Final Term Project

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Introduction

When an unknown painting emerges from a prolific artist's workshop, the question begs to be: is this his work? The painting '*The Reading Hermit*', had been produced in Rembrandt's camp of artists but the original artist remains unknown. It is thought to be Rembrandt himself but it very well could have been one of his many pupils. He carried a large camp with him, totaling to around 40 students[4]. They all followed Rembrandt and seemingly mimicked the artist because his influence was so great over them[6]. With a group of 40 individuals all creating similar artistic styles, the focus shifts when we consider that the painting's creator is unknown. This puts the authenticity of the artwork to question. Could this piece be a replica, fake, or forgery? A paper in the Journal of Analytical Atomic Spectrometry, *Old traces, read anew – 'The Reading Hermit' painting in the light of X-ray fluorescence*, concludes that it is an original piece. Little is known about this piece publicly as it belongs to a private collector[8], but this paper will give insight into the painting and its history. As the topic is delved into and researched, the historical background of the artwork and time period will be discussed. A technical analysis of the painting will also be a talking point. The type of paint, palette, and elements will be relayed.

Rembrandt Harmenszoon van Rijn was born on July 15^{th}, 1606, in the Dutch Republic (now the Netherlands)[6]. His family was a well-off, working-class family and belong to the Catholic religion although his parent's churches differed. This is important as art takes place in the Baroque period, religion is portrayed often and is the main focus of the art for the most part. Rembrandt went to Latin school and studied painting for around 4 years under two painters[6]. In 1624, he then opened the doors to his own workshop taking his many students. Unlike many other famous artists, Rembrandt had much success in his lifetime making money as a portrait artist and collecting fees from students. He would then marry a colleague's cousin, Saskia van Uylenburgh[6]. They would share one child together after many failing to survive. She passed later on due to tuberculosis and Rembrandt would later carry on a relationship with his maid[6]. Despite having multiple sources of income, Rembrandt's tastes lived outside his budget purchasing many artworks and rarities that would lead to his bankruptcy. What was once the marks of a successful man, faded quickly as he downgraded his living standards[6]. He later died, outliving his significant other and his son. Rembrandt died poor, lonely, and in the shadow of who he was.

The painting that has been produced in Rembrandt's academy, was estimated to be produced in the 1630s[8]. In the artwork, there is a lonely old man in what looks to be his den. He sits at a chair holding a book with no desk or anything in front of him. A very distinct and important feature of Rembrandt is his use of light[9]. He commonly applies an almost spotlight like use of light to place importance. In this Hermits life, he has no concern or material possessions. His clothes are tattered and ripped. His living quarters has little organization and cannot afford a desk for himself to sit at. The Hermit has no worry for this though. He places the light in his life into the information and pleasure he takes in reading. Perhaps the Hermit might only let in light into his den to read and would live in the dark if he could. Now, why would a man as well off as Rembrandt paint this in his mid-20s? Rembrandt sees himself as the Hermit. Being born a financially stable family, Rembrandt was able to take value in knowledge and the pursuit of it not having to work[9]. He could paint and study the art in his early to late teens, quickly to open his own academy to teach. This is because he valued knowledge and wanted to relay it to his students. It was also by chance that Rembrandt gained popularity and the jobs he got. The success and pay was noise to him. His vicarious spending was not due to his vain but because did not care for the money. Perhaps this is why he died poor in monetary means.

This painting also came at the beginning of the Age of Enlightenment. Rembrandt grew up in a time where religion was of great importance and reigned supreme. The Age of Enlightenment was a philosophical and intellectual movement that value scientific reason and reductionism over religious tradition[2]. 4 years after Rembrandt's birth, Galileo observes the moons of Jupiter[1]. John Napier discovers logarithms in 1614[1]. Immanuel Kant and Francis Bacon are publishing major philosophical works that shaping the future of thought[2]. Isaac Newton is publishing his monumental *Principia* in the late 1600s. This gives support to the idea that Rembrandt was depicting a man who was rejecting tradition and embracing the Enlightenment movement.

Preparatory Information

Although not much is known about this painting historically and scientifically, information can about its components can be gathered about it from the original paper and by studying his other works. First, an introductory overview of the paint and colours will be done. An analysis of a reference painting will then also be used to gain any other or extra information on the paint. Finally, the information provided by the paper will be used and interpreted. An analysis of a reference painting will then also be used to gain any other or extra information on the paint. If the palette in figure 2 is looked at, it can be seen that the painting contains mostly browns, oranges, blacks, whites, and a small portion of blue. If the paint names are used then they are; Van Dyke Brown, Umber, Madder Lake, Vermilion, Yellow ochre, Lead-tin yellow, Azurite, Smalt, Bone black, Carbon black, and Lead White[5]. By looking at what was used in pigments at the time period, an approximate group of elements can be gathered. In figure 5, it can be seen that the common colours use similar compounds. The rough list of main elements in the painting should be; iron, mercury, sulfur, lead, copper, potassium, carbon, hydrogen, and oxygen[7]. To reduce the gathering of extra information, specific elements will be looked at to stay inline with the main paper and see which elements are useful. The elements that will be looked at are lead, iron/manganese, and copper/calcium[8]. Manganese and calcium were not listed above in the main chemical compound list. This was because the main chemicals were listed and some of the lesser ones were not. Manganese is found in Umber and calcium is found in Bone Black[7].

Before the paper's analysis is discussed, the process of how they did the analyses will be talked about. The team used facilities at Helmholtz-Zentrum Berlin (HZB) and uses the BER II to carry out Neutron-Autoradiography[8]. Neutron-Autoradiography is a technique that uses the fact that neutrons are not charged and they can penetrate a target a few centimeters deep. Exposing the painting to these neutrons would irradiate the painting[3]. The atomic nuclei in the painting are then brought into an excited state and can emit γ- or β-rays. The half-lives of the elements tell us then what is inside the painting. This would then bring in the need for a detector and this is done by Micro-X-Ray Fluorescence[8]. So now that the process is known, the results can be looked at. For lead (figure 6), there is little lead in the paint itself but rather in the material it is painted on. This is a very interesting note. The artwork was painted on wood, rather than the typical canvas. The artist in this case had covered the wood in a thin layer of lead white. This was done to fill the little gaps in the wood and to get a more flat canvas. Regardless of the application, it is seen that it is present in the paint but at a low degree in the darker areas (which is expected). For iron/manganese (figure 7), there is a strong correlation between the two in the same area. They are in a higher concentration around the outline of the man. In a dark area like that this agrees with the expected result because Umber contains the manganese and the Van Dyke Brown contains iron. They both combine to create that dark, dingy colour but not a solid black around the man. For the copper/calcium (figure 8), there is an interesting pattern that arises. The copper follows the outline of the man like iron and manganese. The copper comes from the blue paint and it can be seen that there is a strong mix of blues, and browns to achieve the colour that Rembrandt did. The calcium, however starts to take the opposite shaping of the outline. Calcium is found in the bone black colour. This means that to differentiate between the darkness of the mixed blues and browns, Rembrandt had to achieve a darker colour that would accentuate the man's body but not make it obvious. The calcium bleeds into other areas as well. Bone black is the lighter of the two blacks and can be used to make grey easier. This is why it is seen on the floor in the grey area. With all of the scans agreeing with the expected elements, it can be used as evidence that the painting of '*The Reading Hermit*' is an original. Although it is not a definitive test, it gives evidence of it being authentic[8].

From this paper and the analysis of the painting, it can be seen that the painting is authentic. The scans of the different elements give light of this by agreeing with the expected elements. The expected elements came from using a reference image and taking the common colours. The common colours and the time period were both used to find out common chemical compounds and elements found in the paint at the time. This time period was the Age of Enlightenment. Rembrandt grew up in a time period where the world was changing around him for the better. Traditions were changed, old ideas of the world were thrown away, new philosophies were being accepted. A man who seemingly had a passion for knowledge and sharing it with his students embraced the documenting of tradition through painting religious topics. Perhaps Rembrandt wanted to be poor, and solitary in his old age, and knew that one day he would become '*The Reading Hermit*'.

Calculating the radioactivity of the painting

This next part of the report will be detailing and displaying the calculated radioactivity of the painting after its exposure to the neutron beam. Through this, it is hoped to answer the two umbrella questions: what is the resulting damage to the painting? and will the painting discolour? This analytical portion of the report begins by locating the neutron absorption cross-sectional values for each isotope of the atoms.

To ensure that the values are correct, two sources will be considered. However, in the chance that one source disagrees with the other, the newer and/or more regarded source's values will be taken. The first source comes from the IAEA's, International Nuclear Information System (INIS). The publication dates back to 1964 and was published by N. E. Holden. The second source comes from the National Institute of Standards and Technology's (NIST), Center for Neutron Research. This information is taken from the journal *Neutron News* and dates back to 1992. For the case of a discrepancy, the latter source will be used. The following table contains the information need for the absorption cross-sectional values for each isotope. It should be noted that the cross-sectional measurements will be in barns (1 b = 10^{-28} m^2).

Main Element	Isotope	INIS Source[10], σ	NIST[11], σ
Calcium	^{48}Ca	0.22 ± 0.04	0.41
Manganese	^{55}Mn	13.2 ± 0.1	13.3
Iron	^{60}Fe	2.7 ± 0.2	2.59
Copper	^{63}Cu	4.5 ± 0.1	4.5
Lead	^{208}Pb	0.03	0.0048

Table 1: List of the absorption cross sectional values from INIS and NIST

It can be seen that for most of the isotopes, the results agree with each other. In calcium and lead, they both have differing results to different degrees. Nevertheless, the data from NIST will be considered over the data from INIS. The next step in the analysis of the painting takes place in the instrument that will be irradiating the painting.

The painting was irradiated at the research reactor called the BER II. The BER II reactor worked by following the nuclear fission process. The unfiltered neutrons would be far too strong to apply to the painting so the neutrons are passed through the 'cold source'. The 'cold source' slows down the neutrons by exposing them to extremely cold hydrogen. These hydrogen atoms collide with the neutrons and result in a dramatic loss of energy. The neutrons in this case are cooled until they reach an energy range of 5×10^{-5} eV to 0.025 eV. The paper also gives the information on the flux which turns out to be 10^9 neutrons per cm^2 per second. This matches with the B8 instrument. The B8 is used to guide the neutrons and contains some information about itself that allows for the calculation of the flux by hand. The calculation matches with the papers and the B8 values. It can be found in the appendix under figure 9.

With the two values of the neutron absorption cross-section and flux of the neutrons, the total rate at which the neutrons will be captured can be found. The cross-sectional values give the area at which if a neutron were to pass through then it would be capture and put into an excited state. The flux can be multiplied to each of these values to see the rate at which it will be accepted into the paintings atomic structure. Another interesting calculation will be applied in the following table as well. One of the last columns will contain the total neutron capture production rate (TNCPR) multiplied by the time it ran for. This number will allow the consideration of scaling and having a sense for how much of the painting's atoms will be affected. The cross-sectional values listed will be in barns (1 b $= 10^{-28}\text{m}^2$), the flux is the number of neutrons per cm^2 per second, and the total neutron capture rate will be in neutrons per second.

Element	NIST, σ	Flux, J	TNCPR	Rate of affected atoms
Calcium	0.41	10^9	4.1×10^{-16}	1.7712×10^{-11}
Manganese	13.3	-	1.33×10^{-14}	5.7456×10^{-10}
Iron	2.59	-	2.59×10^{-15}	1.11888×10^{-10}
Copper	4.5	-	4.5×10^{-15}	1.944×10^{-10}
Lead	0.0048	-	4.8×10^{-18}	2.0736×10^{-13}

Table 2: Calculation of the total neutron capture production rate and rate of affected atoms

From this table, it can be observed that the TNCPR values are all quite similar and vary by the difference in the order in their cross-sectional measurements. Furthermore, from the last column it can be seen that manganese will have the most affected atoms (about 6 atoms per every 10^{10}) and lead will have the least affected atoms (about 2 per every 10^{13}). This makes intuitive sense as manganese has the largest cross-sectional area and would have a greater chance of interacting with an incoming neutron. Conversely, lead would be expected to have a lower chance as its cross-sectional value is lower and the calculation follows accordingly.

This all precedes the actual irradiation of the painting. For the next portion, the painting after it has been exposed to the neutrons from the BER II reactor. After the painting is exposed to the radiation it will be placed into a 3D micro x-ray fluorescence to begin investigations. This device is used to monitor and measure the radiation coming from the painting. This will give light to confirm the purpose of the paper and answers to the purpose of this report. Beginning with lead, the reaction follows as Pb(n,β)Bi. The inbound neutron excites the ^{208}Pb to ^{209}Pb. ^{209}Pb has a half-life of around 3 hours and decays into a β^- particle and into ^{209}Bi. ^{209}Bi is not a stable isotope but has an extraordinarily large half-life (2.01×10^9 years) and remain as such for a while. Figure 11 in the appendix shows the drastic difference between the two timescales. Copper has a unique interaction with the neutrons. It is unique in the way that it decays with either a β^- particle and into ^{64}Zn, or with a β^+ particle and into ^{64}Ni. This happens with a half-life of 12 hours. Both of the daughter particles in this experiment are stable and will not decay any further. Iron absorbs the neutrons and is excited to ^{61}Fe. This isotope decays into a β^- particle and ^{61}Co with a half-life of 6 minutes. This isotope of ^{61}Co however is not stable and decays into another β^- particle and ^{61}Ni which is stable. ^{61}Co has a half-life of 2 hours. Manganese gets excited in ^{56}Mn. This isotope decays into a β^- particle and ^{56}Fe at a half-life of 2 hours. ^{56}Fe is a stable isotope and not decay any further. Calcium is trans-mutated into ^{49}Ca. The decays into a β^- particle and ^{49}Sc with a half-life of around 8 minutes. ^{49}Sc is not very stable itself and in turn decays into another β^- particle and into ^{49}Ti, which is a stable isotope. The graph in figure 12 displays the decay of all of these isotopes over the period of one day. This is the time frame that the researchers used for the measurements in the XRF. It can be seen on the graph that most of the decays happen over this time period. Copper would be expected to show the weakest signal as it has the highest half-life (lesser number of decays) and this agrees with the scans in figures 6-8. Another note about these scans is that it seems as if there are some strokes that had been redone. This is evident by the lead-based paint spreading across almost the whole targeted portions. Although, this was to be expected as the wooden canvas used a lead-based white paint across most of the canvas to fill in the small imperfections in the wood. With the overlapping of most of the chemicals and from figure 13 especially, it can be seen that the pentimenti are visible. This is clear in the outline of the hermit and his barely visible shadow below him.

Furthermore in the calculation portion, the exposure of the painting can be found. This is done by estimating the decays per unit time (emittance of β particles per unit time). To solve for the exposure two major stipulations will be taken into account. The first will be that each test of the elements was run in their specific pairings. This means that the painting was irradiated 3 times as per the paper. The second is that the observation period was ran for 24 hours. Assuming that the tests were in the as they appeared and with no layover time, the following values for total time ran will be used respectively: 3-days for lead, 2-days Iron/Manganese, 1-day copper/calcium. The decay equations were used and can be seen in figure 14. For the decay chains with daughter isotopes that also decay, halve of the initial amount of particles will be used for half of the respective time. To gain a measurement for the mass of the painting exposed the density of the wood ($\rho_{\text{oak}} = 593\ \frac{\text{kg}}{\text{m}^3}$), and the amount of exposed wood will be taken into account (eight $3.5 \times 12.5cm^2$ panels). For the volume of the exposed wood, it is known that neutrons can pass through it so it will be considered to be 1 inch thick. Since all of the particles that decay are β particles, the energy is known to be 0.511 MeV. Alternatively, the number of active particles can be thought of as an electron charge. This allows us to convert from the number of electrons into Coulombs and then eventually dividing by the mass affected to find the exposure. The amount of Coulombs turns out to be 0.000081727501878. Finally, it can be seen that the exposure on the painting after the trials is $0.00123054\ \frac{\text{C}}{\text{kg}}$. To further the calculations for the next week, and then a month the time in the half-life decay equations just needs to be changed. One week after the experiment, the painting has its exposure greatly reduced to $1.282021 \times 10^{-7} \frac{\text{C}}{\text{kg}}$. After one month, the exposure rated has decayed so much that it becomes stable once again.

Conclusion

Referring back to the two main questions that have driven this project (How damaged will the painting be? and Will it discolour?), the answer is believed to be no for both of them. The painting is believed to not be damaged to a great deal for a few main reasons. One of the reasons is that most of the neutrons inbound for the painting will pass right through it. The neutrons have a very small target to hit start a nuclear reaction but even at that atomic scale, it is still an extremely small that just anything one neutron will collide with an element. Furthermore, when the neutrons do react, they release a β particle. This is important for when the fact that the β particles do not travel far in dense substances. They will either get stopped by the wood in front of them or take the path of least resistance and not collide with the wood at all. This shows that even for the products of an already rare occurrence will have even less of an impact on damaging the painting. This is all done on purpose. The neutrons that come from the BER II reactor are much higher in energy and easily make quick history of the painting if they used an unfiltered source. The team that is running these experiments will be optimizing the neutron beam (by using the cold source) to do as little damage as possible. Not only would the private collectors hopeful Rembrandt be destroyed (losing out on likely a lot of money), but it would continue to encourage those who have bad narratives for nuclear research. All in all between the instruments being optimized to the highest degree, the reputation at hand, and the incident particles not interfering with the work itself, the painting should have experienced no damage.

For the second question, the resultant stable elements will be looked at. The resultant stable elements all end up being elements that are dark in nature and or already in the painting itself. A few of them decayed in iron, which was already rampant and contained in the painting. Nickel is a darker metal itself and when it decays from one with the same characteristics its clear to see that there will be little to no discolouring. The little discolour could come from a sneaky culprit, however. When lead had decayed into bismuth, this could bring a metallic shine and array of colour to the area of the painting which is supposed to be mainly a shadow. There was some lead in the darker colours but most of the lead was believed to be from the white base coat. If the researchers considered this to be true, then they could continue to irradiate the painting with no worry as the white base was not visible by any means.

It is regarded that the painting did not yield any damage for a number of analytical and numerical reasons. The team working on the painting and the reactor would prioritize the condition the of the painting making sure nothing would comprise the work of art. They even used updated ways of doing this experiment to ensure it was not to be tampered with. Since there is still the mystery of whether Rembrandt did it himself or not it the paper's work would suggest that it is an original from that time period and place. For the sake of the lore and mystery, it would be beneficial to say it was Rembrandt's. A man who envisioned a future where knowledge would surpass monetary means at the Age of Enlightenment.

Bibliography

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Appendix

Figure 1: 'The Reading Hermit', oil on a wooden panel, unknown artist, private property[8]

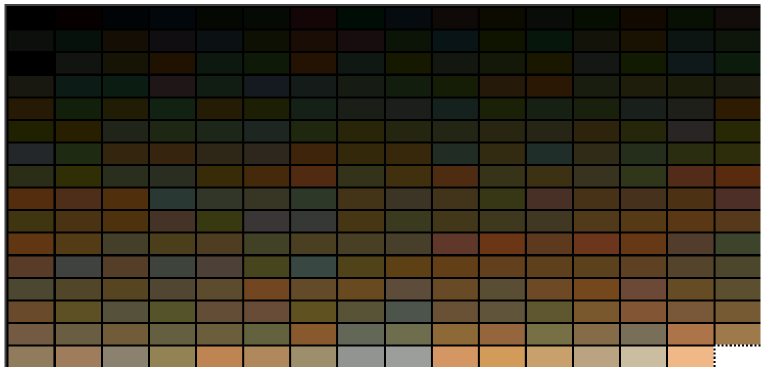

Figure 2: Colour palette of the painting generated by GIMP

Figure 3: ‘Belshazzar’s Feast’, oil on canvas, Rembrandt, National Gallery London[5]

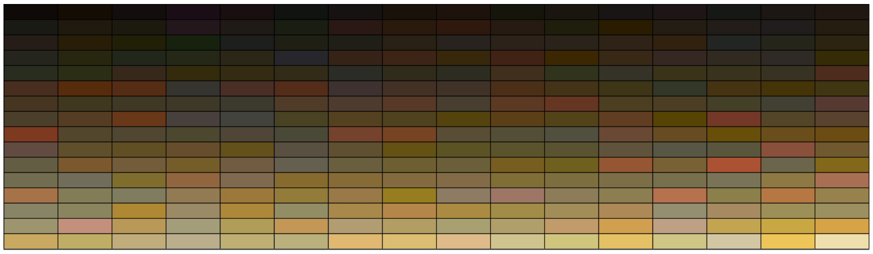

Figure 4: Colour palette of the reference painting generated by GIMP

	Brown	Red	Yellow	Blue	Black	White
Colour Name(s)	Van Dyke Brown	Madder Lake	Yellow ochre	Azurite	Bone black	Lead White
	Umber	Vermillion	Lead-tin yellow	Smalt	Carbon black	
Main Chemical Compunds	Iron(III)-oxides	Alizarin and Purpurin	Iron oxyhydroxide	Basic copper(II)-carbonate	Carbon	Basic lead(II)-carbonate
	Iron(III)-oxide	Mercuric sulfide	Lead stannate (Type I) and lead tin oxide silicate (Type II)	potassium glass containing cobalt	Amorphous carbon	

Figure 5: Rough table of colours and elements

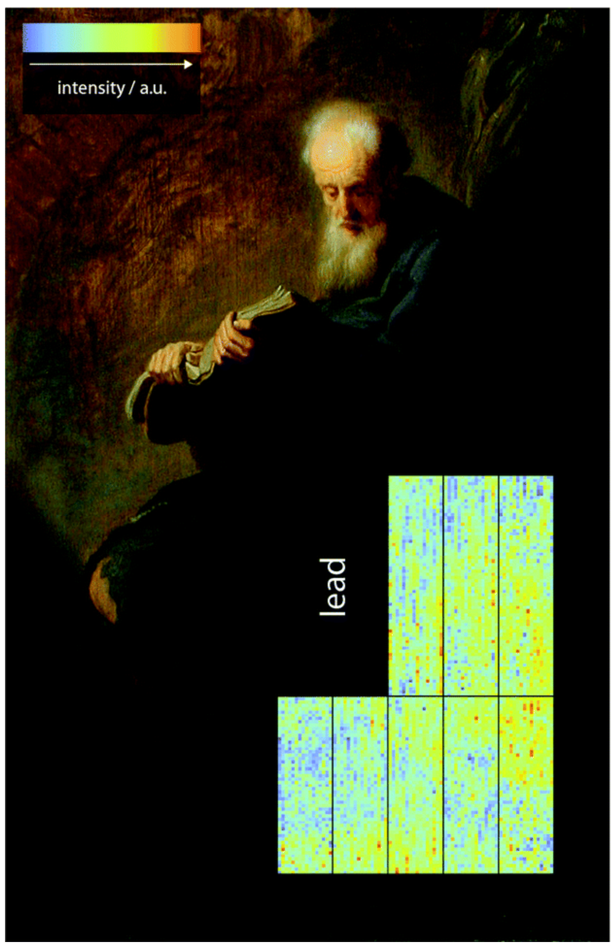

Figure 6: The result of the scan for lead[8]

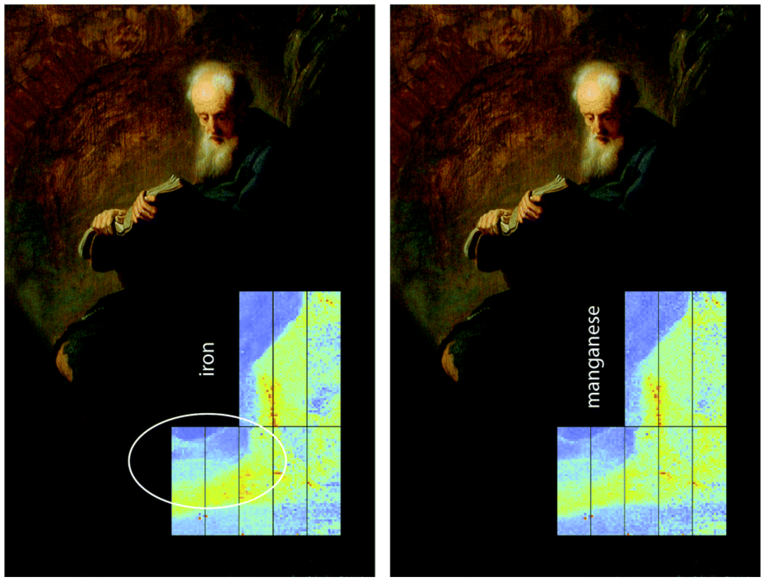

Figure 7: The result of the scan for iron/manganese[8]

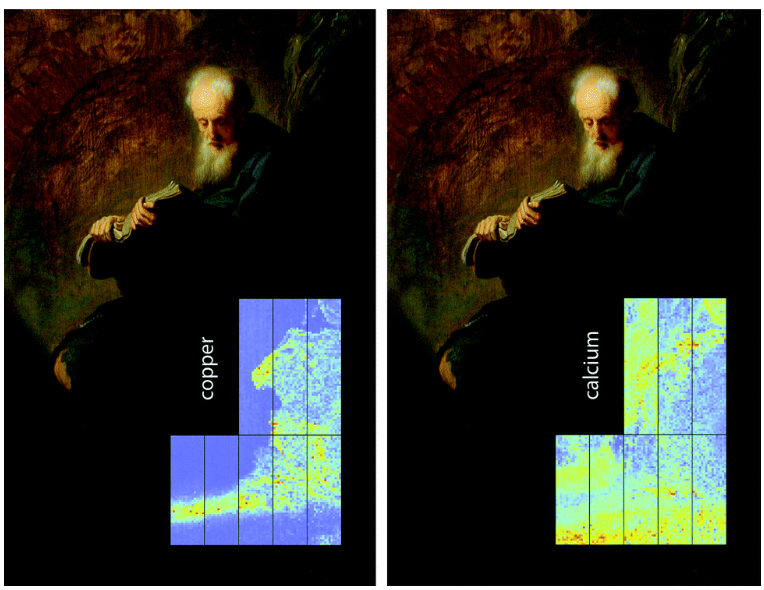

Figure 8: The result of the scan for copper/calcium[8]

By unit analysis, it can be seen that the flux of the neutrons is

$$J = \frac{\text{number of neutrons}}{\text{cm}^2 \times \text{s}} \tag{1}$$

Since, this information is known the calculations will be verified by two ways. By taking the known value of the flux, the number of neutrons can be solved for. The number will then be confirmed by another source. This will verify the validity of the equation and the flux.

$$J = \frac{n}{\sigma t} \tag{2}$$

$$J\sigma t = n \tag{3}$$

$$(10^9 \frac{\text{n}}{\text{cm}^2\text{s}})(3.5\text{cm} \times 12.5\text{cm})(43200\text{s}) = 1.89 \times 10^{15} \text{ neutrons} = n \tag{4}$$

Figure 9: Using the information given to find the total number of neutrons released to confirm the validity

$$\text{Total Neutron Capture Production Rate} = \sigma \times J \tag{5}$$

Figure 10: Calculation of the Total Neutron Capture Production Rate

Figure 11: Displaying the great disparity between the half-life of ^{209}Pb (blue line) and ^{209}Bi (black line) plotted against one years time span

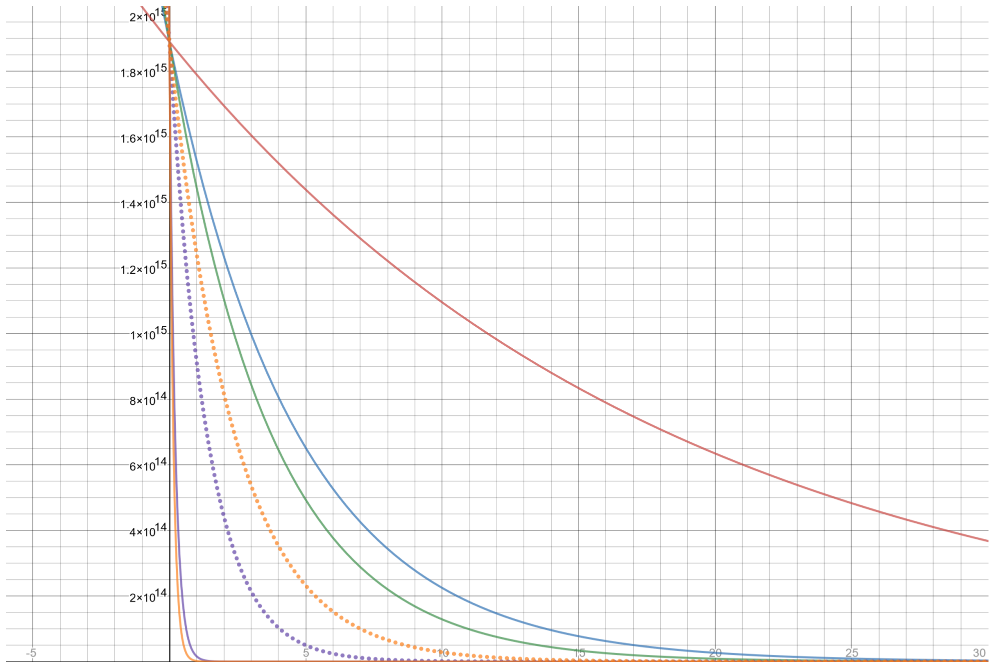

Figure 12: The half-lives of ^{48}Ca (purple), ^{55}Mn (green), ^{60}Fe (orange), ^{63}Cu (red), and ^{208}Pb (blue) across one day
The dotted line indicate the daughter decay of the parent isotope

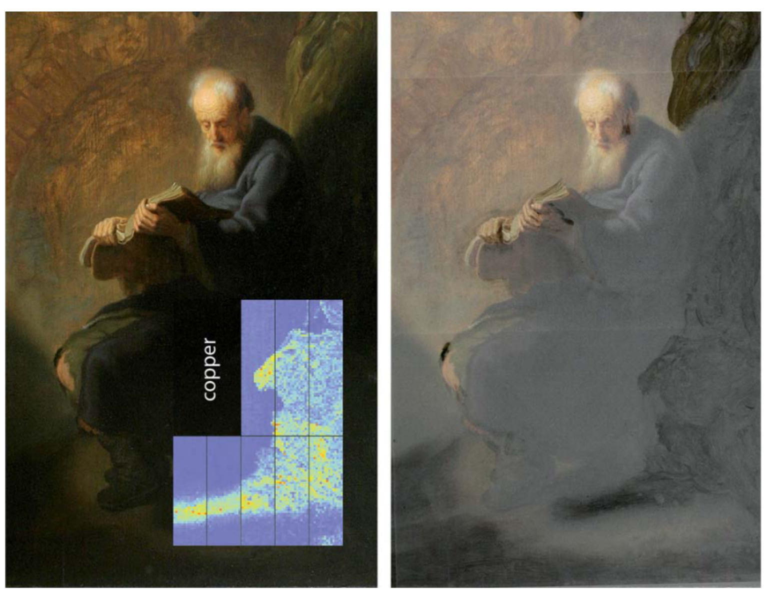

Figure 13: Using the scans to see where strokes were redone or covered up

1

$$y = 1\,890\,000\,000\,000\,000 \cdot e^{-\left(\frac{x}{\frac{3.258}{\ln 2}}\right)}$$

2

$$y = 1\,890\,000\,000\,000\,000 \cdot e^{-\left(\frac{x}{\frac{12.7}{\ln 2}}\right)}$$

3

$$y = 1\,890\,000\,000\,000\,000 \cdot e^{-\left(\frac{x}{\frac{2.5789}{\ln 2}}\right)}$$

4

$$y = 1\,890\,000\,000\,000\,000 \cdot e^{-\left(\frac{x}{\frac{0.1453}{\ln 2}}\right)}$$

5

$$y = 1\,890\,000\,000\,000\,000 \cdot e^{-\left(\frac{x}{\frac{0.9533}{\ln 2}}\right)}$$

6

$$y = 1\,890\,000\,000\,000\,000 \cdot e^{-\left(\frac{x}{\frac{0.09967}{\ln 2}}\right)}$$

7

$$y = 1\,890\,000\,000\,000\,000 \cdot e^{-\left(\frac{x}{\frac{1.650}{\ln 2}}\right)}$$

Figure 14: The decay equations use to calculate the exposure of the painting

$$\text{Mass of wood exposed} = \rho_{\text{oak}} \times Volume = (593)(0.35)(1.25)(0.256) = 0.066416\text{kg} \quad (6)$$

Figure 15: Calculation of the mass of exposed wood

Element	Half-life (hours)	Time (hours)	Particles Left
^{49}Ca	0.1453	24	0
^{49}Sc	0.9533	12	2.45181×10^{7}
^{56}Mn	2.5789	48	4.71536×10^{9}
^{61}Fe	0.09967	48	0
^{61}Co	1.65	24	7.90394×10^{10}
^{63}Cu	12.7	24	5.10019×10^{14}
^{208}Pb	3.5	72	4.10809×10^{8}
Total			5.10103×10^{14}

Table 1: Total amount of particles still active in the painting

Figure 16:

Element	Half-life (hours)	Time (hours)	Particles Left	Time (hours)	Particles Left
^{49}Ca	0.1453	24+168	0	24+730	0
^{49}Sc	0.9533	12+168	-	12+730	-
^{56}Mn	2.5789	48+168	-	48+730	-
^{61}Fe	0.09967	48+168	-	48+730	-
^{61}Co	1.65	24+168	-	24+730	-
^{63}Cu	12.7	24+168	5.31×10^{10}	24+730	-
^{208}Pb	3.5	72+168	0	72+730	-
Total			5.31×10^{10}		0

Table 2: Total amount of particles still active in the painting after one week and one month

Figure 17: