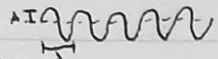
Reflection and Refraction

Propogation of Light.

- As parallel rays that travel in straight lines (rectilinear propogation)

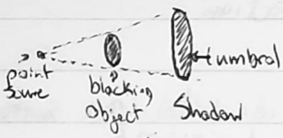
  or

- As waves characterized by amplitude and wavelength (or freq.)
  - 

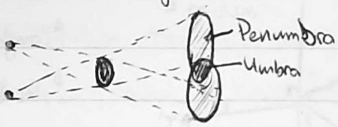
  - freq now denoted by ν, is oscillations per second.
  - speed of propogation $v = \lambda \nu$ ($v = c = 2.998 \times 10^{8} \frac{m}{s}$)
- Light is an EM wave which forms a continuous spectrum

Shadows

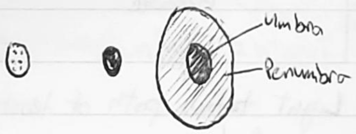
- Consequence of rectilinear propogation
- Single point source of light:



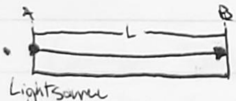
- Two point sources:

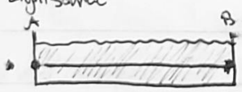


- Extended source:

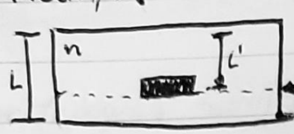


Opitical Path Length:

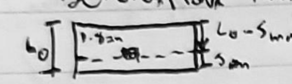
- 

  Light travels from A to B which is distance L in air
- 

  Light still travels L but it take longer since light interacts with the water molecules.
- To account for the delay define the Optical Path Length
  - $S = Ln$ where n is the refractive index.
- Also $n = \frac{c}{v}$
- S is the distance the light would have travelled in same amount of time if it had been in a vacuum.

- Consider an observer viewing an object through a medium of index, n. The object appears closer than if removed
- 

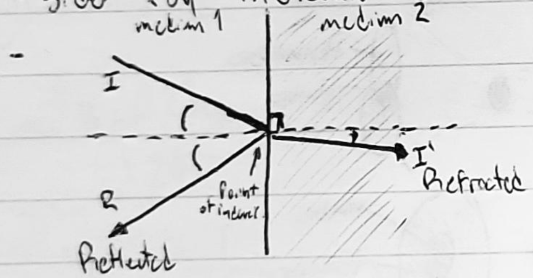
- Object will appear to be at the location which results in the optical path length in air is equal to the optical path length in matter.
- $L n_0 = L' n \Rightarrow L' = \frac{L}{n}$
- Ex. Glass has $n = 1.8$, the writing appears to be 5mm closer than before. How thick is the plate?



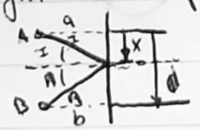
$L' = \frac{L}{n}$, $L_0 - 5 = \frac{L_0}{n}$ → $L_0(1 - \frac{1}{n}) = 5$, $L_0 = \frac{5}{1 - \frac{1}{1.8}}$ → $L_0 = 11.25$ mm

Reflection and Refraction

- Consider ray incidental on surface between 2 media



- angles always measured with normal.
- incident, reflected, & refracted ray are coplanar

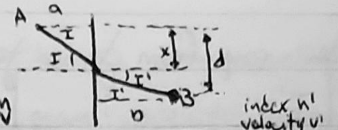
- Fermat's Principle: Light takes path of least time
  - Consider reflection 

  - Total Path Length (A-B): $L = \sqrt{a^2 + x^2} + \sqrt{(d-x)^2 + b^2}$
  - $t = \frac{L}{v} = \frac{\sqrt{a^2+x^2} + \sqrt{(d-x)^2+b^2}}{v}$
  - $\frac{dt}{dx} = 0 \Rightarrow 0 = \frac{1}{2}(a^2+x^2)^{-\frac{1}{2}} 2x + \frac{1}{2}((d-x)^2+b^2)^{-\frac{1}{2}} 2(d-x)(-1)$

    ↳ $\frac{d-x}{\sqrt{(d-x)^2+b^2}} = \frac{x}{\sqrt{a^2+x^2}}$ but when looking at the diagram we see

    $\sin R = \sin I$

• Now consider refraction:



- $t_{TOT} = t_A + t_B = \frac{L_1}{v} + \frac{L_2}{v'} = \frac{\sqrt{a^2+x^2}}{v} + \frac{\sqrt{b^2+(d-x)^2}}{v'}$
- $\frac{dt_{TOT}}{dx} = 0 = \frac{1}{v}\frac{1}{2}(a^2+x^2)^{-\frac{1}{2}}2x + \frac{1}{v'}\frac{1}{2}(b^2+(d-x)^2)^{-\frac{1}{2}}2(d-x)(-1)$

$$\frac{1}{v}\frac{x}{\sqrt{a^2+x^2}} = \frac{1}{v'}\frac{d-x}{\sqrt{b^2+(d-x)^2}}$$

$$\frac{\sin I}{v} = \frac{\sin I'}{v'}$$

$$\frac{\sin I}{\frac{c}{n}} = \frac{\sin I'}{\frac{c}{n'}} \Rightarrow n \sin I = n' \sin I'$$

• Back to $n_0 L_0 = n'L'$

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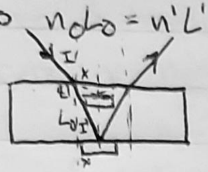


• $\sin\theta \simeq \tan\theta$.
• $\tan I = \frac{x}{L'} = \sin I$
• $\tan I' = \frac{x}{L_0} = \sin I'$

$n_0 \sin I = n' \sin I$
$n_0 \frac{x}{L'} = n' \frac{x}{L_0}$
$n_0 L_0 = n' L'$

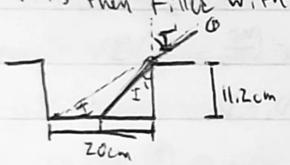
• Note: Index of Refraction changes with wavelength.
- at $\lambda = 588.6\,nm$

Vacuum = 1

Glass = 1.5230

Air = 1.0003.

• Ex. An open cylinder is 20cm in diameter & is 11.2cm tall. Observer is looking from such a direction that they can just see the opposite bottom. It is then filled with liquid, what is the refractive index?

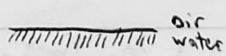


$n_0 \sin I = n' \sin I' \Rightarrow n_0 \frac{20}{\sqrt{20^2+11.2^2}} = \frac{10}{\sqrt{10^2+11.2^2}}$

$n_0' \simeq 1.31$

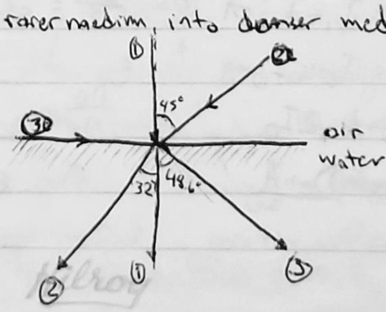
Critical Angles

• Consider boundaries between media of different n
• Eg. Air = 1.0003, water $\frac{4}{3}$



• Consider case of ray passing from rarer medium, into denser medium
•

Ray	I	I'
①	0°	0°
②	45°	32°
③	90°	48.6°

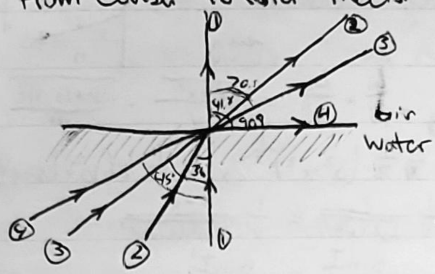


- In case 3, I' is called the limiting angle of refraction
- $I'_{lim} = \sin^{-1}\left(\frac{n}{n'}\right)$

Note:

- Next consider ray from denser to rarer media

Ray	I	I'
①	0°	0°
②	30°	41.8°
③	45°	70.5°
④	48.6°	90°

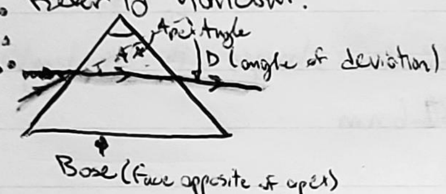


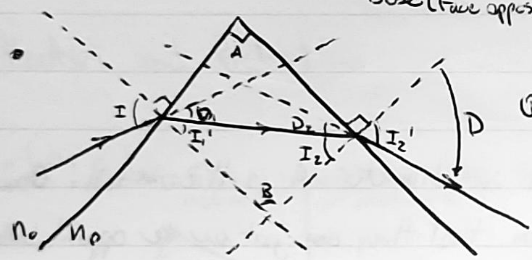
- In case 4 the critical minimum angle of total internal reflection
  - $I_{CRIT} = \sin^{-1}\left(\frac{n'}{n}\right)$
  - $I > I_{CRIT}$, reflected.

- Note: $I'_{lim} = I_{CRIT}$

Prisms

- Reflecting Prisms: Refer to handout.
- Refracting Prisms:





- Follow rays through to find D

① 1st Surface: $n_o \sin I_1 = n_p \sin I_1' \Rightarrow I_1' = \sin^{-1}\left(\frac{n_o}{n_p}\sin I_1\right)$

 $A + B + 180 = 360 \Rightarrow A + B = 180$

 $I_2 + I_1' + B = 180 \Rightarrow I_2 + I_1' + (180 - A) = 180$

② $\Rightarrow I_2 = A - I_1'$

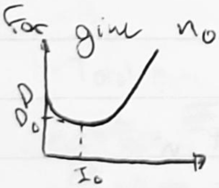
- 2nd Surface: $n_p \sin I_2 = n_o \sin I_2' \Rightarrow I_2' = \sin^{-1}\left(\frac{n_p}{n_o}\sin I_2\right)$ ③

 $D_1 = I_1 - I_1'$, $D_2 = I_2' - I_2$, $D = D_1 + D_2$

$D = I_1 - I_1' + I_2' - A + I_1'$ ← sub in eq ②

④ $D = I_1 + I_2' - A$

- For given n_o, n_p, A get a curve that looks like:



D_o occurs when ray passes symmetrically through prism so $I_2' = I_1 = I_o$ and $I_1' = I_2$.

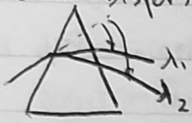
- $D = I_1 + I_2' - A$ ③ $\Rightarrow \frac{D_o + A}{2} = \sin^{-1}\left[\frac{n_p}{n_o}\sin\left(\frac{A}{2}\right)\right]$

$D_o = I_o + I_o - A$

$D_o + A = 2I_o$

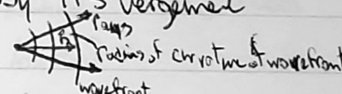
$I_o = \frac{D_o + A}{2}$

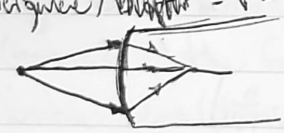
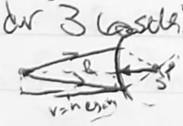
$$\frac{n_p}{n_o} = \frac{\sin\left(\frac{D_o + A}{2}\right)}{\sin\left(\frac{A}{2}\right)} \Big\} \text{Prism Eqn.}$$

- When a bundle of light passes through each wavelength is refracted according to its index for the prism (white light disperses into spectrum)
- Normal Dispersion: at longer wavelengths index is less (true in regions away from absorption)
- 

$\Delta = D\lambda_2 - D\lambda_1$ ← Angular Dispersion
- To characterize glasses with respect to how much dispersion occurs, need to look at behavior at more than one wavelength
- They use 3 wavelengths (red, green, blue) (n_c, n_F', n_c')
- The ratio, $v = \frac{n_c - 1}{n_F' - n_c'}$ ← Abbe's number (v-value)
- Glasses of low dispersion ($n_F' - n_c'$ is small) are called crown glasses and v is large ($v > 55$)
- Glasses of high dispersion ($v < 50$) are called flints

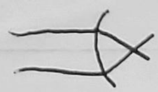
Light on a Curve Surface

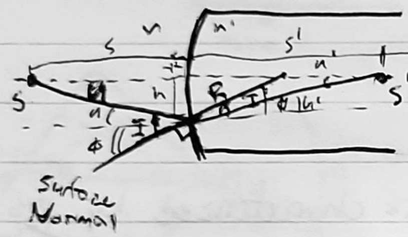
- Light incident on surface is characterized by its vergence
- Light that spreads out is called divergent 

- R is always measured from wavefront to center of curvature
- Since it is opposite direction of propagation of light, R is negative
- Light that comes together is convergent 
- R is positive
- Vergence/~~[illegible]~~ $= V = \frac{n}{R}$ units $= m^{-1}$ or diopter
- 
  - Surface has refractive power, $P = \frac{n}{f}$ where f is focal ~~[illegible]~~ length
- Focal Length is measured from the surface to focal power.
- Consider 3 cases:
  - 

$V' = V + P$ Because $P > |V|$ some excess positive vergence remains, l.t after refraction is convergent
  - 

Could have ~~[illegible]~~ situation where $|V| = P$ the $V' = 0$ and light emerges parallel & it image is projected at ∞ pupil.
  - 
Parallel light incident a positive surface comes out of focus at the second focal point, F_2

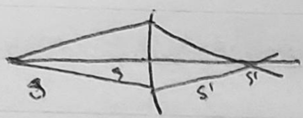


- Object + image can lie on either side of vertex so obj + image spaces overlap completely and extended to infinity in both directions
- Whether a given point is part of object or the image space depends on whether it is part of a ray before or after refraction
- Sign Convention: Rational Cartesian Sign Convention.
- Figures drawn with light propagating left → right if possible
- Distances measured from surface in a direction opposite direction of propagation of light are negative; those in the direction of propagation of light are positive.
- Radii of curvature → measured from surface.
  - Those in direction of propagation of light → positive
  - —"— opposite direction ——"—— → negative
- Gauss' formula and the Surface Power Equation
  - 

    $I = u + \phi$ (1)

    $I' = \phi - u'$ (2)
  - Paraxial Approximation → Assume angles $u + u'$ are small → ray travels close to optical axis.
    - $n \sin I = n' \sin I' \to nI = n'I'$
  - Sub (1) + (2) → $n(u+\phi) = n'(\phi - u')$ (3)
  - For paraxial rays c is small compared to $|s|$ & $|s'|$, so $\tan u \simeq u = \frac{h}{|s|+|c|} = \frac{h}{|s|}$ (4); $\tan u' \simeq u' = \frac{h}{|s'|+|c|} = \frac{h}{|s'|}$ (5)
  - $\sin \phi = \phi = \frac{h}{|R|}$ (6)
  - Sub (4) + (5) + (6) → $n\left(\frac{h}{|s|} + \frac{h}{|R|}\right) = n'\left(\frac{h}{|R|} - \frac{h}{|s'|}\right)$

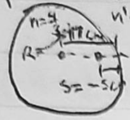
    $$\frac{n'}{|s'|} = \frac{(n'-n)}{|R|} - \frac{n}{|s|}$$
  - For case above; $|s| = -s$, $|s'| = s'$, $|R| = R$
  - $\boxed{\frac{n'}{s'} = \frac{(n'-n)}{R} + \frac{n}{s}}$ Gauss' formula for refraction at a single surface
  - Consider $s = -\infty \to F_2$ $\quad \frac{n'}{f_2} = \frac{n'-n}{R} + \cancel{\frac{n}{\infty}}^{0}$ → $\boxed{P = \frac{n'-n}{R}}$ Surface Power Equation

- 

$V = \frac{n}{s}$ $V' \frac{n'}{s'}$

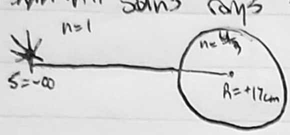
incoming vergence Outgoing Vergence.

- Gauss' formula can be written $V' = P + V$
- Ex ~~A~~ fish is 5cm from edge of bowl, bowl has 34 cm diameter. a) Find ~~appar~~ apparent position of fish



$\frac{n'}{s'} = \frac{n'-n}{R} + \frac{n}{s} \Rightarrow s' = \frac{n'}{\frac{n'-n}{R} + \frac{n}{s}} \Rightarrow s' = \frac{1}{\frac{1-\frac{4}{3}}{-17} + \frac{\frac{4}{3}}{-5}} \simeq -4\text{cm}$

b) Will suns rays focus on fish?

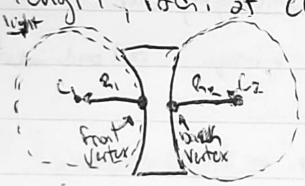


$\frac{n'}{s'} = \frac{n'-n}{R} + \cancel{\frac{n}{s}}^{0} \Rightarrow s' = \frac{n'R}{n'-n} \Rightarrow s' = \frac{(\frac{4}{3})(17)}{\frac{4}{3}-1} \simeq 68\text{cm}$

Don't have to worry about fishes' blindness.

Lenses.

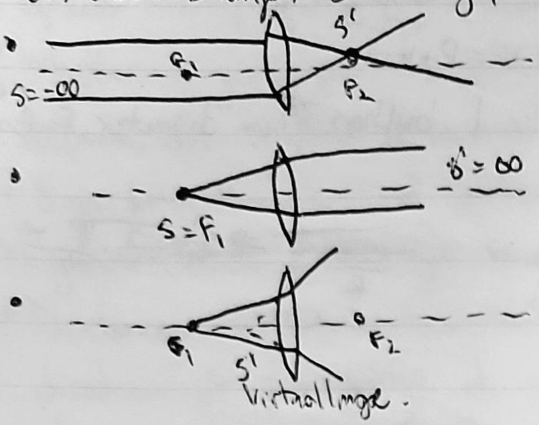
Thin Lenses

- A lens whose thickness is small compared to the focal length, radii of curvature, and s' + s.
- 

  - lens has 2 centres of curvature.

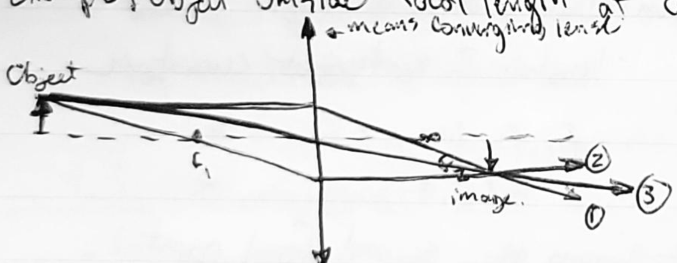
- Incident parallel light forms the second focal point
- Light made parallel by lens comes from 1st focal point
- Generally "the focal point of the lens" refers to F_2
- Positive Lens (Converging)
  - makes parallel light convergent
  - thicker in the middle than at periphery
  - F_2 to right of lens
  -  Bi-Convex  Plano-Convex
- Negative lens (Diverging)
  - makes parallel light diverge
  - thinner in middle than at periphery
  - F_2 to left of lenses
  -  Bi-Concave  Plano-Concave.

- As object is brought in from infinity towards lens; image distances changes accordingly



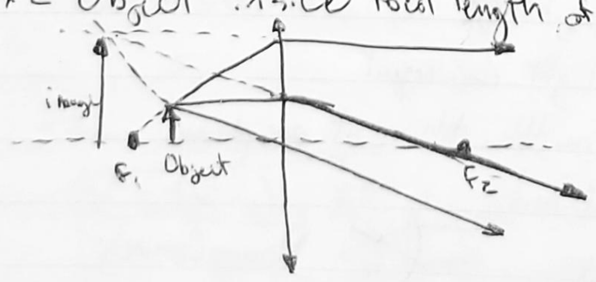
Graphical Ray Tracing

- Need to use at least 2 of 3 principal rays
  1. Parallel Ray - Initially parallel to optical axis, after passing through lens passes through F_2.
  2. Focal Ray - Initially passes through F_1 emerges parallel to optical axis
  3. Undeviated Ray - goes through center of lens with no deviation
- Example 1 Object Outside focal length of converging lens.



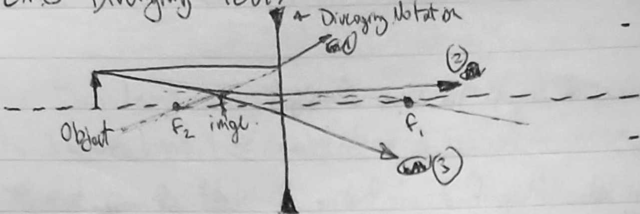
- image is real and inverted

- An ~~image~~ object is virtual if rays converge towards it before passing through the lens.
- Ex 2 Object inside focal length of converging lens.



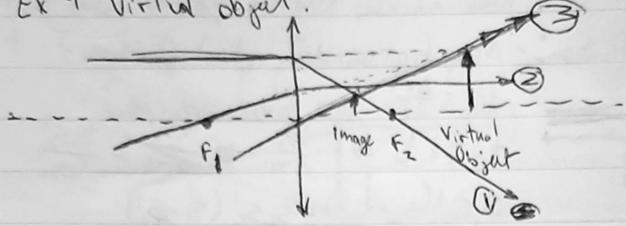
- image is upright, virtual, magnified

- Ex 3 Diverging lens



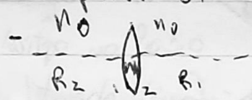
- Image is virtual, upright, reduced in magnification
- True for all real objects.

- Ex 4 Virtual object.



Thin Lens Equations

- (1) Lens Makers formula: - For a thin lens refractive power is sum of powers of 2 surfaces of lens



$$P_L = P_1 + P_2 = \frac{n_L - n_0}{R_1} + \frac{n_0 - n_L}{R_2} = (n_L - n_0)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

  - R_2 is negative.
  - Recall: $P = \frac{n_{surrounding}}{f} = \frac{n_{air}}{f} = \frac{1}{f}$

- (2) Thin Lens Equation: - $V' = V + P$, for $\frac{n}{s}$ → $\frac{n'}{s'} = \frac{n}{s} + P$ ← General form + use if differ
  - If in air $\frac{1}{s'} = \frac{f}{s+f}$

- Example: When obj is placed 75 cm in front of a converging lens, its image is 3 times as far away from lens as when obj is placed at infinity, what is f of lens?

$$\frac{1}{s'} = \frac{1}{s} + \frac{1}{f} \Rightarrow \frac{1}{s'} = \frac{f+s}{sf} \Rightarrow s' = \frac{sf}{f+s}$$

$s_1' = 3s_2'$

$\frac{s_1 \cancel{f}}{f+s_1} = \frac{3s_2\cancel{f}}{f+s_2}$

$s_1(f+s_2) = 3s_2(f+s_1)$

$\frac{s_1}{s_2}(f+s_2) = 3(f+s_1)$

$\frac{s_1}{s_2}f + s_1 = 3f + 3s_1$

$s_1 - 3s_1 = 3f - \frac{s_1}{s_2}f$

$-2s_1 = f\left(3 - \frac{s_1}{s_2}\right)$

$f = \frac{-2s_1}{\left(3 - \cancel{\frac{s_1}{s_2}}\right)}$

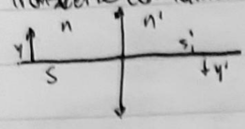
$\simeq \frac{-2}{3}s_1$

$f \simeq \frac{2}{3}(75) \simeq 50\,cm$

Magnification

- Comparison of size of image to that of object
- There are 3 types:

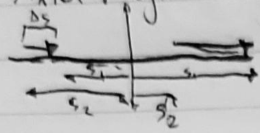
① Transverse (or lateral) Magnification: Ratio of height of image to hight of object.

- 

$$M_T = \frac{y'}{y} = \frac{n}{n'}\frac{s'}{s}$$

when $n = n'$, $M_T = \frac{y'}{y} = \frac{s'}{s}$

② Axial Magnification: - Magnification along optical axis

- 

$$M_x = \frac{\Delta s'}{\Delta s}$$

$$\frac{1}{s} + \frac{1}{f} = \frac{1}{s'} \Rightarrow f = \frac{ss'}{s+s'} \Rightarrow \frac{s_1 s_1'}{s_1+s_1'} = \frac{s_2 s_2'}{s_2+s_2'}$$

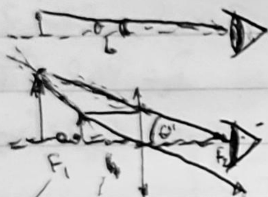
$$s_1 s_1'(s_2+s_2') = s_2 s_2'(s_1+s_1')$$

⋮

$$\frac{\Delta s'}{\Delta s} = \frac{s_1' s_2'}{s_1 s_2}$$

$$M_x = \frac{\Delta s'}{\Delta s} = M_T^2$$

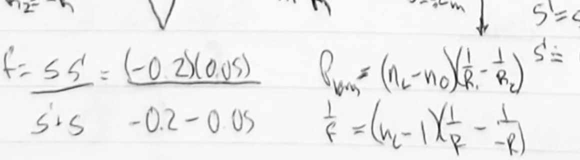
③ Angular Magnification: Refers to a comparison of angles subtended at the eye by an object with and without the aid of an optical device.



- Small object of height y when observed by unaided eye is assumed to be held at normal viewing eye distance (~25 cm)
- To magnify, lens within focal length.
- $M_\theta = \frac{\theta'}{\theta}$

- Ex. object is located 20cm in front of a thin converging lens. Lateral magnification of object is $M = -0.25$; and the index of refraction of the lens material is 1.65. What are the 2 radii of curvature of the lens?

$|R_1| = |R_2| = R$



$s' = s \cdot M_T = -0.2(-0.25)$

$s' = 0.5m$

$$f = \frac{ss'}{s'+s} = \frac{(-0.2)(0.05)}{-0.2-0.05} = 0.04\,m$$

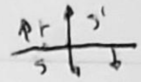
$$P_{lens} = (n_L - n_0)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (n_L - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

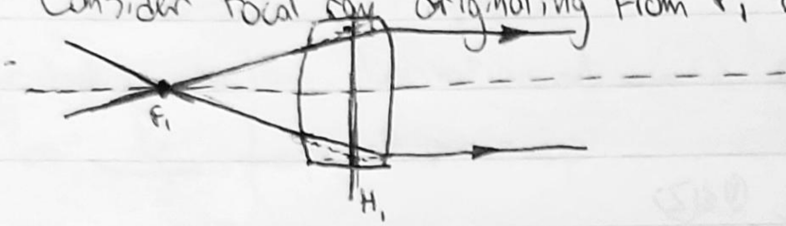
⋮

$R = 0.052\,m$

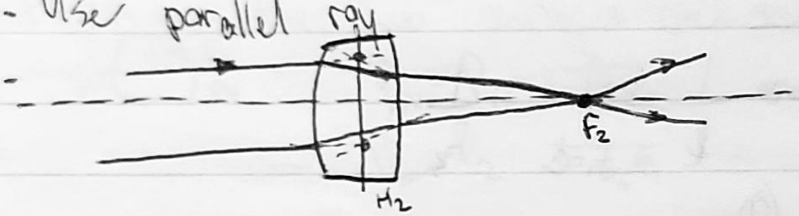
Thick Lenses and Combinations of Lenses

- For thin lens we measure f, s + s' from center of lens 

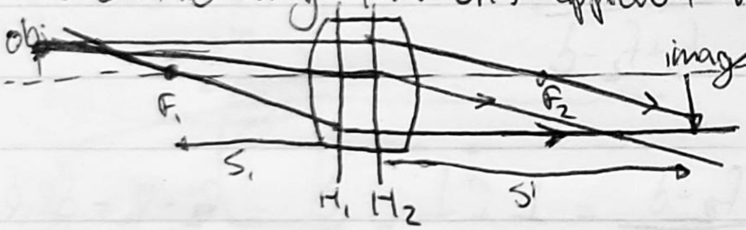
- In thick lens (or combination of lenses) → measure from hypothetical planes called principal planes
- We can continue to use conventional thin lens equations.
- How are Principal Planes determined in a thick lens?
- 1 - First Principal Planes
  - Consider focal ray originating from F_1 of a thick lens



- 2 - Second Principal Plane
  - Use parallel ray



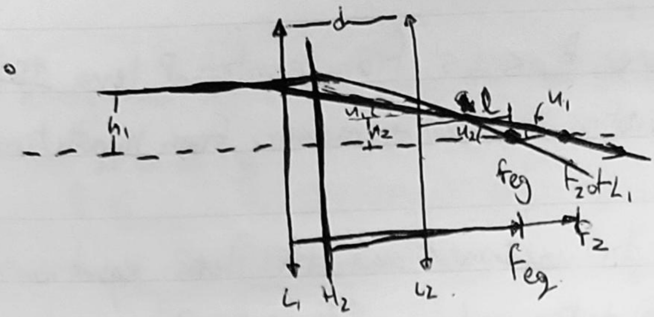
- To continue using thin lens approach we assume the following:



$$\frac{1}{s'} = \frac{1}{s} + \frac{1}{f_2}.$$

  - Principal planes are conceptual only they are located where refraction is assumed to occur so that thin lens equations are possible
  - Lens that are asymmetrical, H_1 & H_2 will move towards larger curvature.

Combinations of Lens

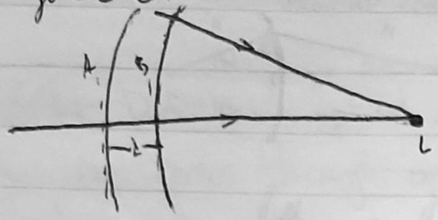


- $\dfrac{h_1}{h_2} = \dfrac{f_1}{f_1 - d}$ ①
- $\dfrac{h_1}{h_2} = \dfrac{f_{eq}}{l}$ ②
- Sub and solve ① & ②
  - $l = \dfrac{f_1 - d}{f_1} f_{eq}$ ③
- $\dfrac{1}{s'} = \dfrac{1}{s} + \dfrac{1}{f} \Rightarrow \dfrac{1}{l} = \dfrac{1}{f_1 - d} + \dfrac{1}{f_2}$ 4
- Sub in ③ to ④

  $f_{eq} = \dfrac{f_1 f_2}{f_1 + f_2 - d}$
- With Power
  - $P = \dfrac{1}{f} = \dfrac{f_1 + f_2 - d}{f_1 f_2} = \dfrac{1}{f_2} + \dfrac{1}{f_1} - \dfrac{d}{f_1 f_2} = P_2 + P_1 - dP_1P_2$
- Principals are inaccurate to use for measurements
- Use vertices of lens instead.

- Power of front Vertex - $P_{fv} = -\dfrac{n_o}{f_{fv}}$
- " " Back " - $P_{bv} = \dfrac{n_f}{f_{bv}}$

• Vergenence



Plane A → $L = \frac{n_0}{V_A}$

Plane B → $V_B = \frac{n_d}{L-d}$

$$V_B = \frac{V_A}{1 - \frac{d}{n_0} V_A}$$

The vergence at a distance d from a point where vergence is V_0 is given by $$V_{eff} = \frac{V_0}{1 - \frac{d}{n_0} V_0}$$ } Change of Vergence Eqn

• Power had a similar derivation and wound up with
- $P_{eff} = \frac{P_0}{1 - \frac{d}{n_0} P_0}$ } change of power eqn.

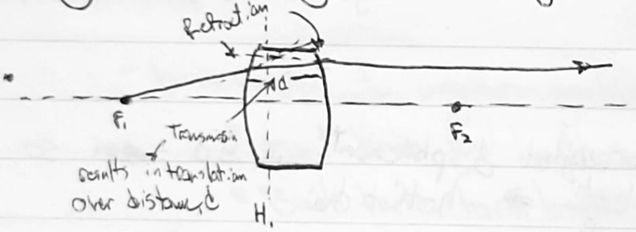
• For thin lens $P = P_1 + P_2$

• For thick lens the two surfaces are separated by d.

$$P_{FV} = P_1 + \frac{P_2}{1 - \frac{d}{n_0} P_2}$$

$$P_{BV} = P_2 + \frac{P_1}{1 - \frac{d}{n_0} P_1}$$

Describing Lens Using Matrix Algebra.



• For refraction: ray changes direction but does not change height above OA.

• For translation: ray does not change in direction but does change its height relative to OA.

• To follow ray, need two operators
↳ one for refraction; one for translation

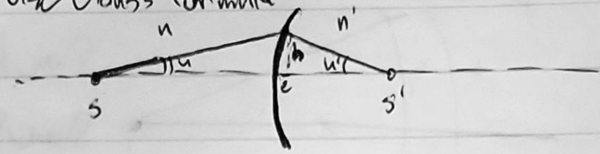
• With matrix algebra can easily trace rays from an object to an image through complicated optical system

⇒ ⇒

- The refraction matrix: Use Gauss' formula

$\frac{n}{s} + \frac{n'-n}{R} = \frac{n'}{s'}$

P



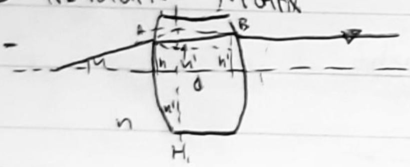
- $\frac{n}{s} + P = \frac{n'}{s'}$
- For paraxial rays: $\tan u \approx u \approx \frac{h}{s} \Rightarrow s = \frac{h}{u}$

  $\tan u' \approx u' \approx \frac{h}{s'} \Rightarrow s' = \frac{h}{u'}$
- $\frac{nu}{h} + P = \frac{n'u'}{h} \Rightarrow nu + Ph = n'u'$
- $nu + Ph = n'u'$ can be written in matrix form
- $\underset{\text{Refraction}}{\begin{pmatrix} 1 & P \\ 0 & 1 \end{pmatrix}} \underset{\text{Input}}{\begin{pmatrix} nu \\ h \end{pmatrix}} = \begin{pmatrix} n'u' \\ h' \end{pmatrix} \Rightarrow$ $nu + Ph = n'u'$ ← Gauss' formula

  $h = h'$ ← No change in height

- $R = \begin{pmatrix} 1 & P \\ 0 & 1 \end{pmatrix}$ ← Refraction Matrix
- $\det R = (1)(1) - (P)(0) = 1$
- for planar surface: $P = \frac{n'-n}{R_{\infty}} = 0$ so,
  - $R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ← unit matrix

- The Translation Matrix



Note: there is a sign convention for angles. an angle measured CW wrt OA is +ive

- For paraxial rays: horizontal displacement is $\approx d$ so
- $\tan u' \approx u' \approx \frac{h-h'}{d} \Rightarrow h - h' = du'$
- $\underset{\text{Translation matrix}}{\begin{pmatrix} 1 & 0 \\ -\frac{d}{n'} & 1 \end{pmatrix}} \underset{\text{Input}}{\begin{pmatrix} n'u' \\ h \end{pmatrix}} = \underset{\text{Output}}{\begin{pmatrix} n'u' \\ h' \end{pmatrix}}$

  $n'u' = n'u'$

  $-\frac{d}{n'}(n'u') + h = h'$

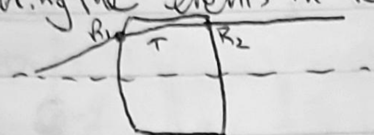
  $h - h' = du'$

  n of medium of which light travels

- $\det T = 1$
- $T = \begin{pmatrix} 1 & 0 \\ -\frac{d}{n} & 1 \end{pmatrix}$

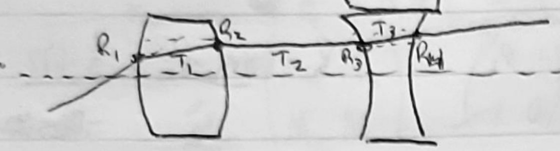
System Matrix

- ~~[illegible]~~ Describes optical system
- when ray passes through optical system it undergoes various refraction & translation process.
- In order to describe the system, we multiply the matrices together representing the events in reverse order of occurrence.
- Ex. Thick lens



$S = R_2 T R_1$

- Ex.



$S = R_4 T_3 R_3 T_2 R_2 T_1 R_1$

- Any system no matter how complex can be represented by one system matrix (2x2 matrix).
- Note: because determinant of a matrix product is equal to the product of the determinants of the matrices, $\det S = 1$
- System matrix is a 2x2 matrix with 4 elements called the gaussian constants, i.e. $S = \begin{pmatrix} b & a \\ d & c \end{pmatrix}$ & represent various parameters of the optical system
  - constant a is $P_{eq} = a$
  - Equivalent focal lengths are $-f_{eq_1} = \frac{-n_0}{a}$
  - $-f_{eq_2} = \frac{n_f}{a}$

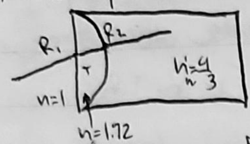
  - b is related to angular magnification
    - Front Vertex focal length is, $f_{FV} = \frac{-b n_0}{a} = b f_{eq_1}$
    - Back Vertex focal length is, $f_{BV} = \frac{c n_f}{a} = c f_{eq_2}$
  - The negative reduced thickness of the lens (or equivalent for system of lenses is $\frac{-d}{n_{lens}}$
  - Now we know where the principal planes are.
  -

~~[illegible]~~

$h_1 = f_{FV} - f_{eq_1}$ $h_2 = f_{BV} - f_{eq_2}$

Example

- Spherical hemisphere of radius 1.25 cm made of glass of index 1.72 is used to seal a tube of water. Light is incident on planar surface from air
- a.) Find system matrix



$S = R_2 T R_1$

$P_1 = \frac{n_a - n_g}{r_1}$

r_1 ← radius of surface 2

$P_1 = 0$

$P_2 = \frac{n_w - n_g}{r_2} = \frac{\frac{4}{3} - 1.72}{-0.0125} = 30.93\ m^{-1}$

$$S = \begin{pmatrix} 1 & P_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{d}{n_g} & 1 \end{pmatrix} \cancel{\begin{pmatrix} 1 & P_1 \\ 0 & 1 \end{pmatrix}}$$

$$S = \begin{pmatrix} 1 & 30.93\ m^{-1} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-0.0125}{1.72} & 1 \end{pmatrix}$$

$$S = \begin{pmatrix} 1-(30.93)\left(\frac{-0.0125}{1.72}\right) & 30.93 \\ -\frac{0.0125}{1.72} & 1 \end{pmatrix} = \begin{pmatrix} 0.7752 & 30.93\ m^{-1} \\ -0.007267\ m & 1 \end{pmatrix}$$

- b.) Find locations of focal points and first and 2nd principal planes.

H_1:

$f_{eq_1} = -\frac{n_0}{a} = -\frac{1}{30.93} = -0.03233\ m$

$f_{FV_1} = b f_{eq_1} = 0.7752(-0.03233) = -0.02506\ m$

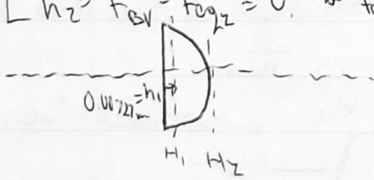
$h_1 = f_{FV} - f_{eq_1} = -0.02506 - (-0.03233) = 0.00727\ m$

H_2:

$f_{eq_2} = \frac{n_f}{a} = \frac{n_w}{a} = \frac{4}{3}\frac{1}{30.93} = 0.043103\ m$

$f_{BV} = c f_{eq_2} = f_{eq_2} = 0.043103\ m$

$h_2 = f_{BV} - f_{eq_2} = 0$ ← tells us H_2 is at the back vertex.



- c.) Obj is 10 cm in front. Where is image?

$\frac{n_0}{s} + P_{eq} = \frac{n_f}{s'}$

$s' = \frac{n_f}{\frac{n_0}{s} + P_{eq}}$

wrt H_1 →

$s' = \frac{\frac{4}{3}}{\frac{1}{10+h_1} + 30.93}$

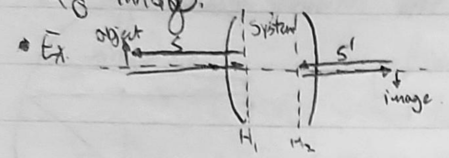
$s' = \frac{\frac{4}{3}}{\frac{1}{-.10\,m - 0.00727} + 30.93}$

$s' = \frac{\frac{4}{3}}{\frac{1}{-0.10727} + 30.93}$

$s' = 0.0617\ m$

↑ wrt H_2.

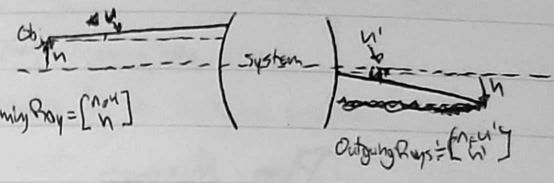
- Next we want to use matrices to connect object and image points.
- Need to translate from object → system and then from system to image.
- Ex.



- Described by: $M = \begin{pmatrix} 1 & 0 \\ -\frac{s'}{n_f} & 1 \end{pmatrix}\begin{pmatrix} b & a \\ d & c \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{s}{n_0} & 1 \end{pmatrix}$

  Object image matrix

  $= \begin{pmatrix} M_b & M_a \\ M_d & M_c \end{pmatrix}$



- In matrix form can write the general transformation from obj to image as:

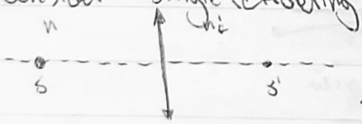
$$\begin{pmatrix} M_b & M_a \\ M_d & M_c \end{pmatrix}\begin{pmatrix} n_0 u \\ h \end{pmatrix} = \begin{pmatrix} n_f u' \\ h' \end{pmatrix}$$

$$M_b n_0 u + M_a h = n_f u' \quad ①$$

$$M_d n_0 u + M_c h = h' \quad ②$$

- Since this transformation connects two conjugate points P & P' all rays that begin at P must end up at P' → implies h' does not depend on angle of incoming ray u. ($M_d = 0$)
- ② then becomes $M_c h = h' \Rightarrow M_c = \frac{h'}{h}$ ⇐ lateral Magnification, M_T
- Thus: $M = \begin{pmatrix} M_b & M_a \\ 0 & M_c \end{pmatrix}$ Generally related to angular magnification

- Ex. Consider single refracting surface (or thin lens)



$s = \mathcal{R}$ ← refraction
↑
System Matrix

$$M = \begin{pmatrix} 1 & 0 \\ -\frac{s'}{n_f} & 1 \end{pmatrix}\begin{pmatrix} 1 & P \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{s}{n_0} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & P \\ -\frac{s'}{n_f} & 1-\frac{s'P}{n_f} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{s}{n_0} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+\frac{s}{n_0}P & P \\ -\frac{s'}{n_f}+\frac{s}{n_0}-\frac{ss'}{n_0 n_f}P & 1-\frac{s'P}{n_f} \end{pmatrix} = \begin{pmatrix} M_b & M_a \\ M_d & M_c \end{pmatrix}$$

$$M_d = 0 = -\frac{s'}{n_f}+\frac{s}{n_0}-\frac{ss'}{n_0 n_f}P$$

$$0 = -\frac{n_f}{s}+\frac{n_f}{s'}-P$$

$$\frac{n_f}{s} = \frac{n_0}{s}+P \quad \text{← Gauss's formula.}$$

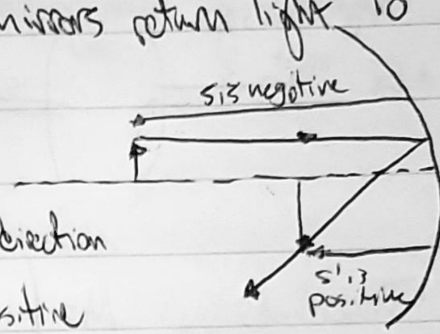
$$M_c = 1-\frac{s'}{n_f}P = 1-\frac{s'}{n_f}\left(\frac{n_f}{s}-\frac{n_0}{s}\right)$$

$$= 1-\frac{s'}{n_f}\left(\frac{n_f s - n_0 s'}{ss'}\right) = \cancel{1}-\frac{n_f}{n_f}+\frac{n_0 s'}{n_f s}$$

$$M_c = M_T = \frac{n_0 s'}{n_f s} \quad \text{← Transverse Magnification.}$$

Mirrors

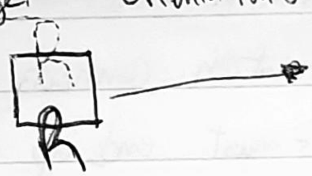
- Can focus light; form images
- Main difference from lenses is ~~that~~ mirrors return light to same medium in which it was travelling
- Sign Convention is adopted to account for this by assuming:
  - Distances measured in the same direction as the light is moving are positive
  - Distances measured in opposite direction of light travel are negative



Plane Mirrors

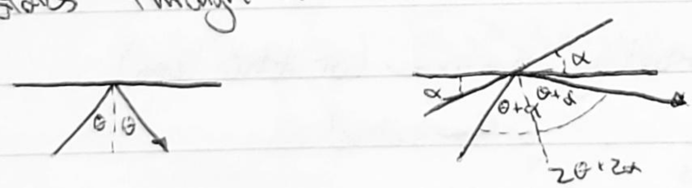
- Reflect light without focusing it.
- A virtual image is created the same ~~distance~~ distance behind the mirror as the object is in front.
-

Orientation of image:
- upright
- right left reversed

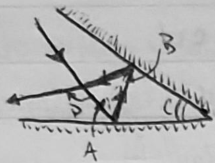


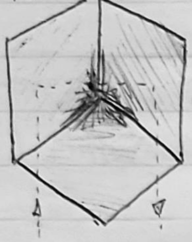
- inverted
- right left reversed

- If mirror is rotated through angle, α, the reflected ray rotates through 2α
-



Multiple Plane Mirrors.

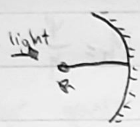
- 

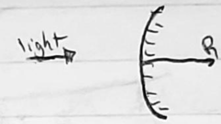
  - D is the angle through which ray is rotated
  - $D = 2A + 2B$ → $C + (90 - B) + (90 - A) = 180$
  $D = 2C$ ← C $= A + B$
- Thus if 2 rays were perpendicular, the ray would return parallel to its initial direction
- Three mirrors at right angles form a corner cube retroreflector
- 

  - has the property that any incoming ray in any direction will return parallel to its original direction

Spherical Mirrors

- Two types:

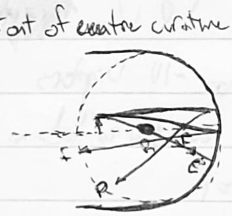




Concave
- R is negative
- analogous to converging lens

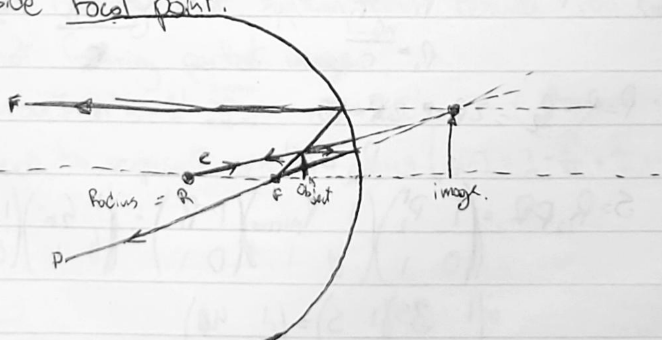
Convex
- R is positive.
- analogous to diverging lens.

- Focal point is $f = -\frac{R}{2}$
- Ray Tracing: - Ex. object out of centre curvature.

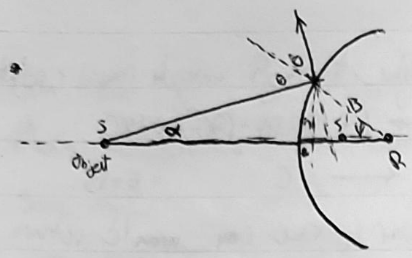


P = Parallel Ray
F = Focal Ray
C = Chief Ray

- Ex. Object inside Focal point.



Derivation of $f = \frac{-R}{2}$.



$A = B + C$

$\theta = \varphi + \alpha$

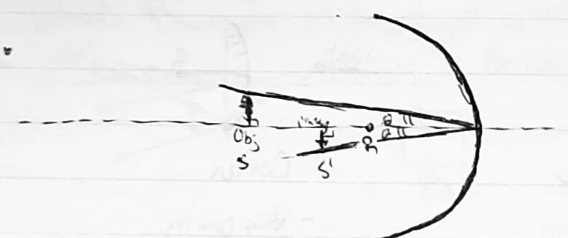
$2\theta = \alpha' + \alpha$

$2(\varphi + \alpha) = \alpha' + \alpha$

$\alpha - \alpha' = 2\varphi$ ①

- $\tan\alpha = \frac{h}{|s|+c} \xRightarrow[\text{Approx}]{\text{Paraxial}} \alpha \approx \frac{h}{|s|}$ / $\tan\alpha' = \frac{h}{|s'|-c} \Rightarrow \frac{h}{|s'|}$?
- $\sin\varphi = \frac{h}{R} \Rightarrow \varphi \approx \frac{h}{|R|}$
- $\frac{h}{|s|} - \frac{h}{|s'|} = 2\frac{h}{|R|} \Rightarrow \frac{1}{-s} + \frac{1}{s'} = \frac{-2}{R} \Rightarrow \frac{1}{s} - \frac{1}{s'} = \frac{1}{R}$.
- Assume $s = -\infty \Rightarrow \frac{1}{-\infty} - \frac{1}{f} = \frac{2}{R} \Rightarrow f = \frac{-R}{2}$
- $-\frac{1}{f} = \frac{1}{s} - \frac{1}{s'} = \frac{2}{R}$

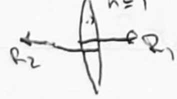
Magnification



$M_T = \frac{y'}{y} = \frac{s'}{s}$

- Power $= \frac{n}{f} \Rightarrow P = \frac{n}{\frac{-R}{2}} \Rightarrow P = \frac{-2n}{R}$
- Reflection Matrix $= F = \begin{pmatrix} 1 & -P \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{2n}{R} \\ 0 & 1 \end{pmatrix}$

- Ex. A sphereical mirror has +10 Diopters and has n=1.5. The back surface of lens coated with metal



$r_1 = -r_2 = r$

$P_1 = \frac{n_G - 1}{r_1}$

$P_1 = \frac{n_G - 1}{r}$

$P_2 = \frac{1 - n_G}{-r} \Rightarrow \frac{n_G - 1}{R}$

$P = P_1 + P_2 = 2P \rightarrow 2P = 10$

$P_{1\&2} = 5\,m^{-1}$ $\qquad P = 30\,m^{-1}$

$S = R_2 F R_1 = \begin{pmatrix} 1 & P_1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & P_{mirror} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & P_1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 5m^{-1} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 30 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 5m^{-1} \\ 0 & 1 \end{pmatrix}$

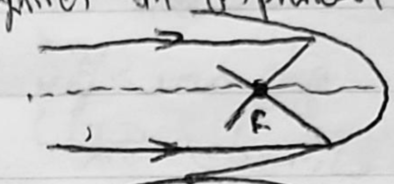
$= \begin{pmatrix} 1 & 35 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 40 \\ 0 & 1 \end{pmatrix}$

Note: We ignore translation matrices because its a thin lens

Aspherical Mirrors

- A spherical will reflect a point object into a perfect point image only if both object and image lie a centre of curvature of mirror
- 

- To image other points perfectly, requires an aspherical mirror
- Ex. Parabolic Mirror
  - Is used when object or image is at infinity



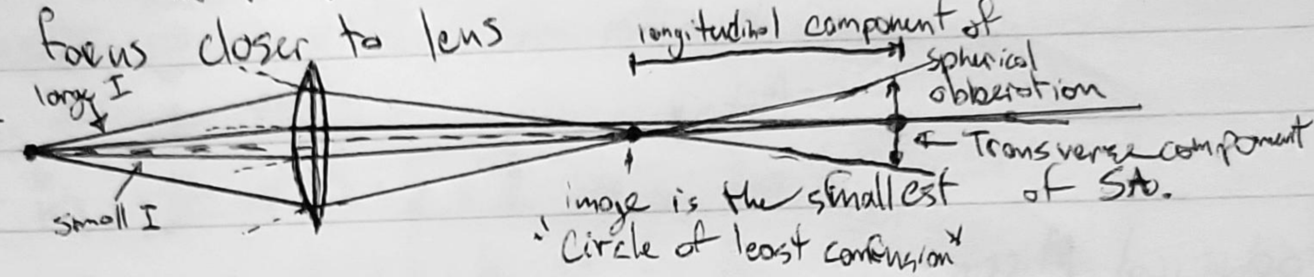
- Ex Ellipsoidal Mirror
  - Object located at one focus provides images at the other

Aberrations

- Inherent short comings of system that results in rays from object not forming perfect image.
- Snell's Law: $n \sin I = n' \sin I' \underset{\text{approx}}{\overset{\text{Paraxial}}{\simeq}} nI = nI'$

- for large I need to represent $\sin$ as a series $\left(\sin I = I - \frac{I^3}{3!} + \frac{I^5}{5!} \dots \right)$
-

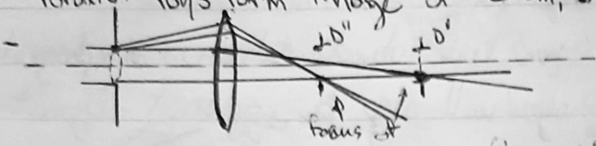
The 5 Primary Abberations

- 1- Spherical Abberation
  - Caused by fact that rays from point object passing through a lens at the periphery (edges) make larger angles than do those passing near OA
  - Those peripheral rays will refract more strongly; focus closer to lens
  - 

  - SA can be minimized by making both surfaces of lens contribute to the refraction equally
  - 

  - In designing lens, $P=(n_{lens}-n_0)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$
    - → make radii of curvature larger (angles will be less) → to get same power would require larger index glass
  - To eliminate SA
    - → Use gradient index lens (n higher at center than periphery)
    - → make surfaces aspherical
    - → Use system of lenses that compensate for each other

· 2- Coma

- is similar to SA with the added complication that object is away from OA.
- Object is circular opening of diam, D
- Paraxial rays form image of diam, D'
- 

peripheral rays (image D'' is larger than D)

- To correct for coma need to make different images coincide
- The optical (or Abbe's) sine theorem states that

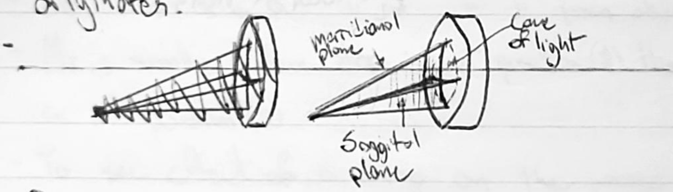
→ $n y \sin u = n' y' \sin u'$ u, u' = angle of rays in object/image space

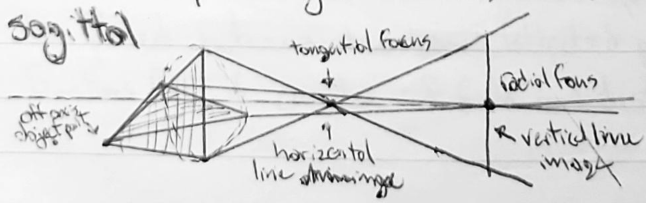
n, n' = indices in object/image space

y, y' = heights of rays in " " "

- To correct for coma, $M_T = \frac{y'}{y}$ must be same for all rays
- Optical system that has been corrected for SA & Coma is called aplanatic

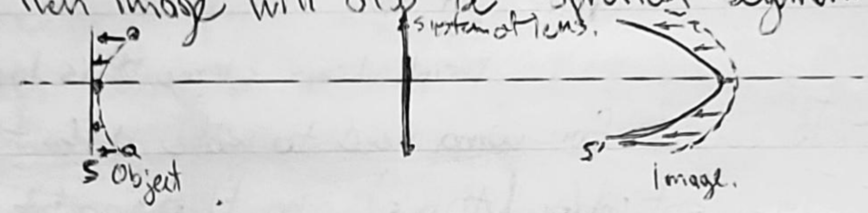
3- Astigmatism

- Object point becomes image line
- Consider off-axis point from which a cone of light originates.
- 

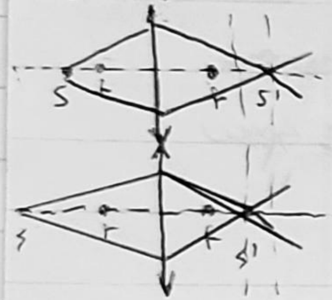
- ~~On average~~ meridional rays form larger angles of incidence with lens than do the saggital rays
- Meridonal rays tangential focus closer to lens than saggital
- 

- Cylindrical lens has astigmatism only along one axis  → surface is part of cylinder instead of a sphere.
- Vertical:  Horizontal: 
- Thus overall result of lines formed by vertical rays at horizontal focus (cylindrical lens correct astigmatism).

• 4 - Curvature of Field.

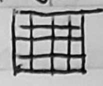
- Assume we have system that has been corrected for SA, coma, and astigmatism
- Now there is 1 to 1 correspondence between object and image point
- Assume we have object which is a spherical segment; then image will also be spherical segment
- 

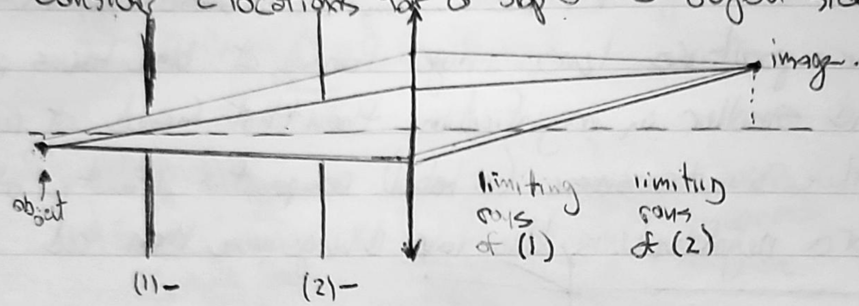
Recall:



- Move each point away from lenses to form an object line
- Thus points on the plane surface object will form a ~~flat~~ curved image and vice versa
- ~~Wrong~~ Very damaging in optical systems where the image is expected to be flat.
- Correcting for curvature of field is called field flattening and is possible by using 2 lenses that meet the Petzval Condition
- $n_1 f_1 + n_2 f_2 = 0$, f_1 = ~~the~~ focal of lens 1
  f_2 = focal of lens 2.

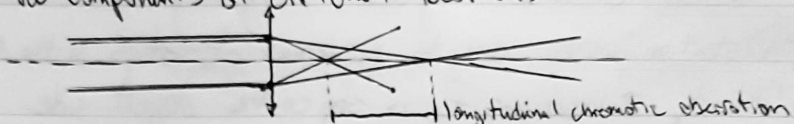
• 5 - Distortion

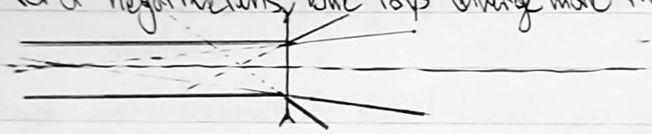
- Results from variation in the lateral (transverse) magnification of object's at different distances from OA.
- Consider object that is a square grid 
- If M_T increases with distance from OA we have a pincushion distortion  ← diagonal corners are further from OA.
- If M_T decreases with distances from OA we have barrel distortion  ← diagonal corners less mag.
- Distortion arises due to presence of stops in the system
- Consider 2 locations for a stop on the object side.



- On average rays travel longer distances from object to lens when stop is in position (1) and shorter distance from lens to image
- Reverse is true for position (2)
- So $M_T \approx \frac{s'}{s}$ ← Average distance rays travel to image
  ← " " " " from object
- M_T is smaller when stop is at position (1) than when it is at position (2)
- To see effect of a stop on the image side, simply reverse role object & image play in diagram above then we find M_T is larger when stop is at (1) than at (2) (more pincushion at (1))
- To eliminate (or reduce) distortion on aperture, stop should be placed between two lenses (effects of position will cancel if done properly)
- Systems free of curvature of field and distortion are orthoscopic.

Chromatic Abberation

- Unlike primary abberations which occur for monochromatic light, chromatic abberation occurs when there is a spectrum of light
- In general, n is higher for blue light (higher freq, shorter λ) than for red (lower freq, higher λ).
- A single lens would thus focus a beam comprised of blue & red components at different locations
- 

- For a positive lens image formed at blue focus is closer and smaller in magnification than that formed at red focus (there is a transverse (or lateral) component of chromatic aberration)
- For a negative lens, blue rays diverge more than red
- 
- Thus a system of lenses could be free of chromatic aberration
- Recall for 2 lenses separated by d
  - $P_{AB} = P_A + P_B - P_A P_B d$ ①
- also Lens-Makers formula
  - $P = (n_L - \cancel{n_0}^{1})\underbrace{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}_{K} \Rightarrow P = (n_L - 1)K$ ②
- $d = 0$, Lens $A = P_A \Rightarrow n_A$, Lens $B = P_B \Rightarrow n_B$.
  - $P_{eq} = P_A + P_B = (n_A - 1)K_A + (n_B - 1)K_B \Rightarrow P_{eq\,Blue} = P_{eq\,Red}$
- Recall Abbe's Number (F' is blue, C' is red).
- $(n_{AF'} - 1)K_A + (n_{BF'} - 1)K_B = (n_{AC'} - 1)K_A + (n_{BC'} - 1)K_B \Rightarrow$
  $\Rightarrow \frac{K_A}{K_B} = \frac{-((n_{BF'} - 1) - (n_{BC'} - 1))}{[(n_{AF'} - 1) - (n_{AC'} - 1)]}$ ③ $\Rightarrow \frac{-n_{BF'} - n_{BC'}}{n_{AF'} - n_{AC'}}$
- Consider green light (e)
  - $P_{Ae} = (n_{Ae} - 1)K_A \quad P_{Be} = (n_{Be} - 1)K_B \Rightarrow \frac{K_A}{K_B} = \frac{P_{Ae}}{P_{Be}}\frac{(n_{Be} - 1)}{(n_{Ae} - 1)}$
- Equate ③ & ④
  - $\frac{K_A}{K_B} = \frac{K_A}{K_B} \Rightarrow \frac{P_{Ae}}{P_{Be}} = -\frac{V_A}{V_B} \Rightarrow \frac{P_A}{P_B} = -\frac{V_A}{V_B}$

- $P_A = P_{eq} - P_B \Rightarrow P_{eq} - \left(-\frac{P_B V_B}{V_A}\right)$
- $P_A = P_{eq}\left(\frac{V_A}{V_A - V_B}\right)$ and $P_B = -P_{eq}\left(\frac{V_B}{V_A - V_B}\right)$
- Note: One positive and one negative is expected.
- Another way of making system achromatic is to use 2 lenses of same optical material separated by an appropriate distance, d.
- Eqn ①: with $n_A = n_B = n$

$$P_{eq} = (n-1)K_A + (n-1)K_B - (n-1)^2 K_A K_B d$$

$$P_{eq} = (n-1)(K_A + K_B) - (n-1)^2 K_A K_B d$$

- for the combination to be achromatic P_{eq} must remain constant even if the wavelength (and hence n) changes.

$$\frac{dP_{eq}}{dn} = 0 = (K_A + K_B) - 2(n-1)K_A K_B d$$

$$d = \frac{K_A + K_B}{2(n-1)K_A K_B}$$

- Recall: $P = (n-1)K$

$$d = \frac{\frac{P_A}{n-1} + \frac{P_B}{n-1}}{2(n-1)\frac{P_A}{(n-1)}\frac{P_B}{(n-1)}} = \frac{P_A + P_B}{2P_A P_B}$$

distance A & B must be separated for achromatic

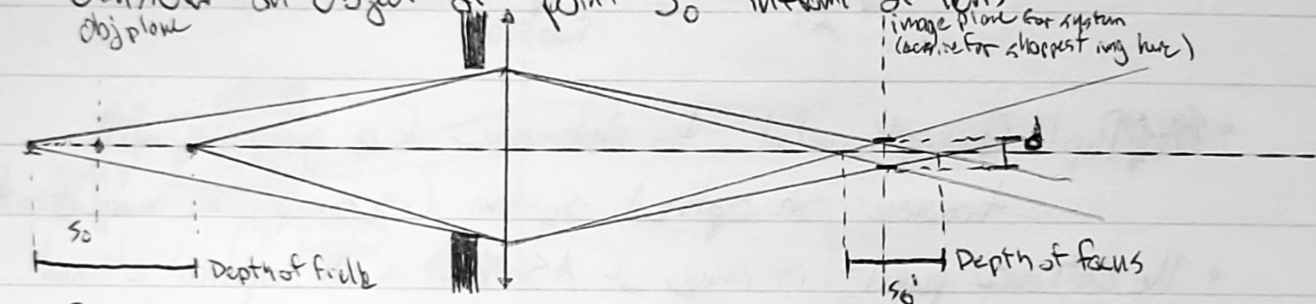
- Recall: $P = \frac{1}{f}$

$$d = \frac{\frac{1}{f_A} + \frac{1}{f_B}}{2\left(\frac{1}{f_A}\right)\left(\frac{1}{f_B}\right)} = \frac{1}{2}(f_A + f_B)$$

Applies for when lenses are of the same material

Stops and Pupils

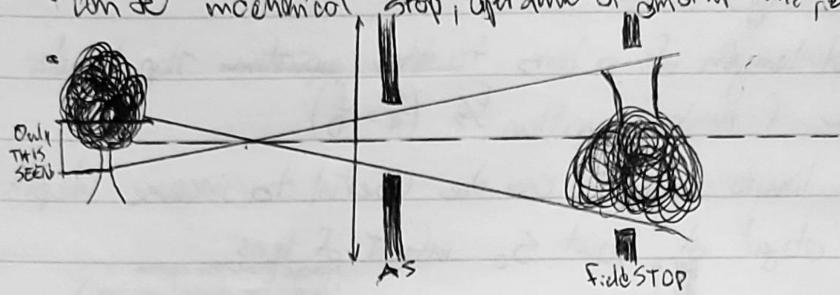
- Aperature Stops → limits light that travels through opitical system (controls brightness)
  → can be rim of one of the lenses in the system or a physical object such as iris
- Ratio of focal length of a lens to the aperature stop diameter is called the f-number, written $f/\#$ $\left(\frac{f}{\#} = \frac{f}{D}\right)$ e.g. "50mm $\frac{f}{1.4}$" on cameras.
- Although AS limits light, can be useful to increase depth-of-field.
- Consider an object at point S_o infront of lens



- Points at object plane are imaged to perfect points in image plane
- Points further and closer to lens than S_o image closer and further respectively from image plane
- Assume if they intercept image plane with "circle of confusion" of diameter 'd' of less they are considered 'focused'.
- Depth-of-field is the 'space' within the object space that results in circles of confusion less than 'd', i.e object space that is in focus
- Corresponding interval in image space is called the depth of focus
- Making AS smaller reduces the angles subtended by rays and allowing rays from a larger depth of field to focus within acceptable range.

Field Stop

- limits the size or angular width (field of view) of the object being imaged
- can be mechanical stop, aperture of another lens, edge of film etc.
-

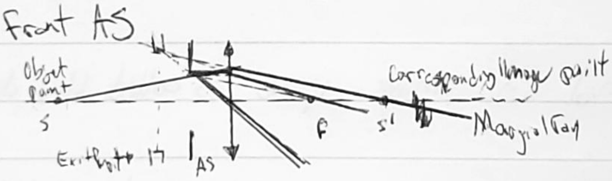


- ~~Stop~~ Pupil: Concept useful to determine if a given ray will traverse an optical system (is simply an image ~~of~~ of AS)
- The entrance pupil - is image of AS ~~~~ in all opitical element that precedes it in the system
  - If nothing proceeds, AS is entrance pupil
- The exit pupil - is image of AS in all opitical element that follow in the system
  - If nothing follows AS, AS is exit pupil
- Cone of light entering opitical system is determined by EP
- " " " exiting " " " " XP

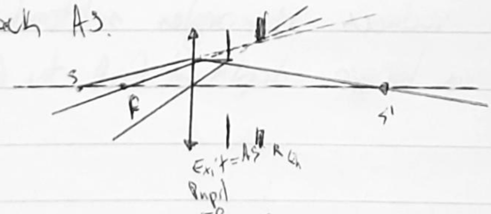
EP = Entrance Pupil

XP = Exit Pupil

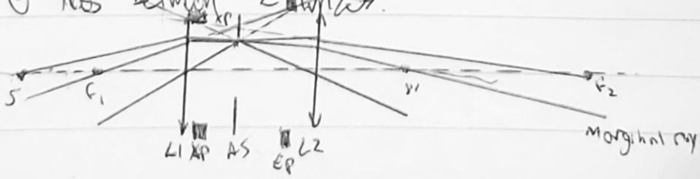
- (1) Front AS



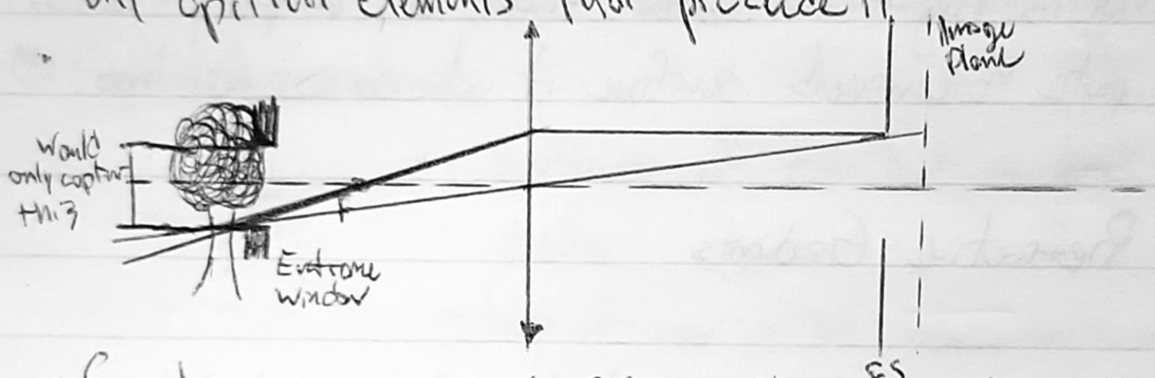
(2) Back AS



(3) AS between 2 lenses



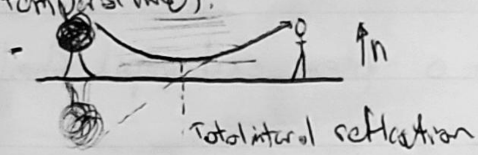
- In a situation where it is unclear which element is the AS you must take each element that could be, and image it all optical elements that precede it. Then the EP that forms smallest angles with object point corresponds to the element that is the actual AS.
- For comfortable viewing of a system exit pupil should correspond to pupil of eye
- The entrance window is the image of a field stop formed by all opitical elements that precede it
-



- Similary the image of FS in all opitical elements that follow is called the exit window
- In ex above, FS is exit window.

Gradient Index Optics (GRIN)

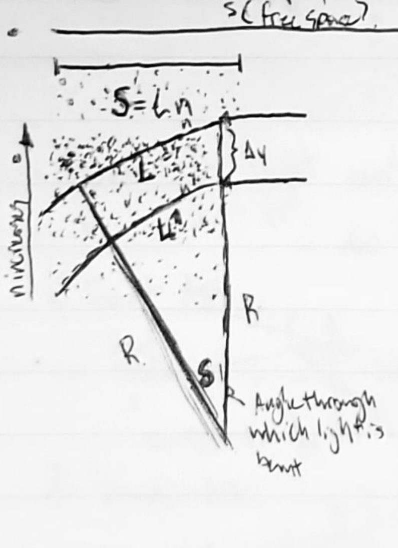
- GRIN refers to variation in refractive index.
- A mirage is an example of a GRIN phenomenon. (caused by changing air temperature).



- A plane parallel circular plate can be made with a refractive index that is higher in center than at periphery → acts as lens
  - If plate has curved surface it acts as a multi-lens

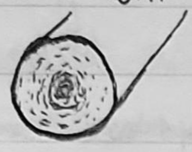
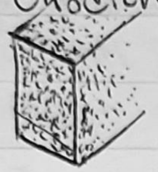
Theory of Refractive Gradients

- Assume that in free space ($n=1$) light travels distance, s, in time, t. In medium of index, n, light travels OPL $S = Ln$ in same time t where L is physical distance traveled in medium n



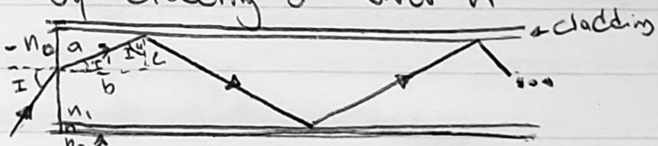
- Upper path: OPL $\sim Ln$ (where n is avg index over the path)
- Lower path: OPL $\sim L'n'$
- time assumed to be same along both paths
- $Ln \sim L'n'$, $L \sim R\delta$, $L' \sim (R-\Delta y)\delta$
- $R\delta n \sim (R-\Delta y)\cancel{\delta} n'$
- $\Delta y\, n' \sim R(n'-n)$, $n'-n = \Delta n$
- $R \sim n'\frac{\Delta y}{\Delta n} = \frac{n'}{\frac{\Delta n}{\Delta y}} \sim \frac{n}{\frac{dn}{dy}}$
- So $R \sim \frac{n}{\frac{dn}{dy}}$ gives radius of curvature of path light takes entering an index gradient

GRIN Materials/Lens Come in Different Forms

- ① Radial Gradient
  -  - varies as a function of r
- ② Axial Gradient
  - 
    - n varies along linear axis
    - bends light doesn't focus
    - need curvature to focus light
- ③ Spherical Gradient
  - 
    - surfaces of constant n are spherical shells
    - lens of eye is an approximate example

Fibre Optics

- ① Step Index Fibre - Consists of transparent core surrounded by cladding of lower n



- light travels through fiber via total internal reflection
- for total internal reflection $\sin I'' = \frac{n_2}{n_1} = \frac{c}{a}$
- $\sin I' = \frac{b}{a} = \sqrt{\frac{b^2}{a^2}} = \sqrt{\frac{a^2-c^2}{a^2}} = \sqrt{1-\frac{c^2}{a^2}} = \sqrt{1-\frac{n_2^2}{n_1^2}}$
- Snell's Law: $n_0 \sin I = n_1 \sin I'$

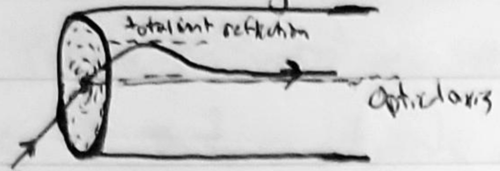
$$= n_1\sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$n_0 \sin I = \sqrt{n_1^2 - n_2^2}$$ ← Numerical Aperture of fibre

- tells you how wide a cone of light travels inside fibre

• ② GRIN Fiber

- Index is higher in center than at periphery

- 

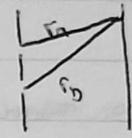
• Advantage of GRIN Fibre is all rays have same OPL so pulse of light injected at one end retains its shape at other

Physical [redacted] or Wave Optics

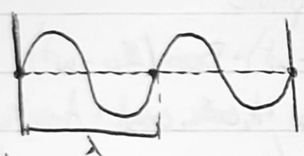
- If λ of light cannot be neglected with respect to dimensions of the optical system the light must be treated as a wave $\rightarrow$ gives rise to interference
- Ex. Youngs Double Slit.
  - Optical Path Difference
    $\Gamma = 0$ (central max)
    $\Gamma = m\lambda$ (m = interger) at m^{th} order max
    $\Gamma = (m - \frac{1}{2})\lambda$ at m^{th} order min.
  - $\Gamma \cong d \sin\theta$ an approximate expression using small angles
  - $\Gamma \cong n d \sin\theta$ in a medium
  - For max - $m\lambda = n d \sin\theta$
  - For min - $(m - \frac{1}{2})\lambda = n d \sin\theta$

  Double Slit

Note: $\Gamma = r_b - r_a$

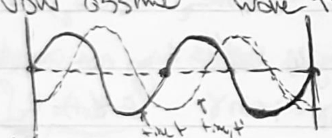


Representing Wave Mathematically

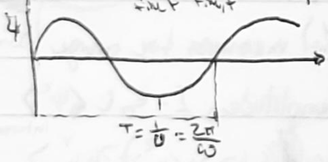
- 

Standing wave represent as: $\Psi(x) = A \sin(2\pi \frac{x}{\lambda})$

- Now assume wave travels at velocity v_0:



Represented by: [redacted] $\Psi(x,t) = A \sin \frac{2\pi}{\lambda}(x_0 - v_0 t)$

- 

- velocity of ~~plane~~: $v_0 = \lambda \upsilon = \lambda \frac{\omega}{2\pi} = \frac{\lambda\omega}{2\pi}$
- Rewritten: $\Psi(x,t) = A \sin(\frac{2\pi}{\lambda} x - \frac{2\pi}{\lambda} v_0 t)$ $K = \frac{2\pi}{\lambda} \equiv$ wave vector, $\omega = \frac{2\pi v_0}{\lambda}$

$\Psi(x,t) = A \sin(Kx - \omega t)$

$\Psi(x,t) = A \sin(Kx - \omega t + \epsilon)$ $\epsilon \equiv$ initial phase

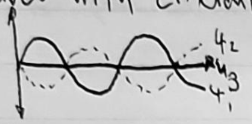
or $\Psi(x,t) = A \cos(Kx - \omega t + \epsilon')$

- Generally interested in time dependence, or spatial dependence of Ψ at time t_0
- $\Psi(\overset{\text{const}}{x_0}, t) = y(t) = A \sin(\Phi_K - \omega t)$ $\Phi_K = Kx_0 + \epsilon \equiv$ constant

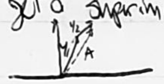
  or $\Psi(x, \overset{\text{const}}{t_0}) = y(x) = A \sin(Kx - \Phi_\omega)$ $\Phi_\omega = \omega t_0 - \epsilon \equiv$ constant
- Argument of the sin/cos fxn is the phase of the wave, φ
  - $\varphi = Kx - \omega t + \varphi$

Superposition of Waves

- ① Waves with same phase φ:
  - 

    $\psi_1 = A\sin\varphi$, $\psi_2 = B\sin\varphi$ $\rightarrow$ $\psi_3 = \psi_1 + \psi_2 = (A+B)\sin\varphi$
- ② Waves with different Φ_k but same frequency.
  - 

    $\psi_1 = A\sin\varphi$, $\psi_2 = B\sin\varphi$ $\rightarrow \psi_3 = 0$. (in this case).
  - i) Vector Addition - represent wave by phasor diagram; length of vector $\equiv$ amplitude; angle vector makes with ref. angle is φ
    - 

      if 2 waves $\psi_1 = A\cos(\Phi - \omega t)$, $\psi_2 = B\cos(\Phi - \omega t)$
    - since ω is same $\rightarrow$ both process at same rate we therefore can do a simple vector addition to get a superimposed
    - 

    - result again is a harmonic wave with same freq, but with a different phase
  - ii) Algebraicly - $y = y_1 + y_2 \rightarrow y = A\cos(\Phi_{k1} - \omega t) + B\cos(\Phi_{k2} - \omega t)$

    $y = A_1\cos\Phi_{k_1}\cos\omega t + A_1\sin\Phi_{k_1}\sin\omega t + A_2\cos\Phi_{k_2}\cos\omega t + A_2\sin\Phi_{k_2}\sin\omega t$

    $y = (A_1\cos\Phi_{k_1} + A_2\cos\Phi_2)\cos\omega t + (A_1\sin\Phi_2 + A_2\cos\theta_2)\sin\omega t$
    - Adding more things/changing makes things more complicated
    - Recall $Z = Ae^{i\gamma} = z = A\cos\gamma + i\sin\gamma$ $\quad$ $(E_x(x,t) = \mathrm{Re}[Ae^{i(kx - \omega t + \varphi)}]$ or $\mathrm{Im}[Ae^{i(kx - \omega t)}]$
    - The intensity or irradiance $(\frac{W}{m^2})$ measures time average of the square of the amplitude, $I = \epsilon_0 c \langle \psi^2 \rangle$
    - $I = \epsilon_0 c \langle (\psi_1 + \psi_2)^2 \rangle = \epsilon_0 c (\langle \psi_1^2 \rangle + \langle \psi_2^2 \rangle + 2\langle \psi_1 \psi_2 \rangle)$ interference term

      $= \langle y_1 y_2 \rangle = A_1 A_2 \cos(\Phi_{k_1} - \omega t)\cos(\Phi_2 - \omega t)$

      $= A_1 A_2 \langle (\cos\Phi_{k_1}\cos\omega t + \sin\Phi_{k_1}\sin\omega t) \times (\cos\Phi_{k_2}\cos\omega t + \sin\Phi_{k_2}\sin\omega t)$

      $= \ldots = A_1 A_2 [\cos\Phi_{k_1}\cos\Phi_{k_2}\langle\cos^2\omega t\rangle + \cos\Phi_{k_1}\sin\Phi_{k_2} + \sin\Phi_{k_1}\cos\Phi_{k_2}\langle\cos\omega t\sin\omega t\rangle$
      $+ \sin\Phi_{k_1}\sin\Phi \langle\sin^2\omega t\rangle]$

    - $\langle\cos^2\omega t\rangle = \sin^2\omega t = \frac{1}{2}$
    - $I = \frac{1}{2}A_1 A_2\cos(\Phi_{k_1} - \Phi_{k_2}) = I = \frac{1}{2}A_1 A_2\cos\delta$
    - $\langle\psi_1^2\rangle = \frac{1}{2}A_1^2$, $\langle\psi_2^2\rangle = \frac{1}{2}A_2^2$
    - $I = \frac{\epsilon_0 c}{2}[A_1^2 + A_2^2 + A_1 A_2\cos\delta]$

      $I = I_1 + I_2 + \sqrt{I_1 I_2}\cos\delta$

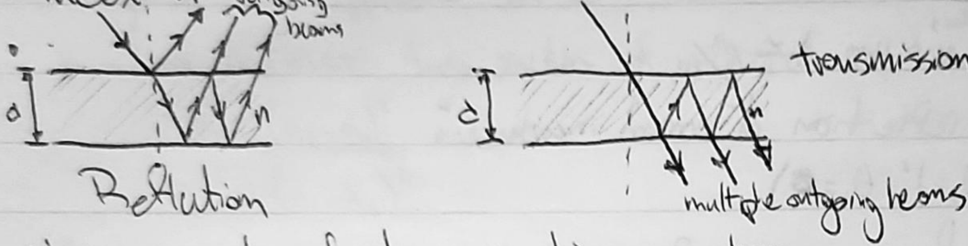
$I_1 = \sqrt{\frac{2I}{\epsilon_0 c}}$, $I_2 = \sqrt{\frac{2I}{\epsilon_0 c}}$ | $I_{max} = 4I_1$, $\delta = 180 = 0$, $I_{between} = 2I_1(1 + \cos\delta)$

The intensity or irradiance $(\frac{W}{m^2})$

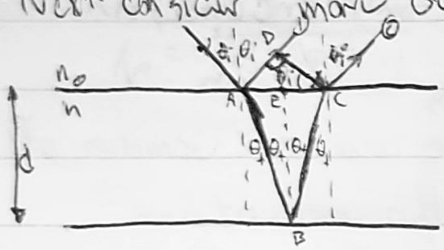
me

Thin Films

- Often exhibit multiple beam interference
- Consider 2 plane surfaces separated by d. Medium between has index, n



- have ignored refraction in diagrams above
- Next consider more accurately:



- want to find path difference of 1 & 2
- Assume detector at ∞
- $\Gamma = n(|\overline{AB}| + |\overline{BC}|) - n_0(|\overline{AD}|)$

$= 2n|\overline{AB}| - n_0|\overline{AD}|$ $\qquad |\overline{AB}| = \frac{d}{\cos\theta_t} \Rightarrow \cos\theta_t = \frac{d}{|\overline{AB}|}$

$= \frac{2nd}{\cos\theta_t} - n_0|\overline{AC}|\sin\theta_i$ $\qquad |\overline{AD}| = |\overline{AC}|\sin\theta_i$

$= \frac{2nd}{\cos\theta_t} - n_0 \frac{2d\sin\theta_t}{\cos\theta_t}\sin\theta_i$ $\qquad |\overline{AC}| = 2|\overline{AE}| \Rightarrow \sin\theta_t = \frac{|\overline{AE}|}{|\overline{AB}|} = \frac{|\overline{AE}|\cos\theta}{d}$

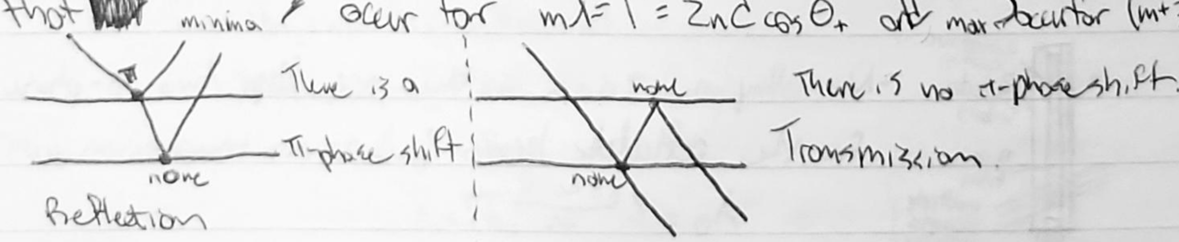
$= \frac{2nd}{\cos\theta_t} - 2n\sin^2\theta_t\, d/\cos\theta_t$

$= \frac{2nd}{\cos\theta_t}(1-\sin^2\theta_t)$

$= \frac{2nd}{\cos\theta_t}\cos^2\theta_t$

$$\Gamma = 2nd\cos\theta_t$$

- Need to analyze π-phase changes
- Saw previously that if there is $\begin{smallmatrix}0\\ \pi\end{smallmatrix}$ phase difference between 2 beams that $\begin{smallmatrix}\text{maxima}\\ \text{minima}\end{smallmatrix}$ occur for $m\lambda = \Gamma = 2nd\cos\theta_t$ and $\begin{smallmatrix}\text{min}\\ \text{max}\end{smallmatrix}$ occur for $(m+\frac{1}{2})\lambda = \Gamma = 2nd\cos\theta_t$.



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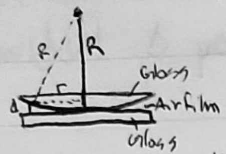
① Newtons Rings

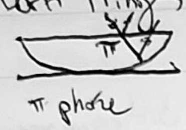
- Occur when spherical surface is in contact with planar surface
- 

  - assume r is radius of a dark ring then

  $R^2 = (R-d)^2 + r^2 = R^2 - 2Rd + d^2 + r^2 \simeq R^2 - 2Rd + r^2 \Rightarrow$

  then $d \simeq r^2/2R$

- Assume reflection at normal incidence:
  - $(\cos\theta_t = 1,\ \theta_t = 0)$
  - Dark fringes occur for



minima for π-phase change $2nd\cos\theta = m\lambda$

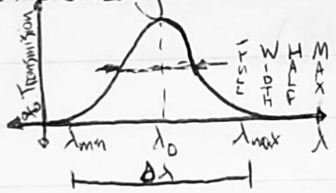
$2n\frac{r^2}{2R} = m\lambda$

  - $R = \frac{nr^2}{m\lambda}$

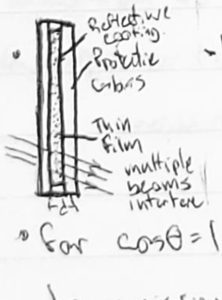
② Interference Filters

- filter → works by absorption or by interference
- Characterizing a filter

  -



Transmission Interference Filter

- 

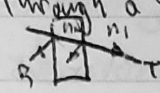
  - Normally $n_{coat} > n_{film}$, so there are two π-phase changes for the reflected beams and so max transmission occurs for:

  $$\lambda_0 = 2\frac{nd\cos\theta}{m}$$

- for $\cos\theta = 1$ (normal incidence) and first order ($m=1$) will get strong transmission at $\lambda_0 = 2nd$ ($n = n_{film}$)(d = thickness of film)

Anti-reflection Coatings.

- Part of light passing through a transparent boundary is lost due to ~~[illegible]~~ reflection

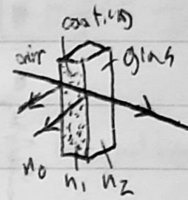


- Using an EM approach, can be shown that the reflectivity of the boundary between two media of indicies n_1 & n_2 is:

$$R = \frac{(n_2 - n_1)^2}{(n_1 + n_2)^2}$$

- Ex. Interface between air and Glass

$$R = \frac{(1.5 - 1)^2}{(1.5 + 1)^2} = 4\% \Rightarrow 4\% \text{ of light lost to reflectance}$$



- To minimize this, use an ~~on~~ anti-reflection coating (min film)
  - need to ensure reflected beams are π out of phase and of equal (approx) in amplitude.
- Set up so $n_0 < n_1 < n_2$ then overall no π phase change due to reflections (i.e. π at both)
- For a minimum $(m - \frac{1}{2})\lambda = 2 n d \cos\theta$ (n film, d thickness of film) which for normal incidence and $m = 1 \Rightarrow \frac{\lambda}{4} = nd$ ← optimal thickness of a quarter-wave coating
- For amplitude condition, since $n_0 < n_1 < n_2$ we can write:
  - $R_{surface1} = R_{surface2}$

$$\sqrt{R_{surface1}} = \sqrt{R_{surface2}}$$

$$\frac{n_1 - n_0}{n_1 + n_0} = \frac{n_2 - n_1}{n_2 + n_1}$$

$$(n_1 - 1)(n_2 + n_1) = (n_2 - n_1)(n_1 + 1)$$

$$n_1^2 + \cancel{n_1 n_2} - n_1 - n_2 = \cancel{n_1 n_2} - n_1^2 + n_2 - n_1$$

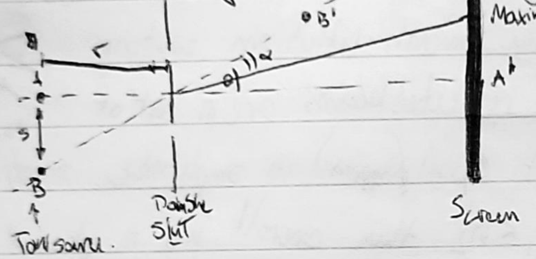
$$2n_1^2 = 2n_2$$

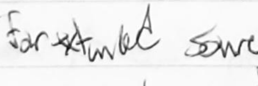
$$n_1 = \sqrt{n_2}$$

- Thus with coating satisfying $n_1 d = \frac{\lambda}{4}$ and $n_1 = \sqrt{n_2}$ there will be complete destructive interference & all light must pass through (due to cons. of E).

Coherence

- Refers to a comparison of the correlation between phases of em-waves
- Beams with random phase relations are 'incoherent' → do not exhibit interference
- Beams with a constant phase relationship are 'coherent' and will exhibit interference
- ~~Two~~ Types of Coherence
  - ① Transverse Spatial Coherence
    - Refers to phase relation between beams travelling concurrently
    - Assume 2 point sources illuminating a freaking double slit

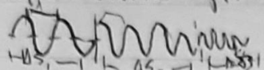


    - Point source A produces pattern centered at A', while B produces pattern at B' both by ~~[illegible]~~ $d\sin\alpha = m\lambda$
    - $\alpha = \theta = \frac{\lambda}{2d}$ ①
    - Max on A & B: $\alpha = \frac{\lambda}{d} = \theta$
    - $\Gamma = \frac{\lambda}{2} = \theta d \Rightarrow \frac{\lambda}{2} = \theta \frac{s}{r} \Rightarrow S = \frac{r\lambda}{2d}$, where S is distance between sources which fringes disappear. POINT
    - For a continuous line source fringes do not disappear until: $S = \frac{r\lambda}{d}$, where S is the length of a continuous line source LINE
    - Only for $S < \frac{r\lambda}{d}$ will fringes be observed.
    - Max slit distance is: $d_{max} = \frac{r\lambda}{s} = \frac{\lambda}{\theta}$ ⇒ spatial coherence width
    -  place single slit so $d < \frac{r\lambda}{s}$
    - For extended sources there are geometrical factors

$$c = d_{max} \frac{r}{} = \frac{1.22}{\theta}$$

← angular width of object

② Temporal Coherence

- Refers to predictability of phase as a fxn of time.
- Is a measure of how monochromatic the light it. (no perfectly monochromatic sources)
- Sources considered 'monochromatic' are const of wavetrains of finite length separated from each other by a discontinuous phase change
- 

- length of wave trains: coherence length, $\Delta s = N\lambda$
- Time $= \frac{\Delta s}{c}$ ①
- finite lifetime means one must represent wave train as
- a frequency band of width $\Delta f = \frac{1}{\Delta t}$ centered at $f = \frac{c}{\lambda}$
- $\Delta s = c\Delta T \Rightarrow \Delta s = \frac{c\Delta f}{6}$ $\quad \frac{\Delta f}{\Delta \lambda} = \left|\frac{df}{d\lambda}\right| = \left|\frac{d}{d\lambda}\left(\frac{c}{\lambda}\right)\right| = \left|-\frac{c}{\lambda^2}\right| = \frac{c}{\lambda^2}$

  so $\Delta f = \frac{c}{\lambda^2}\Delta\lambda$

  and $\Delta s \Delta \lambda = \lambda^2 \leftarrow$ form of inequalities.
- $\Delta s = c\Delta t \Rightarrow \Delta t = \frac{\Delta s}{c} = \frac{\lambda^2}{\Delta\lambda\, c}$
- For a photon: $E = \frac{hc}{\lambda}$

  $\frac{\Delta E}{\Delta \lambda} = \left|\frac{dE}{d\lambda}\right| = \frac{d}{d\lambda}(hc\lambda^{-1}) = \left|\frac{-hc}{\lambda^2}\right| = \frac{hc}{\lambda^2}$

  $\Delta\lambda = \frac{\lambda^2 \Delta E}{hc} \xRightarrow{t = \frac{\lambda^2}{\Delta\lambda c}} \Rightarrow \Delta E \Delta t = h$

Partial Coherence

- So far have assumed we beams are "coherent" they overlap in full extent, this would produce high contrast fringes.
- Define Contrast in pattern by:

  $\gamma = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100\%$ ①

- Ex. $Max = 4I \quad min = 0$ $\qquad$ $Max = 2I \quad min = 2I$

  $\frac{4I - 0}{4I + 0} = 100\%$ $\qquad$ $\frac{2I - 2I}{2I + 2I} = 0\%$

- How does contrast relate to coherence.
  - 2 bundles of light $I = I_2 = I_1$ (and A: $I_{max} = 4I_A$ & $I_{min} = 0$), B: $I_{min} = I_{MAX} = 2I_B$
  - Assume each consists of 2 parts $I = I_A + I_B$
  - Parts A are coherent and Parts B are incoherent
  - C = degree of coherence $\quad I_A = CI \,;\, I_B = (1-C)I$
  - $I_{max} = I_{AMAX} + I_{BMAX} = 4CI + 2(1-C)I = 2I(1+C)$

    $I_{min} = I_{Amin} + I_{BMIN} = 0 + 2(1-C)I = 2I(1-C)$.
  - $\gamma = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100\% = \frac{2I(1+C) - 2I(1-C)}{2I(1+C) + 2I(1-C)} \times 100\% = C$

degree of contrast = degree of coherence

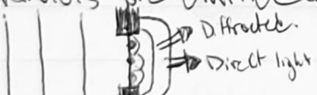
Dirty Diffraction

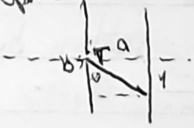
- Geometrical Optics → shadow completely, uniform dark

- In reality, if one looks at shadow in detail there are fringes, light spots, dark spots.
- Due to Diffraction – Huygens Principle – explains how waves bend around object
- Represent wave by a wave front (hypothetical surface connecting points of equal phase).
- Each point on wave front acts as a source of secondary spherical (circular) wavelets.
- Envelope of wavelets forms next wave front
- 2 Types of Diffraction: ① Fraunhoffer
  ② Frenell

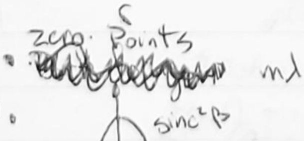
① Fraunhoffer Diffraction (far field diffraction)

- Occurs when source and detector (screen) are far apart and light is considered parallel
- Consider light incident on a single slit of width b, Huygen's wavelets are emitted across aperture
-





- Integrate ds, $-\frac{b}{2}$ to $\frac{b}{2}$
- Rays will not be in phase
- Consider each interval ds to the intensity at θ
- $A = \frac{A_0}{r_0} b \,\text{sinc}\,\beta\, e^{i(kr_0-\omega t)}$; $I = \frac{A^2}{r_0^2} b^2 \,\text{sinc}^2\beta \,\frac{\epsilon_0 c}{2} = I_0 \,\text{sinc}^2\beta$
- zero points $m\lambda = b\sin\theta \Rightarrow m\lambda = b\frac{y}{a}$
-



- red light diffracted more than blue

$s=0$ integrate from $-\frac{b}{2}$ to $\frac{b}{2}$

$\Psi = A_0 e^{i(kr-\omega t)}$

$I = \epsilon_0 c \Psi^2$

$I = \cancel{\epsilon_0} A^2$

$d\Psi = \frac{A_0 e^{i(kr-\omega t)}}{r} ds$

$d\Psi = \frac{A_0 e^{i(k(r_0+r)-\omega t)}}{r} ds$

$\delta \to r$

$r = s\sin\theta$

$d\Psi = \frac{A_0 e^{i(k(r_0+s\sin\theta)-\omega t)}}{r_0} ds$

$\int \frac{A_0 e^{i(k(r_0+s\sin\theta)-\omega t)}}{r_0} ds$

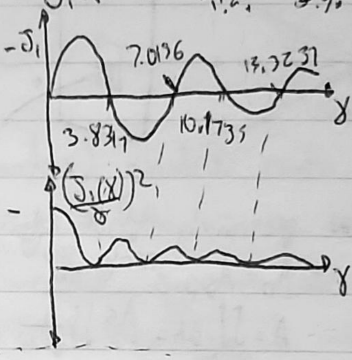
$A = \frac{A_0}{r_0}\left[\int_{-\frac{b}{2}}^{\frac{b}{2}} e^{iks\sin\theta} ds\right] e^{i(kr_0-\omega t)}$

$A = \frac{A_0}{r} b\sin\beta\, e^{i(kr_0-\omega t)}$

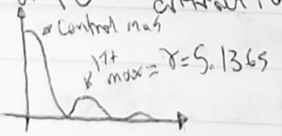
$\beta = \frac{1}{2} kb\sin\theta$

$I = \frac{A_0^2}{r_0^2} b^2 \sin^2\beta \frac{\epsilon_0 c}{2}$

- Circle aperture – Concentric rings
  - $I = I_{\theta=0}\left[\frac{2J_1(\gamma)}{\gamma}\right]^2$ – $I_{\theta=0}$ is intensity of central max
  - $\gamma = \frac{1}{2}kD\sin\theta$
  - $J_1(\gamma)$ = 1st order Bessel function of 1st kind
  - $J_1(\gamma) = \frac{\gamma}{2} - \frac{(\frac{\gamma}{2})^3}{1!\,2!} + \frac{(\frac{\gamma}{2})^5}{2!\,3!} \ldots$



- 1st Min → $D\sin\theta = 1.22\lambda$ ✱
- The minimum angle of resolution (θ_{min})
  - θ_{min} occurs when central max of one object falls on first min of the other
  - Gives angular separation of objects for which they can be resolved.
  - From ✱: $\theta_{min} = \sin^{-1}\left(\frac{1.22\lambda}{D}\right)$
    - for small angles $\theta_{min} \simeq \frac{1.22\lambda}{D}$ Rayleigh Criterion
  - Note: Short λ give better resolution than long λ.
  - Ex. Lycopodium seeds are spherical & uniform. They are placed on glass plate. For parallel of 640nm, the angular radius of the 1st diffraction is 2°, how large are seeds?



$\gamma = \frac{1}{2}kD\sin\theta$

$\frac{2\gamma}{kD} = \sin\theta$

$\frac{2\gamma\lambda}{2\pi D} = \sin\theta$

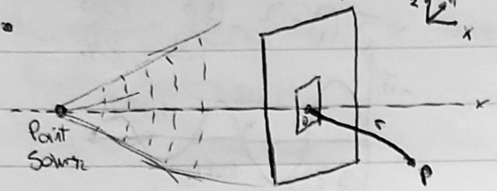
$\frac{2\gamma\lambda}{2\pi\sin\theta} = D$

$3\times10^{-5}\,m = D$

$30\,\mu m = D$

Hilroy

Fresnel Diffraction

- Occurs when source & detector are not infinitely far away
- Point source emits spherical waves which encounter an aperture
- 

  - Examine diffraction at field point P by integrating contributions across the slit at points like O

- Saw previously that spherical wave has form: $dA = \frac{A_0\, da}{r} e^{i(kr-\omega t)}$ ① (amplitude) $I \propto A^*A$ allows no time dependence
  - $A_0 = \frac{A_s}{r'} e^{ikr'}$ ② $\xrightarrow{\text{sub } ①, ②}$ $dA = \frac{A_s}{rr'} e^{ik(r+r')} da$
  - $A = \iint_{\text{aperture}} dA = A_s \iint_a \frac{1}{rr'} e^{ik(r+r')} da$
- have neglected 2 things: ① there is 90° shift between primary light & diffracted light
  ② Because direction from various aperture points O to field pt P is no longer constant, there is an additional obliquity factor $\Rightarrow$ $f(\theta) = \frac{1+\cos\theta}{2}$
- The corrected integral is called the Fresnel-Kirchoff Diffraction formula: $A = \frac{-ikA_3}{2\pi} \iint f(\theta) \frac{e^{ik(r+r')}}{rr'} da$
- Neglecting these corrections we get: $A = C \iint \frac{e^{ik(r+r')}}{rr'} da$ (all constants)
- Assume rectangular aperture in yz plane, divide into areas that are $w\,dz = da$.
- After algebra $A = c \int e^{ik(D + \frac{z^2}{2L})} w\, dz$

$$A = cwe^{ikD} \int e^{\frac{ikz^2}{2L}} dz$$

$$A = c' \int e^{\frac{i2\pi z^2}{\lambda 2L}} dz = c' \int e^{\frac{iz^2}{\lambda L}} dz \qquad v = \sqrt{\frac{2}{L\lambda}}\, z \Rightarrow z = \left(\frac{L\lambda}{2}\right)^{\frac{1}{2}} v \qquad \frac{dz}{dv} = \left(\frac{L\lambda}{2}\right)^{\frac{1}{2}}$$

$$A = c' \int e^{i\frac{\pi}{\lambda L}\frac{L\lambda}{2}v^2} \left(\frac{L\lambda}{2}\right)^{\frac{1}{2}} dv$$

$$A = A' \int e^{\frac{i\pi v^2}{2}} dv$$

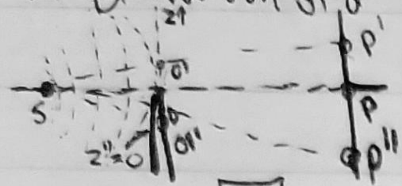
Fresnel Integrals $\rightarrow$ $A = A'\left[\underbrace{\int \cos\left(\tfrac{\pi}{2}v^2\right) dv}_{x(v)} + i \underbrace{\int \sin\left(\tfrac{\pi}{2}v^2\right) dv}_{y(v)}\right]$

$$X(v_0) = \int_0^{v_0} \cos\left(\tfrac{\pi}{2}v^2\right) dv \qquad Y(v_0) = \int_0^{v_0} \sin\left(\tfrac{\pi}{2}v^2\right) dv$$

- Intensity $= I = \frac{\epsilon_0 c}{2} A^*A = \frac{\epsilon_0 c}{2} [x(v_0) + iy(v_0)][x(v_0) - iy(v_0)]$

$$= \frac{\epsilon_0 c}{2} A'^2 \left[x(v_0)^2 + y(v_0)^2\right]$$

· Ex. Diffraction of a knife edge.



- O is origin for waves emitted by S and detected at P
- At P contribution is from $z=0$ to $z=+\infty$

$v=0$ to $z=\infty$

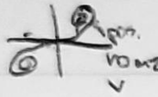
$v = \sqrt{\frac{2}{L\lambda}}\, z$

$I_P = I_0 [\Delta x^2 + \Delta y^2]$

$= I_0 [0.5^2 + 0.5^2]$

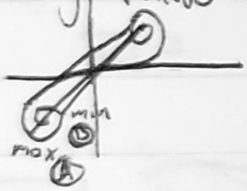
$I_P = \frac{I_0}{2}$

· For a point P'' in the geometrical shadow centre of wavefront occurs at positive value occurs at finite positive value of $v(z)$

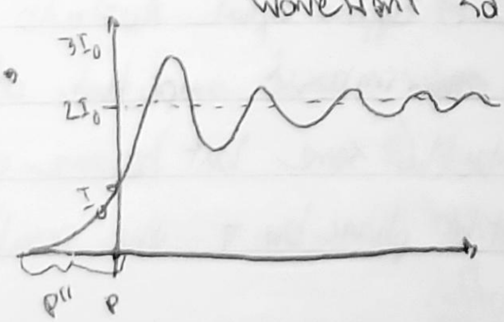


· as P'' moves further into geom. shadow line segment which is from point v_0 to the +'ve eye will become increasingly shorter, so intensity decreases monotonically.

· For a point P', due to location of O' the contribution is from a neg. value of v to $v=+\infty$

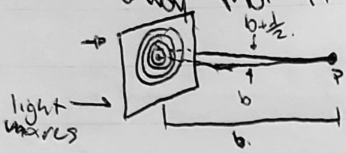


· As P' moves further up the screen the contribution starts at increasingly more -'ve values of v, thus as you move along the lower spiral you will get a series of maxima and minima and will oscillate about the Eye to Eye contribution which corresponds to $v=-\infty$ to $v=+\infty$ and represents the unobstructed wavefront so intensity is $I_P = I_0[1^2+1^2] = 2I_0$.



Fresnel Diffraction from a Circular Aperature

- Ex Zone Plate
  - → consists of a series of concentric rings, each spaced in such a way that it is $\frac{\lambda}{2}$ further from the field point P.



  - each represents a half-period zone and is out of phase with the neighbouring zones by π
  - → Light passing through the zone plate is diffracted at each of the zone edges
  - → Consequently the zone plate acts as a lens concentrating the light at P.
  - → find focal length b:
    - Radius of first zone boundary: $R_1 = \sqrt{(b+\frac{\lambda}{2})^2 - (b)^2} = \sqrt{\lambda b + \frac{\lambda^2}{4}}$ → negligible
      $\simeq \sqrt{\lambda b}$
    - Area of 1st zone: $A_1 = \pi R_1^2 = \pi \lambda b$
    - Radius of nth zone: $R_n = \sqrt{(b+n\frac{\lambda}{2})^2 - b^2}$
    - Radius of second zone: $R_2 = \sqrt{(b+\lambda)^2 - b^2} = \sqrt{2\lambda b + \lambda^2} = \sqrt{2\lambda b}$
    - Area of 2nd zone: $A_2 - A_1$
      $A_2 = 2\pi\lambda b - \pi\lambda b = \pi\lambda b$
    - Find that area of each zone is approx equal $A_N \simeq \pi\lambda b$
    - Thus contribution to the superimposed amplitude at P from each zone will nearly be the same but because each successive boundary is out of phase by π the resultant intensity at P is $\simeq 0$.
    - $A_P = a_1 - a_2 + a_3 - a_4 + \ldots \simeq 0$
    - If however we block out every 2nd zone, then only zones 1,3,5 contribute and each contribution is out of phase by 2π; P will be bright.
    - To find b: $b = \frac{R_1^2}{\lambda}$
      $b = \frac{R_1^2}{\lambda}$ is the first order focus.
    - Note: There are higher order foci which one can show are given by $f_n = \frac{R_1^2}{(2M-1)\lambda}$ $M = 1, 2, 3$
    - Can show: $d_{min} = \frac{1.22 R_N \lambda}{2N(2M-1)}$

tells us to resolve small details must make R_N smaller

index of λ.

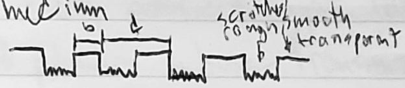
• Poisson's Spot: - From Fresnel Theory if instead of an open aperture one uses a solid circular obstacle that there will be a bright spot in the centre of that geometrical shadow

- 

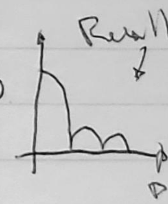
all rays from edge to point on OA travel same distance
∴ are in phase
- cancel due to the infinite no. of rays from circumference that ~~converges~~ converges on axis in phase

• Babinet's Principle - The Fraunhofer diffraction patterns generated by objects that are the photographic negative of each other (slit and wire, or open aperture) are the same ~~except~~ except in the zeroth order.

• Diffraction Grating - In simplest form consists of series of parallel, evenly spaced grooves "scratched" onto a transparent medium

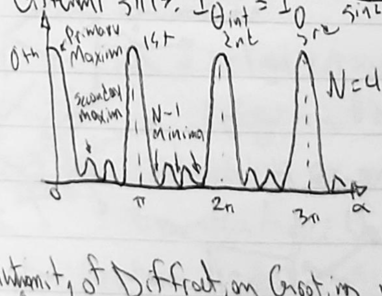
- 

- Characterized by b & d.
- Grating obeys $d \sin\theta = m\lambda$
- Intensity profile is affected by 2 contributions;

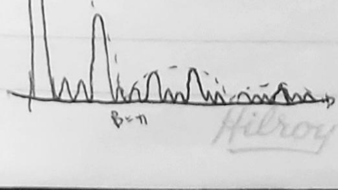


① The finite width of slits - for a single slit of width b we saw that: $I_{\theta_{diff}} = I_0 \left(\frac{\sin\beta}{\beta}\right)^2$ where $\beta = \frac{\pi}{\lambda} b \sin\theta$ ①

② The multiplicity of the slits - Interference will occur between rays that pass through equivalent points on different slits. $I_{\theta_{int}} = I_0 \frac{\sin^2(N\alpha)}{\sin^2(\alpha)}$ ② where $\alpha = \frac{\pi}{\lambda} d \sin\theta$, N is # of slits
$\alpha = \frac{\beta}{b} d$



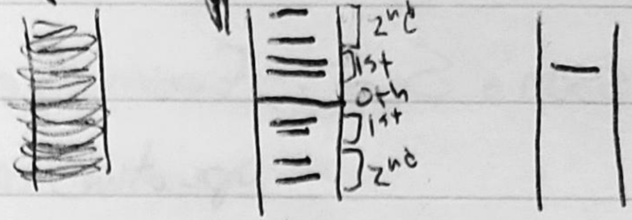
• Intensity of Diffraction Grating is multiplying ① & ②



$\beta = \pi \Rightarrow \alpha = \frac{\pi d}{b}$ if $d = 2b$, then $\alpha = 2\pi$

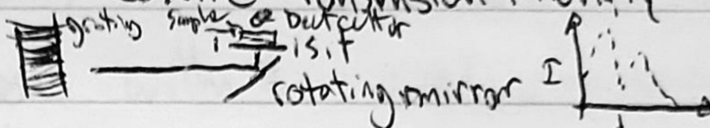
most of the visible diffracted light is in the 1st order peak.

· Spectroscopy

- 

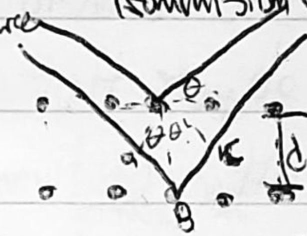
- Can place slit to achieve one specific λ

- Place sample between slit & detector to measure transmission intensity

- 

- Replace sample by air and take ratio

- Transmission $(T) = \frac{I_{sample}}{I_{air}}$

· 3-D Grating (Crystal lattice) - 

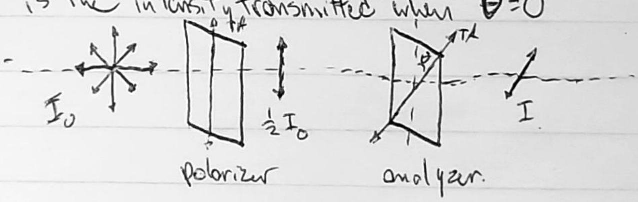
$\Gamma = \overline{AB} + \overline{BC} \Rightarrow 2d\sin\theta = m\lambda$

- Bragg's Law $2d\sin\theta = m\lambda$

Polarization of Light.

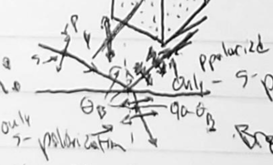
- "types" → ① Unpolarized - light from most sources is unpolarized.
  - due to fact that light is emitted by atoms oscillate independently
  - E vector is related to osc. of electric dipole.
- ② Linearly Polarized - E-vector oscillates in a given constant orientation
- ③ Partially Polarized -
  - E-vector is predominantly but not completely in one direction.
- ④ Circularly Polarized - E vector has constant magnitude but changes direction
  - Rotates in a helix around propagation.
- ⑤ Elliptical - E vector rotates & changes its magnitude → most general form of polarization
  - circular & linear are two extremes of elliptical

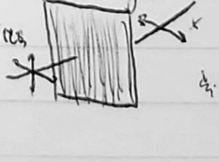
- Malus' Law: - Intensity of linearly polarized light transmitted through two linear polarizers whose transmission axes (TA) are rotated through θ is given by $I = I' \cos^2\theta$, where I' is the intensity transmitted when $\theta = 0$



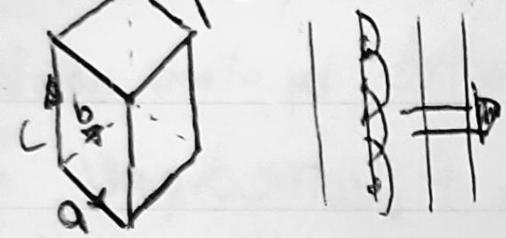
Production of Polarized Light.

- (i) By Scattering: 

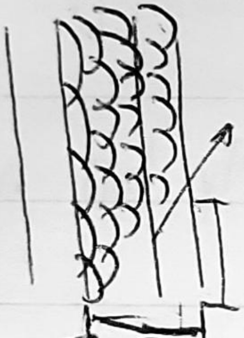
- light scattered at 90° to the direction of propagation will be linearly polarized. particle in suspension.
- (ii) By Reflection: 

- s-polarized light osc. out of plane, p in plane.
- Brewster's Angle = $\theta_B = \tan^{-1}\left(\frac{n_1}{n_2}\right)$ ← angle to get polarized reflected light.
- (iii) Polarization by Selective Absorption: Grid of wires 
- Component parallel to wires ~~absorbed~~ 10 axes ~~absorbed~~ & is polarized.

Polarization by Double Refraction

- Isotropic material: $n_a = n_b = n_c$ ← refractive index
- 

light would form spherical wavelets & travel through same direction
- Uniaxial Material: $n_a = n_b \neq n_c$
- $v = \frac{c}{n}$



ellipitical wavelets that travel faster in one medium than the other.

- 

ordinary ray "o"

extraordinary ray "e"

- Nicol Prism Polarizer.