What Electronics Deals with?

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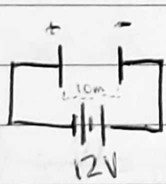
- Current(I), Electric Potential (V)
- These go through the active elements
  - Active Elements - Resistors (R)
    - Capacitors (C)
    - Inductors (L)
    - Diodes
    - Transistors
- Unit of Charge is Coulombs (C)
- Charge is quantized and the unit of charge |e| is the charge of one electron
- Charge $= q = ne$ where $n = 1, 2, 3 \ldots$
- Force $= F = ma$ units $\frac{kg\,m}{s^2}$ or N.
- Work $= W = F\Delta l$ ~~W=Fdl~~ $W = \int F\,dl$ unit = J or $\frac{kg\,m^2}{s^2}$
- Power $= \frac{Energy}{Time} = \frac{dW}{dt}$ units = Watts or $\frac{Joules}{second}$
- Coulomb Force $= F_c = \frac{k q_1 q_2}{r^2}$ $k = \frac{1}{4\pi\epsilon_0}$ ϵ_0 is permitivity of free space $= 8.82 \times 10^{-12} \frac{C^2}{m^2 N}$
- Electric field $= \vec{E} = \frac{F}{Q} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$ units $= \frac{N}{C}$
- Voltage or Potential Difference $dV = \frac{dW}{Q} = \vec{E} \cdot dl$ $V = -\int \vec{E} \cdot dl$
  - Units of Voltage $= \frac{J}{C} = V$ (volts)

negative wants to make work positive.

- 

$\Delta V = \vec{E}\,\Delta d$

$\frac{\Delta V}{\Delta d} = \vec{E}$

$\frac{12}{0.01m} = \vec{E}$

$1200 \frac{V}{m} = \vec{E}$

$F = ma$

$F = qE$

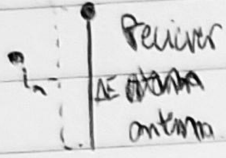
$F = (1.6 \times 10^{-19})(1200)$

$F = 1.9 \times 10^{-16}$ N

$ma = 1.9 \times 10^{-16}$

$a = \frac{1.9 \times 10^{-16}}{9.11 \times 10^{-31}}$

$a \approx 2 \times 10^{14} \frac{m}{s^2}$



$\Delta E = \frac{\Delta V}{\Delta d}$ receives a $1 \frac{mV}{m}$ field

$\Delta V = \Delta E \cdot \Delta d$

$\Delta V = 1 \frac{mV}{m} \cdot 1 m$

$\Delta V = 1 mV \approx 0.001 V$

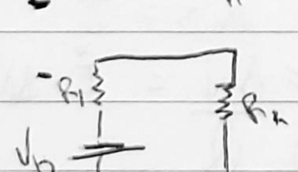
- Current - the time rate at which charge (Q) pass a given cross section in time, t.
  - $I = \frac{\Delta Q}{\Delta t} = \frac{C}{s} = $ Amps

- Power $= \frac{\Delta W}{\Delta t} = \frac{\Delta W}{\Delta Q} \cdot \frac{\Delta Q}{\Delta t} = V \cdot I = VI$

- Resistor   Inductor
  Capacitor   Battery

- Ohms Law $R = \frac{\Delta V}{\Delta I}$ in Ω $R = \rho \cdot \frac{L}{A}$ ρ = resistivity in $\Omega \cdot m$
  L = Length   A = Area

- Battery - Chemical Energy converted to electrical energy
  - Ideal battery has no internal resistance.
  - " " has constant voltage.



$V_b = R_1 I + \underbrace{R_2 I}_{V_L}$

$V_L = V_b - R_1 I$

$I = \frac{V_b}{R_2 + R_1}$ $V_L = IR_2 = \frac{V_b}{1 + \frac{R_1}{R_2}}$ $R_2 = \infty$

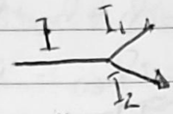
$V_L = V_B$

$V_B = R_1 I + R_2 I$

- Joules Heating Law: $P = VI = \frac{V^2}{R} = RI^2$ $\int dQ = Q = \int_{t_1}^{t_2} P\,dt$

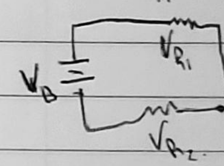
Kirchhoffs

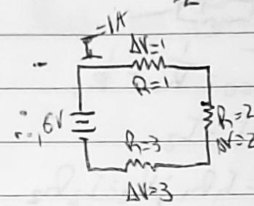
- Kirchhoffs Law - Current Law -



$I_1 + I_2 = I$

- 2nd Law - $V_{R_1} + V_{R_2} + V_B = 0$

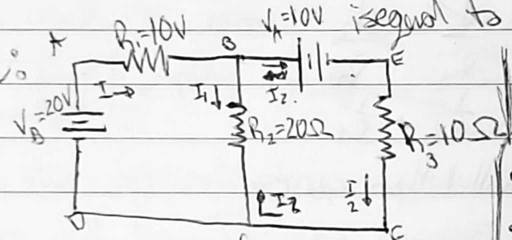


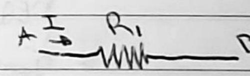


$\Delta V = 1 + 2 + 3 = 6V$

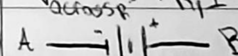
- In any closed loop, potential energy is equal to zero.

- Example:





- $V_{across R_1} = -R_1 I$ (from A to B)
- $V_{across R_1} = +R_1 I$ (from B to A)



- $V_{AB} = +V$
- $V_{BA} = -V$

Loop ABCDA → $-R_1 I - R_2 I_1 + V_B = 0$
$-10I - 20I_1 + 20 = 0$

Loop BEFCB → $-10V - 10I_2 + 20I_1 = 0$

$I - I_1 - I_2 = 0$

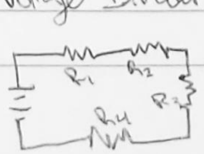
$\begin{cases} -10I + 20I_1 = 20 \\ -10I_2 + 20I_1 = +10 \\ I = I_1 + I_2 \end{cases}$ ~ $\begin{cases} -10(I_1 + I_2)\ 20I_1 = 20 \\ -10I_2 + 20I_1 = 10 \end{cases}$ and

$I_1 = 3A$
$I_2 = 5A$

$\begin{cases} -10I_1 - 10I_2 + 20I_1 = 20 \\ -10I_2 + 20I_1 = +10 \end{cases}$ ~ $\begin{cases} -10I_2 + 10I_1 = 20 \\ -10I_2 + 20I_1 = 10 \end{cases}$

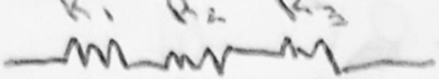
- Easier Way - $20V = 10I_1 + 20I_1 - 20I_2$
$-10V = 10I_2 + 20I_2 - 20I_1$

- Voltage Divider (Resistors in Series) - $V_{R_3} = \frac{R_3}{R_1 + R_2 + R_3 + R_4} \cdot V$



- Add all resistors in denom, voltage across a desired resistor on top with voltage.

- Resistor in Series and Parallel.
  - Series -



    - $R_{EQ} = R_1 + R_2 + R_3$.
    - $V = iR_{EQ} = i(R_1 + R_2 + R_3)$.
  - Parallel -

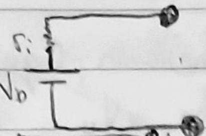
    - $\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2}$.
    - $I = I_1 + I_2$
    - $V_1 = I_1 R_1$; $V_2 = I_2 R_2$
    - $\cancel{\Delta V = V_1 + V_2 = I_1 R_1 + I_2 R_2}$ $\Delta V = I R_{EQ}$
    - $\frac{\Delta V}{R_{EQ}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2}$.

$= \frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2}$

- Solve parallel then series.

Voltage Source vs. Current Source

- Voltage Source -



battery is the voltage source here.

- Current Source -

Used when a constant current is needed.

Network Simplification

- This is used to make complicated circuits simple
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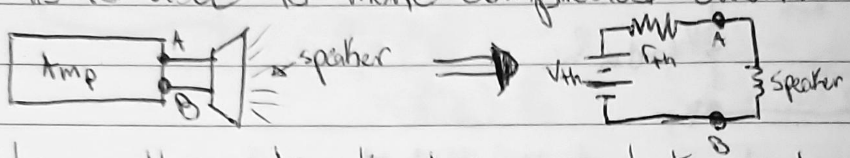
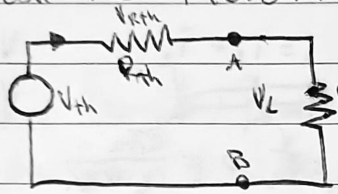


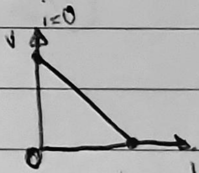
- We say this network has equivalent resistance and voltage which we call the Thevenin Equivalent.
- Example:



- Experimental determination of V_{th} and R_{th}: Remove load and measure with ohm-meter & measure voltage with voltmeter.

$$V_{th} + V_{Rth} + V_L = 0$$
$$V_{th} - iR_{th} - V_L = 0$$
$$V_L = V_{th} - iR_{th}$$

When $i = 0$, $V_L = V_{th}$ ⇒
When $V_L = 0$, $V_{th} = iR_{th}$



- Theoretical Calculation of V_{th} & R_{th}
  - V_{th} - Remove the Load and measure the potential difference from A-B
  - R_{th} - To determine R_{th}, one short circuit the voltage circuit, then calculate the equivalent resistor of the circuit.
- Find the Thevenin Equivalent of the following circuit (V_{th} & R_{th})

$V_{AB} = V_{th}$

$V_A = V_{BAT} \cdot \frac{R_B}{R_A + R_B}$

$V_A = 2 \cdot \frac{3}{4}$

$V_A = \frac{3}{2} V$

$V_B = V_0 \cdot \frac{R_D}{R_C + R_D}$

$V_B = 2 \cdot \frac{4}{6}$

$V_B = \frac{4}{3} V$

$V_{th} = V_A - V_B$

$V_{th} = \frac{3}{2} - \frac{4}{3}$

$V_{th} = \frac{9}{6} - \frac{8}{6}$

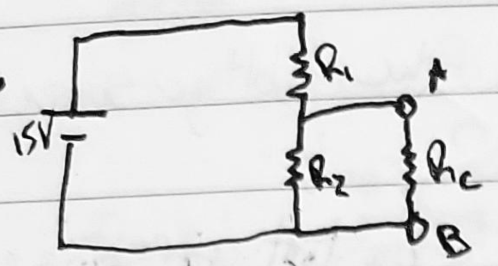
$V_{th} = \frac{1}{6} V$

$R_{th} = R_{eq_1} + R_{eq_2}$

$R_{th} = \frac{3}{4} + \frac{4}{3}$

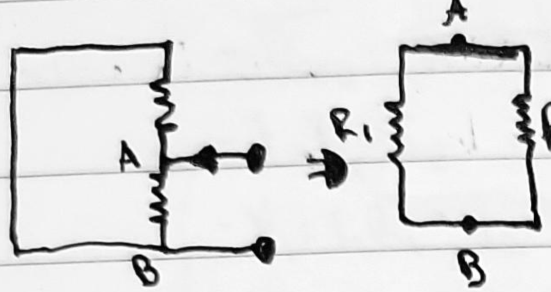
$R_{th} = \frac{25}{12} \Omega$

Example

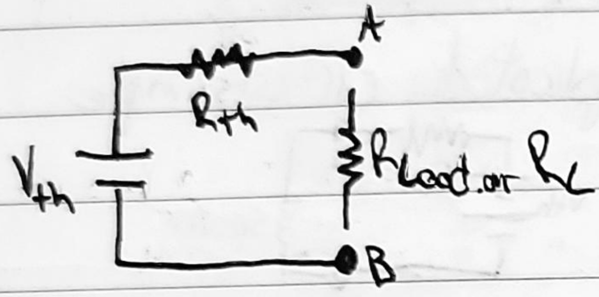


- Find Thevenin Equiv.

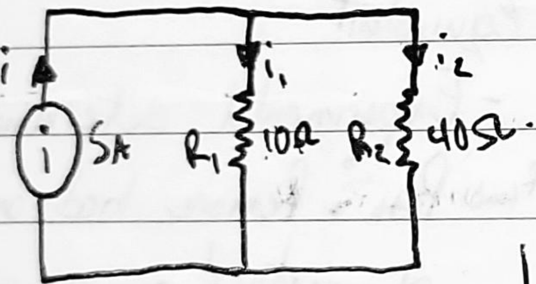
$V_{AB} = V\frac{R_2}{R_1+R_2} = V_{th}$ } Thevenin Voltage



$R_{eq} = \frac{R_1 R_2}{R_1+R_2} = R_{th}$ } Thevenin Resistance.



- Determine the current through both resistors



$i = i_1 + i_2$

$\cancel{i = \frac{V}{R_1} + \frac{V}{R_2}}$

$I_1 = \frac{V}{R_1} \Rightarrow I_1 = i\frac{R_1R_2}{(R_1+R_2)R_1}$

$I_1 = i\frac{R_2}{R_1+R_2}$

$V = IR_{eq}$ $R_{eq} = \frac{R_1R_2}{R_1+R_2}$

$V = I\frac{R_1R_2}{R_1+R_2}$

$I_2 = \frac{iR_1}{R_1+R_2}$

$i = \frac{iR_2}{R_1+R_2} + \frac{iR_1}{R_1+R_2}$

Maximum Power transfer

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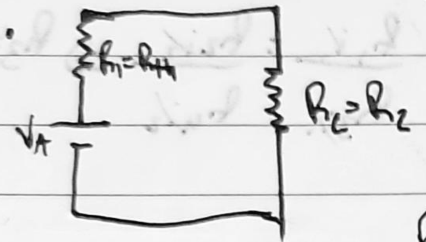
- We want to connect a load to the network, what should be the network resistance (impedance) in order to have the maximum power transfer to the load

- 

$$P = \frac{V^2}{R} = I^2 R = V_A I \qquad I = \frac{V_A}{R_1 + R_2}$$

$$P_L = I^2 R_2 = \left(\frac{V}{R_1 + R_2}\right)^2 R_2 = \frac{V^2}{(R_1 + R_2)^2} R_2$$

$y = f(x) \rightarrow y' = f'(x)$

$$\frac{dP}{dR_2} = V_A^2 \left[\frac{-2R_2(R_1+R_2)}{(R_1+R_2)^4} + \frac{1}{(R_1+R_2)^2} \right]$$

$$= V_A^2 \left[\frac{-2R_2}{(R_1+R_2)^3} + \frac{1}{(R_1+R_2)^2} \right]$$

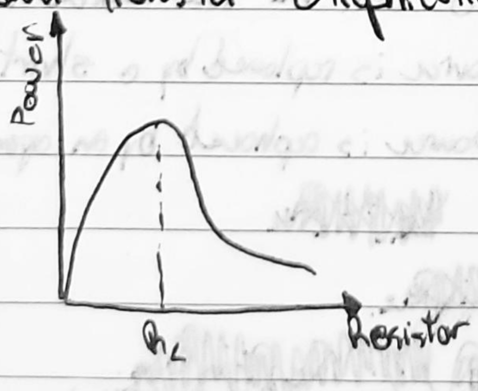
$$= V_A^2 \left[\frac{-2R_2 + R_1 + R_2}{(R_1+R_2)^3} \right]$$

$$= V_A^2 \left[\frac{R_1 - R_2}{(R_1+R_2)^3} \right] \quad R_1 = R_2$$

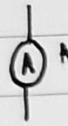
The P_{max} transfers from source to Load when the Load resistance is equal to source resistance.

$$P = i_2 R_L = \frac{V^2}{4R_L}$$

- Power Transfer Graphically



The Wheatstone Bridge.

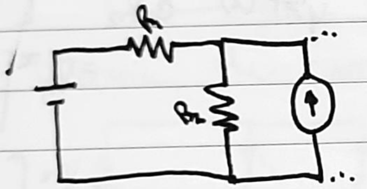


- R_4 is a variable resistor
- We change R_4 til no current flows through ammeter
  - $\to i_2 = 0,\ i_1 = i$
- Potential across CD, $V_{CD} = V_{Battery}$.
- $V_B - V_C = V_B - V_D$

$R_1 i_1 = R_2 i_2 \Rightarrow \frac{R_1 \cancel{i_1}}{R_3 \cancel{i_1}} = \frac{R_2 \cancel{i_2}}{R_4 \cancel{i_2}} \Rightarrow R_3 = \frac{R_1 R_4}{R_2}$

$R_3 i_1 = R_4 i_2$

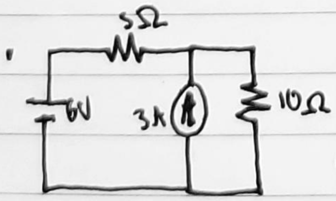
Solving Complex Circuits



Q: Measure current through R.

- Superposition Principle: Tells us that one can solve a network problem by shutting off all sources with the exception of one. Solve the problem for this source, then continue for all other sources. Finally, add up the results of all ~~th~~ the sources.

(Voltage Source is replaced by a short circuit, Current Source is replaced by an open circuit)



$V = V_1 + V_2$

$6 = iR_1 + iR_2$ $\qquad V_5 = V_{10}$

$6 = i15$ $\qquad i_1 R_5 = i_2 R_{10}$

$\frac{6}{15} = i_1$ $\qquad i_1 = i_2 \cdot 2$

$i_1 + i_2 = i \Rightarrow i_1 = 2(3) - 2i_1 \Rightarrow i_1 = 2A$

$= 2 - \frac{6}{15}$

$= \frac{30}{15} - \frac{6}{15} \Rightarrow \frac{24}{15} \Rightarrow \frac{8}{5} \Rightarrow 1.6A$

Example

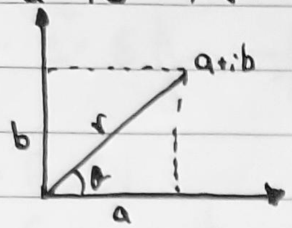
- A resistor at 20°C $R(20) = 1000\ \Omega$. What is the resistance at 100°C? ; $\alpha_c = -0.0005\ \frac{\Omega}{°C}$ (; α_c is the coefficient of thermal conductivity).

$R = R_0(1 + \alpha\Delta T)$.

$= 1000(1 + (-0.0005)(80))$

$= 960\ \Omega$.

- $a + ib \quad i = \sqrt{-1}$



$a + ib = r(\cos\theta + i\sin\theta) = re^{i\theta}$

$\theta = \arctan\left(\frac{b}{a}\right) \qquad r = \sqrt{a^2 + b^2}$

$\frac{1}{i} = \frac{1}{i} \cdot \frac{i}{i} = -i$

$\frac{1}{a+ib} = \frac{1}{a+ib} \cdot \frac{a-ib}{a-ib} = \frac{a-ib}{a^2+b^2}$

- $\sin^2\theta + \cos^2\theta = 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

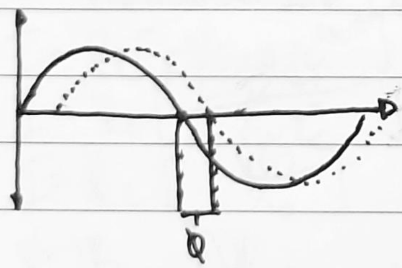
- $f(\theta) = \sin\theta \qquad f'(\theta) = \cos\theta$

$f(\theta) = \cos\theta \qquad f'(\theta) = -\sin\theta$

$f(\theta) = \tan\theta \qquad f'(\theta) = \sec^2\theta$.

AC Circuits

$V = V_s \sin(\omega t) \qquad \omega = 2\pi f \Rightarrow 2\pi \frac{1}{T} \left(\frac{rad}{s}\right)$

- 

$V = V_s \sin(\omega t - Q)$

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$V = V_s \sin(\omega t - Q) \qquad \omega = \frac{2\pi}{10} = 0.63$

$V = V_s \sin(\omega t - Q)$

$10 = 10 \sin(0 - Q)$

$1 = \sin(-Q)$

$-\frac{\pi}{2} = Q$

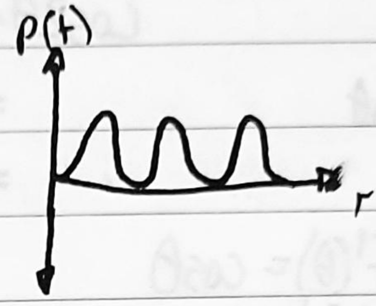
$V = V_s \sin(\omega t - Q)$

$V = 10 \sin(0.63t + \frac{\pi}{2})$

Resistor in AC Circuit.

- $R = \frac{V}{I}$ $\quad R = \frac{\rho L}{A}$

- Series $\rightarrow R_{eq} = R_1 + R_2 + R_3 \dots$
  Parallel $\rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \dots$
- $V = V_s \sin(\omega t)$
  $\frac{V}{R} = \frac{V_s}{R} \sin(\omega t)$
  $I = I_s \sin(\omega t)$
- $P(t) = \frac{V_s^2}{2R}(1 - \cos 2\omega t) \rightarrow$

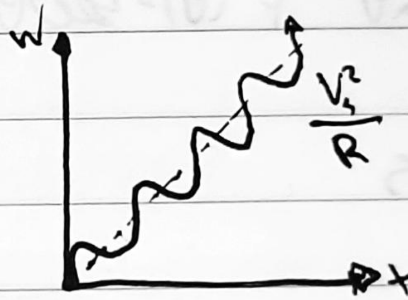


- $\int_0^t P(t)\,dt = \int_0^t \frac{V_s^2}{R}\left(\frac{1}{2} - \cos 2\omega t\right)dt$

$$\Downarrow$$

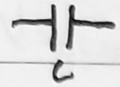
$$W = \frac{V_s^2}{R}\left[\frac{t}{2} - \frac{1}{2}\sin 2\omega t \frac{1}{200}\right]$$

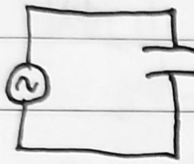
$$W = \frac{V_s^2}{R}\left[\frac{t}{2} - \frac{1}{2}\sin t\right]$$



- $\overline{P(t)} = \frac{V_s}{2} \cdot I_s$
  $V_{rms} = \frac{V_s}{\sqrt{2}}$
  $I_{rms} = \frac{I_s}{\sqrt{2}}$
  $\overline{P(t)} = V_{rms} I_{rms} = \frac{V_s I_s}{2}$

Capacitors in AC Circuit

-  f, $mf = 10^{-3} f$, $\mu f = 10^{-6} f$, $nf = 10^{-9} f$, $pf = 10^{-12} f$
- $C = \varepsilon \frac{A}{d}$  $\varepsilon_0 = \varepsilon_0 \varepsilon_r$, A = area of plates, d = distance between the capacitors.
- $Q = CV$
- Series $\rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \ldots$

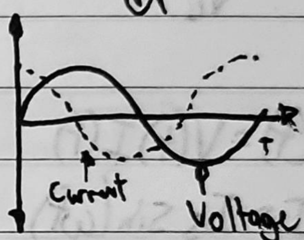
  Parallel $\rightarrow C_{eq} = C_1 + C_2 \ldots$
- 

$$i = \frac{dQ}{dt} = \frac{d(CV)}{dt} = C\frac{dV}{dt} = C\frac{d\,V_s\sin(\omega t)}{dt}$$

$$= CV_s\cos(\omega t)\omega$$

$$= C\omega V_s\cos(\omega t)$$

$$i = C\omega V_s\sin\left(\omega t + \frac{\pi}{2}\right)$$



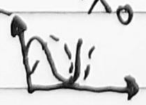
Capacitance Reactance

- $V = V_s\cos(\omega t) = Re\left[V_s e^{i\omega t}\right]$
- $I = CV_s\,\omega\cos\left(\omega t + \frac{\pi}{2}\right)$

  $I = Re\left[CV_s\omega e^{i\omega t + \frac{\pi}{2}}\right]$

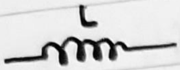
  $I = Re\left[CV_s\omega\, i e^{i\omega t}\right]$

  $I = Re\left[\frac{CV_s e^{i\omega t}}{i\omega C}\right]$

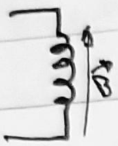
  $I = Re\left[\frac{V(t)}{X_0}\right]$
- $V = IR$ 

  $V = IX_0$ where $X_0 = \frac{1}{i\omega C}$

Inductors in AC Circuits

-  $L = \frac{\Phi}{I}$ [=] H

  $V = \frac{d\Phi}{dt}$ $\quad \Phi = \int_0^t V\,dt \quad I = \frac{\Phi}{L}$

- 

- $I = \frac{\Phi}{L} = \frac{1}{L}\int_0^t V\,dt = \frac{1}{L}\int_0^t V_s \cos(\omega t)\,dt = \frac{V_s}{L}\left(\sin(\omega t)\frac{1}{\omega}\right)$

- $I = \frac{V_s}{\omega L}\sin(\omega t) = \frac{V_s}{\omega L}\cos\left(\omega t - \frac{\pi}{2}\right)$

- $P(t) = V(t)I(t)$

  $I = I_s \sin(\omega t)$

  $V = \frac{d\Phi}{dt} \quad L = \frac{\Phi}{I}$

  $V = \frac{d(LI)}{dt}$

  $V = \frac{d(I_s \sin \omega t)}{dt}.$

  $V = L I \omega \cos(\omega t)$

- $W = \int_0^t P(t)\,dt = \int_0^t V(t)I(t)\,dt$

  $W = L\int_0^t \frac{dI}{dt}(I(t))\,dt.$

  $W = \frac{LI^2}{2}.$

Inductance Reactance

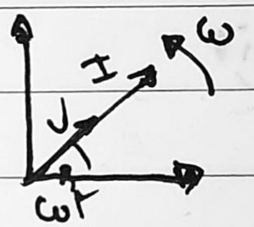
- $V = V_s\cos(\omega t)$ $V = \frac{dQ}{dt}$ $L = \frac{Q}{t}$ $I = \frac{Q}{L} = \frac{1}{L}\int_0^t V\,dt = \frac{1}{L}\int_0^T V_s\cos(\omega t)$

- $I = \frac{V_s}{\omega L}\sin(\omega t) = -\frac{V_s}{\omega L}\cos(\omega t - \frac{\pi}{2})$

$I = \mathrm{Re}\left[\frac{V_s}{\omega L} e^{i\omega t}\right]$, $I = \mathrm{Re}\left[\frac{V_s}{\omega L}\right]$

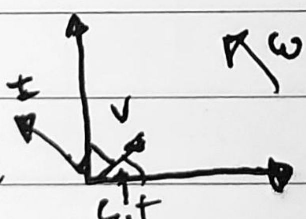
$I = \frac{\mathrm{Re}[\ ]}{i\omega L}$ $I = \mathrm{Re}\left[\frac{V(t)}{X_0}\right]$

- R  $V = IR$



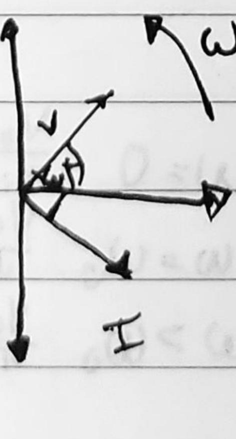
C  $V = IX_C$

$X_C = \frac{1}{i\omega C}$



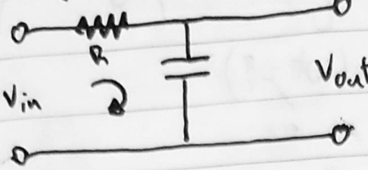
L  $V = IX_L$

$X_L = i\omega L$



RC Filters

- Lowpass RC Filter



$V_{in} = IR + V_{out}$

$V_{in} = IR + \frac{I}{i\omega C}$

$V_{in} = I\left(R + \frac{1}{i\omega C}\right)$

$\frac{V_{in}}{R + \frac{1}{i\omega C}} = I$

$$\frac{V_{out}}{V_{in}} = \frac{1}{j\omega RC + 1}$$

$a = 1$

$b = \omega RC$

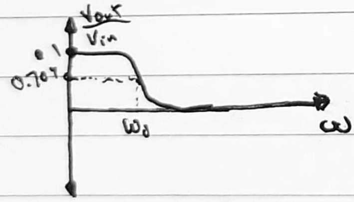
$\omega_0 = \frac{1}{RC}$

$$= \frac{1 - j\omega RC}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$$= \frac{\sqrt{1 + \omega^2 R^2 C^2}}{1 + \omega^2 R^2 C^2} e^{i\theta}$$

$$= \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} e^{i\theta}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}} e^{i\theta}$$



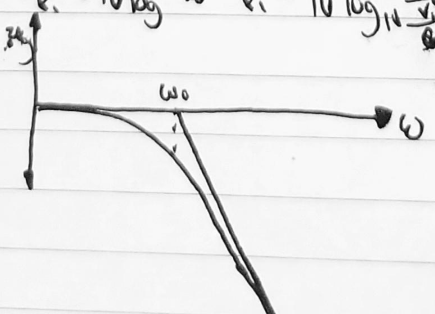
$\omega = 0 \quad \frac{V_{out}}{V_{in}} = 1$

$\omega = \omega_0 \quad \frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{2}} = 0.707$

$\omega > \omega_0 \quad \frac{V_{out}}{V_{in}} = 0$

Power Gain

- $\frac{P_2}{P_1} = 10 \log \Rightarrow \frac{P_2}{P_1} = 10 \log_{10} \frac{V_2^2 / R}{V_1^2 / R} = 20 \log_{10} \frac{V_2}{V_1}$



$$\left|\frac{V_{out}}{V_{in}}\right|^2 = \left|\frac{V_{out}}{V_{in}}\right| \cdot \left|\frac{V_{out}}{V_{in}}\right|$$

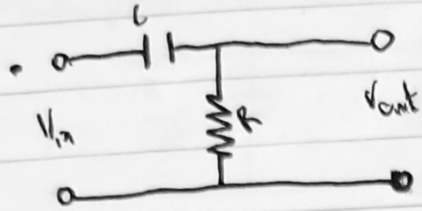
$$= \frac{1}{1 - i\omega RC} \cdot \frac{1}{1 + i\omega RC}$$

$$= \frac{1}{1 + \omega^2 R^2 C^2}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{\frac{\omega}{\omega_0}}}$$

HighPass RC Filter



$$V_{in} = \frac{I}{i\omega C} + V_{out} \Rightarrow \frac{I}{i\omega C} + IR$$

$$V_{in} = I\left(\frac{1}{i\omega C} + R\right)$$

$$I = \frac{V_{in}}{\frac{1}{i\omega C} + R}$$

$$\frac{V_{out}}{V_{in}} = \frac{IR}{V_{in}}$$

$$= \frac{\frac{\cancel{V_{in}}}{\frac{1}{i\omega C}+R}\cdot R}{\cancel{V_{in}}}$$

$$= \frac{R}{\frac{1}{i\omega C}+R}$$

$$\frac{V_{out}}{V_{in}} = \frac{i\omega RC}{1+i\omega RC}\cdot\frac{1-i\omega RC}{1-i\omega RC}$$

$$= \frac{i\omega RC + \omega^2 R^2 C^2}{1+\omega^2R^2C^2}$$

$$= \frac{\omega RC(i+\omega RC)}{1+\omega^2R^2C^2}$$

$$\frac{V_{out}}{V_{in}} = \frac{\omega RC}{1+\omega^2R^2C^2}\sqrt{1+\omega^2R^2C^2}\, e^{i\theta}$$

$$= \frac{\omega RC}{\sqrt{1+\omega^2R^2C^2}}\cdot e^{i\theta}$$

$$= \frac{1}{\sqrt{1+\frac{1}{\omega^2R^2C^2}}}\cdot e^{i\theta}$$

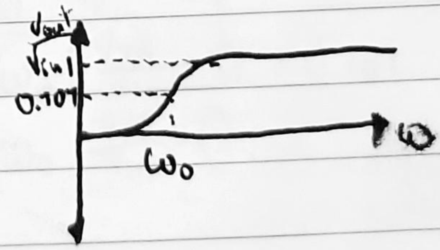
$$= \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}\, e^{i\theta}$$

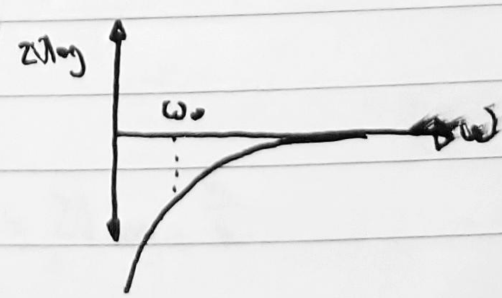
$$\left|\frac{V_{out}}{V_{in}}\right|^2 = \left|\frac{-i\omega RC}{1-iRC\omega}\right|\cdot\left|\frac{i\omega RC}{1+i\omega RC}\right|$$

$$= \frac{\omega^2R^2C^2}{1+\omega^2R^2C^2}$$

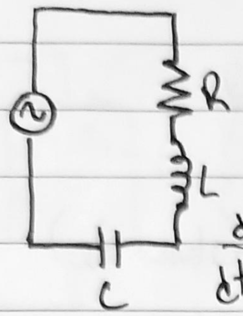
$$= \frac{1}{1+\frac{1}{\omega^2R^2C^2}}$$

$$= \frac{1}{\sqrt{1+\frac{1}{\omega^2R^2C^2}}}$$





RLC Circuits



$V = V_0\cos(\omega t)$

$V = IR + I i\omega L + \frac{I}{i\omega C}$

$V = IR + L\frac{dI}{dt} + \frac{Q}{C}$

$\frac{d}{dt}\left(V_0\cos(\omega t) = IR + L\frac{dI}{dt} + \frac{Q}{C}\right)$

$-V_0\sin(\omega t) = R\frac{dI}{dt} + L\frac{d^2I}{dt^2} + \frac{1}{C}\frac{dQ}{dt}$

$-V_0\sin(\omega t) = R\frac{dI}{dt} + L\frac{d^2I}{dt^2} + \frac{1}{C}I$

- $I = I_0\cos(\omega t)$

$I = \frac{V_0\cos(\omega t)}{R + i(\omega L + \frac{1}{\omega C})}$

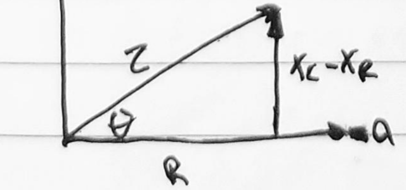
$= Re\left[\frac{V}{R + i(\omega L + \frac{1}{\omega C})}\right] \cdot \frac{R - i(\omega L - \frac{1}{\omega C})}{R - i(\omega L - \frac{1}{\omega C})}$

$= \frac{V(R - i(\omega L - \frac{1}{\omega C}))}{R^2 + (\omega L - \frac{1}{\omega C})^2}$

$= \frac{V\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}{R^2 + (\omega L - \frac{1}{\omega C})^2} e^{i\theta}$

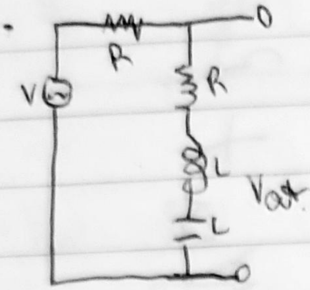
$= \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} e^{i\theta}$

- X_C X_L $Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$



RLC in Series

• $\omega_0 = \frac{1}{\sqrt{LC}}$



$V_{in} = V_{R_1} + V_R + V_L + V_C$

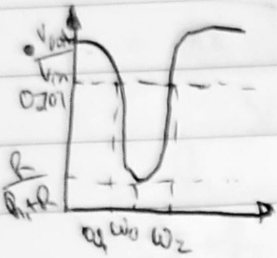
$V_{in} = IR_1 + IR + IX_L + IX_C$

$\frac{V_{in}}{V_{out}} = \frac{IR_1 + IR + IX_L + IX_C}{IR + IX_L + IX_C}$

$\frac{V_{out}}{V_{in}} = \frac{R + X_L + X_C}{R_1 + R + X_L + X_C}$ $\quad X_L + X_C = i(\omega L + \frac{1}{\omega C})$

$\frac{V_{out}}{V_{in}} = \frac{R + i(\omega L - \frac{1}{\omega C})}{R_1 + R + i(\omega L - \frac{1}{\omega C})}$

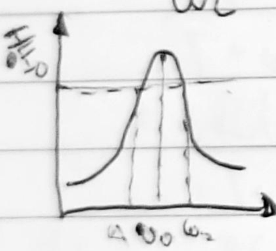
$\frac{V_{out}}{V_{in}} = \frac{R}{R_1 + R}$



$\Delta\omega = \omega_2 - \omega_1 = B =$ Bandwidth.

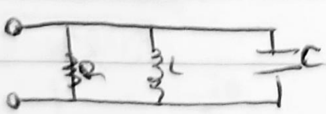
• Q = Quality factor

$Q = 2\pi \frac{\omega_s}{\omega_L} = \frac{\omega_0}{\Delta\omega}$



Admittance

• $y = \frac{1}{Z}$



$y = \frac{1}{R} + \frac{1}{i\omega L} + i\omega C$

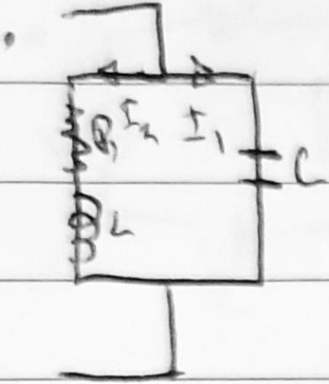
$y = \frac{1}{R} + i(\omega C - \frac{1}{\omega L})$

$y = \frac{G + iB}{G^2 + B^2}$

G - Conductance

B - Susceptance.

Example



$R_1 = 20$

$L = 1\,mH$

$C = 1\,nF$

$Z_0 = ?$ $\quad \omega = \omega_0$

$$I = I_1 + I_2$$

$$\frac{V}{Z} = \frac{V}{R + X_L} + \frac{V}{X_C}$$

$$\frac{1}{Z} = \frac{1}{R + i\omega L} + i\omega C.$$

$$Y = i\omega C + \frac{R - i\omega L}{R^2 + \omega^2 L^2}$$

$$Y = i\omega C + \frac{R}{R_1^2 + \omega^2 L^2} - \frac{i\omega L}{R^2 + \omega^2 L^2}$$

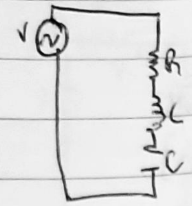
$$Y = \frac{R_1}{R^2 + \omega^2 L^2} + i\left[\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right]$$

$\vdots$

$$Z_0 = 50.02\,k\Omega.$$

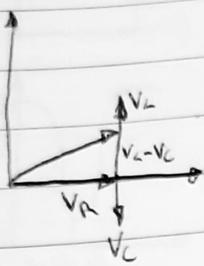
Power Factor

- RLC



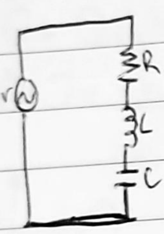
$$\overline{P(t)} = V_{RMS} I_{RMS} \underbrace{\cos(\phi)}_{\text{Power Factor}}$$

$$\overline{P(t)} = V_{rms} I_{rms} \frac{R}{2}$$

- 

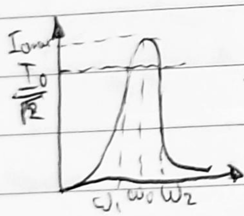
$V_R \Rightarrow \phi = 0 \quad \cos\phi = 1 \Rightarrow \bar{P} = \max$

$V_{L \text{ or } C} \Rightarrow \phi = \pm\frac{\pi}{2} \quad \cos(\frac{\pi}{2}) = 0 \Rightarrow \bar{P} = 0$

- 

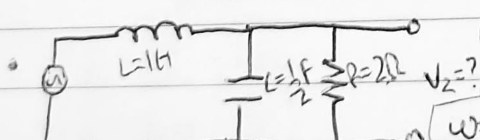
$$I = \frac{V_0 e^{i\omega t}}{R + i\omega L + \frac{1}{i\omega C}} = I e^{i\omega t}$$

$$I_0 = \frac{V_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \qquad Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{\omega_2 - \omega_1}$$



$$I_0 = \frac{I_{0max}}{\sqrt{2}} = \frac{\frac{V_0}{R}}{\sqrt{2}} = \frac{V_0}{R\sqrt{2}}$$

Example

- 

$$V = \cos(2t)$$

$\omega = 2 \quad V_2 = ?$

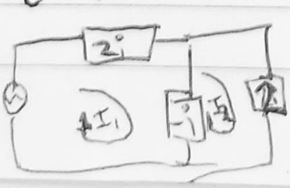
$\tan^{-1}\left(\frac{-1}{-1}\right) = \frac{\pi}{4}$

$V = Re[e^{i2}] = Re[e^{i0}] = 1$

$X_L = i\omega L = 2i$

$X_C = \frac{1}{i\omega C} = \frac{1}{i(2)(\frac{1}{2})}$

$X_C = -i$



$I = 2iI_1 - iI_1 + I_2$

$I = I_1 i + I_2 i$ ①

$0 = -iI_2 + 2I_2 + iI_1$

$0 = iI_1 + (2+i)I_2$ ②

① - ② $= 2iI_2 + 2I_2$

$I_2 = \frac{i}{4}(-1-i)$

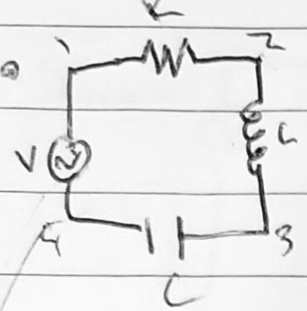
$V_2 = Re[V e^{i\omega t}]$

$= Re[R I e^{i2t}]$

$= Re[\frac{\sqrt{2}}{2} e^{i(2t + \frac{\pi}{4})}]$

$= \frac{\sqrt{2}}{2} \cos(2t + \frac{\pi}{4})$

Example



$V_0 = 150V$ $\quad L = 80mH$ $\quad V_{R0} = ?$ $\quad V_{24} = ?$

$\omega = 100 \frac{rad}{s}$ $\quad C = 50\mu F$ $\quad V_{L0} = ?$

$R = 40\Omega$ $\quad V_{C0} = ?$

$X_L = \omega L = (100)(80\times10^{-3}) = 8\Omega$ $\qquad Z = \sqrt{R^2 + (8-200)^2}$

$X_C = \frac{1}{\omega C} = \frac{1}{(100)(50\times10^{-6})} = 200\Omega$ $\qquad Z = 196.12\Omega$

$I_0 = \frac{V_0}{Z} = \frac{150}{196.12} = 0.765A$ $\qquad V_{R0} = RI_0 = 30.6V$ $\qquad V_{L0} = X_L I_0 = 6.12V$ $\qquad V_{C0} = X_C I_0 = 153V$

$V_{24} = 6.12 \cancel{+} 153$
$= 146.88V$

$R = 40\Omega$ $\quad V_0 = 40V$ $\qquad z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

$L = 160mH$ $\quad \omega = 200 \frac{rad}{s}$ $\qquad z = 43.86\Omega$

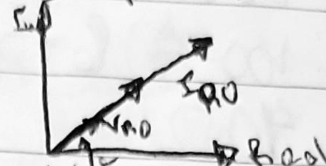
$C = 100\mu F$ $\quad z = ?$

$I_0 = \frac{V_0}{z} = 0.912A$

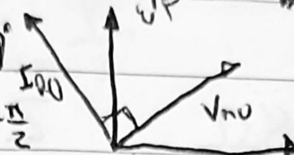
$\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) = -24.2°$

Summary

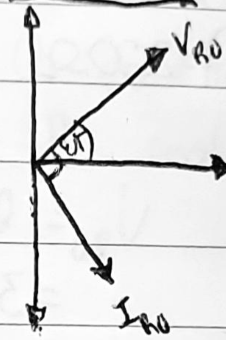
- AC $V(t)=V_S\sin(\omega t)$ $I=I_S\sin(\omega t-\phi)$
- R: $I_{R0}=\frac{V_{R0}}{R}$ $\phi=0$



- C: $I_{R0}=\frac{V_{R0}}{X_C}$ $\phi=-\frac{\pi}{2}$



- L: $I_{R0}=\frac{V_{R0}}{X_L}$ $\phi=+\frac{\pi}{2}$

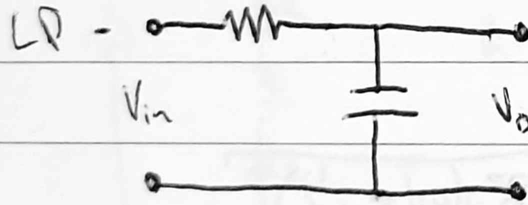


$V=\frac{dQ}{dt}$, $L=\frac{\phi}{I}$

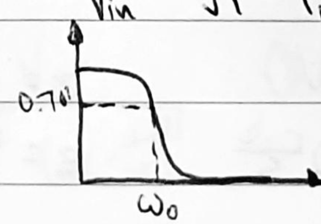
$V=\frac{d(LI)}{dt}=L\frac{dI}{dt}$

- RC Filters

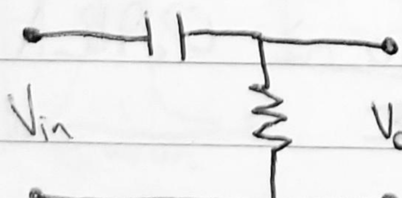
LP -



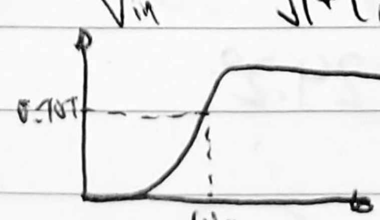
$\frac{V_{out}}{V_{in}}=\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}\cdot e^{i\theta}$ $\theta=\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$



HP -

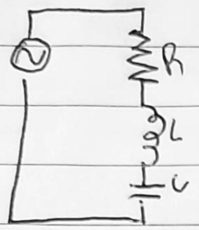


$\frac{V_{out}}{V_{in}}=\frac{1}{\sqrt{1+\left(\frac{\omega_0}{\omega}\right)^2}}\cdot e^{i\theta}$ $\theta=\tan^{-1}\left(\frac{\omega_0}{\omega}\right)$



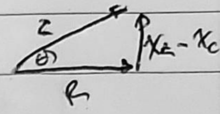
RLC Circuit

- Series -

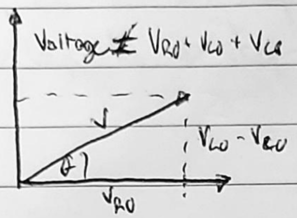


$$I = \frac{V e^{i\theta}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

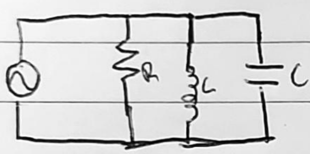


$$\theta = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

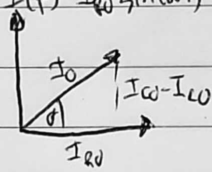


$$V = \sqrt{V_{R0}^{\,2} + (V_{L0} - V_{C0})^2}$$

- Parallel -



$$I(t) = I_{R0}\sin(\omega t) + I_{L0}\sin\left(\omega t - \frac{\pi}{2}\right) + I_{C0}\sin\left(\omega t + \frac{\pi}{2}\right)$$

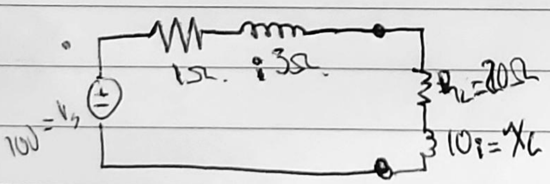


$$I_0 = \sqrt{I_{R0}^{\,2} + (I_{C0} - I_{L0})^2}$$

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$$

$$\theta = \tan^{-1}\left(R\left(\cancel{X_C} - \frac{1}{X_L}\right)\right)$$

Example



$$100 = 1I + i3I + 20I + i10I$$

$$I = \frac{100}{21 + 13i}$$

$$I = \frac{100}{21+13i} \cdot \frac{(21-13i)}{(21-13i)} = \frac{100(21-13i)}{21^2 + 13^2}$$

$$I = 3.44 - 2.13i$$

$$I = c e^{i\theta} \qquad c = \sqrt{a^2 + b^2}$$

$$I = 4.04 e^{i31.8°} \qquad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$P_{line} + P = 171.61$$

$$V_{load} = I Z_{LOAD}$$
$$= (3.44 - 2.13i)(20 + i10)$$
$$= 90.1 - 8.2i$$

$$V_{load} = 90.47 e^{i5.2°}$$

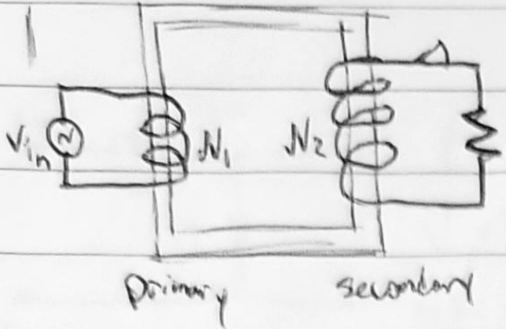
$$P = \frac{1}{2} I^2 R$$
$$P = \frac{1}{2} I I^* R$$
$$P = \frac{1}{2} V I^*$$
$$P = \frac{1}{2}\mathrm{Re}\left[V_{load} I^*\right]$$
$$P = \frac{1}{2}\mathrm{Re}\left[(90.47e^{i5.2})(4.04e^{i31.8})\right]$$

$$P = \frac{1}{2}\mathrm{Re}\left[365.5 e^{i26.6}\right]$$
$$P = 163.41\,W$$

$$P_{line} = \frac{1}{2} I^2 R^{line}$$
$$P_{line} = \frac{1}{2} 4.04^2$$
$$= 8.2\,W$$

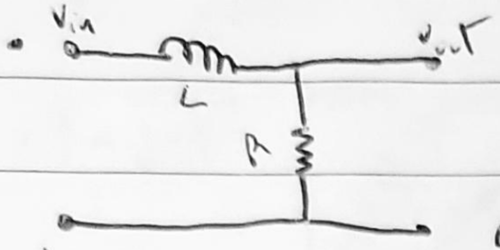
$$P_{source} = \frac{1}{2}\mathrm{Re}\left[V_s I^*\right]$$
$$P_{source} = 171.65\,W$$

Transformers



- $V_{in} = -N_1 \frac{d\Phi}{dt}$ $\quad V_{out} = -N_2 \frac{d\Phi}{dt}$.
- $\frac{V_{in}}{V_{out}} = \frac{N_1}{N_2} \Rightarrow \frac{I_{in}}{I_{out}} = \frac{N_2}{N_1} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{I_{in}}{I_{out}}$.
- $\frac{V_{out}}{V_{in}} = \frac{N_2}{N_1}$, $N_2 > N_1 \Rightarrow V_{out} > V_{in} \Rightarrow$ step-up transformer
  , $N_2 < N_1 \Rightarrow V_{out} < V_{in} \Rightarrow$ step-down "
-

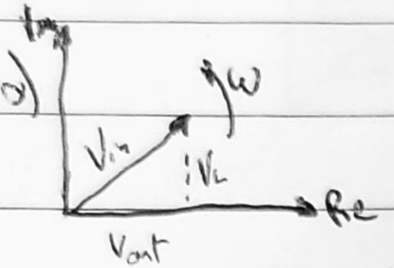
RL Filter

- 

$L = 400\,mH$ a) R when 30° lag?

$V_{in} = 20 \sin \omega t$ b) $\frac{V_{out}}{V_{in}}$?

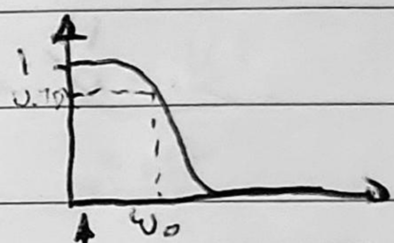
$\omega = 200 \frac{rad}{s}$ c) $\frac{V_{out}}{V_{in}}$ for swapped R & L?

- a) 

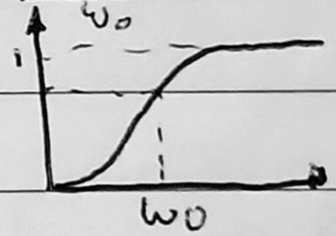
$$\tan 30 = \frac{V_L}{V_{out}} = \frac{I X_L}{I R} = \frac{\omega L}{R}$$

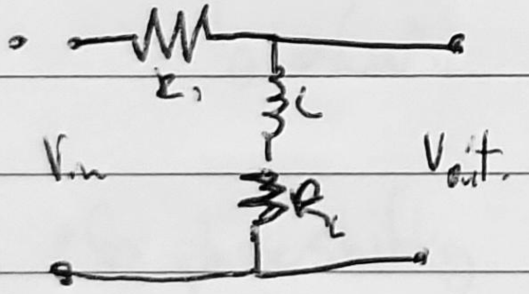
$$R = \frac{\omega L}{\tan 30} \qquad R = 139\,\Omega.$$

- b) $$\frac{V_{out}}{V_{in}} = \frac{IR}{I\sqrt{R^2 + X_L^2}} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{R}{R} \approx 1.$$



- c) $$\frac{V_{out}}{V_{in}} = \frac{I X_L}{I\sqrt{R^2 + X_L^2}} = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} \approx \frac{1}{\sqrt{1 + \frac{R^2}{\omega^2 L^2}}}$$



- 

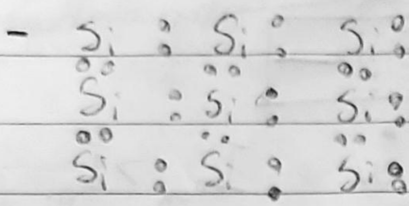
$\frac{V_{in}}{V_{out}} = ?$ $L = 250\,mH$

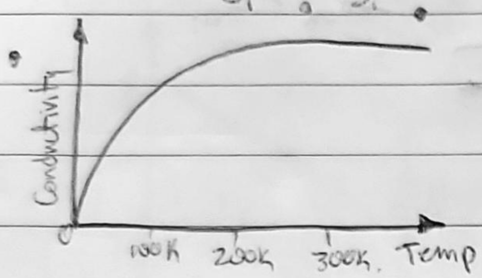
$R_1 = 15\,\Omega$ $f = ?$

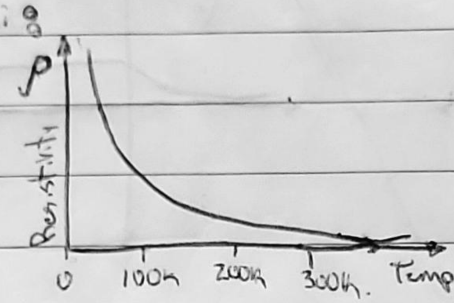
$R_2 = 10\,\Omega$

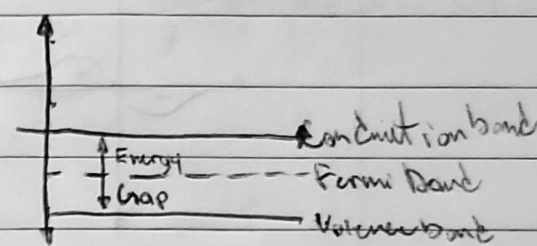
Semi Conductors

- 2 main elements → Silicon, and Germanium
- In metals each [redacted] atom contribute one free electron
- This free electron conducts electricity
- In metals there are ~10^{23} free electrons per mole.
- Si, Ge both have 4 valence electrons

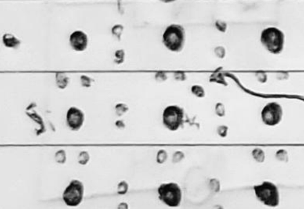




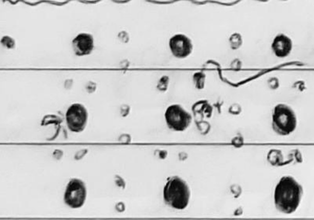




- There is conduction due to excitation of electrons by temperature in Si, Ge can carry current which we call is intrinsic current and its very small in applied field.



~~If you dop~~ replace 1% of Si with As. Since it is As^{15} has an extra electron. This is called n type doping. } N-Type

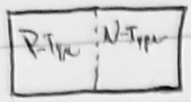


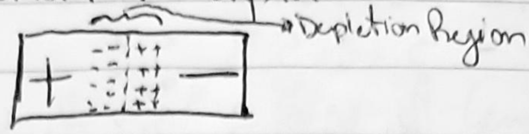
Empty place where an electron should be is called a hole. This leaves an extra electron and is called p-type doping. } p-Type

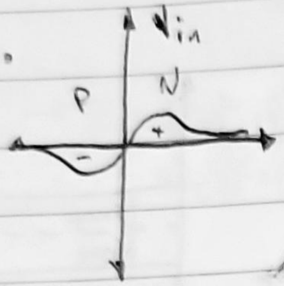
- N_e is the density of conducting negative electrons, $n_e >> n_{int}$
- N_h is the " " " hole $n_h >>> n_e$
- N_{int} " " " " " electrons due to temp. $n_h >> n_{int}$

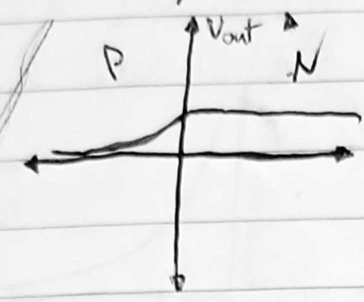
P-N Junction

- A single crystal of a semiconductor may be grown with acceptor (hole doping) impurities (p-type) and doped with donor impurities (n type) such that a sharp transition is formed between n & p type material in the crystal

- 

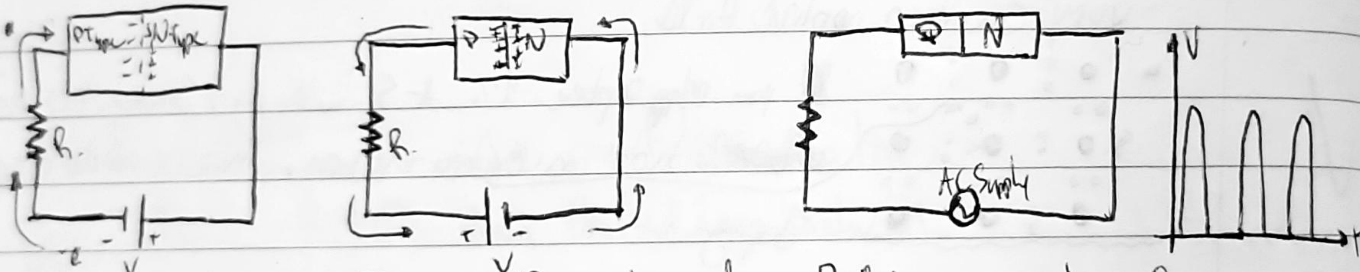
 - P-N Junction . 

- 



$I_{nem} > I_{ni}$

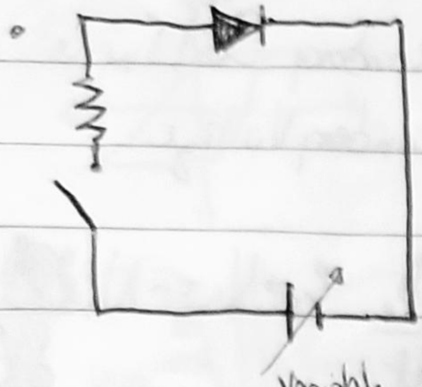
Forward and Reverse Bias

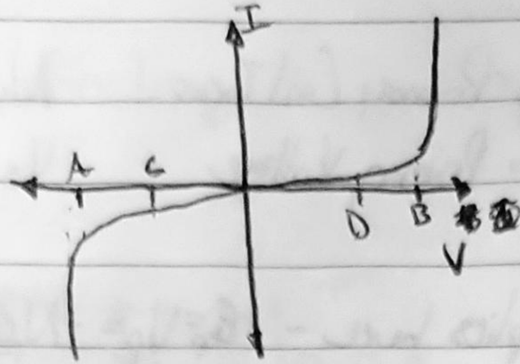


- Depletion Region increased	- Depletion Region decreased	- Rectifier properties of p-n junction which we call a diode
- Doesn't allow flow	- Allows flow.	
- $I = 0$	- $I > 0$.	
- Reverse Bias	- Forward Bias	

- Symbol = 

IV Characteristics





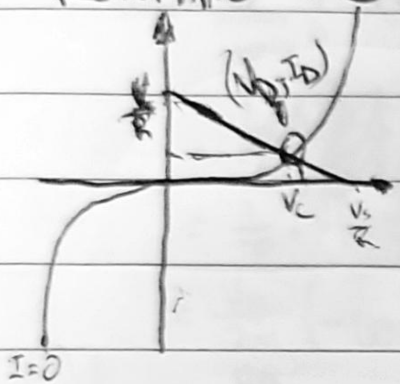
A= Critical Reverse Voltage or Breakdown Voltage.

B= Forward Current

C= Reverse Current (Small)

D= V_{CUT}

• Determine the voltage of I-V for a given Source voltage, V_s, in forward Bias

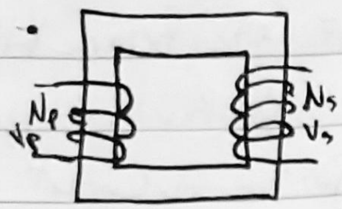


$V_R = V_s - V_D$

$IR = V_s - V_D$

$I = \frac{V_s}{R} - \frac{V_D}{R}$

Transformers

- 

N_p = Primary Coil Turns $\quad N_s$ = Secondary Coil Turns

V_p = Primary Voltage $\quad V_s$ = Secondary Voltage

- Faraday's Law Applies here - $\varepsilon_p = V_p = -N_p \frac{d\Phi_p}{dt} \quad \varepsilon_s = V_s = -N_s \frac{d\Phi_s}{dt}$

- The flux through both coils are identical $\rightarrow \frac{d\Phi_p}{dt} = \frac{d\Phi_s}{dt}$.

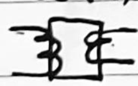
- $\frac{V_s}{V_p} = \frac{-N_s \frac{d\Phi}{dt}}{-N_p \frac{d\Phi}{dt}} \qquad \frac{V_s}{V_p} = \frac{N_s}{N_p}$

- Input power = Output Power $\Rightarrow P_p = P_s$

$$V_p I_p = V_s I_s$$

$$\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p}$$

- Note: If you need high voltage output, you decrease the output current & vice versa.

- Circuit Symbol - 

Example

- 

$V_p = 3600\,V \quad V_s = 120\,V$

$N_p = ? \quad N_s = 60$

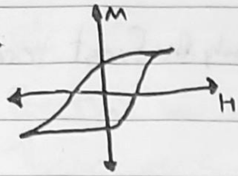
$I_p = ? \quad I_s = ?$

$\frac{V_s}{V_p} = \frac{N_s}{N_p} \Rightarrow 1800 \text{ turns} = N_p$

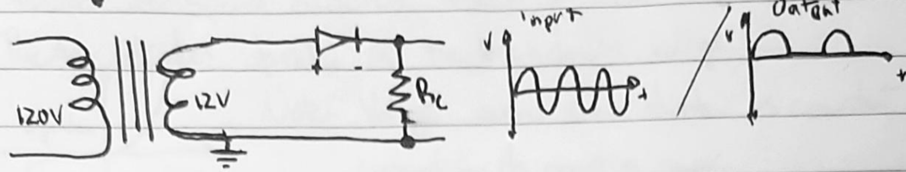
$I_p = \frac{I_s}{30}$

$I_s = I_p 30$

Transformers

- 

Hystrisis Loop.

- 

-HALF Wave Rect

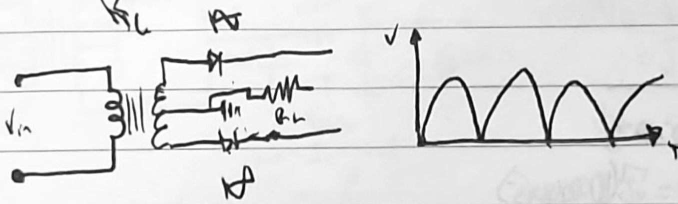
$$V_{AVG} = \frac{1}{T}\int_0^{\frac{T}{2}} V_p \sin\omega t \, dt$$

$$= \frac{1}{T}\cdot V_p \cdot \frac{1}{\omega}(-\cos\omega t)\Big|_0^{\frac{T}{2}}$$

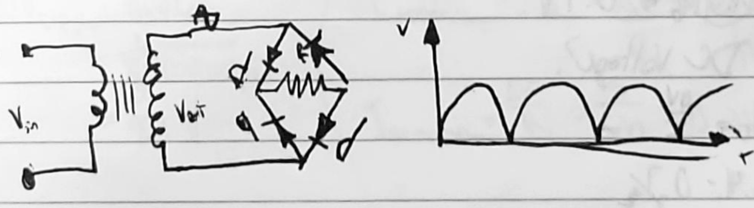
$$= \frac{1}{T}\cdot V_p \cdot \frac{T}{2\pi}(-\cos\omega t)\Big|_0^{\frac{T}{2}}$$

$$= \frac{V_p}{2\pi}(-\cos\omega t_{\frac{T}{2}} - \cos\omega 0$$

$$V_{AVG} = \frac{V_p}{\pi}$$

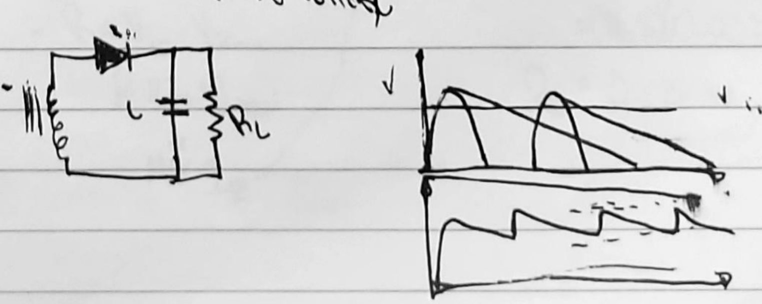
- 0.6-0.7 is required to turn on the diode.
- $I_{DC} = \frac{V_{AVG}}{R_L}$
- 

-Full Wave Rectifier

- 

-FULL WAVE Rectifier

-Total remaining voltage = 1.4V.

- 

Ripple voltage

- $V(t) = V_0 e^{-\frac{t}{RC}}$

$= V_0 [1 - \frac{t}{RC} + \frac{t^2}{2!(RC)^2} - \frac{t^3}{3!(RC)^3} \dots]$

- Then we choose an RC such that $T << RC$ so only the first term is need
- $V(t) = V_0 [1 - \frac{T}{RC}]$

Example

- Ripple Voltage

$\Delta V = V_0[1 - (1 - \frac{T}{RC})]$

$= V_0 \frac{T}{RC}$ $\quad T = \frac{1}{2f}$

$\Delta V = \frac{V_0}{2fRC}$ } Full Wave Rectifier

- For a circuit with $100 \mu F = C$, $R = 1 k\Omega$, $6V_{rms} = V_S$, $f = 60Hz$

$\Delta V = \frac{V_0}{2fRC}$

$= \frac{6V_{rms}}{2(60)(1000)(100 \times 10^{-6})}$

$= V_{rms}(0.08333) = \sqrt{2}(6)(0.08333)$

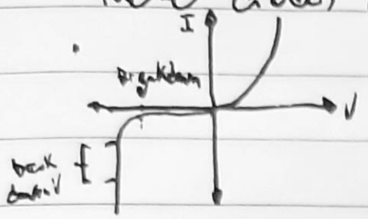
$\Delta V = 0.7 V.$

- What is the DC Voltage?
  - $V_{avg} = V_0 - \frac{\Delta V}{2}$

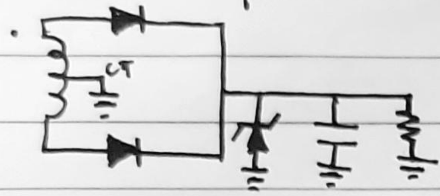
$= 8.4 - 0.7/2$

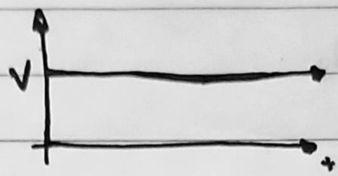
$= 8.05 V.$

Zener Diode

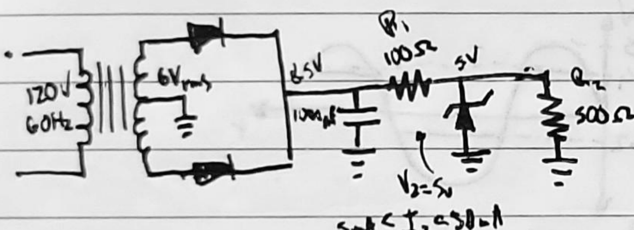
- We want a circuit with noripple voltage ($\Delta V = 0$).
- For this we use Zener diodes
- These are special diodes which can recover from a breakdown
- These diodes operate in the breakdown voltage
- 

Note: Voltage across zener diode is constant for any current in its operating range.

- Circuit Symbol: 
- 



Example



$$V_p = \sqrt{2}\, V_{rms}$$
$$= \sqrt{2}\, 6$$
$$V_p = 8.4V$$

$$\text{Approx. } \Delta V_{ripple} = \frac{1}{2fRC}$$
$$= \frac{1}{2(60)(100)(1000\mu F)}$$
$$= \frac{1}{24} V$$

$$I_{across R_1} = I_Z = \frac{\Delta V}{R_1} = \frac{8.5-5}{100} = 0.035 = 35\,mA$$

$$I_{across R_2} = \frac{\Delta V}{R_2} = \frac{5}{500} = 0.01A = 10\,mA.$$

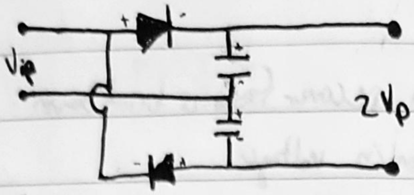
- How do we choose R_1?
  - $R_1 = \dfrac{V_{in} - V_Z}{(0.02 I_Z + I_{(max)})}$

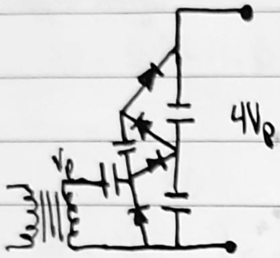
    ↓

    MAX I_Z

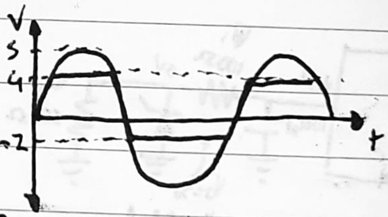
$$\left.\begin{aligned} P_Z &= V_Z I_Z \\ &= (\cancel{8}.5)(0.035) \\ P_Z &= 0.175\,W \end{aligned}\right\} \text{Power Across Zener.}$$

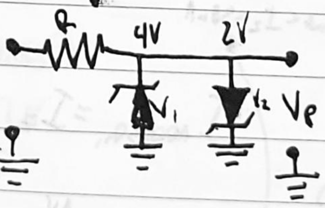
"Transformer" Circuit with Diodes

- This circuit deals with AC current
- 

- This is a voltage doubler.
- To quadruple your voltage make this
- 

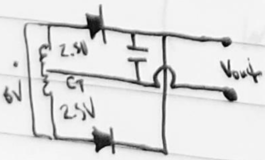
Cutoff Voltages

- To obtain a graph like this: 

- set up a circuit like this: 

-6? 

$V_{out} = 5\sqrt{2} \approx 7.05V$



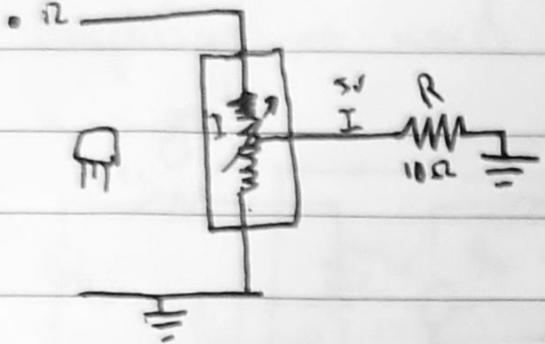
$V_p = \sqrt{2}\, 2.5 - 0.6$

$\approx 3.5 - 0.6$

$V_1 \approx 2.9V.$

$$I_{DIODE} = K e^{\left[\frac{-e(V_0 - 1)}{K_B T}\right]}$$

Three Terminal Voltage Regulator

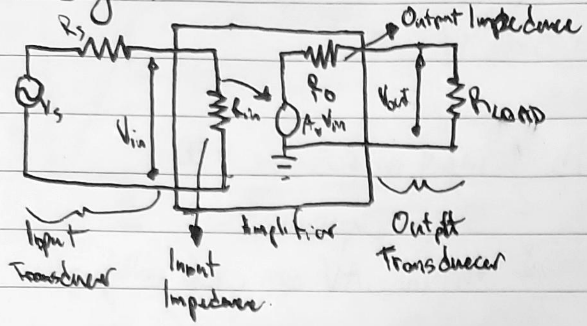


$12 - 5 = 7$ ← Voltage across regulator (variable resistor

$\frac{V}{R} = I \Rightarrow \frac{5}{10} = I$ ← Current in the circuit

$P = VI \Rightarrow 7(0.5) = 3.5W$ ← Power used in Regulator.

Amplifiers

- Amplifiers are used to increase the voltage power level of a signal.
- 

R_{in} - Input Impedance
V_s, R_s - Source Voltage/Impedance
R_L - External Circuit Load
A_v - Open Circuit Amplification of Amplifier
R_o - Output Impedance.

- Example: Microphone - $R_S = 10000\Omega$, $V_s = 0.2\,V_{RMS}$ Amplifier - $R_o = 200\Omega$, $R_i = 1000\Omega$, $A_v = 2000$ Speaker - $R_L = 8\Omega$

$$V_{in} = V_s \frac{R_{in}}{R_{in}+R_s} = \frac{(0.2)\,1000}{10000+1000} = 0.0182\,V_{rms}$$

$$A_v V_{in} = 2000 \cdot 0.0182 = 36.4\,V_{rms}$$

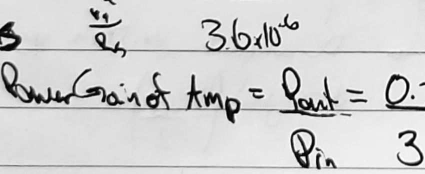
$$V_o = A_v V_{in} \frac{R_L}{R_o + R_L} = 36.4\left(\frac{8}{200+8}\right) = 1.4\,V_{rms}.$$

$$\text{Voltage Amplification} = \frac{1.4}{0.2} = 7$$

$$P_{out} = \frac{V_o^2}{R_L} = \frac{1.4^2}{8} = 0.245\,W$$

$$\frac{P_{out}}{P_s} = \frac{0.245}{\frac{V_s^2}{R_s}} = \frac{0.245}{3.6\times10^{-6}} = 68600 \quad \text{in dB} \Rightarrow 48\,dB.$$

$$A_p = \text{Power Gain of Amp} = \frac{P_{out}}{P_{in}} = \frac{0.245}{3.31\times10^{-7}} = 74260 = 58.7\,dB.$$

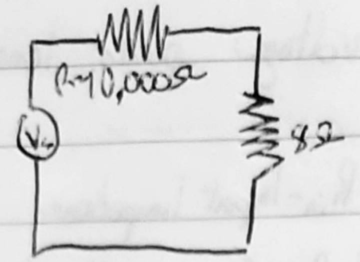
- There are amplifiers that we call 'voltage follower'.
  - This is an amplifier such that the amplification is 1.
  - Input Impedance $= \infty$, Output impedance $=$ ~~load impedance~~ Almost 0
  - 

$$V_{in} = V_s \frac{R_i}{R_s + R_i} = V_s \frac{1}{\frac{R_s}{R_i} + 1} = V_s$$

$$AV_i = (1)(0.2) = 0.2V$$

$$V_{RL} = AV_i \frac{R_L}{R_o + R_L} = 0.2 \cdot \frac{R_L}{R_L} = 0.2V$$

$RI^2 = P$
$R_L\left(\frac{V_s}{8}\right)^2 = P$
$8\left(\frac{0.2}{8}\right)^2 = P$
$5\times10^{-8} = P$

- Consider direct connection from Microphone to speaker
- 

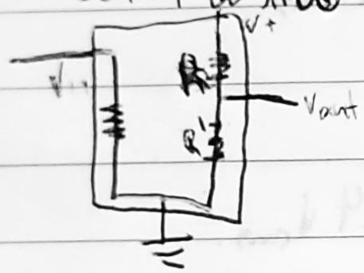
$P = RI^2$

$= 8\left(\frac{0.2}{10000}\right)^2$

$= 3 \times 10^{-9}$ W

Note: Normally the output impedance of amp should match R_L for max power transfer

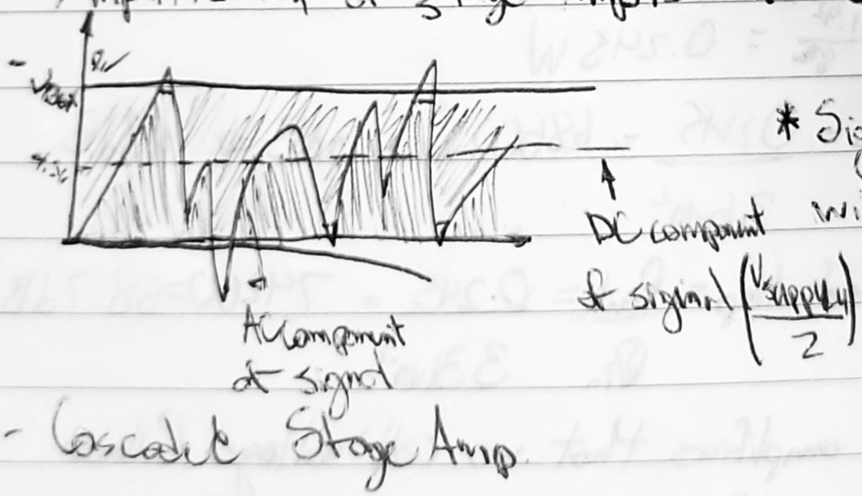
- An ideal amp is one with 1- input impedance = ∞
  2- Output impedance = 0
  3- Very large AV open circuit voltage gain

- Stage Model of Amp
  - Transistor or vaccum tube form an amp
  - Amplification is not too high
  - Several transistors should be cascaded in order to obtain desired amplification.
  - 

$V_{out} = V_{in} \frac{R'}{R+R'}$

$= \frac{V_{in}}{\frac{R}{R'}+1}$

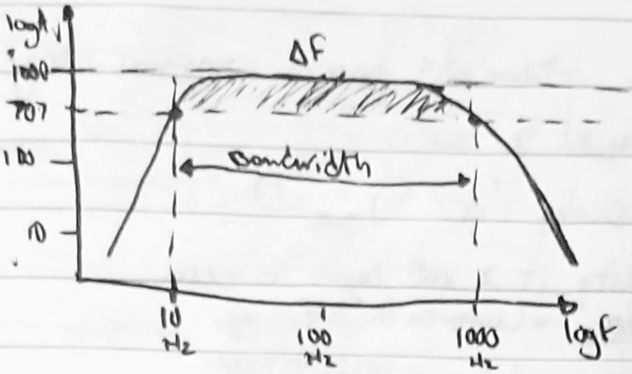
- Amplification of stage Amp is low ≈ 3-5 times



* Signal always stays within V_{supply} *

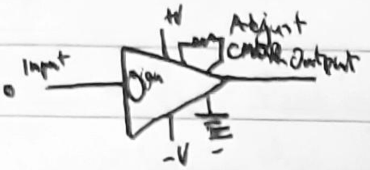
- Cascade Stage Amp

Amplification Bandwidth



- Any amplifier cannot price to all frequencies. The range of frequencies response of an amplifier is called bandwidth.

Inverting and Non-Inverting Amplifier

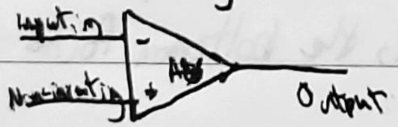


- For inverting amplifier the sign is (−)
- For non-inverting " " " " (+)

- Properties
  - Inverting
    The phase of the output changes by a 180° rotation
  - Non-Inverting
    The phase of the input signal is the same as the output

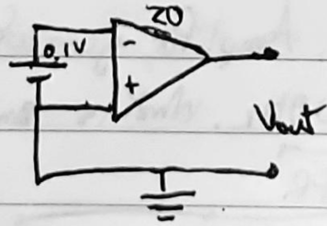
Difference or Differential Amplifier

- This amplifier has two inputs one inverting and the other non-inverting

- 

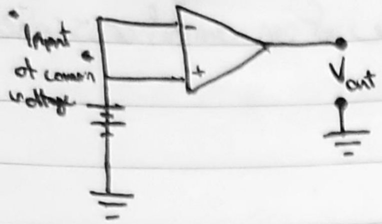
$$V_{out} = A_v(V_{in+} - V_{in-}) = A_v V_d$$

- Example



$$V_{out} = A_v(V_+ - V_-)$$
$$= 20(0 - 0.1)$$
$$V_{out} = -2V$$

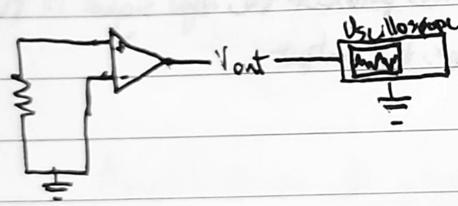
Real Amplifier

- In real amplifier, when both terminals have a common voltage the output voltage is not equal to zero.
- 

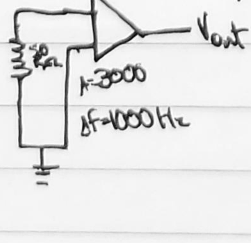
  - Theoretically the $V_{out}=0$
  - In reality, it is not equal to zero
  - $A_{vc} = \frac{V_{oc}}{V_{ic}}$ → Common Input Voltage / → Common Input Voltage.

- $A_{vd} = \frac{V_{od}}{V_{id}}$ → Differential Output Voltage / → " Input "
- CMRR = Common Mode Rejection Ratio = $\frac{A_{vc}}{A_{vc}}$

Johnson Noise

- 

- Johnson noise is the uncertainty created by free electrons inside a resistor
- $P = I_{N_{rms}}^2 R = 4 k_B T \Delta f$, where Δf is the bandwidth, T is the temperature(K) & k_B is the boltzman factor
- $V_{N_{rms}} = \sqrt{4 k_B T \Delta f R}$
- Example → 50 kΩ resistor, 20°C, Amplifier gain of 3000, frequency response of 1000 Hz. What is amp output



$V_{N_{rms}} = \sqrt{4 k_B T \Delta f R}$

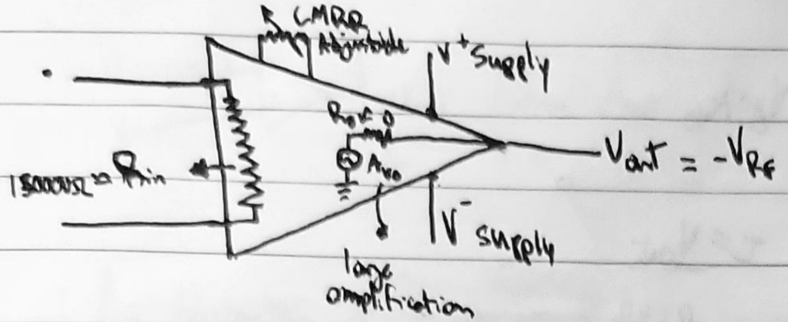
$= \sqrt{4(1.38\times10^{-23})(273+20)(1000)(50000)}$

$V_{N_{rms}} = 0.9\,\mu V$

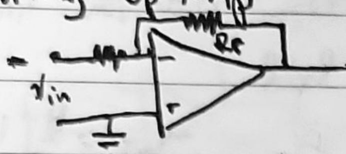
$V = \_\_ \pm 0.0027V$

$V_{out} = A_{vd} V_{N_{rms}} = 3000 \cdot 0.9\,\mu V \approx 2.7\, mV_{rms}$

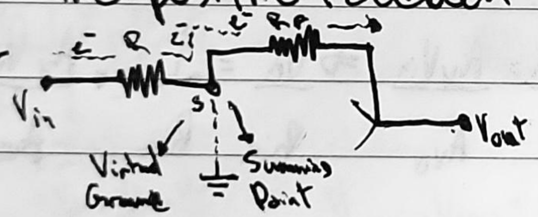
Operational Amplifier



- Inverting Op-Amp



- When the input is connected to the inverting port of amplifier the op-amp is called inverting op-amp
- R_f = Feedback
- When the output is connected to ⊖ terminal of op-amp the feedback is called negative feedback
- The positive feedback causes oscillating.
-



$V_{R_f} = I R_f$

$V_{out} = -V_{Rf}$

$V_{out} = -I R_f$

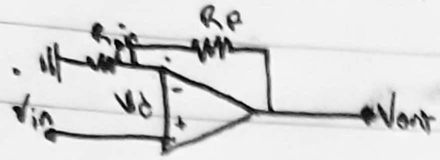
$V_{out} = \frac{V_{in}}{R} R_f$

$\frac{V_{out}}{V_{in}} = -\frac{R_f}{R}$

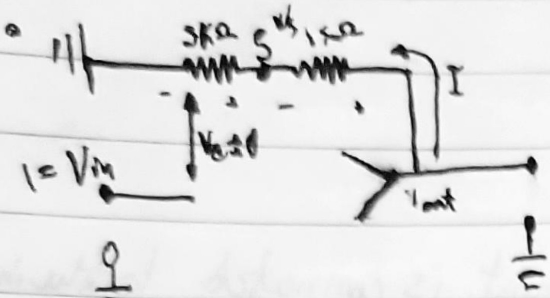
Notes

- In an op-amp $V_{out} = A_{vo} V_d$

Non-Inverting Op-Amp



$V_{out} = V_d \cdot A_{vo}$



$$I = \frac{V_{out}}{R_F + R}$$

$$V_{in} = V_S = IR = \frac{V_{out}}{R_F + R} \cdot R \Rightarrow V_{out} = V_{in} \frac{R + R_F}{R}$$

$$V_{out} = V_{in}\left(1 + \frac{R_F}{R}\right)$$

$A_v = 1 + \frac{R_F}{R}$ $\quad V_{out} = V_{in} A_v$

$A_v = 1 + \frac{15}{3}$ $\quad V_{out} = (1)(6)$

$= 6$ $\quad V_{out} = 6V.$

- Input Impedance
  - This is a very value
  -  Input Impedance.

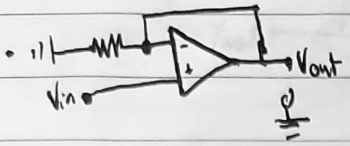
- $V_d = I_{in} R_{in} \Rightarrow V_d = \frac{V_{out}}{A_{vo}} \Rightarrow V_d = \frac{A_v V_{in}}{A_{vo}} \Rightarrow \frac{V_d}{R_{in}} = \frac{1}{R_{in}} \cdot \frac{A_v V_{out}}{A_{vo}} = I_{in}$

$$I_{in} = \frac{V_d}{R_{in}} = \frac{A_v}{A_{vo}} \cdot \frac{V_{out}}{R_{in}} \Rightarrow I_{in} = \frac{A_v}{A_{vo}} \frac{V_{in}}{R_{in}} \Rightarrow R_{input} = \frac{A_{vo} R_{in}}{A_v}$$

- Ex. $A_{vo} = 200{,}000$, $A_v = 100$, $R_{in} = 2M\Omega$,

$$R_{input} = \frac{200000}{100} \cdot 200\,000 = 4\,000\,000\,000\ \Omega$$

Voltage Follower

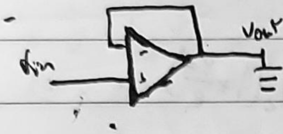


$V_{in} = V_{out}$ by $V_{out} = V_{in}\left(1 + \frac{R_f}{R}\right)$

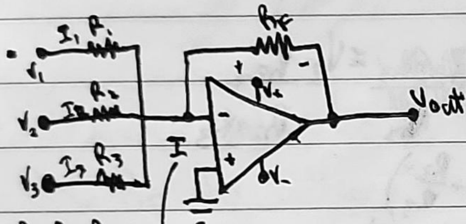
$= V_{in}\left(1 + \frac{0}{R}\right)$

$= V_{in}$.

- An Advantage of this circuit is input impedance $= R_{in} \approx \infty$
- It also doesn't need that resistor



Summing Op-Amp



$R_1 = R_2 = R_3$

Output signal of $V_{out} \propto (V_1 + V_2 + V_3)$

$\to V_{OUT^1} = V_1\left(1 + \frac{R_f}{R_1}\right)$

$V_{OUT^2} = V_2\left(1 + \frac{R_f}{R_2}\right)$

$V_{OUT^3} = V_3\left(1 + \frac{R_f}{R_3}\right)$

$= I_1 + I_2 + I_3$

$= \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$

$V_{OUT} = -V_{Rf} = I R_f$

$= -\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right) R_f$

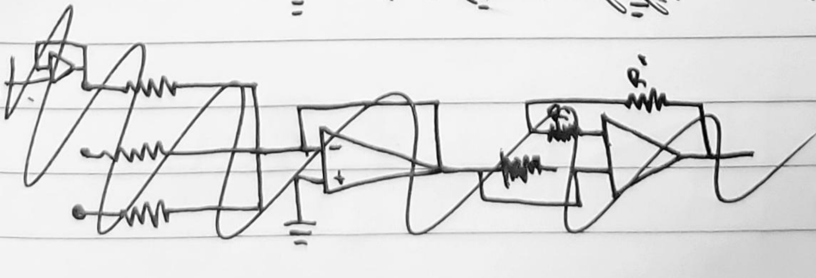
$= (V_1 + V_2 + V_3)\frac{R_f}{R}$ ← if $R_1 = R_2 = R_3 = R$

$V_{OUT} = (V_1 + V_2 + V_3)$ ← If $R_f = R$

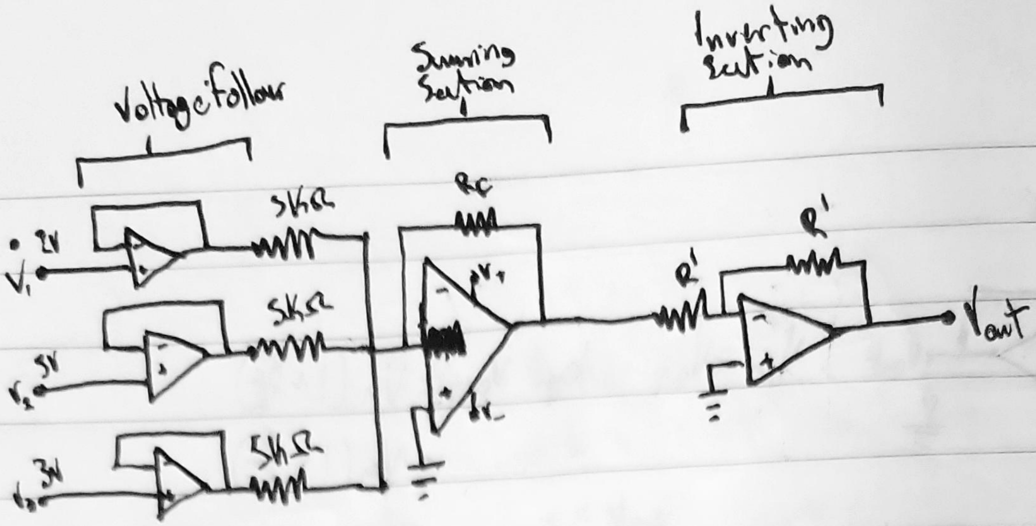
- Design Summing Device to add 3 Voltages $\begin{pmatrix} V_1 = 2V \\ V_2 = 5V \\ V_3 = 3V \end{pmatrix}$

$\cancel{R_1 + R_2 + R_3 = R,\ R = R_f}$

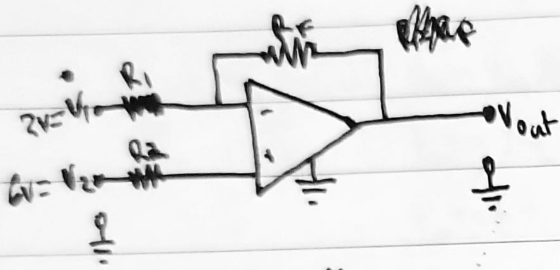
$\cancel{V_{out} = (V_1 + V_2 + V_3)}$



Next Page

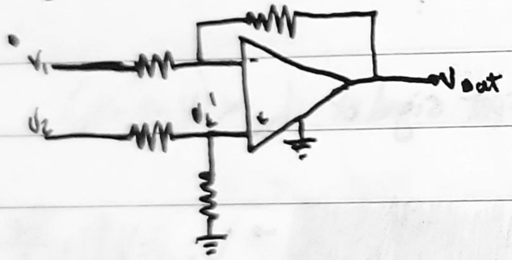


Difference Op-Amp



$$V_{out} = V_1\left(-\frac{R_f}{R_1}\right) + V_2\left(1+\frac{R_f}{R_1}\right)$$

We need to get $V_{out} = -V_1 + V_2$



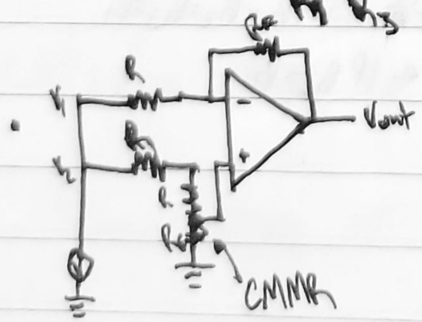
$$V_2' = V_2 \frac{R_3}{R_2+R_3}$$

$$V_2 = V_2'\left(1+\frac{R_2}{R_3}\right)$$

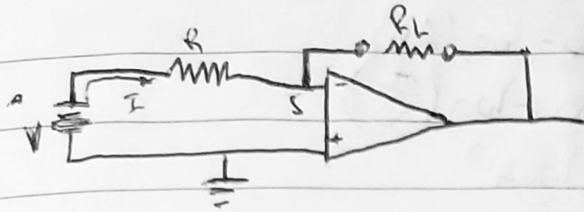
$V_{out(2)} = V_2'\left(1+\frac{R_f}{R_1}\right)$ and we want $V_2\left(\frac{R_f}{R_1}\right)$

$$= V_2 \frac{R_3}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)$$

$$\frac{R_f}{R_1} = \frac{R_2}{R_3} \rightarrow \quad = V_2\left(\frac{R_f}{R_1}\right)$$



Constant Current Source.



- $I = \frac{V}{R}$
- Any load will receive constant currents.

- Current to voltage converter

$V_{out} = -V_{R_F}$
$= -I R_F$

~~Virtual Ground~~
Virtual Ground

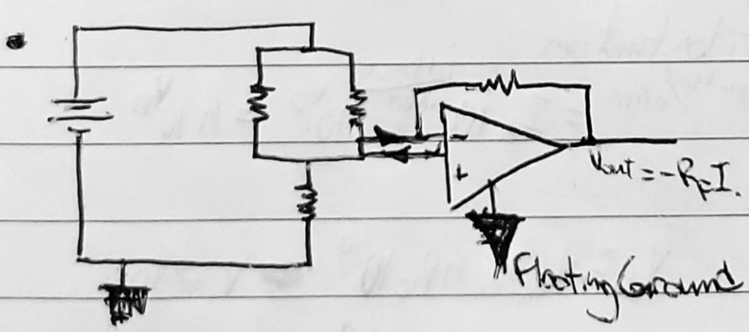
Example

- Voltmeter reads 1 volts, construct an ammeter that reads 10 μA, find R_F also, voltage that appears between the ~~micro ammeter~~ micro ammeter leads if you use 741 op amp, $V_{out} = 1V$

$V_{out} = -I R_F$ $\quad |1V| = |10\mu||R_F|$ $\quad V_{out} = V_d A_{vo}$

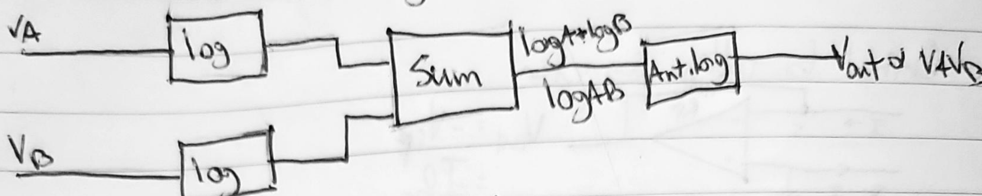
$|1V| = |I R_F|$ $\quad \frac{1}{10\mu} = 100k\Omega$ $\quad 1 = V_d \, 200000$

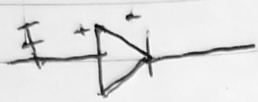
$5\mu V = V_d$



Logarithimic Function Block.

we know - $\log A + \log B = \log(AB)$
- $\log A - \log B = \log\left(\frac{A}{B}\right)$
- $n \log A = \log A^n$





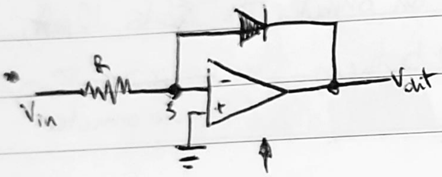
$I = I_R e^{|e|V_D/nK_BT}$

$\frac{I}{I_R} = e^{|e|V_D/nK_BT}$

$\log \frac{I}{I_R} = \frac{|e|V_D}{nK_BT}$

$V_D = 0.0059n \log\left(\frac{I}{I_R}\right)$

|e| → electron charge
V_D → Voltage across diode
n → Fudge Factor Si=1 Ge=2
K_B → Boltzman Factor
T → Absolute temperature
I_R → reverse leakage voltage.

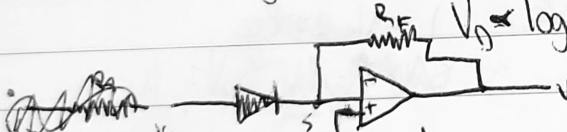


This is our log function.

$V_D = 0.0059n \log\left(\frac{I}{I_R}\right) = 0.0059n \log\left(\frac{V_{in}}{RI_R}\right)$

$= 0.0059n \log V_{in} - \underbrace{0.0059(RI_R)}_{\text{constant}}$

$V_D \propto \log V_{in}$



$V_{out} \doteq -V_{Rf} = IR_f$

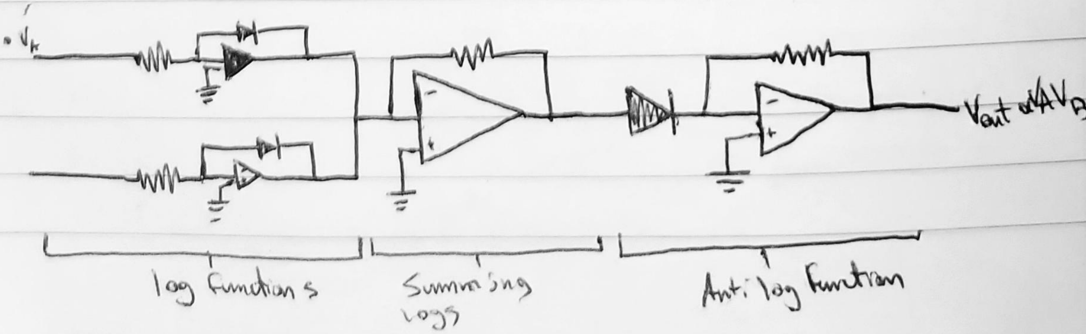
Anti log function

$I = I_R \exp\left[\frac{|e|V_D}{nK_BT}\right] = I_R 10^{|e|\log_{10}e V_D/nK_BT} = I_R 10^{\frac{|e|\log_{10}e}{nK_BT}} 10^{V_D} = K 10^{V_D}$

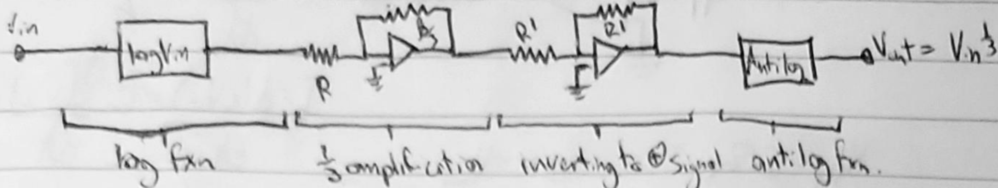
$V_{Rf} = IR_f = K 10^{V_D} R_f$

$V_{out} \doteq -V_{Rf} = KR_f 10^{V_D}$ $V_{in} \approx V_D$

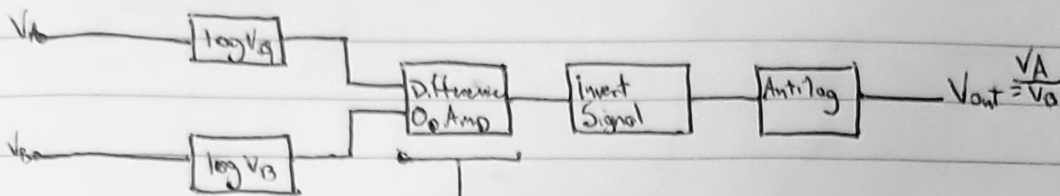
$= KR_f 10^{V_{in}}$

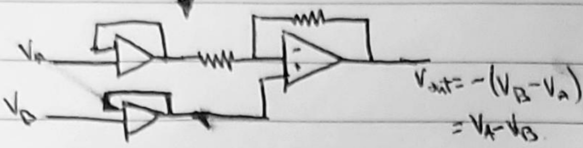


• Make a circuit with $V_{out} = V_{in}^{\frac{1}{3}}$

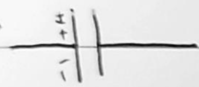


• Make a circuit with $V_{out} = \frac{V_A}{V_B}$





Differential and Integrating Circuits

• 

$q = VC \qquad I = \frac{dq}{dt} \Rightarrow I\,dt = dq$

$\int_{t_0}^{t} I\,dt = \int_{q_0}^{q} dq$ when $t = t_0 \to q_0 = 0$, $t_0 = 0$

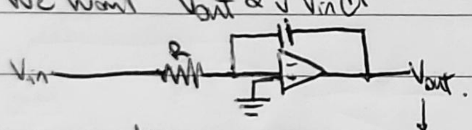
$\int_0^t I\,dt = \int_0^q dq$

$\int_0^t I\,dt = q$

Note:
If V_{in} is constant
$V_{out} = \frac{-V_{in}t}{RC}$

INTEGRATION

• We want $V_{out} \propto \int V_{in}\,dt$



$I = \frac{V_{in}}{R} \qquad I = \frac{dq}{dt} \Rightarrow \int_{q_0}^{q} dq = \int_0^t I\,dt$

$q - q_0 = \int_0^t \frac{V_{in}}{R}\,dt$

$q = \frac{1}{R}\int_0^t V_{in}\,dt \qquad q = VC$

$q = \frac{1}{R}\int_0^t V_{in}\,dt.$

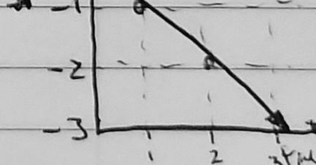
$V_{out} = -V_C = -\frac{q}{C}$
$= \frac{-1}{RC}\int_0^t V_{in}\,dt$
$= \frac{-1}{RC}\int_0^t V_{in}\,dt.$

• Make a circuit that output voltage changes linearly as a function of time

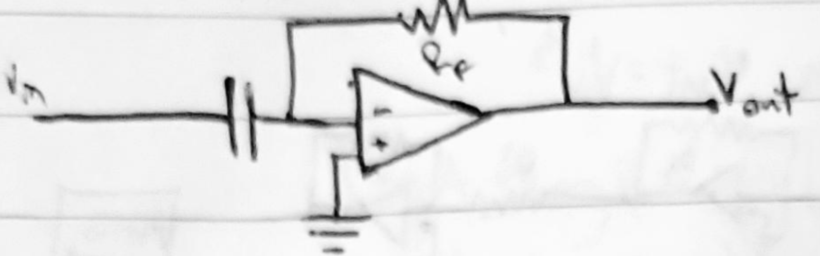
$V_{out} = \frac{-1}{RC}\int_0^t V_{in}\,dt.$

$= \frac{-1}{(10\times10^{-6})(10^{5})}\int_0^t 1\,dt.$

$V_{out} = -t \; \frac{\text{volts}}{\text{seconds}}.$



• Differentiator



$I = \frac{dq}{dt}$ $\quad q = CV_{in}$

$I = \frac{d(CV_{in})}{dt} = C\frac{d(V_{in})}{dt}$

$V_{R_F} = I R_F$

$= C\frac{d(V_{in})}{dt} R_F$

$V_{out} = -V_{R_F}$

$= -CR_F \frac{dV_{in}}{dt}$