}{\text{Total Time}}$ or $a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$ (m/s^2)
- When an object slows down, its acceleration is opposite to the direction of motion, This is known as deceleration.
- Acceleration - Time Graph.

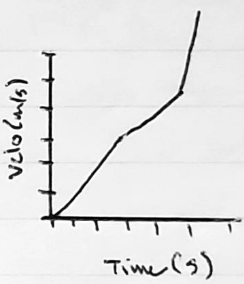
Positive Direction



Negative Direction



← would look like this → on a velocity time graph



2.5 Motion Equations for Constant Acceleration in One Dimension.

- 5 Kinematic Equations.
- t = time; $\vec{v}$ = velocity; $\Delta\vec{v}$ = average velocity; d = distance; $\vec{d}$ = displacement; $\Delta\vec{x}$ = total displacement; a = acceleration,
- 1- $\Delta \vec{d} = \left(\frac{v_f + v_i}{2}\right)t$ → Doesnt have (a)

  2- $v_f = v_i + a_{av}t$ → Doesnt have (Δx)

  3- $\Delta \vec{d} = v_i t + \frac{1}{2} a_{av} t^2$ → Doesn't have (v_f)

  4- $v_f^2 = v_i^2 + 2ad$ → Doesn't have (t)

  5- $\Delta \vec{d} = v_f t - \frac{1}{2} a t^2$ → Doesnt have (v_i)

★ To convert ~~km/h~~ m/s to km/h multiply by 3.6 ★

- $v_{av} = \frac{v_0 + v}{2}$ → finds Average velocity.

Gravity

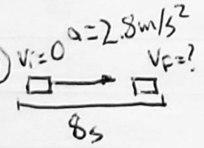
- Objects in freefall experience f the force of gravity.
- Gravity is a force that accelerates an object
- It accelerates at a rate of $9.8\,m/s^2$
- $9.8\,m/s^2$ ~~is the gravitational constant and is~~ is represented by 'g'

2.6 Problem Solving in One Dimensional Kinematics.

Problem Solving Steps.

1- Examine the situation and asses the physical properties involved. Make a sketch of those properties to determine things like direction.
2- Make a list of the information provided. Remember things a rest can be assumed as 0.
3- Identify what needs to be found in the question.
4- Find an equation ~~of~~ or set of equations that will find the answer. Your list will help here. Keep in mind that max the 1 unknown will call for several equations.
5- Substitite the known variables into the appropriate equation and obtain your result with units.
6- Check the magnitude, signs, and directions of your answer to make sure the make sense logically.

• Ex. A car accelerates from rest at a rate of $2.8 m/s^2$ for a time interval of 8s. Determine the final speed of the car in km/h

① 

② $v_i = 0 m/s$
$a = 2.8 m/s^2$
$t = 8s$
$v_f = ?$

③ Need to find v_f
④ $v_f = v_i + at$
⑤ $v_f = 0 + (2.8)(8)$
$v_f = 22.4 m/s$
⑥ ∴, final velocity of the car is 22.4 m/s.

$$v_f = \frac{22.4 m}{s} \times \frac{1 km}{1000 m} \times \frac{3600 s}{1 h}$$

$v_f = 80.64 km/h$

Ex. A car is travelling at 18 m/s when the driver sees the traffic light turn red ahead. The driver reacts at a speed of .62 s. The car slows down at a rate of 5.8 m/s². Determine the stopping distance from when he saw the red light.

$v_f^2 = v_i^2 + 2ad_2$

$v_f^2 - v_i^2 = 2ad_2$

$\frac{v_f^2 - v_i^2}{2a} = d_2$

$\frac{0^2 - 18^2}{2(-5.8)} = d_2$

$27.93\text{ m} = d_2$

$v = \frac{d_2}{t}$

$18(.62) = d$

$11.16\text{m} = d_1$

$\Delta d = d_1 + d_2$

$\Delta d = 11.16 + 27.93$

$\Delta d = 39.09$

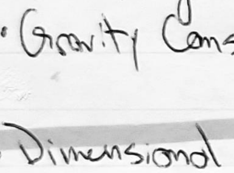
2.7 Falling Objects.

Gravity

- If air resistance and friction are neglible, then all objects fall toward Earth at the same constant acceleration, independent of their mass.
- Heavier objects fall faster than lighter objects because of air resistance and friction
- An object that falls with no air resistance and no friction is said to be in 'free-fall'
- The objects fall toward the center of the Earth due to the force of gravity and this is acceleration due to gravity
- Gravity Constant, g, is $9.8\,m/s^2$.

One Dimensional Motion Involving Gravity.

- When an object is dropped the initial velocity is 0
- When an object reaches max height $v=0$.
- Ex. A person throws a rock straight up a 13m/s and the object falls down the cliff. Calculate the position at 1s, 2s, & 3s.



$v_i = 13\,m/s$

$a = -9.8\,m/s^2$

(1s) $\Delta d = v_i t + \frac{1}{2}at^2$

$\Delta d = (13)(1) + \frac{1}{2}(-9.8)(1)^2$

$\Delta d = 13 + 4.9$

$\Delta d = 8.1\,m$

(2s) $\Delta d = v_i t + \frac{1}{2}at^2$

$= 13(2) + \frac{1}{2}9.8(2)^2$

$= 26 + 19.6$

$\Delta d = 6.4\,m$

(3s) $\Delta d = v_i t - \frac{1}{2}at^2$

$= (13)(3) + \frac{1}{2}(-9.8)(3)^2$

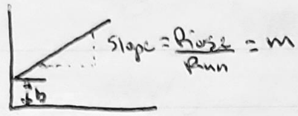
$= 39 + 44.1$

$\Delta d = -5.1\,m$

2.8 Graphical Analysis of One-Dimensional Motion

Slope and General Relationship

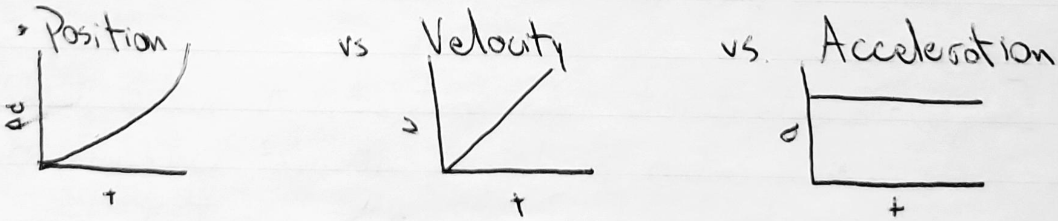
- When 2 physical quantities are plotted on a graph, the horizontal axis is usually considered to be the independent variable and the vertical axis is the dependent variable.
- $y = mx + b$



Graphs of Displacement vs. Time

- Time is almost always the independent (x) variable while displacement depends on time so it is the dependent (y) variable.
- When displacement / velocity is constant, then acceleration is 0
- The equation $d = d_0 + vt$ come from these graphs
- Rearranged $d = d_0 + vt$ look like $d = vt + d$ or $y = mx + b$.

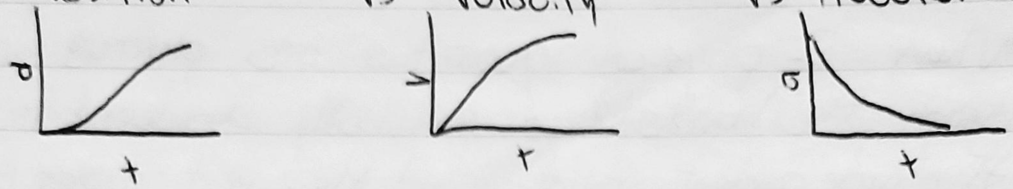
Graphs of Motion when 'a' is Constant



- All 3 graphs show the same thing
- In the velocity vs time graph the line is straight which tells use that slope can be taken to find the constant acceleration.
- $v = v_0 + at$ is then derived.

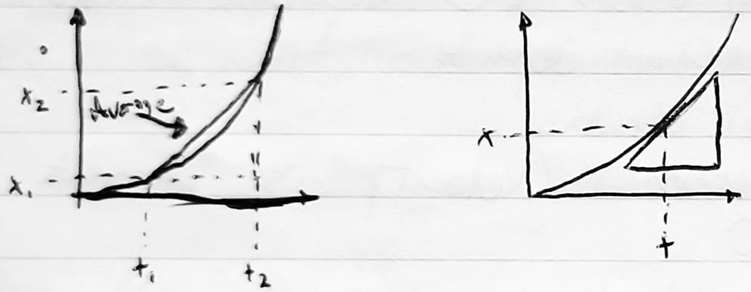
Graphs of Motion Where Acceleration is Not Constant

- Position vs Velocity vs Acceleration



- All 3 graph show the object slowing down.
- The equations will be inaccurate in the calculations be we are calculating an average rather than instantaneous rates.

Instanteous vs. Average



Average Instantaneous.

- To make a curved graph, make small tangent lines and join the lines.

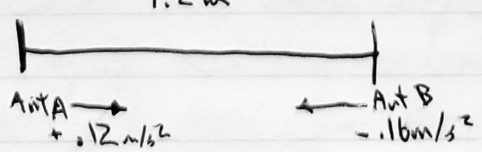
Example - 2 ants at rest 1.2m apart. Ant A acceles at .12m/s² and ant b acceles at .16 m/s². Determine the meeting position interms of on ant a.

$\Delta d = 1.2$

$a_a = .12 m/s^2$

$a_b = .16 m/s^2$

$d_{2a} = d_{2b}$



Glossary

- Acceleration - the rate of change in velocity; the change in velocity over time.
- Acceleration due to gravity - Acceleration of an object as a result of gravity
- Average Acceleration - the change in velocity divided by time over which it changes.
- Average Speed - distance travelled divided by time over which motion occurs (scalar)
- Average Velocity - displacement divided by time over which displacement occurs (vector)
- Deceleration - acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity.
- Dependent Variable - the variable measured; usually plotted on the y-axis
- Displacement - the change in position of an object
- Distance - the magnitude of displacement between 2 points
- Distance Travelled - the total length of path traveled between 2 points
- Elapsed Time - the difference of the ending time and the begining time.
- Free-Fall - the state of movement that results from gravitational force only
- Independent Variable - the variable

3.1 Introduction to Kinematics: Two Dimensions.

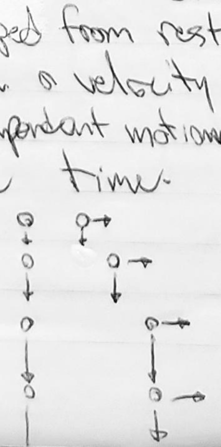
- In one-dimensional Kinematics, arrows were used to denote vectors.
- In two-dimensional Kinematics, an magnitude can have 3 parts: horizontal vector, vertical vector, and average vector.
- The addition of the vertical and horizontal vectors (vector addition - tip to tail) equate to the average vector

Vector can be represented by its components $\vec{v}=(4,5)$

- In most cases pythagoreus' theorm could be used to solve for average velocity.

The Independence of Perpendicular Motions.

- When a person walks eastward, their position can only move when there is movement in that plane.
- Vice Versa when you walk northward, your position in that plane is only affected by movement in the southern or northern direction
- Independence of Motion - The horizontal and vertical components of two-dimensional are independent of each other; Any motion in the vertical direction doesn't affect the horizontal and vice versa
- Ex. 2 balls are dropped down a cliff. 1 is dropped from rest and the other is thrown with a velocity of 10m/s. For the reason of independant motion, both will hit the ground at the same time.



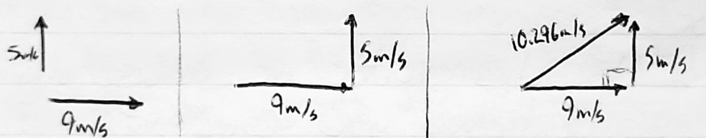
- Analyzing a problem where the path is curved (horizontal and vertical) component, is called projectile motion

3.2 Vector Addition and Subtraction: Graphical Methods.

Vectors in Two Dimensions

- A vector is a quantity with a magnitude and a direction
- In one dimensional motion this could be represented by a plus or minus
- In two dimensions however, vectors will need reference frames (coordinate systems)

Vector Addition: Tip-to-Tail

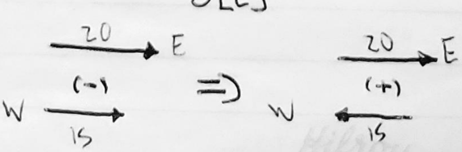
- This is a graphical way to add vectors.
- The tip of one vector lines up with the tail of the next vector
- 

$$V_{av} = \sqrt{9^2+5^2}$$
$$= 10.296$$

- If the vector make a right angle, pythagorean theorem can be used and so can the trigonometric properties.

Vector Subtraction

- Vector subtraction is adding the negative of a vector.
- $A - B = A + (-B)$
- The substraction changes the direction of the vector and leaves the magnitude alone.
- $20[E] - 15[E] = 20[E] + 15[W]$
  $= 5[E]$



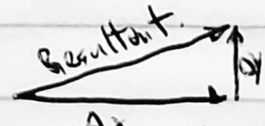
Multiplication of Vectors and Scalars.

- When a vector 'x' is multiplied by the scalar 'z'
  - the magnitude of the vector becomes the absolute value of x.
  - if 'z', the scalar, is positive, the direction of the vector doesnt change
  - if 'z', the scalar, is negative, the direction is reversed

Resolving a Vector into Components.

- When we add vectors we achieve a resultant vector
- To get to this vector or reverse engineer the resultant, we use components.
- These are the x and y components
- $\Delta x = x_1 + x_2 + x_3 \dots$

  $\Delta y = y_1 + y_2 + y_3 \dots$



- You can also use the coordinates to add vectors. $\vec{A} + \vec{B} = (x_A + x_B, y_A + y_B)$
- Ex. $\vec{A} = (2,4)$

  $\vec{B} = (3,-2)$ $\quad\quad \vec{A} + \vec{B} = (2+3, 4+(-2))$

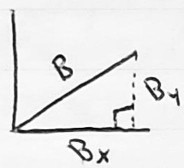
  $\vec{A} + \vec{B} = (5, 2)$

3.3 Vector Addition and Subtraction: Analytical Methods.

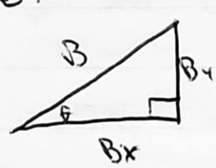
- Analytical methods of vector addition use geometry and simple trigonometry

Resolving a Vector into Perpendicular Components.

- Right angle triangles are a necessity in analytical methods
- for example, vector B is made up of the components B_x and B_y. These make a right angle triangle.

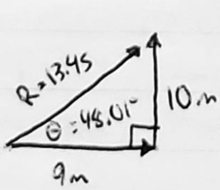


- Now that we have this we see that the pythagorean can be used to find missing components/variables.
- We can also see that trigonometric properties can be used here.



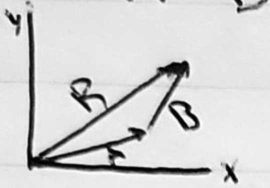
$$B_x = B\cos\theta \qquad B = \sqrt{B_x^2 + B_y^2}$$
$$B_y = B\sin\theta \qquad = \frac{B_y}{\sin\theta}$$
$$= \frac{B_x}{\cos\theta}$$

- Angle θ can also be found in these cases aswell
- Ex. You walked 9 meters right an 10 meters up. Calculate the resultant vector and the angle at which it points.

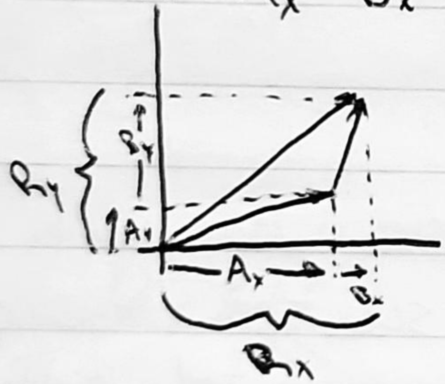


$$R = \sqrt{9^2 + 10^2} \qquad \tan\theta = \frac{10}{9}$$
$$R = 13.45 \qquad \theta = \tan^{-1}\left(\frac{10}{9}\right)$$
$$\theta = 48.01°$$

Adding Vectors Using Analytical Methods

- Vectors A+B equate to the resultant R.
- 

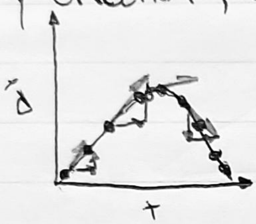
- A and B represent 2 different displacements and R is the total displacement.
- We know that if we have R_x and R_y we can find R and θ.
- To do this we need to break A & B up into components
- $R_x = A_x + B_x$ & $R_y = A_y + B_y$



- R can then be solved by the formula $R = \sqrt{R_x^2 + R_y^2}$, and θ can be solved by $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

3.4 Projectile Motion

- Projectile Motion - the motion of an object or projected into the air which is only subjected to gravity
  - The object is a projectile and its motion is called the trajectory
- The key to analyzing projectile motion problem is to break up the motion into x and y components.
- It should also be noted that acceleration in the y-direction, a_y, is $-9.8\,m/s^2$.
- Ex



- The red pen displays that velocity is tangent to the motion.
- The components of the velocity can be broken down. (Blue Pen)

- Ex. A firework is shot into the air at 70 m/s and at an angle of 75°. The firework detonates at maximum height. At what height will it explode.

$$v_{fy}^2 = v_{iy}^2 + 2ad$$

$$0^2 = 70^2_{\sin 75} + 2(-9.8)(y - y_0)$$

$$\frac{-4900}{-19.6} = -19.6\,y$$

$$233.6\,m = y$$

- The maximum Range is found at angle $45°$.
- When dealing with projectile motion, make an x and y plane table
-

x	y
$v_x=$	$v_y=$
$d_x=$	$d_y=$
$t=$	$a_y=g$
	$t=$

  - This helps find unknowns and find formulas.

4.1 Developement of Force Concept.

- Dynamics - the study of the forces that cause objects and systems to move.
- Force - A push or push with a magnitude and a direction (vector)
- Recall Free Body Diagrams (FBD)

  -

- If $\vec{F} = 0$, then $\vec{a} = 0$ and velocity is constant.

Four Fundamental Types of Forces

1- Strong Nuclear Force
2- Weak Nuclear Force
3- Electromagnetic Force
4- Gravitational Force

4.2 Newtons First Law of Motion: Inertia.

- Newtons First Law - A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by net external force.

Mass

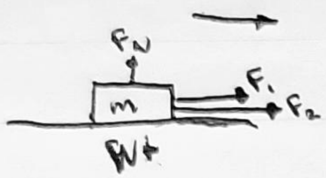
- The inertia of an object is measured by its mass
- Mass is the measure of the amount of 'stuff' (or matter) in something.
- Mass in constant in the universe, while weight changes depending on gravity
- Weight = mg

Reference Frames.

- Your body is not an accurate reference frame
- Overhead / Side Views using imagination are better.

4.3 Newtons Second Law of Motion; Concept of a System

- Newtons Second Law is closely related to his first law
- His second is more quantitative.
- The second law contains the relationship of force, mass, and acceleration.
- The net force causes a change in motion (acceleration) of a mass
- Ex



- Here the normal force (F_N) balance in the y direction.
- In the x direction, Net force is $F_1 + F_2$
- We can gather that acceleration is directly proportional to the net force. - $a \propto F_{net}$ ($\propto$ means directly proportional)
- We find net force by vector addition.
- We can also gather that mass is inversely proportional to the ~~acceleration~~ acceleration - $a \propto \frac{1}{m}$
- $\therefore$ the formula is $a = \frac{F_{net}}{m}$ or $F_{net} = ma$

- Force is measured in N (Newtons) and is broken down in $1N = 1\ Kg \cdot m/s^2$
- Weight can also be found by $W = mg$, where W is the force of the surface and 'g' is the acceleration of gravity.

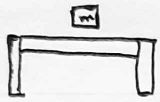
4.4 Newtons Third Law of Motion: Symmetry in Forces.

- Whenever one body exerts a force on a second body, the first experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.
- $F_{A\,on\,B} = -F_{B\,on\,A}$
-

4.5 Normal, Tension, and Other Examples of Force

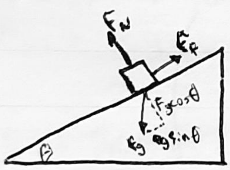
Normal force

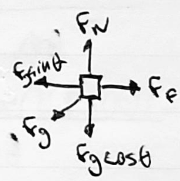
- Weight or force of gravity is a constant force that is always there.
- Some other force must counteract it to keep it from falling.
- This is the normal force.
- Ex. A table sags when a mass is put on it. It acts like a trampoline.



$\Longrightarrow$

- When objects dont move because of normal force, the net force is zero (equilibrium).
- Ex. Incline Planes Diagram.





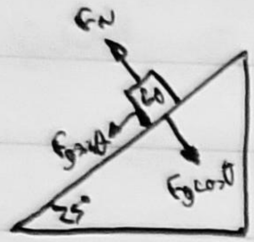
Tension

- Tension is the force along a length of a medium
- Any flexible connector (rope, string, cable) can exert pulls parallel to its length
- Tension a a pull in a connector.
-

- Tension must equal the force of gravity or else it will break
- $T = F_g = W$

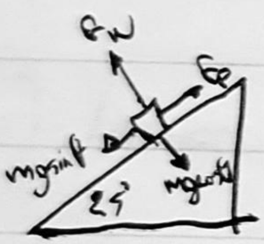
Examples of Normal force.

- A Skier skis down a with mass 60 kg. What is her acceleration if there is no friction? If there is a friction force of 45N?



$$F_{net_x} = ma$$
$$mg\sin\theta = ma$$
$$\frac{\cancel{m}g\sin\theta}{\cancel{m}} = a$$
$$9.8\sin 25 = a$$
$$4.142\ m/s^2$$

without friction

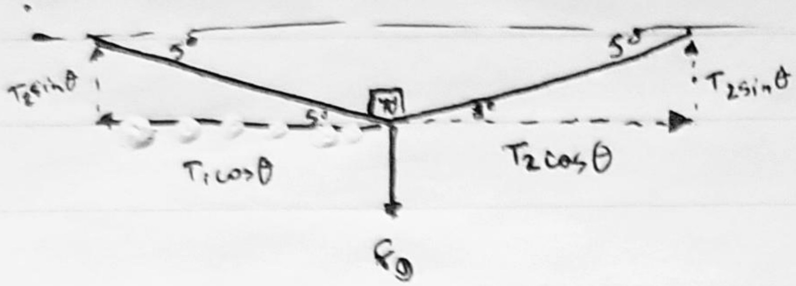


$$F_{net_x} = ma$$
$$mg\sin\theta - F_f = ma$$
$$\frac{mg\sin\theta - F_f}{m} = a$$
$$\frac{(60)(9.8)\sin 25 - 45}{60} = a$$
$$3.392\ m/s^2 = a$$

with friction.

Examples of Tension

- A 70 kg person causes a wire to bend. Calculate the tension in the rope.



$F_{net_x} = ma \rightarrow ma = 0$ cause acceleration is 0

$T_2 \cos\theta - T_1 \cos\theta = 0$

$T_2 \cos\theta = T_1 \cos\theta$

$F_{net_y} = 0$

$T_1 \sin\theta + T_2 \sin\theta - F_g = 0$

$T_1 \sin\theta + T_2 \sin\theta = mg$ → because Tension is equal

$2T\sin\theta = mg$

$$T = \frac{mg}{2\sin\theta}$$

$$T = \frac{70(9.8)}{2\sin 5}$$

$T = 3935.5\,N.$

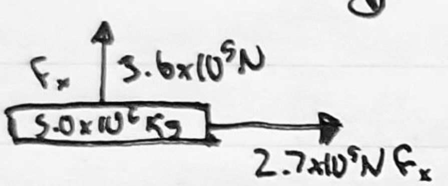
4.6 Problem Solving Strategies

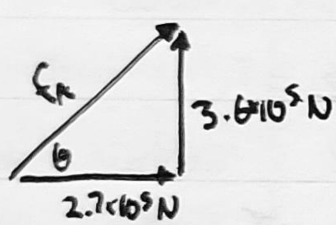
1- Draw a diagram; select a coordinate system
2- Identify relevant objects/agents
3- Sketch an interaction scheme
4- Draw a free body diagram
5- Apply Newtons 2^{nd} Law to each relevant object.

4.7 Further Applications of Newton's Laws of Motion

Drag force

· 2 two tug boats push on a barge. One with a force of 2.7×10^5 N in the x-direction and one with a force of 3.6×10^5 N in the y-direction. If the barges mass is 5.0×10^6 kg and acceleration is 7.5×10^{-2} m/s^2 Find the drag force.



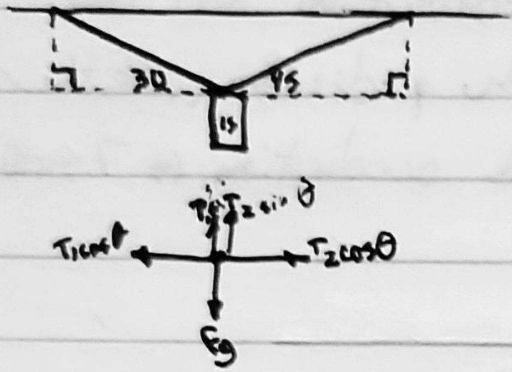


$$F_A = \sqrt{F_x^2 + F_y^2}$$
$$= \sqrt{(2.7\times10^5)^2 + (3.6\times10^5)^2}$$
$$F_A = 4.5\times10^5 \text{ N}$$
$$\theta = \tan^{-1}\left(\frac{3.6\times10^5}{2.7\times10^5}\right)$$
$$\theta = 53.13°$$

$$F_{net} = ma$$
$$F_A - F_{Drag} = ma$$
$$F_A - ma = F_D$$
$$4.5\times10^5 - (5.0\times10^6)(7.5^{-2}) = F_D$$
$$75\,000 \text{ N} = F_D$$

Tension

- A traffic light hangs with a mass of 15 kg. Find the ~~mass~~ tension of the wires.



$$F_{net_y} = 0$$
$$T_1 \sin\theta + T_2 \sin\theta - F_g = 0$$

$$F_{net_x} = 0$$
$$-T_1 \cos\theta + T_2 \cos\theta = 0$$
$$T_2 \cos 45 = T_1 \cos 30$$
$$\frac{T_2 \cos 45}{\cos 30} = T_1$$
$$(.816) T_2 = T_1$$

$$T_1 \sin\theta + T_2 \sin\theta - mg = 0$$
$$((.816) T_2) \sin 30 + T_2 \sin 45 = mg$$
$$.408 T_2 + .707 T_2 = (15)(9.8)$$
$$1.115 T_2 = 147$$
$$T_2 = \frac{147}{1.115}$$
$$T_2 = 131.84 N$$

$$T_1 = (.816) T_2$$
$$T_1 = (.816)(131.84)$$
$$T_1 = 107.58$$

4.8 The 4 Basic forces.

- The fact that there are only 4 forces in physics is one of the most remarkable simplifications.
- Nearly all of the forces we experience are just the electro magnetic force even.
- Action at a distance, like gravity, is the exsistance of a force field rather than contact.
- The 4 are gravitional, electromagnetic, weak nuclear, andstrong nuclear force.
-

Force	Approx Relative Strength	Range	Attraction/Repulsion	Particle
Gravitional	10^{-38}	∞	Attraction only	Graviton
Electromagnetic	10^{-2}	∞	Attraction & repulsion	Photon
Weak nuclear	10^{-13}	$< 10^{-18}$ m	"	W^+, W^-, W^0
Strong nuclear	1	$< 10^{-15}$ m	"	gluons

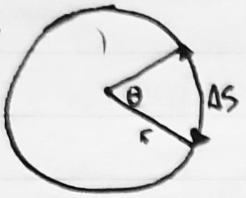
5.1 Friction

- Friction - a force that opposes relative motion between systems in contact.
- There are 2 types of friction, static and kinetic.
- Static is usually greater than kinetic friction
- Magnitude of static friction: $Ff_s \leq \mu_s N$
- An object begins to move when the force of friction is equal or greater than the other side ($\mu_s N$)
- $\therefore f_{fs} = \mu_s N$ but since it is moving $F_{fk} = \mu_k N$.
- Magnitude of kinetic friction: $f_{fk} = \mu_k N$.

5.2 Drag force.

- Drag force is very complicated, like friction, so there is nt a good definition form it.
- The formula for drag force is $D = .3Av^2$, where A is the surface area, v is velocity and D is Drag force.

6.1 Rotation Angle and Angular Velocity

- When an object rotates about an axis it follows a circular arc.
- There is also a rotation angle that is present.
- $\Delta\theta = \frac{\Delta s}{r}$ where θ is the rotation angle, Δs is arc length, and r is radians.
- 

- Since we know that for one revolution the arc length $= 2\pi r$, then $\Delta\theta = \frac{2\pi \cancel{r}}{\cancel{r}}$, $\Delta\theta = 2\pi$.
- And ~~thats the~~ thats the basis for measuring angles (radians)
- 2π radians $= 1$ revolution

Angular Velocity

- Angular velocity is the rate of change of an angle
- $\omega = \frac{\Delta\theta}{\Delta t}$ where $\Delta\theta$ is angular rotation, Δt is time, and ω is the angular velocity
- The units here are rad/s (radians per second)
- Angular velocity is also closely related to linear velocity.
- $v = \frac{\Delta s}{\Delta t} \Rightarrow v\Delta t = \Delta s$, consider $\Delta\theta = \frac{\Delta s}{r} \Rightarrow \Delta\theta = \frac{v\Delta t}{r}$

tangential velocity.

$\Rightarrow \frac{r\Delta\theta}{\Delta t} = v$, since $\omega = \frac{\Delta\theta}{\Delta t}$, $v = r\omega$

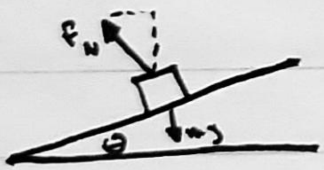
6.2 Centripetal Acceleration

- When an object moves in a circular path its direction is constantly changing
- Since the direction is changing is has centripetal acceleration (a_c)
- Centripetal Acceleration moves around an axis and its direction always faces in to the axis.
- $a_c = \frac{v^2}{r}$, where a_c is centripetal acceleration, v^2 is instantaneous velocity, and r is radius.
- a_c can also be calculated by $a_c = r\omega^2 \left(\frac{(r\omega)^2}{r} = \frac{v^2}{r} \right)$

6.3 Centripetal Force

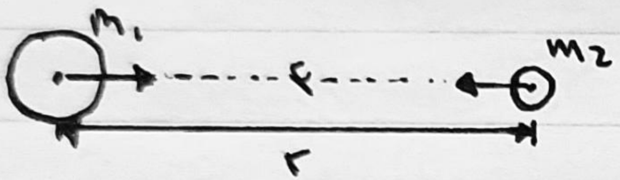
- Any net force dealing with uniform circular motion is called a centripetal force.
- $F_c = ma_c$
- Since we have 2 formulas for a_c, $\frac{v^2}{r}$, $r\omega^2$, we can sub those in
- $F_c = \frac{mv^2}{r}$ or $F_c = mr\omega^2$
- It should be noted when friction is applied, the force of friction points toward the center as well.

Banked Curves

- For ideal banking, the net force is equal to the horizontal centripetal force.
- 

- These cases are similar to inclined planes.

6.8 Newton's Universal Law of Gravitation

- Gravity is a simple force, it always attracts and on mass and the space between them
- Newton's Universal Law of Gravitation states that every particle in the universe attracts every other particle with a force joining them
- 

- r measures to the center of mass (CM)
- The formula of Gravitational force = $F = \frac{Gm_1m_2}{r^2}$
- Where G is the gravitational constant $6.674 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$, m_1 & m_2 are the masses, and r is the distance between them

6.6 Satellites and Kepler's Laws: An Argument for Simplicity

- Orbits have 2 very important characteristics.
  1. A small mass, m, orbits a much larger mass M. This allows us to view the motion as if M were stationary - in fact, as if from an inertial frame of reference placed on M - without significant error. Mass m is the satellite of M, if the orbit is gravitationally bound
  2. The system is isolated from other masses. This allows us to neglect any small effect due to outside masses.

Kepler's Laws of Planetary Motion

- First Law - The orbit of each planet about the Sun is an ellipse with the Sun at one focus
- Second Law - Each planet moves so that an imaginary line drawn from the Sun sweeps out equal areas in equal times
- Third Law - The ratio of the squares of the periods of any two ~~periods~~ planet about the Sun is equal to the ratio of the cubes of their average distances from the Sun. $\frac{T_1^2}{T_2} = \frac{r_1^3}{r_2^3}$

Other Important Formulas

- $T^2 = \frac{4\pi^2}{GM} r^3$
- $\frac{r^3}{T^2} = \frac{G}{4\pi^2} M$

7.1 Work: The Scientific Definition.

- Work is done when there is a transfer of energy
- Alternatively, a force must be exerted and there must be a displacement in the direction of the force.
- For one dimension work, it is defined as:

$$W = Fd\cos\theta$$

- The units of work are, J, or Joules or Neton meters or N·m or $Kg \cdot \frac{m^2}{s^2}$

- Work-Energy Theorem - $\Delta E = W + Q$

7.2 Kinetic Energy and the Work-Energy Theorem

Net Work and the Work-Energy Theorem

- Where there is net force, there is net work.
- When force changes (not constant) most of the $W = Fd\cos\theta$ will not work.
- In some cases $W = F_{net}\, d \cos\theta$ will work.
- When speed changes we can derive this formula

$$W = F_{net}\, d$$
$$= mad$$
$$= m\left(\frac{v_2^2 - v_1^2}{2\cancel{d}}\right)\cancel{d}$$

$$W_{net} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

- This allows us to see the $KE = \frac{1}{2}mv^2$

7.3 Gravitational Potential Energy

Work Done Against Gravity

- When you do work in the y-axis you are either working against ~~g~~ or with gravity.
- This is also known as "gravitational potential energy"
- This is calculated by $W = fd \Rightarrow W = mgh$
- $PE = mgh$

7.4 Conservative Forces and Potential Energy

Potential Energy and Conservative Forces.

- Work is done by forces and some forces have special characteristics
- A conservative force is one
  - This ~~is~~ where only the starting and ending points of motion matter and the path taken isn't considered.
  - An example would be gravity, or a spring.
- Potential energy - the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable

Potential Energy of a Spring

- For potential energy in a spring, we use Hooke's laws.
- $W_3 = \frac{1}{2}kx^2$, where k is the spring force's constant and x is the displacement of it original position.

Conservation of Mechanical Energy

- When dealing with conservative forces, energy isn't lost.
- ∴, the initial work and energy should equal the final
- This equation then can be used.
  - $W_2 = W_1$

    $\frac{1}{2}mv_2^2 + mgh_2 = \frac{1}{2}mv_1^2 + mgh_1$

    $KE_2 + PE_2 = KE_1 + PE_1$
- When kinetic energy and potential energy is added, this is known as mechanical energy

7.5 Nonconservative Forces

Nonconservative forces and friction

- forces are either conservative or nonconservative.
- A non conservative force is one where work depends on the path taken
- Friction is a nonconservative forces
- Because of its dependence on the path taken, there is no potential energy associated with it.
- Nonconservative forces either add or remove mechanical energy from a system.
- For example energy could be lost as heat

How the Work-Energy Theorem Applies

- Since kinetic energy is equal to the net work we can say $W_{net} = \Delta KE$
- But also since the net work is equal to sum of the conservative and nonconservative forces, we can say $W_{nc} + W_c = \Delta KE$, where W_{nc} is the work done by nonconservative forces and W_c is the work done by conservative forces.
- It is also known that conservative force work, W_c is equal to the loss of potential energy or $-\Delta PE$
- This gives us $W_{nc} - \Delta PE = \Delta KE \Rightarrow W_{nc} = \Delta KE + \Delta PE$
- Now if we were to apply the initial and final aspects we achieve the formula $KE_1 + PE_1 + W_{nc} = KE_2 + PE_2$

7.6 Conservation of Energy

- 'Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same'

Other Forms of Energy than Mechanical Energy

- We consider all other forms of energy in one grouping because of our level in physics
- We denote other energy by, OE,
- This gives us another variable in the equation $KE_1 + PE_1 + W_{nc} + OE_1 = KE_2 + PE_2 + OE_2$

Some Other Forms of Energy

- Some other forms of energy are electrical, chemical, radiant energy, nuclear energy, or thermal energy.

Efficiency

- Efficiency is the output of useful work
- It is calculated by, $EF = \dfrac{\text{useful energy/work output}}{\text{total energy input}} = \dfrac{W_{out}}{E_{in}}$

7.7 Power

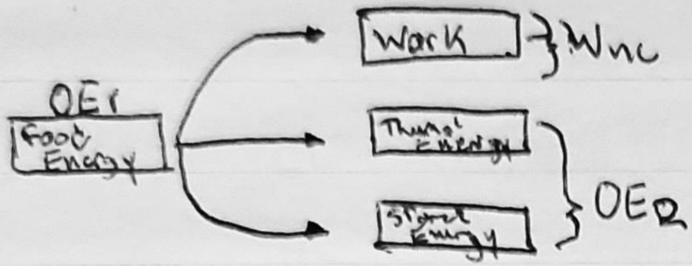
- Power is defined as the rate at which work is done
- Power is calculated by, $P = \frac{W}{t}$, where W is work and t is time.
- The SI units for power are J/s or W (watts)
- An important number is 1 MW which is 1000,000 W.

Power and Energy Consumption

- Energy companies measured energy in KW·h
- This can be found through the power equation
- $P = \frac{W}{t} \Rightarrow Pt = W \Rightarrow Pt = E$, where P is measured in KW and t is in hours.

7.8 Work, Energy, and Power in Humans

Energy Conversion in Humans

- 

$$OE_1 + W_{nc} = OE_2$$

Power Consumed at Rest

- The rate at which the body uses food energy to sustain life and do different activities is called the metabolic rate.

8.1 Linear Momentum and Force

Linear Momentum

- Defined as the product of a system mass multiplied by its velocity.
- Expressed as $p = mv$, where p is known as momentum.

Momentum and Newtons Second Law

- Momentum was once called the 'quantity of motion'
- The net force equals the change in momentum of a system divided by the time over which it changes
- $F_{net} = \frac{\Delta p}{\Delta t}$, F_{net} is net force, Δp is the change in momentum and, Δt is the change in time

8.2 Impulse

- The effect of a force depends on the magnitude of the force and how long it acts.
- Impulse is written $\Delta p = F_{net} \Delta t$
- Impulse can also be found as $mv_2 - mv_1$ by the proof,

$$= F_{net} \Delta t$$

$$= m a \Delta t$$

$$= m\left(\frac{v_2 - v_1}{\cancel{t}}\right)\cancel{\Delta t}$$

$$= m(v_2 - v_1)$$

$$= mv_2 - mv_1$$

8.3 Conservation of Momentum

- Momentum is an important quantity because it is conserved.
- This means that Δp must be constant.
- $\Delta p_1 = -\Delta p_2$

  $\Delta p_1 + \Delta p = 0$
- If this were wrote in the conservation of momentum form, it would look like $P_{TOT} = \text{constant}$

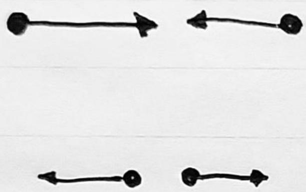
  $P_{tot} = P'_{tot}$
- Isolated system - defined to be one for which the net external force is zero

  - $F_{net} = \frac{P_{\cancel{tot}}}{\Delta t}$

  $= \frac{0}{\Delta t}$

  $F_{net} = 0.$

8.4 Elastic Collisions in One Dimension

- Consider 2 objects moving along a line and they collide.
- When they collide and seperate, they create an elastic collision.
- An elastic collision is where internal kinetic energy is conserved.
- Internal Kinetic energy is the sum of the Kinetic energies of the objects system.
- 

$\Delta p_1 = p_2 + p_1$

$\Delta KE_1 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$\Delta p_2 = p_1' + p_2'$

$\Delta KE_2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$

- $\Delta p_1 = \Delta p_2 \Rightarrow \Delta KE_1 = \Delta KE_2$
- "Hit and split"

Using Momentum & Energy to Find Final Velocities.

- ① $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$ ← momentum

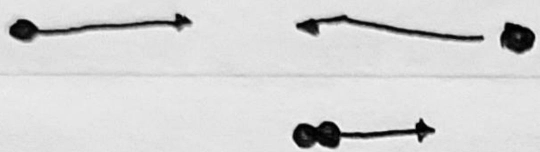
$2\left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2\right)$ ← Kinetic Energy

② $m_1v_1^2 + m_2v_2^2 = m_1v_1'^2 + m_2v_2'^2$ ←

- With the two equations, you can solve equations with two unknowns.

8.5 Inelastic Collisions in One Dimension.

- In elastic collisions, energy is conserved.
- In inelastic collision, energy is not conserved.
- Inelastic Collision - A collision in which internal Kinetic energy changes (not conserved)
- Perfectly Inelastic Collision - "Hit and stick"
-



$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$\frac{1}{2}(m_1+m_2)v_3^2$.

8.6 Collisions of Point Masses in Two Dimensions

- When dealing with a problem in two dimensions x and y components are brought out.
- 

- X: $\Delta p_{1x} = \Delta p_{2x}$

  $m_1 v_{1x} + m_2 v_{2x} = m_1 v_1 \cos\theta_1 + m_1 v_1 \cos\theta_2$

  Y: $\Delta p_{1y} = \Delta p_{2y}$

  $m_1 v_{1y} + m_2 v_{2y} = m_1 v_1 \sin\theta_2 + m_2 v_2 \sin\theta_2$

9.1 The First Condition for Equilibrium

- The first condition to ~~a~~achieve equilibrium is the net force must be 0.
- $F_{net} = 0$
- This means that it must be zero in both the x and y directions.
- There are two types of equilibrium, static (motionless) and dynamic (constant velocity)

9.2 The Second Condition for Equilibrium

- Torque - The second condition to achieve equilibrium involves avoiding accelerated rotation (maintaining constant angular velocity).
- Torque is defined mathematically as $\tau = rF\sin\theta$, where τ is torque, r is distance from the pivot point, ~~and~~, F is the magnitude of the force, and θ is the angle between the force and the vector.
- $\Delta\tau = \tau_1 + \tau_2$

9.3 Stability

- There are 3 types of equilibrium: stable, unstable, and neutral
- Stable Equilibrium - When displaced from equilibrium, it experiences a net force or torque in a direction opposite to displacement.
- Unstable Equilibrium - When displaced from equilibrium, it experiences a net force or torque is the same direction as it was diplaced in.
- Neutral Equilibrium - is where its equilibrium is independent of its displacement from its original position

9.4 Applications of Statics, Including Problem-Solving Strategies

- 1- The first step is to determine whether or not the system is in static equilibrium. This condition is always the case when the acceleration of the system is zero and accelerated rotation does not occur

2- Draw an FBD

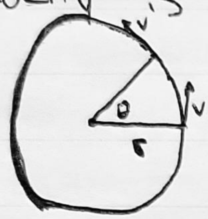
3- Solve the problem by applying either or both of the ~~equation~~ conditions for equilibrium ($F_{net} = 0$ and $\Delta\tau = 0$)

4- Check to see if your answer is reasonable.

10.1 Angular Acceleration

- Uniform Circular Motion and Gravitation used only uniform motion (constant speed)
- The equation for angular velocity is defined as

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- The relation between angular velocity and linear velocity is $v = r\omega$
- 

- When there is no constant velocity the object is accelerating.
- Angular Acceleration looks like $\alpha = \frac{\Delta\omega}{\Delta t}$ where the units are rads/s^2
- If $\Delta\omega$ is increasing, α is positive
- If $\Delta\omega$ is decreasing, α is negative
- There is also tangential acceleration present
- 
- Linear or tangential acceleration is the in on velocities magnitude represented by a_t.
- Centripetal acceleration is the change in direction a_c.
- $a_t = \frac{\Delta v}{\Delta t} \Rightarrow a_t = \frac{\Delta r\omega}{\Delta t} \Rightarrow a_t = r\frac{\Delta\omega}{\Delta t} \Rightarrow a_t = r\alpha$
-

10.2 Kinematics of Rotational Motion

- The kinematics of rotational motion describe relationships among rotation angle, angular velocity, angular acceleration and time.
- Using the equation $v_2 = v_1 + at$, where a is constant we can see that it can be used for rotational motion
- $r\omega_2 = r\omega_1 + r\alpha t \Rightarrow \omega_2 = \omega_1 + \alpha t$
- This can be done for a few of the kinematic equations.
-

Rotational	Translational	
$\theta = \omega t$	$d = vt$	
$\omega_2 = \omega_1 + \alpha t$	$v_2 = v_1 + at$	constant α, a
$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$	$d = v_1 t + \frac{1}{2}at^2$	constant α, a
$\omega_2^2 = \omega_1^2 + 2\alpha\theta$	$v_2^2 = v_1^2 + 2ad$	constant α, a

10.3 Dynamics of Rotational Motion: Rotational Inertia

- Using Newton's 2nd Law, $F_{net} = ma$, and the expression $a = r\alpha$ we can see that. $F_{net} = mr\alpha$ for rotational motion
- $F_{net} = mr\alpha$
- Since we know torque is rF, if we multiply both sides by r, we get

$$rF = mr^2\alpha$$

$$\tau = I\alpha$$

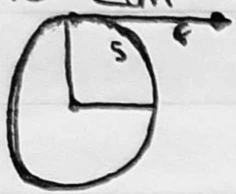
where τ is torque and I is the moment of inertia.

Rotational Inertia and Moment of Inertia.

- I is the sum of all mr^2 for all point masses which is composed of.
- $I = \sum mr^2$
- Seen previously, $\tau_{net} = I\alpha$, or, $\alpha = \frac{\tau_{net}}{I}$

10.4 Rotational Kinetic Energy: Work and Energy Revisited

- Work must be done to rotate objects
- This can be written as $W_{net} = fs$.



- Some algebra will be carried out:

$$W_{net} = fs$$

$$rW_{net} = rfs$$

$$W_{net} = \tau_{net}\frac{s}{r}$$

$$W_{net} = I\alpha\theta$$

- Kinetic rotational energy also has an equation.
- $W_{net} = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2$
- $KE_{rot} = \frac{1}{2}I\omega^2$

10.5 Angular Momentum and Its Conservation

- Every rotational momentum has a direct translational analog
- Angular momentum, L, is defined as $L = I\omega$, where the units are $kg \cdot \frac{m^2}{s}$

- There is also a relationship between torque and angular momentum
- $\tau_{net} = \frac{\Delta L}{\Delta t}$

Conservation of Angular Momentum

- $L = L'$

$I\omega = I'\omega'$