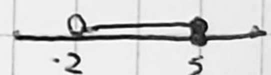
Chapter 1 Fundamentals.

- Natural Number - All whole, positive numbers ($\mathbb{N}$) 1,2,3...
- Integers - All negative, zero, and positive numbers ($\mathbb{Z}$) -2,-1,0,1,2...
- Rational Numbers - A rational number is a ratio of 2 integers ($\frac{m}{n}$) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, n \neq 0$ ($\mathbb{Q}$)
- Irrational Numbers - Cannot be written with 2 integers. π, e
- The set of all real numbers, $\mathbb{R}$, is natural, integer, rational, and irrational numbers.
- 0 is the origin.
- Greater than $>$; Less than $<$
- Set - A collection of objects.
- $\underbrace{A=\{1,2,3,4,5,6,7,8,9,10\}}_{\text{Roster notation.}}$ or $\underbrace{A=\{x \mid x \in \mathbb{N}, 1 \le x \le 10\}}_{\text{Set Builder Notation.}}$

A $C=\{x \mid x \in \mathbb{Z}, 8 \le x \le 15\}$.

- Union of Set A and C = $A \cup C = \{x \mid x \in \mathbb{N}, 1 \le x \le 15\}$.
- Union the set of all elements that belong to set A or C or both.
- Intersection of A and C = $A \cap C = \{x \mid x \in \mathbb{Z}, 8 \le x \le 10\}$
- Intersection is the set of all elements that belong to both A and C.

- Interval Notation - $(-2, 5]$ ↖ Less than or equal to.
  ↑ Greater than

Graphically: 

- $\emptyset$ denotes empty set.

- Absolute Value → Always positive $|x|$

- $D = d(x,y)$ or $|x-y|$ or $|y-x|$

1.2 Exponents and Radicals

- Let $a \in \mathbb{R}$ and $n \in \mathbb{N}$, then $a^n = \underbrace{a \cdot a \cdot a \cdots}_{n \text{ factors}}$ — base, Exponent
- Anything to the power of 0 $(7^0) = 1$
- Anything with the base of 0 $(0^2) = 0$
- $a^{-n} = \dfrac{1}{a^n}$

Properties of Exponents

1. Multiplication – $a^m \cdot a^n = a^{m+n}$
2. Division – $\dfrac{a^m}{a^n} = a^{m-n}$
3. Exponent – $(a^m)^n = a^{m \cdot n}$
4. Distriubtion #1 – $(ab)^n = a^n b^n$
5. Distribution #2 – $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

- Let x be a nonnegative real number, then the square root of x is $\sqrt{x}$. The number also satisfies the following:

$$x = y^2 \,,\; y > 0$$

Ex.

$4 = 2^2$ $\quad y = 2$

$x = 4$

- Principal of n^{th} root of a number, x, is the number y, satisfying the following:

$$y^n = x$$

If n is even, then we require that $x \geq 0, y \geq 0$

Ex. ① $\sqrt[n]{x} = y$, $\sqrt[3]{8} = 2$

② $\sqrt[n]{x} = y$, $\sqrt[4]{16} = 2$

Rational Exponents.

- $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m \rightarrow a^{\frac{m}{n}}$ is the simplest terms.
- $a^{\frac{1}{n}} = \sqrt[n]{a}$
- Ex. Put the following in lowest terms.

$$= \sqrt[4]{24}\,\sqrt[4]{54}$$
$$= \sqrt[4]{8\cdot 3}\,\sqrt[4]{9\cdot 6}$$
$$= \sqrt[4]{8\cdot 3\cdot 9\cdot 6}$$
$$= \sqrt[4]{8\cdot 3\cdot 3\cdot 3\cdot 2\cdot 3}$$
$$= \sqrt[4]{8\cdot 2\cdot 3\cdot 3\cdot 3\cdot 3}$$
$$= \sqrt[4]{16\cdot 3^4}$$
$$= \sqrt[4]{2^4\cdot 3^4}$$
$$= 6.$$

$$\frac{9(st)^{\frac{3}{2}}}{(27s^3t^{-4})^{\frac{2}{3}}} \cdot \left(\frac{3s^{-2}}{4t^{\frac{1}{3}}}\right)^{-1}$$
$$= \frac{9^{\frac{3}{2}} s^{\frac{3}{2}} t^{\frac{3}{2}}}{27^{\frac{2}{3}} s^{3(\frac{2}{3})} t^{-4(\frac{2}{3})}} \cdot \left(\frac{4t^{\frac{1}{3}}}{3s^{-2}}\right)$$
$$= \frac{27\, s^{\frac{3}{2}}\, 4\, t^{\frac{3}{2}+\frac{1}{3}}\, \cancel{s^2}}{9\, \cancel{s^2}\, t^{-\frac{8}{3}}\, 3}$$
$$= \frac{108\, s^{\frac{3}{2}}\, t^{\frac{11}{6}}}{[illegible]} \cdot t^{\frac{8}{3}}$$
$$= 4\, s^{\frac{3}{2}}\, t^{\frac{11}{6}+\frac{16}{6}}$$
$$= 4\, s^{\frac{3}{2}}\, t^{\frac{27}{6}}$$
$$= 4\, s^{\frac{3}{2}}\, t^{\frac{9}{2}}.$$

1.3 Algebraic Expressions

- $(x+y)^2 = x^2 + 2xy + y^2$
- $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
- $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
- $(x+y)(x-y) = x^2 - y^2$

Pascals Triangle

① 1
② 1 1
③ 1 2 1
④ 1 3 3 1
⑤ 1 4 6 4 1
⑥ 1 5 10 10 5 1

① $\cancel{(x+y)}^0$
② $x+y$
③ $x^2 + 2xy + y^2$
④ $x^3 + 3x^2y + 3xy^2 + y^3$
⑤ $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
⑥ $x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Coefficients match Pascals Triangle

Variables sequence like

$(x+y)^n = (x^n y^0 + x^{n-1} y^1 + x^{n-2} y^2 \ldots)$

Sum or Difference of Cubes

- $A^3 + B^3 = (A+B)(A^2 - AB + B^2)$
- $A^3 - B = (A-B)(A^2 + AB + B^2)$

Polynomials

- A polynomial in 'x' is an expression of the form $a_n x^n + a_{n-1} x^{n-1} + \ldots a_1 x + a_{10}$, where n is a nonnegative integer an $a_0, a_1, \ldots, a_{10}$ are real numbers. If $a_n \neq 0$, then the degree of the polynomial is n.
- $x^3 \rightarrow$ Degree $\rightarrow 3 \rightarrow$ Cubic
- $x^2 \rightarrow$ Degree $\rightarrow 2 \rightarrow$ Quadratic.
- $x^4 \rightarrow$ Degree $\rightarrow 4 \rightarrow$ Quartic.
- $(\sqrt{y} + \sqrt{2})(\sqrt{y} - \sqrt{2}) \longrightarrow x^2 - y^2 = (x-y)(x+y)$
  $= y - 2$
- $16z^2 - 24z + 9$
  $= (4z - 3)^2$

• Factor

$$3(2x-1)^2(2)(x+3)^{\frac{1}{2}} + (2x-1)^3\left(\tfrac{1}{2}\right)(x+3)^{-\frac{1}{2}}$$

$$= (2x-1)^2\left((3)(2)(x+3)^{\frac{1}{2}} + (2x-1)\left(\tfrac{1}{2}\right)(x+3)^{-\frac{1}{2}}\right)$$

$$= (2x-1)^2\left(6(x+3)^{\frac{1}{2}} + \frac{(2x-1)\left(\frac{1}{2}\right)}{(x+3)^{\frac{1}{2}}}\right)$$

$$= (2x-1)^2\left(\frac{6(x+3)}{(x+3)^{\frac{1}{2}}} + \frac{(2x-1)\left(\frac{1}{2}\right)}{(x+3)^{\frac{1}{2}}}\right)$$

$$= (2x-1)^2\left(\frac{6(x+3) + (2x-1)\left(\frac{1}{2}\right)}{(x+3)^{\frac{1}{2}}}\right)$$

$$= \frac{(2x-1)^2}{(x+3)^{\frac{1}{2}}}\left(6(x+3) + \tfrac{1}{2}(2x-1)\right)$$

$$= \frac{(2x-1)^2}{(x+3)^{\frac{1}{2}}}\left(\tfrac{12}{2}(x+3) + \tfrac{1}{2}(2x-1)\right)$$

$$= \frac{(2x-1)^2}{2(x+3)^{\frac{1}{2}}}\left(12(x+3) + (2x-1)\right)$$

$$= \frac{(2x-1)^2}{2(x+3)^{\frac{1}{2}}}\left(12x + 36 + 2x - 1\right)$$

$$= \frac{(2x-1)^2}{2(x+3)^{\frac{1}{2}}}\left(14x + 35\right)$$

$$= \frac{7(2x-1)^2(2x+5)}{2(x+3)^{\frac{1}{2}}}$$

Scientific Notation

• A positive number is said to be in scientific notation when written in the form $y \cdot 10^n$

1.4 Rational Expressions

- A rational expression (or rational function) is a ratio of two polynomials
  - Ex. $\frac{x^2-2x-1}{x+1}$, $\frac{x^2+1}{2x^3+x-5}$, x^5-x^2-2

$\frac{2t^2-5}{3t+6}$ ① Finding Domain - Find the undefined (denominator = 0)

$t=-\frac{6}{3}$ $D:(-\infty,-2)\,(-2,\infty)$

$t=-2$

$\frac{x^2-x-2}{x^2-1}$ ② Simplifying

$=\frac{(x-2)(x+1)}{(x-1)(x+1)}$

$=\frac{x-2}{x-1}$

$\frac{5}{2x-3}-\frac{3}{(2x-3)^2}$

$=\frac{5(2x-3)}{(2x-3)^2}-\frac{3}{(2x-3)^2}$

$=\cancel{\frac{25-3}{(2x-3)^2}}$

$=\cancel{\frac{22}{(2x-3)^2}}$

$\frac{10x-15-3}{(2x-3)}$

$=\frac{10x-18}{(2x-3)^2}$

$=\frac{2(5x-9)}{(2x-3)^2}$.

$\frac{\left(\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right)}{h}$

$=\frac{\left(\frac{\sqrt{x}}{\sqrt{x+h}\sqrt{x}}-\frac{\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right)}{h}=\sqrt{x}-\sqrt{x+h}$

$=\frac{\left(\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right)}{h}$

$=$

$=\left(\frac{(7-3x)^{\frac{1}{2}}+\frac{3}{2}x(7-3x)^{-\frac{1}{2}}}{(7-3x)}\right)\frac{2}{2}$

$=\left(\frac{2(7-3x)^{\frac{1}{2}}+3x(7-3x)^{-\frac{1}{2}}}{2(7-3x)}\right)\cdot\frac{(7-3x)^{\frac{1}{2}}}{(7-3x)^{\frac{1}{2}}}$.

$=\frac{2(7+3x)+3x}{2(7-3x)^{\frac{3}{2}}}$

$=\frac{14+6x+3x}{2(7-3x)^{\frac{3}{2}}}$

$=\frac{14-3x}{2(7-3x)^{\frac{3}{2}}}$

$$\frac{2(x-y)}{\sqrt{x}-\sqrt{y}} \cdot \frac{(\sqrt{x}+\sqrt{y})}{(\sqrt{x}+\sqrt{y})}$$

$$= \frac{2(x-y)(\sqrt{x}+\sqrt{y})}{x-y}$$

$$= 2(\sqrt{x}+\sqrt{y})$$

$$\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \cdot \frac{(\sqrt{x}+\sqrt{x+h})}{(\sqrt{x}+\sqrt{x+h})}$$

$$= \frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$$

$$= \frac{-h}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$$

$$= \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$$

$$= \frac{-1}{x\sqrt{x+h}+\sqrt{x^2}\,x+h}$$

1.5 Equations

Linear Equation

- in the variable, x, is an equation that can be written in the form $ax+b=0$, where a & b are real numbers and $a \neq 0$.
  - $ax+b=0$

    $x=-\frac{b}{a}$.

- Ex- $\frac{2}{3}y+\frac{1}{2}(y-3)=\frac{y+1}{4}$

  ~~$\frac{2}{3}y+\frac{1}{2}y-\frac{3}{2}-\frac{y+1}{4}=0$~~

  ~~$12\left(\frac{2}{3}y+\frac{1}{2}y-\frac{y+1}{4}\right)=\frac{3}{2}$~~

  $\frac{2y}{3}+\frac{1(y-3)}{2}-\frac{y+1}{4}=0$

  $12\left(\frac{2y}{3}+\frac{1(y-3)}{2}-\frac{y+1}{4}\right)=0$

  $\frac{24y}{3}+\frac{12(y-3)}{2}-\frac{12(y+1)}{4}=0$

  $8y+6y-18-3y+3=0$

  $11y \quad =18+3$

  $y=\frac{21}{11}$

Quadratic Equation

- A quadratic equation in variable x is an equation that can be written as $ax^2+bx+c=0$, where a,b,c are real number and $a \neq 0$.
  - Methods of Solution - Factoring
    - Completing the Square
    - Use the Quadratic formula.

- Ex- $x^2+8x+12=0$

  $(x+2)(x+6)$

  - ~~$x^2+2bx+c=0$~~

    ~~$(x$~~

- $x^2-4x+2=0$

  $x^2-4x=-2$

  $x^2-4x+4=-2+4$

  $(x-2)^2=2$

  $x-2=\pm\sqrt{2}$

  $x=2\pm\sqrt{2}$.

$ax^2+bx+c=0$

$x^2+\frac{b}{a}x+\frac{c}{a}=0$

$x^2+\frac{b}{a}x=-\frac{c}{a}$

$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$

$\left(x+\frac{b}{2a}\right)^2=-\frac{b^2}{4a^2}-\frac{c}{a}$

$\left(x+\frac{b}{2a}\right)^2=\frac{-b^2-4ac}{4a^2}$

$x+\frac{b}{2a}=\frac{\pm\sqrt{b^2-4ac}}{\sqrt{4a^2}}$

$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$

$=\frac{4\pm\sqrt{4^2-4(1)(2)}}{2(1)}$

$=\frac{4\pm\sqrt{16-8}}{2}$

$=\frac{4\pm\sqrt{8}}{2}$

$=\frac{4\pm\sqrt{4(2)}}{2}$

$=\frac{4\pm\sqrt{4}\sqrt{2}}{2}$

$=2\pm\sqrt{2}$

$3x-1$ $x-4$

3

Ex. $\frac{6\pm\sqrt{6^2-4(-1)(3)}}{2(3)}$

$=\frac{6\pm\sqrt{36+12}}{6}$

$=\frac{6\pm\sqrt{48}}{6}$

$=\frac{6\pm\sqrt{16}\sqrt{3}}{6}$

$=\frac{6\pm4\sqrt{3}}{6}$

$=\frac{2(3\pm2\sqrt{3})}{6}$

$=\frac{3\pm2\sqrt{3}}{3}$

$$\sqrt{5-x}+1=x-2$$
$$\sqrt{5-x}=x-3$$
$$5-x=(x-3)^2$$
$$5-x=x^2-3x-3x+9$$
$$=x^2-6x+x+9-5$$
$$=x^2-5x+4$$
$$=(x-1)(x-4)$$

$$x^4-5x^2+4=0$$
$$t=x^2$$
$$t^2-5t+4=0$$
$$(t-1)(t-4)$$
$$(x^2-1)(x^2-4)=0$$

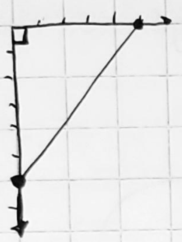
$$\sqrt{x}-3\sqrt[4]{x}-4=0$$
$$x^{\frac{1}{2}}-3x^{\frac{1}{4}}-4=0$$
$$t=x^{\frac{1}{4}}$$

$$|x-6|=-1$$

1.8 Coordinate Geometry

- 
  - Coordinate Plane
  - Cartesiana Plane
- Can find mid point of a line by $x = \frac{x_1 + x_2}{2}$ and $y = \frac{y_1 + y_2}{2}$.

- Ex. $(0,-6)(5,0)$



Slope $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{6}{5}$

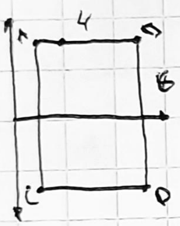
$c = \sqrt{6^2 + 5^2}$

$c = 7.81$

$c = \sqrt{61}$

Midpoint $= (\frac{5}{2}, -3)$

- Ex $A = (1,3)$, $B(5,3)$, $C(1,-3)$, $D(5,-3)$



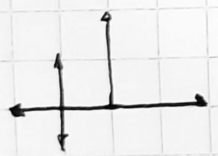
$A = LW$

$A = (4)(6)$

$A =$ [illegible] 4

$A = 24$

- Ex $\{(x,y) \mid x = -1\}$



- Ex $\{(x,y) \mid |y| \leq 2\}$

- Ex $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Find intercepts

$\frac{0^2}{9} + \frac{y^2}{4} = 1$

$y = \pm\sqrt{4}$

$y = \pm 2$

$\frac{x^2}{9} + \frac{0^2}{4} = 1$

$x = \pm\sqrt{9}$

$x = \pm 3$

Symmetry

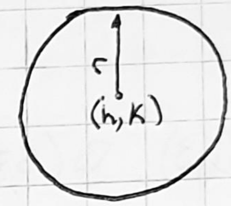
- Replace x with $(-x)$, and if the result is the same then it is symmetrical about the y axis.
- Replace y with $(-y)$, and if the result is unchanged then it is symmetrical about the x axis.
- If you replace x and y by $(-x, -y)$ and the eqn. doesn't change, the equation is symmetrical about the origin.

Circle

- Standard Form: $(x-h)^2 + (y-k)^2 = r^2$

Coordinates to the center point.

Radius



- Ex $x^2 + y^2 + \frac{1}{2}x + 2y + \frac{1}{16} =$

$$x^2 + \frac{1}{2}x + y^2 + 2y = -\frac{1}{16}$$

$$x^2 + \frac{1}{2}x\left(\frac{1}{4}\right)^2 + y^2 + 2y + 1 = -\frac{1}{16} + \left(\frac{1}{4}\right)^2 + 1$$ ← Making Squares.

$$\left(x + \frac{1}{4}\right)^2 + (y+1)^2 = 1$$

Center $= \left(-\frac{1}{4}, -1\right)$

Radius $= 1$

- Ex $3x^2 + 3y^2 + 6x - y = 0$

$$\cancel{3}x^2 + y^2 + 2x - \frac{1}{3}y = 0$$

$$x^2 + 2x + 1 + y^2 - \frac{1}{3}y + \frac{1}{36} = 1 + \frac{1}{36}$$

$$(x+1)^2 + \left(y - \frac{1}{6}\right)^2 = \frac{37}{36}$$

- Ex- $\{(x, y) \mid x^2 + y^2 \leq 1\}$

1.10 Lines

- $\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$
- Point Slope form of a line $= y_2 - y_1 = m(x_2 - x_1)$
- Slope Intercept form $- \; y = mx + b$.

General Equation of a Line

- $ax + by + c = 0$, at least one of (a/b) will be non Zero.
- 2 nonvertical lines are parallel if slopes are equal
- If slope m_1 and m_2 are perpendicular, then $m_1 m_2 = -1$ or equivalent to $m_2 = \frac{-1}{m_1}$

- Ex $(1,2)$ $(3,3)$

$$m = \frac{3-2}{3-1}$$

$$m = \frac{1}{2}$$

- Ex $(-3,-5)$ $m = -\frac{7}{2}$

$$y - (-5) = -\frac{7}{2}(x - (-3))$$

$$y + 5 = -\frac{7}{2}(x + 3)$$

$$2y + 10 = -7x - 21$$

$$7x + 2y + 31 = 0$$

Chapter 2

2.1 What is a function?

- A function from the set A to the set B is a rule, that assigns to each element in A exactly one element in B.
- Ex - $A=\{1,2,4,5\}$ $B=\{1,4,-6,-5\}$

  Function

- $g: \mathbb{R} \to \mathbb{R}$ domain $= \mathbb{R}$

  $g(x)=x^2$ Codomain $= \mathbb{R}$

  Range - $[0,\infty)$

- $g(a)=a^2=b$

  b is the image of a

  a is the preimage of b

- $f: \mathbb{R} \to \mathbb{R}$

  $f(x)=\frac{x}{7}-4$

- $f(x)=\frac{|x|}{x}$ $\quad f(4)=\frac{|4|}{4}=1$ $\quad f(x^2)=\frac{|x^2|}{x^2}=1$ if $x\neq 0$ $\quad f(\frac{1}{x})=\frac{|\frac{1}{x}|}{\frac{1}{x}}=1$ $\quad$ domain $=(-\infty,0)(0,\infty)$ Range $=\{-1,1\}$

  $f(-2)=\frac{|-2|}{-2}=-1$

- $f(x)=\begin{cases} 3x & \text{if } x<0 \\ x+1 & \text{if } 0\leq x\leq 2 \\ (x-2)^2 & \text{if } x>2 \end{cases}$ ← Piecewise function.

  Domain $= \mathbb{R}$

  ~~Range~~

  Codomain $= \mathbb{R}$

  $f(-5)=\begin{cases} 3(-5) \\ \cancel{x+1} \\ \cancel{(x-2)^2} \end{cases} = \{-15\}$ $\quad f(0)=\{0+1\}=1$

• Ex $f(x) = \frac{2x}{x-1}$

$f(a) = \frac{2a}{a-1}$

$f(a+h) = \frac{2a+2h}{a+h-1}$

$$\frac{f(a+h) - f(a)}{h}$$

$$= \frac{\frac{2a+2h}{a+h-1} - \frac{2a}{a-1}}{h}$$

$$= \frac{\frac{(a-1)(2a+2h)}{(a-1)(a+h-1)} - \frac{(a+h-1)2a}{(a-1)(a+h-1)}}{h}$$

$$= \frac{(a-1)(2a+2h) - (a+h-1)2a}{h(a-1)(a+h-1)}$$

$$= \frac{2(a-1)(a+h) - (a+h-1)a}{h(a-1)(a+h-1)}$$

$$= \frac{2(a^2 + ah - a - h - a^2 - ah + a)}{a^2h + ah^2 - ah - ah - h^2 + h}$$

$$= \frac{2(\quad -h)}{h(a-1)(a+h-1)}$$

$$= \frac{2(-h)}{h(a-1)(a+h-1)}$$

$$= \frac{-2}{(a-1)(a+h-1)}$$

• $f(x) = \frac{x^4}{x^2 + x - 6}$

$= \frac{x^4}{(x+3)(x-2)}$

D: $\{x \mid x \in \mathbb{R}, x \neq -3, 2\}$

D: $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$.

• $f(x) = \sqrt[4]{x-9}$ D: $\{x \mid x \in \mathbb{R}, x \geq 9\}$

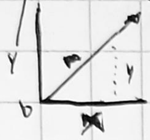
• $f(x) = \frac{x^2}{\sqrt{6-x}}$ D: $x < 6$

• $f(x) = \frac{(x+1)^2}{\sqrt{2x-1}}$ $x = \frac{1}{2}$ D: $x > \frac{1}{2}$

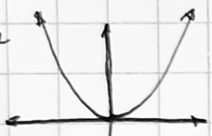
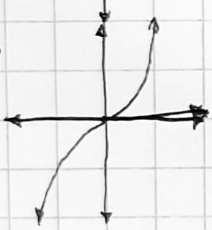
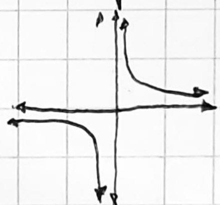
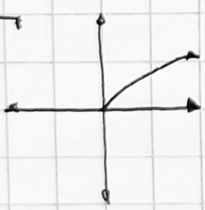
2.1 Graphs of Functions.

- The graph of a function $y=f(x)$ is the set of all points whose coordinates satisify the equation $y=f(x)$.

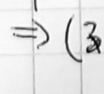
Linear Functions

- $y = mx + b$.
- 
- if $m = 0$ 

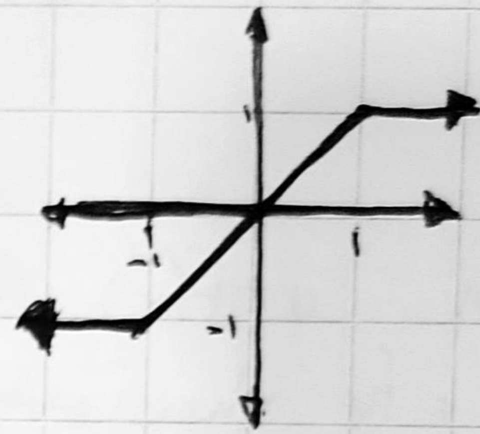
Other functions.

- x^2 
- $|x|$ 
- x^3 
- $\frac{1}{x}$ 
- $\sqrt{x}$ 
- 2^x 

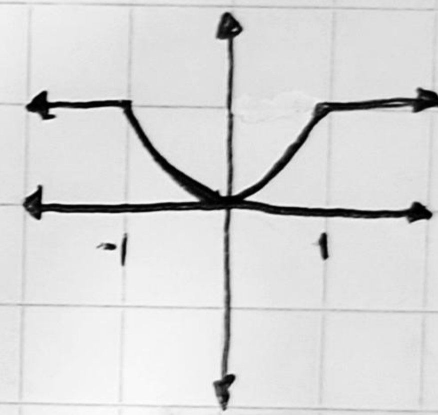
Rules of a Function

- Should only by one y value for one x value.
- VLT  : not a function.
- $(x, y) \Rightarrow (3, 4)\ (3, 7)$: not a function

- $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ x & \text{if } -1 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$

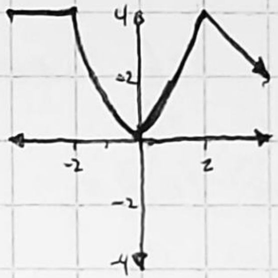


- $f(x) = \begin{cases} x^2 & \text{if } |x| \leq 1 \\ 1 & \text{if } |x| > 1 \end{cases}$



2.2 Graphs of Functions.

- Ex. $f(x) = \begin{cases} 4 & \text{if } x < -2 \\ x^2 & \text{if } -2 \le x \le 2 \\ -x+6 & \text{if } x > 2 \end{cases}$



D: $\{x \mid x \in \mathbb{R}\}$
D: $(-\infty, \infty)$.
R: $\{y \mid y \le 4\}$
R: $(-\infty, 4]$

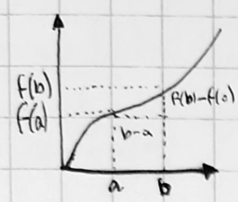
- Ex. Is $x^2 + (y-1)^2 = 4$ a function?
  - No, it is a circle.

- Ex $2x + |y| = 0$

$|y| = -2x$

Not a function, $2(-1) + |2| = 0$, 2 different y values.
$2(-1) + |-2| = 0$

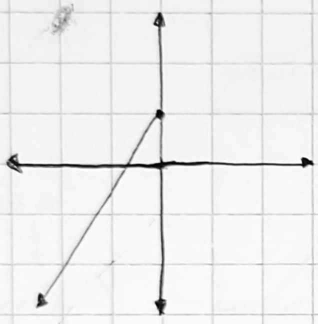
2.4 Average Rate of Change of a Function

- 

Average Rate of Change $= \dfrac{\Delta y}{\Delta x}$

$$= \frac{f(b)-f(a)}{b-a} \implies \frac{y_2-y_1}{x_2-x_1}$$

- Can be called average rate of change or secant line.
- ~~A~~ Average Rate of Change in a line is constant.
- Ex. $f(z) = 1-3z^2 \qquad z=-2,\ z=0$



$$\text{Avg Rate of Change} = \frac{f(0)-f(-2)}{0-(-2)}$$
$$= \frac{1-(-11)}{2}$$
$$= 6$$

- Ex. $f(x) = x + x^4 \qquad x=-1,\ x=3.$

$$\text{Average Rate of Change} = \frac{f(3)-f(-1)}{3+1}$$
$$= \frac{84-0}{4}$$
$$= 21$$

- Ex $f(x) = \dfrac{2}{x+1} \qquad x=0,\ x=h.$

$$= \frac{f(h)-f(0)}{h-0}$$
$$= \frac{\frac{2}{h+1}-2}{h-0}$$
$$= \frac{\frac{2}{h+1}-\frac{2(h+1)}{h+1}}{h-0}$$
$$= \frac{\frac{2-2(h+1)}{h+1}}{h}$$
$$= \frac{\frac{2-2h-2}{h+1}}{h}$$

$$= \frac{-2h}{h(h+1)}$$
$$= \frac{-2}{(h+1)}.$$

• Ex $f(t)=\sqrt{t}$ $\quad t=a, \; t=a+h$

$$= \frac{f(a+h)-f(a)}{a+h-a}$$

$$= \frac{\sqrt{a+h}-\sqrt{a}}{h}$$

$$= \frac{(\sqrt{a+h}-\sqrt{a})(\sqrt{a+h}+\sqrt{a})}{h(\sqrt{a+h}+\sqrt{a})}$$

$$= \frac{a+h-a}{h(\sqrt{a+h}+\sqrt{a})}$$

$$= \frac{\cancel{h}}{\cancel{h}(\sqrt{a+h}+\sqrt{a})}$$

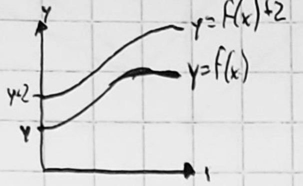
$$= \frac{1}{\sqrt{a+h}+\sqrt{a}}$$

2.5 Transformations of Graphs.

- $y = f(x)$

  $\downarrow$

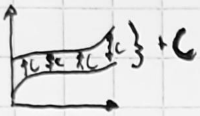
  $y = f(x) + 2$



- $y = f(x)$

  $\downarrow$

  $y = f(x) + c$, then the graph undergoes a vertical shift of c.

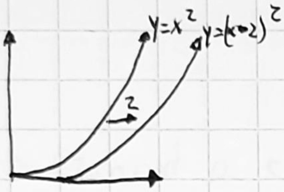


Vertical Shift

- $y = x^2$

  $\downarrow$

  $y = (x-2)^2$



- $y = f(x)$

  $\downarrow$

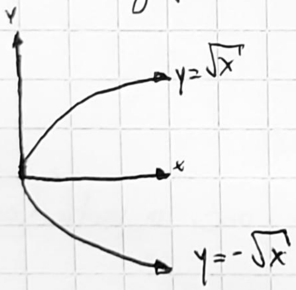
  $y = f(x-c)$, then the graph undergoes a horizontal shift of c.

Horizontal Shift.

- $y = \sqrt{x}$

  $\downarrow$

  $y = -\sqrt{x}$



- $y = f(x)$

  $\downarrow$

  $y = -f(x)$, then the graph undergoes a reflection in the x axis

Reflection in the x axis.

- $y = f(\cancel{x})$

  $\downarrow$

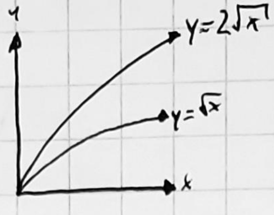
  $y = f(-x)$, then the graph undergoes a reflection in the y axis

Reflection in the y axis.

- $y = \sqrt{x}$

  $\downarrow$

  $y = 2\sqrt{x}$

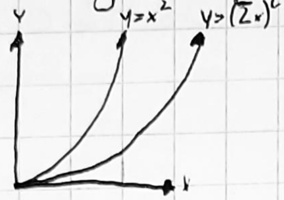


Vertical stretch or compression

- $y = f(x)$

  $\downarrow$

  $y = cf(x)$, then the graph under goes a vertical stretch by c



- $y = (\ x)^2$

  $\downarrow$

  $y = (\frac{1}{2}x)^2$

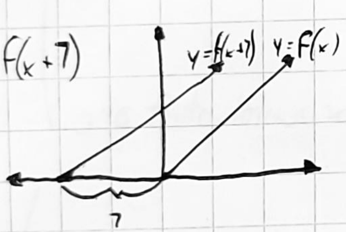
horizontal stretch or compression

- $y = f(x)$

  $\downarrow$

  $y = f(cx)$, then the graph undergoes a horizontal stretch or compression by c

- Ex $y = f(x+7)$



- Ex $y=f(x)$

  $\downarrow$

  $y = -\frac{1}{2}f(x)$

  Reflect in the x axis, and a vertical compression.

- Ex $y = 2 - f(-x)$ Reflect in x axis, Reflect in y axis, shift vertically.

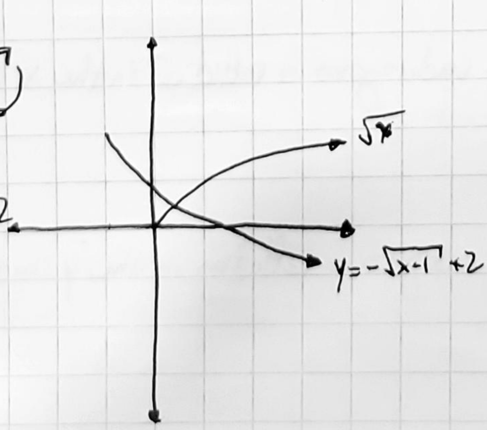
  $\downarrow$

- Ex $y = 2 - \sqrt{x+1}$

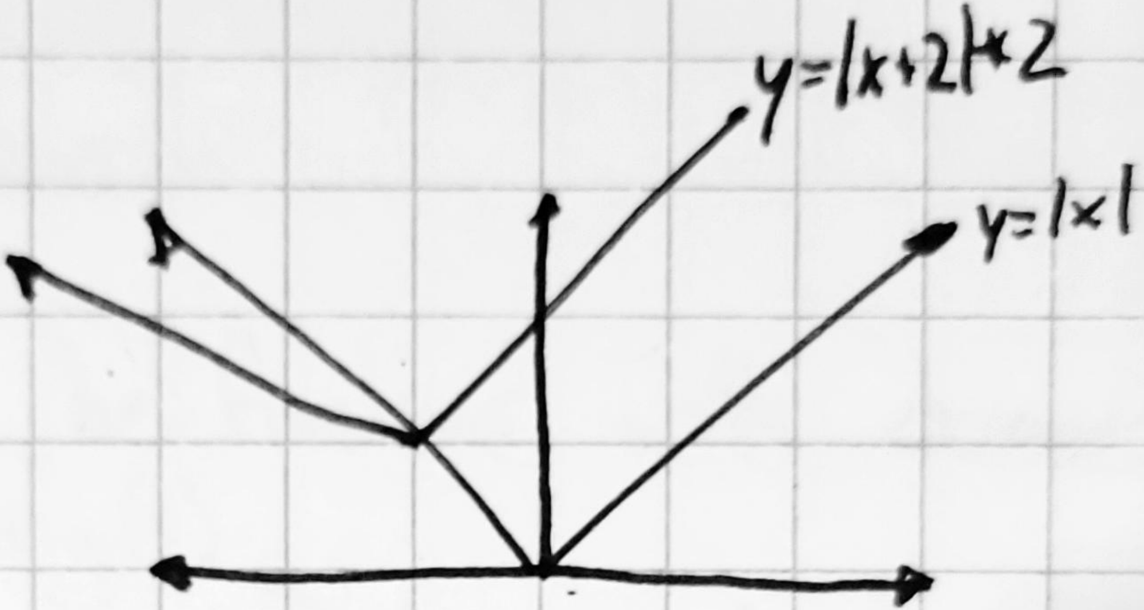
  $y = \sqrt{x+1}$

  $y = -\sqrt{x+1}$

  $y = -\sqrt{x+1} + 2$



$y = |x+2| + 2$



2.6 Combining functions.

- Let f and g be functions with domains A and B, respectively.
- $(f+g)(x) = f(x) + g(x)$ ← domain $= A \cap B$
- $(f-g)(x) = f(x) - g(x)$ domain $= A \cap B$
- $(fg)(x) = f(x)g(x)$
- Ex. $f(x) = \sqrt{x}$, $g(x) = \dfrac{1}{x-1}$ $D_A = [0,\infty)$, $D_B = (\infty, 0)\cup(0,\infty)$

$(f+g)(x) = \sqrt{x} + \dfrac{1}{x-1}$ $D_{A+B} = (1,\infty) \cup [0,1)$

$(f-g)(x) = \sqrt{x} - \dfrac{1}{x-1}$ $D_{A-B} = [0,1) \cup (1,\infty)$

$(fg)(x) = \dfrac{\sqrt{x}}{x-1}$

- $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$ Domain = the set of all elements common to both A & B such that $g(x) \neq 0$. $\{x \mid x \in A\cap B \text{ and } g(x) \neq 0\}$
- Ex $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x}}{\frac{1}{x-1}}$ $D_{\frac{f}{g}(x)} = A \cap B$
$= \sqrt{x}(x-1)$ $= [0,1) \cup (1,\infty)$

$\dfrac{g}{f}(x) = \dfrac{g(x)}{f(x)} = \dfrac{\frac{1}{x-1}}{\sqrt{x}}$

$= \dfrac{1}{\sqrt{x}(x-1)}$ $D_{\frac{g}{f}(x)} = \{x \mid x \in A\cap B,\ f(x) \neq 0\}$
$= (0,1) \cup (1,\infty)$

☆
- $(f+g)(x) = f(x) + g(x) \rightarrow D = A \cap B$
- $(f-g)(x) = f(x) - g(x) \rightarrow D = A \cap B$
- $(fg)(x) = f(x) \cdot g(x) \rightarrow D = A \cap B$
- $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} \rightarrow D = A \cap B,\ g(x) \neq 0$

Ex. $f(x)=\sqrt{9-x^2}$ $\qquad$ $g(x)=\sqrt{x^2-4}$

$\sqrt{(3-x)(3+x)}$ $\qquad$ $\sqrt{(x+2)(x-2)}$

Domain $=[-3,3]$ $\qquad$ Domain $=(\infty,-2][2,\infty)$

Domain of $(f+g)(x)=[-3,-2]\cup[2,3]$

" $(f-g)(x)=[-3,-2]\cup[2,3]$

" $(fg)(x)=[-3,-2)\cup[2,3]$

" $\left(\frac{f}{g}\right)(x)=[-3,-2)\cup(2,3]$

Composition of functions

Important for Calc.

- Let f and g be functions. The composition of f and g, is denoted by $f\circ g$ is defined by
$$(f\circ g)(x)=f(g(x))$$

- Ex $f(x)=3x-5$
  $g(x)=2-x^2$

$$(f\circ g)(x)=f(g(x))$$
$$=f(2-x^2)$$
$$=3(2-x^2)-5$$
$$=6-3x^2-5$$
$$=-3x^2+1$$

- Domain of $f\circ g$ is the set of all x such that x is in the domain of $g(x)$ such that x in the domain of $f(x)$
- Domain of $f(x)=x\in\mathbb{R}$
  Domain of $g(x)=x\in\mathbb{R}$
  Domain of $(f\circ g)(x)=x\in\mathbb{R}$

- $f(x)=3x-5$) $(f\circ f)(-1)=f(f(-1))$
$$=f(3(-1)-5)$$
$$=f(-8)$$
$$=3(-8)-5$$
$$=-29$$

· Ex $f(x) = x^2$ $g(x) = \sqrt{x-3}$

Domain $= x \in \mathbb{R}$ Domain of $g(x) = x \geq 3$

$(f \circ g)(x) = f(g(x))$
$= f(\sqrt{x-3})$
$= (\sqrt{x-3})^2$
$= x-3$
Domain $= x \geq 3$

$(g \circ f)(x) = g(f(x))$
$= g(x^2)$
$= \sqrt{x^2-3}$
Domain $= (-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$

$(f \circ f)(x) = f(f(x))$
$= f(x^2)$
$= (x^2)^2$
$= x^4$
Domain $= x \in \mathbb{R}$

$(g \circ g)(x) = g(g(x))$
$= g(\sqrt{x-3})$
$= \sqrt{\sqrt{x-3}-3}$
Domain $= x \geq 12$

· Ex $f(x) = \frac{2}{x}$ $g(x) = \frac{x}{x+2}$

$(f \circ g)(x) = f(g(x)$
$= f(\frac{x}{x+2})$
$= \frac{2}{\frac{x}{x+2}}$
$= \frac{2(x+2)}{x}$
$= \frac{2x+4}{x}$
$D = \{x \mid x \neq 0, -2 \ x \in \mathbb{R}\}$

$(g \circ f)(x) = g(f(x))$
$= g(\frac{2}{x})$
$= \frac{\frac{2}{x}}{\frac{2}{x}+2}$
$= \frac{2}{x} \div \frac{2x+2}{x}$

$(f \circ f)(x) = f(f(x)$
$= f(\frac{2}{x})$
$= \frac{2}{\frac{2}{x}}$
$= 2 \cdot \frac{x}{2}$
$= x$

$(g \circ g)(x) = g(g(x)$
$= g(\frac{x}{x+2})$
$= \frac{\frac{x}{x+2}}{\frac{x}{x+2}+2}$
$= \frac{\frac{x}{x+2}}{\frac{x+2x+4}{x+2}}$
$= \frac{x}{x+2} \cdot \frac{x+2}{3x+4}$
$= \frac{x^2+2x}{3x^2+4x+6x+8}$
$= \frac{x^2+2x}{3x^2+10x+8}$
$= \frac{x(x+2)}{3x^2+10x+8}$

$\cdot\ f(x) = \sqrt{x} + 1 \Rightarrow fg(x) = f(g(\sqrt{x})$

$= f(\sqrt{x})$

$= \sqrt{x} + 1$

$\cdot\ H(x) \sqrt{1 + \sqrt{x}} = fg(x) = f(g(x))$

$= f(1 + \sqrt{x})$

$= \sqrt{1 + \sqrt{x}}$

$\cdot\ G(x) = \dfrac{2}{(3+\sqrt{x})^2}$

$= f(g(h(x)))$

$= f(g(3+\sqrt{x}))$

$= f((3+\sqrt{x})^2))$

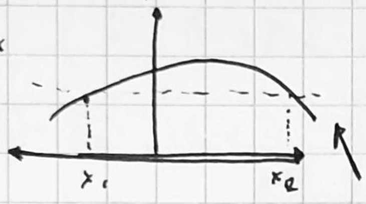
$= \cancel{f}\ \dfrac{2}{(3+\sqrt{x})^2}.$

2.7 One-to-One functions on their inverses

- Let, f, be a function with domain A. When you take 2 points from the domain are they never equal than the function is ~~[illegible]~~ one to one." We say that f is one-to-one if the following condition is satisfied, whenever x_1 and x_2 are 2 distinct elements $f(x_1)$ and $f(x_2)$ must also be distinct.
- $x_1 \neq x_2$, $f(x_1) \neq f(x_2)$
- Ex $f(x_1) = x^2$ $\quad f(x_2) = -x^2$ $\quad \therefore$, x^2 is not a one to one fxn

$$= 2^2 \qquad = -2^2$$
$$= 4 \qquad = 4$$

- Ex

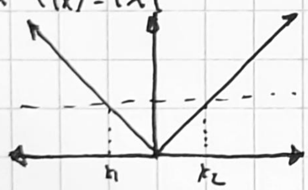


$x_1 \neq x_1$ but $f(x_1) = f(x_2)$ so it is not a one to one fxn.

Horizontal Line Test.

- Horizontal Line Test - When a graph passes through a horizontal line only once, then it is a one to one fxn
  - When a graph passes throug a horizontal line more than once, then it is not a one to one fxn.

- Ex $f(x) = |x|$



Not one to one fxn.

$f(x) = x^3$

one to one fxn.

Inverse One to One Functions.

- Let f be a one to one function with the domain = A and range = B. Then the inverse of f, denoted by f^{-1}, is defined by, $f^{-1}(x) = y$ if and only if $x = f(y)$, whenever x is in B and y is in A.
- Ex $f(x) = 2x - 5$ $\quad f(y) = x$

$D = \mathbb{R}$ $\quad f^{-1}(f(x)) = f^{-1}(x) = y$

$R = \mathbb{R}$ $\quad f^{-1}(f(y)) = y$ for all domain in f

$f^{-1}(f^{-1}(x)) = x$ for all ~~domain~~ Range in x

$f^{-1}(x) = \frac{x+5}{2}$

① write fxn ($y = x \ldots$)
② switch x & y
③ Solve for y to get $f^{-1}(x)$

- $f(5) = 18 \qquad f^{-1}(18) = 5$

- $f^{-1}(4) = 2 \qquad f(2) = ? \qquad f(f^{-1}(x)) = x$

$$x = 4$$
$$f(f^{-1}(4)) = x$$
$$f(2) = 4.$$

- The graph of inverse is a reflection about the line $y = x$.

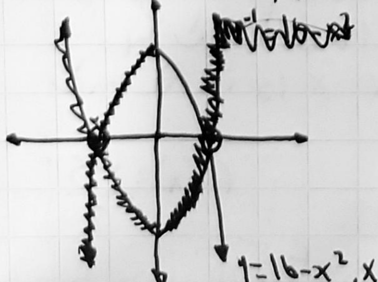
- Domain (f) = Range (f^{-1})
- Range (f) = Domain (f^{-1})

- 

- $f(x) = \dfrac{x-2}{x+2} \; : \; x = \dfrac{y-2}{y+2} \qquad f^{-1}(x) = \dfrac{-2(x+1)}{x-1}$

$$x(y+2) = y-2$$
$$xy + 2x = y - 2$$
$$xy - y = -2x - 2$$
$$y(x-1) = -2x - 2$$
$$y = \frac{-2x-2}{x-1}$$
$$y = \frac{-2(x+1)}{x-1}$$

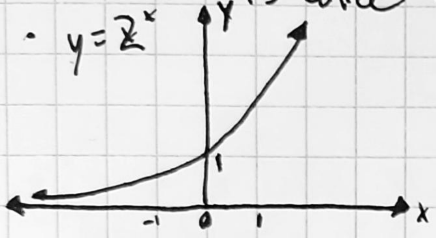
- $f(x) = 16 - x^2, \; x \geq 0$



$$y = -x^2 + 16$$
$$x = -y^2 + 16$$
$$x - 16 = -y^2$$
$$-x + 16 = y^2$$
$$\sqrt{-x+16} = y$$

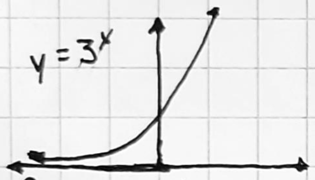
Chapter 4 Exponential and Logarithmic Functions

- $y = 2^x$ – this is a exponential function with base 2.
- $y = 10^x$ – " " " " " " " 10.
- Definition: Let $a > 0$, $a \neq 1$. Then the function $f(x) = a^x$ is called the exponential function with base a
- $y = 2^x$



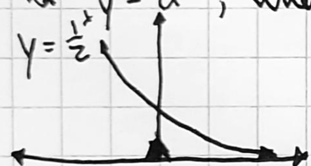
Domain $= \{x \mid x \in \mathbb{R}\}$
Range $= \{y \mid y > 0\}$.

- $y = 3^x$



Domain $= \{x \mid x \in \mathbb{R}\}$
Range $= \{y \mid y > 0\}$.

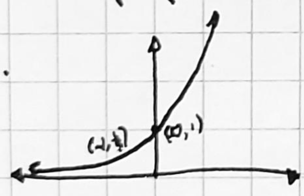
- for $y = a^x$, when $a > 1$, the graphs have the same basic properties.
- $y = \frac{1}{2}^x$



Domain $= \{x \mid x \in \mathbb{R}\}$
Range $\{y \mid y > 0\}$

- For $y = a^x$, when $a > 0$ and $a < 1$, the graphs share the same basic properties.

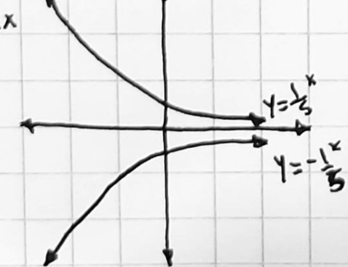
- Ex.



Coordinates $(0,1)$, $(-1, \frac{1}{5})$. what is the graphs eqn.

$y = 5^x$

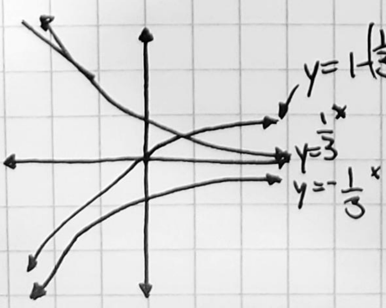
- Ex



$y = -(\frac{1}{3})^x$ Graph the transformations.

Domain $= \{x \mid x \in \mathbb{R}\}$
Range $= \{y \mid y < 0\}$.

- Ex

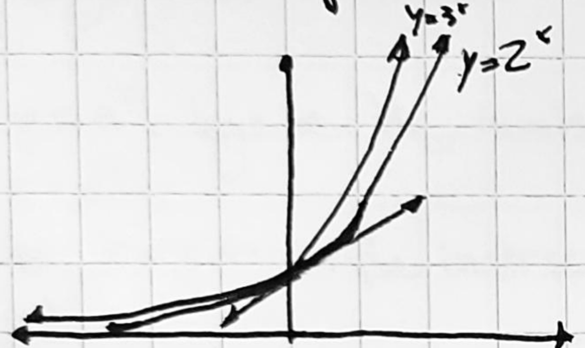


$y = 1 - \frac{1}{3}^x$

Domain $= \{x \mid x \in \mathbb{R}\}$
Range $= \{y \mid y > 1\}$.

4.2 Natural Exponential function.

•



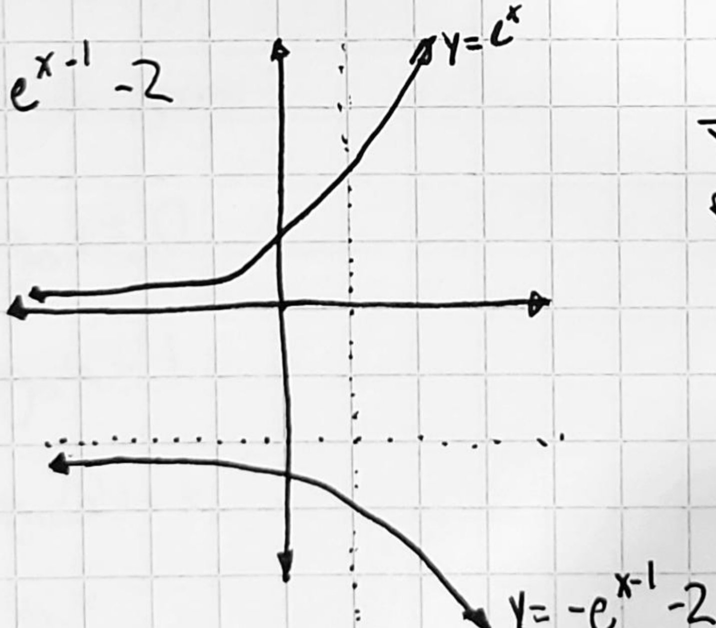
Tangential Slope of $y=2^x$ at $x=0$ is less than 1

Tangential Slope at $y=3^x$ at $x=0$ is greater than 1.

Tangential Slope ~~of~~ with a value of 1 is equal to $y=e^x$

- $e \simeq 2.718$
- $y=e^x$ is called the natural exponential function.
- The derivative of e is itself.

- $y=-e^{x-1}-2$



Domain = $(-\infty, \infty)$

Range = $(-\infty, -2)$

HA = $y=-2$.

4.3 Logarithmic Functions

- $y = a^x$, logs are the inverse of exponential
- $x = a^y$

$\log_a x = y$ ← is the inverse of $x = a^y$

- Let $a > 0$, and $a \neq 1$. Then we write $\overbrace{y = \log_a x}^{\text{Logarithimic form}}$ if and only if $\underbrace{x = a^y}_{\text{Exponential form}}$. We call $\log_a x$, the logarithim of x with base a.

x is the argument of the logarithim.

- $\log_a a = 1$; $\log_a 1 = 0$

Properties of Logs.

- ① $\log_a 1 = 0$

② $\log_a a = 1$

③ $\log_a (a^x) = x$

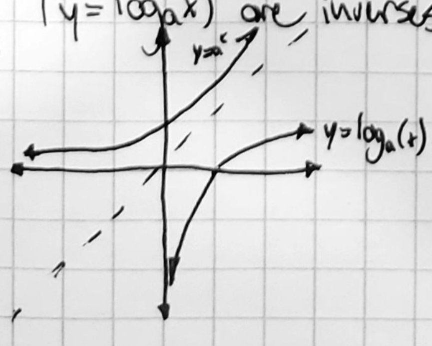
④ $a^{\log_a x} = x$

- Ex. $f(x) = a^x$, $g(x) = \log_a x$

$$(f \circ g)(x) = f(g(x)) = f(\log_a x) = a^{\log_a x} = x$$

$$(g \circ f)(x) = g(f(x)) = g(a^x) = \log_a (a^x) = x$$

- The exponential function $(y = a^x)$ and the logarithmic function $(y = \log_a x)$ are inverses of eachother



Domain = $\{x \mid x > 0\}$

Range = $\{y \mid y \in \mathbb{R}\}$

Common Logarithim

- This is the logarithim of a base 10
- $y = \log_{10} x = \log x$
- Ex - $\log 10 = 1$

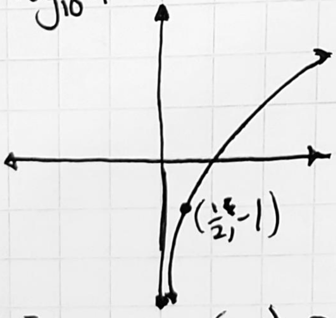
Natural Logarithim

- This is the logarithmin of a base e
- $\log_e x = \ln(x)$
- Ex, $\ln(e) = 1$, $\ln(1) = \log_e(1) = 0$
- $y = \ln x = \underbrace{\log_e x}_{\text{Log form}} = \underbrace{e^y = x}_{\text{Exp form}}$

7_4

- $\log_3 81 = 4 \iff 3^4 = 81$
- $\log_8 4 = \frac{2}{3} \iff 8^{\frac{2}{3}} = 4$
- $\log_5 x = 4 \iff 5^4 = x$

625

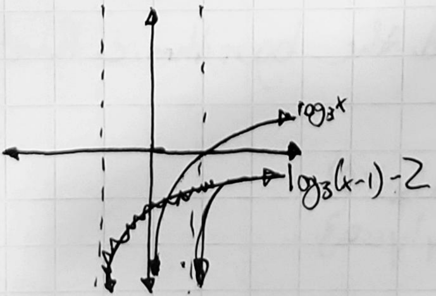
- $\log_{10} .1 = x \iff 10^{-1} = .1$
- 

$-1 = \log_a\left(\frac{1}{2}\right)$

$a^{-1} = \frac{1}{2}$

$2^{-1} = \frac{1}{2}$

- Ex $y = \log_3(x-1) - 2$



$D = \{x \mid x > +1\}$

$R = \{y \mid y \in \mathbb{R}\}$

HA = $x = +1$

• Ex $f(x) = \log_5(8-2x)$

$D = \{x \mid x > 4\}$

$R = \{y \mid y \in \mathbb{R}\}$.

• Ex $f(x) = \sqrt{x-2} - \log_5(10-x)$

$x \geq 2$

$10 - x > 0$

$10 > x$

$D = [2, 10)$

• Ex $f(x) = 3^x$

$g(x) = x^2 + 1$

$(f \circ g)(x) = f(g(x))$

$= f(x^2+1)$

$= 3^{x^2+1}$

$D = \{x \mid x \in \mathbb{R}\}$

$(g \circ f)(x) = g(f(x))$

$= g(3^x)$

$= (3^x)^2 + 1$

$= 3^{2x} + 1$

$D = \{x \mid x \in \mathbb{R}\}$

4.4 Laws of Logarithims

- Let $a>0$ and $a\neq 1$. Let A, B, and C be real numbers such that $A>0$ and $B>0$

1. $\log_a(AB) = \log_a A + \log_a(B)$
2. $\log_a\left(\frac{A}{B}\right) = \log_a A - \log_a B$
3. $\log_a(A^c) = c\log_a A$

- For the first law, $\log_a A = x \implies A = a^x$ $\quad AB = (a^x)(a^y)$
  $\log_a B = y \implies B = a^y$ $\quad AB = a^{x+y}$
  $\Downarrow$
  $\log_a(AB) = \log_a A + \log_a B$

4. $\log_a x = \dfrac{\log_b x}{\log_b a}$

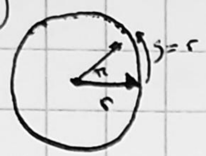
- Ex. $\log_2(160) - \log_2(5)$
  $= \log_2\left(\frac{160}{5}\right)$
  $= \log_2(32)$
  $5 = \log_2(32)$

- Ex. $\log_2(8^{33})$
  $= 33(\log_2 8)$
  $= 33(3)$
  $= 99$

- Ex. $\log_a\left(\frac{x^2}{yz^3}\right)$
  $= \log_a(x^2) - \log_a(yz^3)$
  $= 2\log_a(x) - (\log_a y + 3\log_a z)$

- Ex. $\log_5\left(\sqrt{\frac{x-1}{x+1}}\right)$
  $= \frac{1}{2}\log_5\left(\frac{x-1}{x+1}\right)$
  $= \frac{1}{2}\log_5(x-1) - \frac{1}{2}\log_5(x+1)$

Trigonometry (Chapters 5&6)

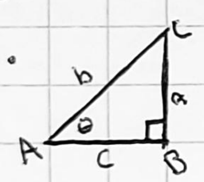
- Angle measures in Radian
  - 

    - If a circle has a radius r, and the arc length is equal to the radius, then the angle is 1 radian.
    - $\theta = \frac{s}{r}$

- Area of a Sector
  - 

    - For any angle θ, $A = \frac{1}{2} r^2 \theta$

Trigonometric Functions

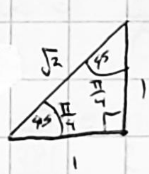
- 

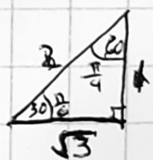
  $\sin\theta = \frac{a}{b}$, $\cos\theta = \frac{c}{b}$, $\tan\theta = \frac{a}{c}$

  $\frac{\sin\theta}{\cos\theta} = \tan\theta$

-

fxn \ θ	0	$\frac{\pi}{6}$ 30°	$\frac{\pi}{4}$ 45°	$\frac{\pi}{3}$ 60°	$\frac{\pi}{2}$ 90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{1}$	$\frac{\sqrt{3}}{1}$	und.

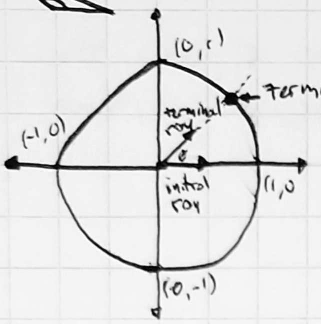




Cosecant Function, Secant, and Cotangent

- $Csc(\theta) = \frac{Hyp}{opp} = \frac{1}{\sin\theta}$
- $Sec(\theta) = \frac{Hyp}{adj} = \frac{1}{\cos\theta}$
- $\cot(\theta) = \frac{Adj}{Opp} = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

Finding Angles larger than 90°

- 
- 

- 2 angles are said to be coterminal if the terminal rays of the 2 angles are the same, when the angles are in standard position.
- Ex. 30°, 390°

These angles are coterminal!

- Ex 54° to radians

$54° \div \frac{180}{\pi}$

$\frac{\pi 54}{180} =$

$\frac{3\pi}{10}$

$\frac{11\pi}{3}$ to degrees.

$\frac{11\pi}{3} \div \frac{\pi}{180}$

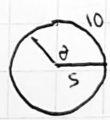
$\frac{1980}{3} =$

$660°$

- Ex -100° Find an angle between 0° and 360° coterminal to -100°

$-100° + 360$

$= 260°$

- Ex 

$\theta = \frac{s}{r}$

$\theta = \frac{10}{5}$

$\theta = 2$ radians.

- Ex $\frac{11\pi}{6}$ find coterminal

$\frac{11\pi}{6} + 2\pi$

$= \frac{23\pi}{6} + 2\pi$

$= \frac{35\pi}{6}$

= [redacted]

$\frac{11\pi}{6} - 2\pi$

$= \frac{-\pi}{6} - 2\pi$

$= \frac{-13\pi}{6}$

- Ex Find area of a sector 60° with r=3

$60° \div \frac{180}{\pi}$

$\frac{60\pi}{180}$

$\frac{\pi}{3}$

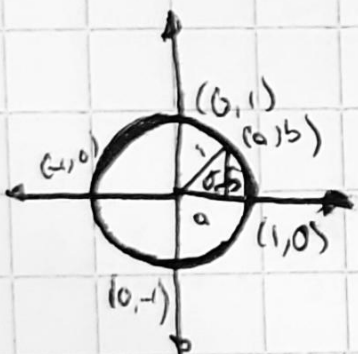
$A = \frac{1}{2} r^2 \frac{\pi}{3}$

$= 4.71238898.$

- Orbit of the earth, find distance in 1 day

$r = 93\,000\,000$

$$s = r\theta$$
$$= (93000000)\left(\frac{2\pi}{365}\right)$$
$$= \frac{186000000\pi}{365}$$
$$= 1600921.188$$

- 

$\sin\theta = \dfrac{b}{1}$

$\underbrace{\sin\theta = b}$

y-coordinate of the terminal point

$\cos\theta = \dfrac{a}{1}$

$\underbrace{\cos\theta = a.}$

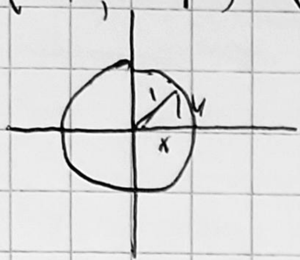
x coordinate of the terminal point.

$\tan\theta = \dfrac{b}{a}$

ratio of the y-coordinate to the x-coordinate.

5.1

- $\left(-\frac{5}{7}, \frac{-2\sqrt{6}}{7}\right)$ Prove that this is a point on the unit circle.
-



$$a^2 + b^2 = 1$$
$$\left(-\frac{5}{7}\right)^2 + \left(\frac{-2\sqrt{6}}{7}\right)^2 = 1$$
$$\frac{25}{49} + \frac{24}{49} = 1$$
$$\frac{49}{49} = 1$$

This is a point on the unit circle.

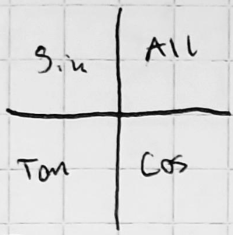
- $P = \left(x, \frac{1}{3}\right)$, point is in the 2nd quadrant on the unit circle

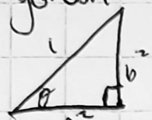
$$x^2 + y^2 = 1$$
$$x^2 + \frac{1^2}{9} = 1$$
$$x^2 = 1 - \frac{1}{9}$$
$$x = \sqrt{\frac{8}{9}}$$
$$x = \frac{\sqrt{8}}{3}$$
$$x = \frac{\sqrt{4(2)}}{3}$$
$$x = -\frac{2\sqrt{2}}{3}$$

- $P = \left(\pm x, -\frac{1}{3}\right)$

$$x^2 + y^2 = 1$$
$$x^2 = 1 - \frac{1}{9}$$
$$x = \sqrt{\frac{8}{9}}$$
$$= \pm\frac{2\sqrt{2}}{3}$$

5.2 Trigonometric Function of Real Numbers.

- 

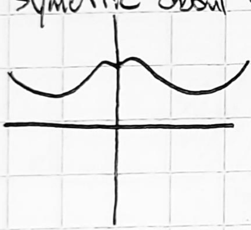
- Pythagorean Identities
  - 

    $a^2 + b^2 = 1$

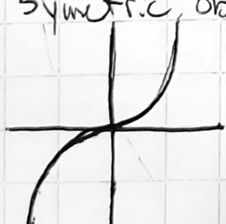
    $(\cos\theta)^2 + (\sin\theta)^2 = 1 \nearrow \cos^2\theta + \sin^2\theta = 1$ ①
  - $\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$

    $1 + \tan^2\theta = \sec\theta$ ②
  - $\dfrac{\cos^2\theta}{\sin^2\theta} + \dfrac{\sin^2\theta}{\sin\theta} = \dfrac{1}{\sin\theta}$

    $\cot^2\theta + 1 = \csc^2\theta$. ③
- f is even when $f(-\theta) = f(\theta)$
  - symetric about the y axis



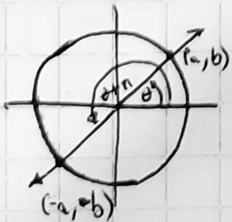
- f is ~~odd~~ when $f(-\theta) = -f(\theta)$
  - symetric about the origin



$f(-x) = f(-x)^3$

$f(-x) = -x$

- Cosine is an even function $\cos(-\theta) = \cos\theta$
- Sine is an odd function $\sin(-\theta) = -\sin\theta$

- 

$\sin(\pi + \theta) = -b$

$\cos(\pi + \theta) = -a$

- $\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$ $\qquad \cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$ $\qquad \cos\left(\frac{7\pi}{3}\right) = \cos 2\pi + \cos\frac{\pi}{3} = +\frac{1}{2}$

- $\cos\left(\frac{7\pi}{6}\right) = \cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$ $\qquad \sec\left(\frac{7\pi}{6}\right) = \frac{1}{\cos\frac{\pi}{6}} = -\frac{2}{\sqrt{3}}$ $\qquad \csc\left(\frac{7\pi}{6}\right) = \frac{1}{\sin\frac{\pi}{6}} = -2$

- $\sin\left(\frac{5\pi}{4}\right) = \frac{1}{\sqrt{2}}$ $\qquad \sec\left(\frac{5\pi}{4}\right) = \frac{1}{\cos\frac{\pi}{4}} = -\sqrt{2}$ $\qquad \tan\left(\frac{5\pi}{4}\right) = 1$

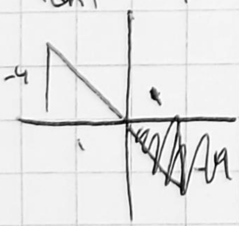
- $\tan t = \frac{1}{4}$ terminal Point of t is in quadrant 3

$1 + \tan^2 t = \sec^2 t$
$1 + \frac{1}{16} = \sec^2 t$
$\sqrt{\frac{17}{16}} = \sec t$
$-\frac{\sqrt{17}}{4} = \sec t$

$\sec t = \frac{1}{\cos t}$
$\cos t = -\frac{4}{\sqrt{17}}$

$\tan t = \frac{\sin t}{\cos}$
$\frac{1}{4} \cdot \frac{-4}{\sqrt{17}} = \sin t$
$-\frac{1}{\sqrt{17}} = \sin t.$

- $\tan t = -4 \qquad \csc t > 0$



$\cot t = \frac{1}{-4}$
$\csc^2 t = 1 + \cot^2 t$
$\csc^2 t = 1 + \frac{1}{16}$
$\csc t = \sqrt{\frac{17}{16}}$
$\csc t = \frac{\sqrt{17}}{4}$

$\sin t = \frac{4}{\sqrt{17}}$
$\cos t = -\frac{1}{\sqrt{17}}$
$\sec t = -\sqrt{17}$

- $f(-x) = f(x) \Rightarrow$ even $\Rightarrow x^2, x^4, x^6$
  $f(-x) = -f(x) \Rightarrow$ odd $\Rightarrow x^1, x^3, x^5$

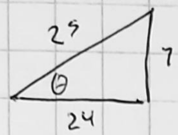
- $f(x) = \sin x + \cos x$ $\qquad$ neither odd or an even fxn.
  $f(-x) = \sin(-x) + \cos(-x)$
  $= -\sin x + \cos x$

- $\left.\begin{aligned} f(x) &= \cos(\sin(x)) \\ f(-x) &= \cos(-\sin x) \\ &= \cos(\sin x) \end{aligned}\right\}$ Even.

  $\left.\begin{aligned} f(x) &= \sin(\cos x) \\ f(-x) &= \sin(\cos(-x)) \\ &= \sin(\cos x) \end{aligned}\right\}$ Even.

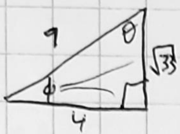
6.2 Trigonometry of Right Triangles

- Ex



$\sin\theta = \frac{7}{25}$ $\cos\theta = \frac{24}{25}$ $\tan = \frac{7}{24}$

- Ex



$a^2 + b^2 = c^2$

$a = \sqrt{c^2 - b^2}$

$a = \sqrt{49 - 16}$

$a = \sqrt{33}$

$\sin\phi = \frac{\sqrt{33}}{7}$

$\cos\phi = \frac{4}{7}$

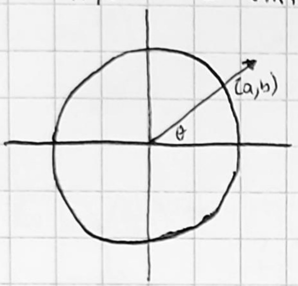
$\tan\phi = \frac{\sqrt{33}}{4}$

$\sin\theta = \frac{4}{7}$

$\cos\theta = \frac{\sqrt{33}}{7}$

$\tan\theta = \frac{4}{\sqrt{33}}$

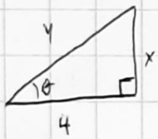
- Consider the Unit Circle



$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

-



~~$\sin\theta = \frac{x}{4}$~~ ~~$\cos\theta = \frac{4}{4}$~~

~~$\sin\theta\, y = x$~~

$\tan\theta = \frac{x}{4}$

$4\tan\theta = y$

$\cos\theta = \frac{4}{y}$

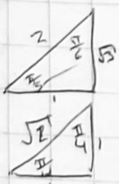
$y = \frac{4}{\cos\theta}$

- $\sin(30)\csc(30)$

$= \sin(30)\frac{1}{\sin(30)}$

$= 1$

- $(\sin(60))^2 + (\cos(60))^2 = 1$



- $\left(\sin\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{\pi}{3}\right)^2$

$= \left(\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)\right)^2$

$= \left(\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)^2$

$= \left(\frac{\sqrt{3} - 1}{2\sqrt{2}}\right)^2$

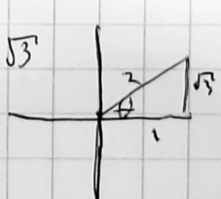
$= \cancel{\frac{3-1}{8}}\ \frac{3 - 2\sqrt{3} + 1}{8}$

$= \cancel{\frac{2}{8}} = \frac{4 - 2\sqrt{3}}{8}$

$=$

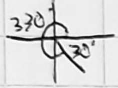
$= \frac{2 - \sqrt{3}}{4}$

- $\tan\theta = \sqrt{3}$



6.3 Trigonometric Functions of Angles

- Ex 330°



$360-330=30^\circ$

$\sin 330 = -\sin 30 = -\frac{1}{2}$

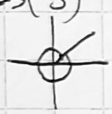
$\cos 330 = \cos 30 = \frac{\sqrt{3}}{2}$

- Ex. $\sin 225$

$225-180 = 45^\circ$

$\sin 225 = -\sin 45 = \frac{1}{\sqrt{2}}$

- Ex $\cos\left(\frac{7\pi}{3}\right)$



$\cos\frac{\pi}{3} = \frac{1}{2}$

- Ex $\csc\left(\frac{5\pi}{4}\right) = \frac{1}{\sin\frac{5\pi}{4}} = \frac{1}{-\sin\frac{\pi}{4}} = -\frac{1}{\frac{1}{\sqrt{2}}} = -\sqrt{2}$

- Ex $\sin\left(\frac{11\pi}{6}\right)$

$\sin\left(\frac{11\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$

- Ex $\tan\theta < 0$ $\sin\theta < 0 \Rightarrow$ when is tan (-) and sin (-)

$\rightarrow$ the 4th quadrant.

- Ex $\cot\theta$ in terms of $\sin\theta$ in quadrant II

$\sin^2\theta + \cos^2\theta = 1$

$1 + \cot^2\theta = \csc^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

$\cot^2\theta = \frac{1}{\sin^2\theta} - 1$

$\cot^2\theta = \frac{1-\sin^2\theta}{\sin^2\theta}$

$\cot\theta = -\sqrt{\frac{1-\sin^2\theta}{\sin^2\theta}}$

- Ex. $\cos\theta$ in terms of $\sin\theta$ θ is in quadrant IV

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = +\sqrt{1-\sin^2\theta}$

- Ex $\sec\theta$ $\tan\theta$ Quad II

$\cos^2\theta + \sin^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$-\sqrt{1+\tan^2\theta} = \sec\theta$

• Ex $\cos\theta = -\frac{7}{12}$ $\quad\theta$ is in quadrant III

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta + \left(\frac{-7}{12}\right)^2 = 1$$
$$\sin^2 + \frac{49}{144} = 1$$
$$\sin^2\theta = \frac{95}{144}$$
$$\sin\theta = -\sqrt{\frac{95}{144}}$$
$$\sin\theta = -\frac{\sqrt{95}}{12}$$

• ~~$\cot(\theta) = \frac{1}{4}$ $\sin\theta < 0$~~ $\cot\theta = \frac{1}{4}$ $\quad \sin\theta < 0$

~~$\tan\theta = 4$~~ $\quad \tan\theta = 4$

$$1 + \cot^2\theta = \csc^2\theta$$
$$1 + \left(\frac{1}{4}\right)^2 = \csc^2$$
$$1 + \frac{1}{16} = \frac{1}{\sin^2}$$
$$\frac{17}{16} = \frac{1}{\sin^2\theta}$$
$$\sin^2\theta = \frac{16}{17}$$
$$\sin\theta = -\frac{4}{\sqrt{17}} \qquad \csc = -\frac{\sqrt{17}}{4}$$

$\csc\theta = 2$ $\quad\theta$ is in I

$\sin\theta = \frac{1}{2}$

$$\cos^2\theta + \sin^2\theta = 1$$
$$\cos^2\theta + \left(\frac{1}{2}\right)^2 = 1$$
$$\cos^2\theta = 1 - \frac{1}{4}$$
$$\cos\theta = +\sqrt{\frac{3}{4}}$$
$$= +\frac{\sqrt{3}}{2}$$

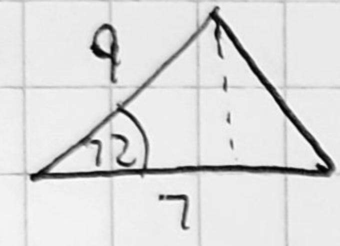
$$\sec\theta = +\sqrt{\frac{4}{3}}$$
$$= \frac{2}{\sqrt{3}}$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$
$$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$
$$\sqrt{3} = \tan\theta \qquad \cot\theta = \frac{\sqrt{3}}{1}$$

$$\text{Area} = \frac{1}{2}bh \longrightarrow = \frac{1}{2}bh$$

$$\sin\theta = \frac{h}{c}$$
$$\sin\theta\, c = h$$

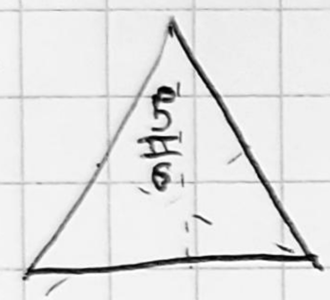
$$= \frac{1}{2}b\sin\theta c$$



$A = \frac{1}{2}bh$

$= \frac{1}{2}(7)(\sin(72)9)$

$= 29.958 \text{ units}^2$



$A = 24$

$24 = \frac{1}{2} b \sin h$

$24 = \frac{1}{2} a \sin a$

$\frac{2(24)}{\sin\frac{5\pi}{6}} = a^2$

$\sqrt{96} = a$

$9.79795897 = a$

Find area of shaded region

$\text{Area} = \frac{1}{2}bh$

$= \frac{1}{2}(12)(12)(\sin\frac{\pi}{3})$

$\text{Area} = 72$

$A = \frac{72\sqrt{3}}{2}$

$A = 36\sqrt{3}$

Area of sector

$= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(12)^2\left(\frac{5\pi}{3}\right)$

$= \frac{1}{2}144\left(\frac{5\pi}{3}\right)$

$= 72\left(\frac{5\pi}{3}\right)$

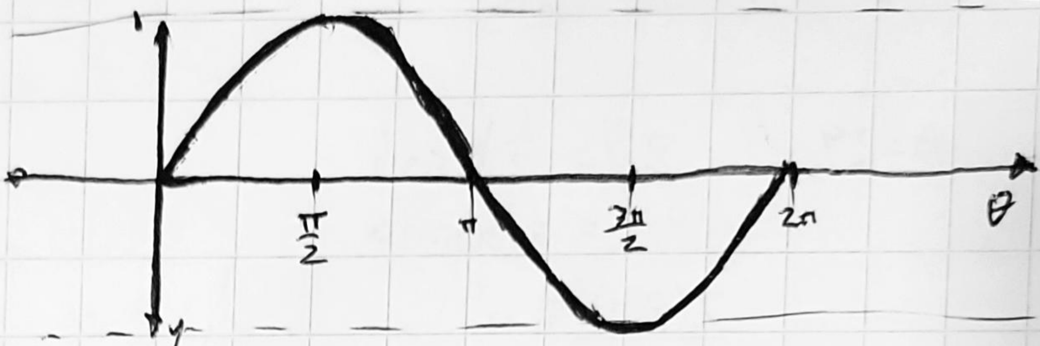
$= 24(5\pi)$

$= 120\pi$

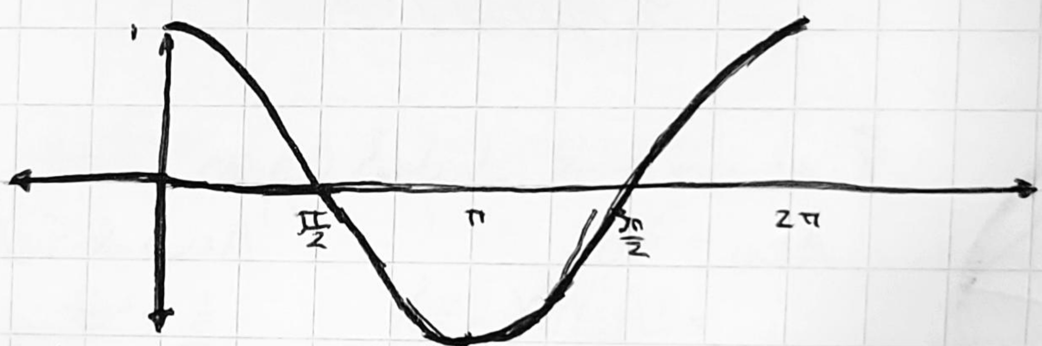
$A_T + A_S = A_{SHADED}$

$36\sqrt{3} + 120\pi = A_{SHADED}$

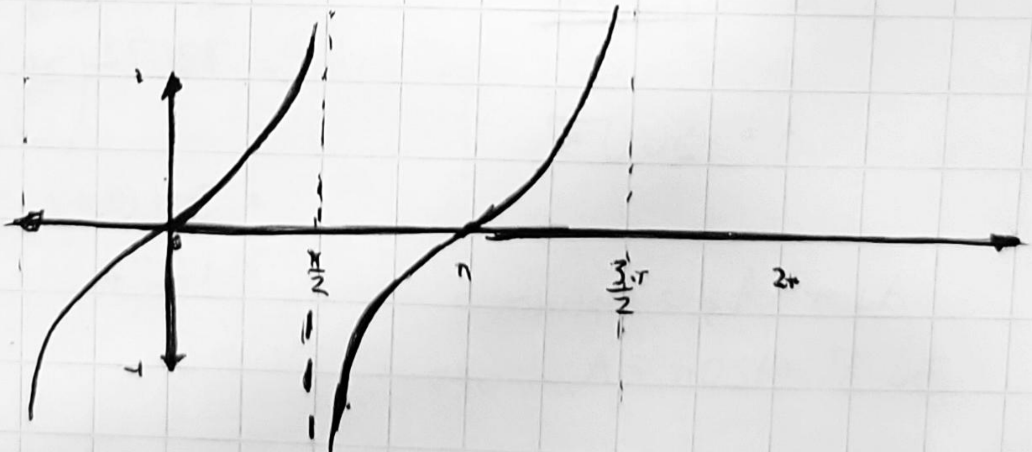
5.3 TRIGONOMETRIC GRAPHS

- $y = \sin(\theta)$
- 


- $y = \sin\theta \rightarrow -1 \leq \sin\theta \leq 1$

- $y = \cos(\theta)$
- 


- $y = \cos\theta \longrightarrow -1 \leq \cos\theta \leq 1$

- $y = \tan x$
- 


- Domain

Range $= (-\infty, +\infty)$

6.4 INVERSE TRIGONOMETRIC FUNCTIONS

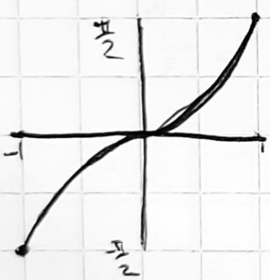
- $y = \sin x$
- sin is not a one to one function cause it fails the horizontal line test
- To get passed this restrict the domain
- $y = \sin x$ + $-\frac{\pi}{2} \leq \sin\theta \leq \frac{\pi}{2}$
- $x = \sin y$

  $\sin^{-1} x = y$, if and only if $x = \sin y$

- $\sin^{-1}(1) = \frac{\pi}{2}$
- $\sin^{-1}(0) = 0$
- $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$
- $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$
- $\sin^{-1}\left(\sin\frac{7\pi}{3}\right) =$

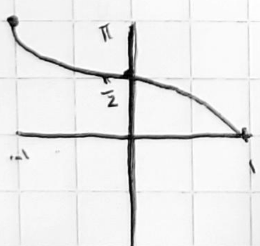
  $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

- $y = \sin^{-1} x$



domain $[-\frac{\pi}{2}, \frac{\pi}{2}]$ of $y = \sin x$
domain $[-1, 1]$ of $y = \sin^{-1} x$

- $y = \cos^{-1} x$



domain = $[0, \pi]$ of $y = \cos x$
domain: $[-1, 1]$ of $y = \cos^{-1} x$

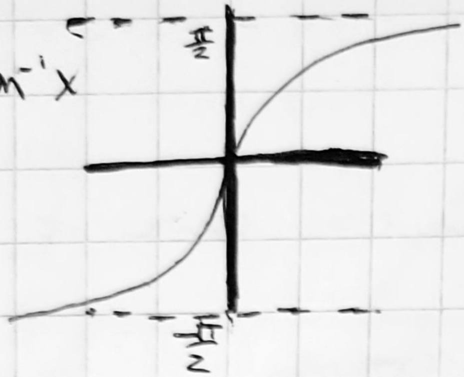
- $\cos^{-1}(0) = y$ $\quad$ $\cos^{-1}(1) = y$ $\quad$ $\cos^{-1}\left(\frac{1}{2}\right) = y$

  $\cos^{-1}(0) = \frac{\pi}{2}$ $\quad$ $\cos^{-1}(1) = 0$ $\quad$ $\cos^{-1}\left(\frac{1}{2}\right) =$ [redacted] $\frac{\pi}{3}$

  $\cos^{-1}\left(-\frac{1}{2}\right) = y$

  $\cos^{-1}\left(-\frac{1}{2}\right) =$ [redacted] $\frac{2\pi}{3}$

• $y = \tan^{-1} x$



Domain of $\tan^{-1} x = (-\infty, \infty)$
Range $= \{ -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

• $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ $\qquad$ $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ $\qquad$ $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

• $\tan\left(\sin^{-1}\left(\frac{4}{5}\right)\right) = \tan(x)$

$x = \sin^{-1}\left(\frac{4}{5}\right)$

$\sin x = \frac{4}{5}$

$\sin^2 x + \cos^2 x = 1$

$\left(\frac{4}{5}\right)^2 + \cos^2 x =$

$\cos^2 x = 1 - \frac{16}{25}$

$\cos x = \pm\sqrt{\frac{9}{25}}$

$\cos x = \pm \frac{3}{5}$

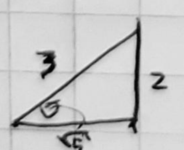
$\tan x = \frac{\sin x}{\cos x}$

$= \frac{\frac{4}{5}}{\frac{3}{5}}$

$= \frac{20}{15}$

$= \frac{4}{3}$

• $\cot\left(\sin^{-1}\left(\frac{2}{3}\right)\right) =$



$2^2 + x^2 = 3^2$

$4 + x^2 = 9$

$x = \sqrt{9-4}$

$x = \sqrt{5}$

$\cot = \frac{\cos\theta}{\sin\theta}$

$= \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}}$

$= \frac{\sqrt{5}}{2}$

• $\sin(\tan^{-1}(x))$ $\qquad$ $\sin x$

$x = \tan^{-1}$

- $\sin(\tan^{-1}(x)) \rightarrow$ Let $\tan^{-1}x = \alpha$, $\tan\alpha = x$

$$\frac{\sin\alpha}{\cos\alpha} = \tan\alpha$$

$$\sin\alpha = \tan\alpha \cos\alpha$$

$$\sin\alpha = x \left(\frac{1}{\sqrt{1+x^2}}\right)$$

$$\sin\alpha = \frac{x}{\sqrt{1+x^2}}$$

$$\sec^2\alpha = 1 + \tan^2\alpha$$

$$\frac{1}{\cos^2\alpha} = 1 + x^2$$

$$\frac{1}{1+x^2} = \cos^2\alpha$$

$$\frac{1}{\sqrt{1+x^2}} = \cos\alpha$$

- $\tan(\sin^{-1}x) \rightarrow$ Let $\sin^{-1}x = \alpha$, $\sin\alpha = x$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$\tan\alpha = \frac{x}{\sqrt{1-x^2}}$$

$$\cos^2\alpha + \sin^2\alpha = 1$$

$$\cos^2 x = 1 - x^2$$

$$\cos^2 x = \sqrt{1-x^2}$$