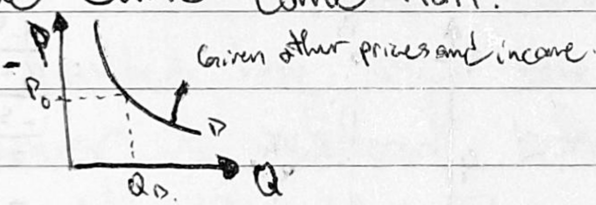
Chapter 4 Consumer Behaviour

Introduction

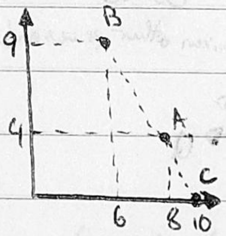
- How do consumers decide how much to demand?
- Where do the "Quantity Demanded" shown by the Demand Curve come from?



- Buyers demand curves are derived from the Theory of consumer behavior or choice
  - Theory says consumers solve a constrained optimization problem.
- Consumers "optimize" in that they choose the bundles of goods that they like best.
  - Depends on preferences over goods.
- But they are "constrained": they can only what they can afford to buy.
  - What they can afford is determined by their income and prices of the goods. which defines the budget constraint.
- Consumers optimize (do the best they can) given their scarce resources.

Consumers Preferences: The Concept of Utility 4.1

- Assume just 2 goods: X and Y
- Consider bundles of the two goods
  - Each consumption bundle has so much X and so much ~~X~~ Y.
- Example: A = (8, 4)
  B = (6, 9)
  C = (10, 0)
  UF: U = (X, Y).



$= \frac{4-9}{8-6} = \frac{-5}{2} = -2.5$

- Utility Function (UF) assigns a utility or ranking number to ~~assig~~ every consumption bundle
  - based on how much X and Y are in the bundle
- Example: Jay ⇒ UF: $U = U(X, Y) = x + y$ $\quad U_A^J = 8 + 4 = 12$
  $U_B^J = 15$ $\quad U_C^J = 10$

  Jay ⇒ B ≻ A (Bundle B is preferred to bundle A) etc.

  Doug ⇒ $U = U(x, y) = 2x + \sqrt{y}$ $\quad U_A^D = 18$ $\quad U_B^D = 15$ $\quad U_C^D = 100$
  ⇒ A ≻ B.
- Utility/ranking number shows how much consumers like the bundle compared to all the other bundles
  - a relative (not absolute) ranking.
- A bundle ~~w/ the~~ with a higher utility number is preferred to all bundles with lower utility numbers.
- But how much higher utility # is, is irrelevant
- Utility provides an ordinal ranking
  - only gives a ranking, only says which bundles are better or worse than other bundles.
- Utility doesnot ~~pro~~ provide cardinal ranking which would also tell you how much better or worse a bundle is, compared to other bundles

- Ordinal ranking → many different utility functions can show the same preferences (not unique).
  - → Can do a monotonic transformation of these UF and preferences stay the same.
  - → Preferences stay same if UF keeps same ordering → occurs if you do a PMT (Positive Monotonic Transformation)
- If 2 bundles have the same utility number then the consumer is indifferent between the bundles.
  - → $U_A = 12$, $U_B = 12$ $U_A \sim U_B$ (Utility A is just as good as Utility B).

- Marginal Utility (MU): is the increase in the utility number assigned to a bundle if the bundle gets one more unit of either X or Y and the other unit stays stagnant.

$A = (8,4) \Rightarrow U_A = 12$
$A = (9,4) \Rightarrow U_{A'} = 13$ $\} \rightarrow MU = 1$

- MU formula → $MU = \frac{\Delta U}{\Delta X}\Big|_{Y} = \frac{dU}{dx}\Big|_{Y} \rightarrow \frac{\partial U(x,y)}{\partial x} =$

- Ex $U(x,y) = x + y$ $MU_x \rightarrow \frac{\partial U(x,y)}{\partial x} \rightarrow \frac{\partial(x+y)}{\partial x} \rightarrow 1 + 0 \rightarrow 1.$

- MU of X, Y are independent of amounts of X & Y
- ~~Marginal Rate~~
- Ex $U = 2x + \sqrt{y}$ $MU_x \rightarrow \frac{\partial U}{\partial x} \rightarrow 2$ - More y dough has the smaller the increase in utility from an increase in Y.

$MU_y \rightarrow \frac{\partial U}{\partial y} \rightarrow \frac{1}{2y^{\frac{1}{2}}}$ - "diminishing MU for Y"

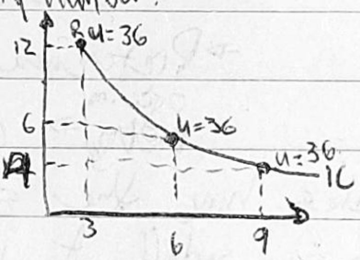
Indifference Curves 4.2.

- An indifference Curve (IC) plots out all bundles of goods that have the same utility number.
- Ex $U(x,y) = xy$

  $R=(3,12) \Rightarrow U_R^K = 36$.

  $S=(6,6) \Rightarrow U_S^K = 36$

  $T=(9,4) \Rightarrow U_T^K = 36$.



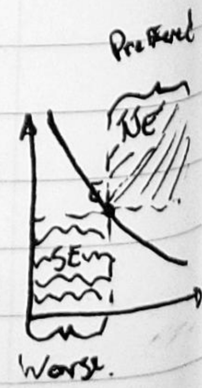
* all points on that curve have the same Utility number *

- Bundles on higher IC are preferred to all bundles on a lower IC and vice versa.

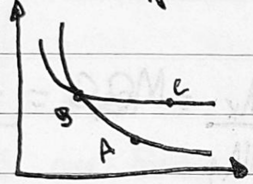
Four Assumptions about Consumer Preferences

1- Completeness
- Consumers can compare all possible bundles of goods and rank them
- All bundles are assigned a utility number
- We can always draw IC
- Every bundle has an IC (Fill up the whole x,y plane)

2- More is better
- Non-satiation
- Bundles with more of both goods (North East) are better/preffered with higher Utility #'s.
- Bundles with less of both goods (southwest) are worse with lower utility #'s.
- ∴, IC have to be downward sloping



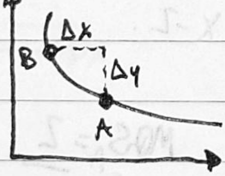
3- Transitivity

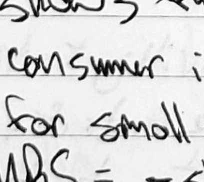
- A, B, and C are bundles of X & Y
- If $A > B$ and $B > C$, then $A > C$
- If $A \sim B$ and $B \sim C$, then $A \sim C$
- Imposes consistency on rankings.
- It also implies that IC curves cannot cross.
- 

$C > A$
$A \sim B$
$B \sim C$

} Contradict, ∴ this cannot be true (curves cannot cross).

4- Convexity

- The more a consumer has of a good, the less she is willing to give up (trade away another good) to get even more of that good
- IC are convex to the origin
- 

- Marginal Rate of Substitution
  - shows the rate of trade (x for y) that the consumer is willing to do.
  - for small trades only. (MRS is the tangent line to the IC).
  - MRS = − slope of the tangent line
  - MRS is a positive number
  - 

MRS at B = 3 so you'd be willing to trade 1 x for 3 y at this point.

- The Rate at which your willing to trade decreases.
- The trade is willing cause they have the same utility.
- Given a UF, $U = U(x,y)$, derive the eqn for MRS as a function of x & y

$$dU = \frac{\partial U(x,y)}{\partial x} \cdot \Delta x + \frac{\partial U(x,y)}{\partial y} \cdot \Delta y$$

- Set $\partial U = 0$
  - $0 = \frac{\partial U(x,y)}{\partial x} \Delta x + \frac{\partial U(x,y)}{\partial y} \Delta y \Leftarrow MU_x = \frac{\partial U(x,y)}{\partial x}$

$$0 = MU_x \Delta x + MU_y \Delta y$$

$$-MU_y \Delta y = MU_x \Delta x$$

$$-\Delta y = \frac{MU_x \Delta x}{MU_y}$$

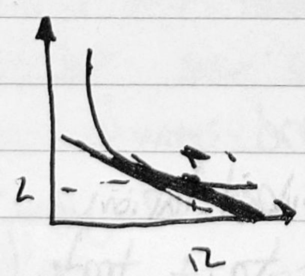
$$\left.\frac{-\Delta y}{\Delta x}\right|_u = \frac{MU_x}{MU_y} \Rightarrow \frac{MU_x}{MU_y} = MRS = \frac{-\Delta y}{\Delta x}$$

- Example: $U = xy - 2y$

$$MU_x = \frac{\partial U}{\partial x} = y \qquad MRS = \frac{MU_x}{MU_y}$$

$$MU_y = \frac{\partial U}{\partial y} = x - 2 \qquad = \frac{y}{x-2}$$

$A = (12, 2)$



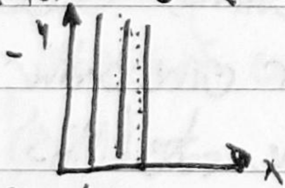
$$MRS_A = \frac{2}{10} = 0.2$$

- If $MRS = 2$, the consumer is willing to trade
  - one good of x for 2 units of y
  - one good of y for $\frac{1}{2}$ units of x
  - If x is graphed horizontally (independent variable).
  - As y in increase in the x direction MRS decreases.

Properties of Indifference Curves

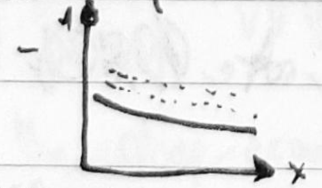
- If IC's are steep, the consumer cares mostly about x.

- If the IC is vertical the consumer only cares about the X and is indifferent to the amount of y

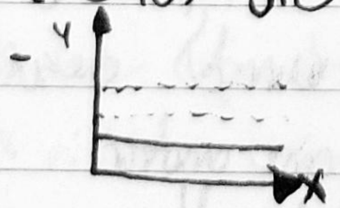


violate assumption #2

- If the ICs are flatter, the ~~???~~ consumer cares mostly about y

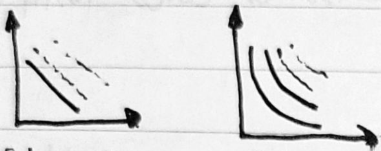


- If the ICs are horizontal, the consumer cares only about y



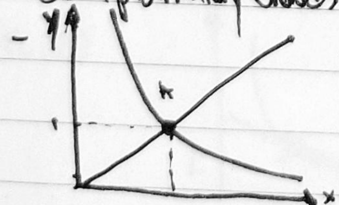
The Curvature of Indifference Curves

- I.E.

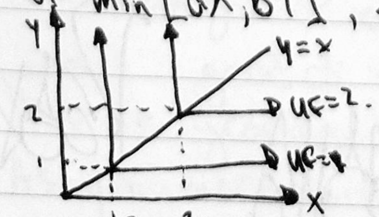


Straight IC   Curvy I.C.

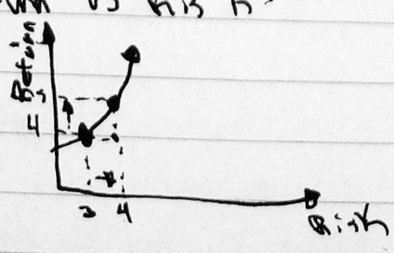
- Relatively straight indifference curves describe goods that are easily substitutable for one another
  - Trade X & Y at roughly the same rate (MRS) regardless of ~~how~~ much X & Y you have.
- Extreme case: Perfect substitution
  - Consumer trades at a fixed rate (Constant MRS).
  - IC are straight lines.
  - Uf must be liner $U(x,y) = aX + bY$ $(MRS = \frac{a}{b})$
- IC's that are more convex (more curved) describe goods that are more complementary to one another
  - Complementary Goods - Goods used together in some proportion
  -



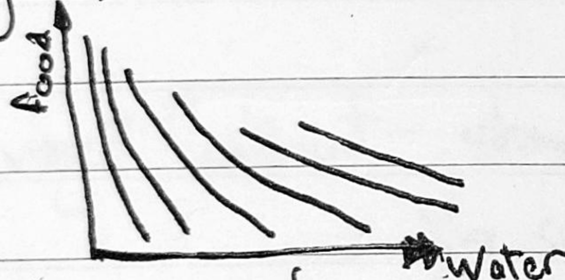
  - trades take done mostly around said point A.
- Extreme Case: Perfect Complements
  - Goods used in a fixed proportion
  - $U = \min[aX, bY]$, a/b dependent on your proportion.
  -



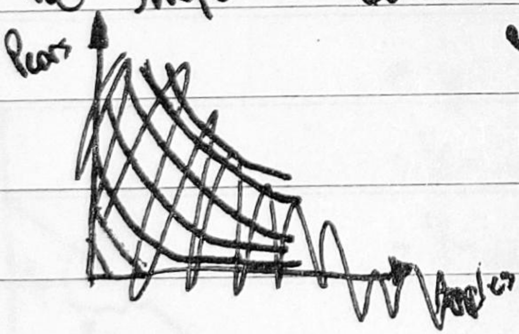
- Return vs Risk

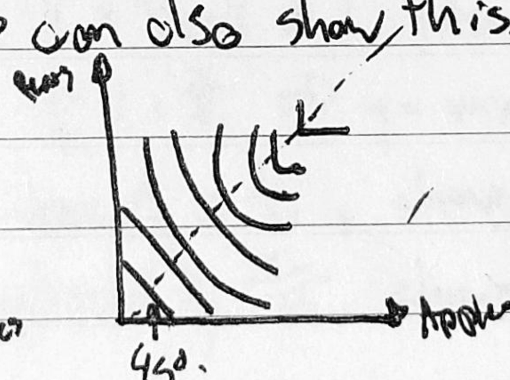


- Indifference Curves do not ~~[illegible]~~ have to be parallel.
- The can have different Slopes for different combination of goods

- 

- The shapes of curves can also show this.



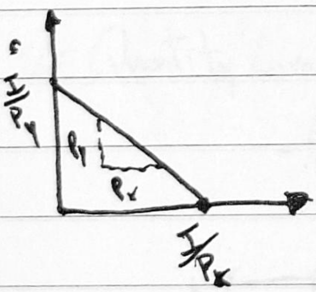


- All the above examples has been for goods, but you can all so have 'bads' i.e. (return vs. risk).
- Bads have upwards slopes.
- This violates assumption 2 (more better).

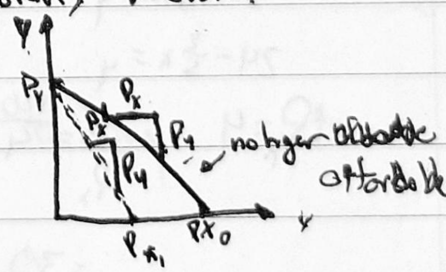
Consumer's Income and Budget Constraint

- Assume Consumer - has a fixed income, I, to buy X and Y.
  - is a price taker for X & Y (expenditure $= P_x X$)
  -

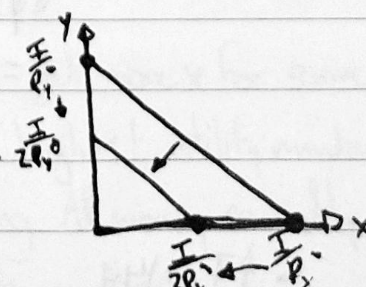
- Budget Contraint - shows all of the bundles of X & Y that the consumer can afford to buy
  - $P_x \cdot X + P_y \cdot Y = I$ (Assumes consumer spends all I).
  - $y = -\frac{P_x}{P_y} X + \frac{I}{P_y} \Rightarrow y = mx + b$
  - X-intercept $= \frac{I}{P_x}$, shows max amount of x you can buy
  - Y-intercept $= \frac{I}{P_y}$, shows max amount of y you can buy.



- If P_x changes; (holding income, and P_y constant) vice versa.
  - If $P_x \uparrow$ - max $X \downarrow$
    - " Y stays the same
    - BC rotates in

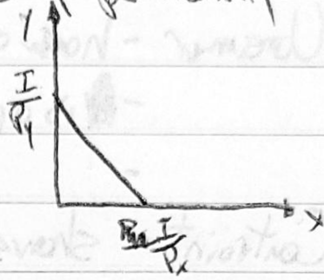


- If I changes; (holding P_x, P_y constant)
  - Both intercepts shift up or down
  - Slope stay the same
  - BC shifts parallelly.
- If both prices $\uparrow$ proportionally
  - an $\downarrow$ in real income but actual I stays the same
  - P_x^0, P_y^0, I_0
  - Slope stays the same.



BL: $P_{x0}X + P_{y0}Y = I$

- If income (I), P_x, P_y all increase proportionally
  - Real income stays the same.
  - Now = $2P_{x0}$, $2P_{y0}$, $2I$
  - Now: $\frac{2I}{2P_{x0}}$, $\frac{2I}{2P_{y0}}$, $\frac{\cancel{2}P_{y0}}{\cancel{2}P_{y0}}$
  - Everything stays stagnant (slope, intercept etc.)



Example

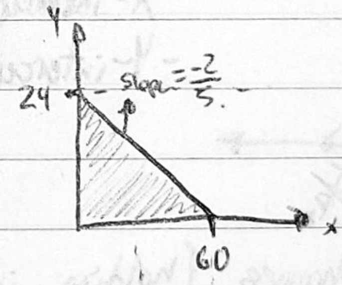
- $I = 120$, $P_x = 2$, $P_y = 5$

$I = P_x X + P_y Y$

$120 = 2x + 5y$

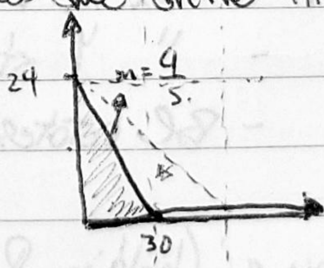
$\frac{120}{5} - \frac{2}{5}x = y$

$24 - \frac{2}{5}x = y$



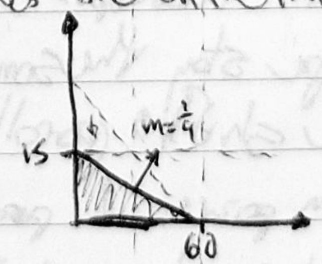
Feasible bundles are on the line or inside BL.

- ↑ $P_x = 4$ $\quad \frac{I}{P_x} = \frac{120}{4} = 30$ $\quad m = \frac{P_x}{P_y} = \frac{-4}{5}$



Feasible bundles are on the line or inside BL.

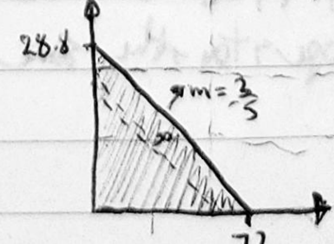
- ↑ $P_y = 8$ $\quad \frac{I}{P_y} = \frac{120}{8} = 15$ $\quad m = \frac{P_x}{P_y} = \frac{2}{8} = \frac{-1}{4}$



Feasible bundles are on the line or inside BL.

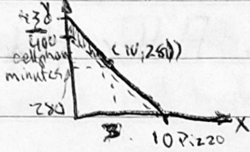
- ↑ $I = 144$

$\frac{I}{P_y} = \frac{144}{5} = 28.8$ $\quad \frac{I}{P_x} = \frac{144}{2} = 72$ $\quad m = \frac{-2}{5} =$

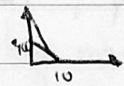


Nonstandard Budget Constraint

- 1 - Quantity Discounts - After a minimum quantity of a good is purchased
  - This results in a kink in budget contraint.



$P_y \downarrow$ to 0.2 after 280 mins.

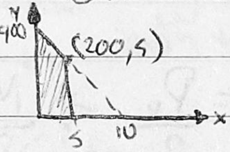


$0.25 \cdot 280 = 70$

$100 - 70 = \$30$

$280 + \frac{30}{0.2} = 430.$

- 2 - Quantity Limits - Max amount of good you can buy
  - This also results in a kink.
  -



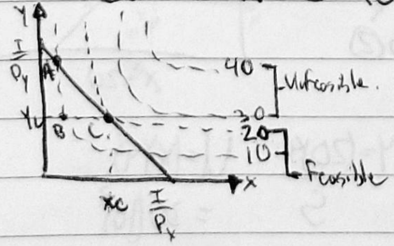
max 5 pizzas

$= 50$ $100 - 50 = 50$

$50 \cdot 25 = 200$

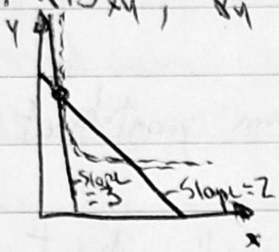
Combining BC & Preferences: Deciding What to Buy

- BC shows affordable/feasible bundles
- Choose feasible X, Y bundle consumer likes best.
  - Feasible bundle with highest utility number
  - " " on highest IC
  - Bundle where IC is tangent to BC.



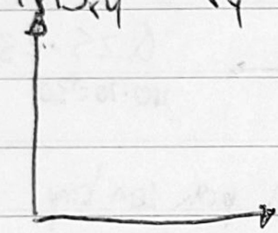
- Point B - No good, can get more x for same y & vice versa.
- Point A - No good, not highest utility number.
- Point C - Best, Spending All money correctly and have the highest utility number.
- A > B, C > A, C > B
- Tangent also implies the same slope.
  - $IC = MRS = \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$ = Slope of BC.
  - Rate of Willing Trade = Rate of Able Trade.

] or U_{max} (Point C)

- If $MRS_{xy} > \frac{P_x}{P_y}$, IC steeper than BC.
  - 

    $MRS = 3$ - Willing to give up 3_y for $1x$

    $\frac{P_x}{P_y} = 2$ - Should buy more X, less Y in market

Slides
- If $MRS_{xy} < \frac{P_x}{P_y}$, IC Flatter than BC
  - 

- Optimal Bundle
  - $MRS_{xy} = \frac{MU_x}{MU_y} = \frac{P_x}{P_y} \Rightarrow \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$
  - Marginal Utility per dollar spent is equalized across all goods.

Example

- $U = XY^2$ $\quad P_x = 2$ $\quad MRS = \frac{Y^2}{2XY} = \frac{Y}{2X}$

  $I = 120$ $\quad P_y = 5$

- $\frac{Y}{2X} = \frac{2}{5}$ $\quad BC: P_x X + P_y Y = I$

  $2X + 5Y = 120$ ②

  $Y = \frac{4X}{5}$ ① sub and solve ① into ②

  $2X + 5\left(\frac{4X}{5}\right) = 120$

  $6X = 120$

  $X^* = 20$

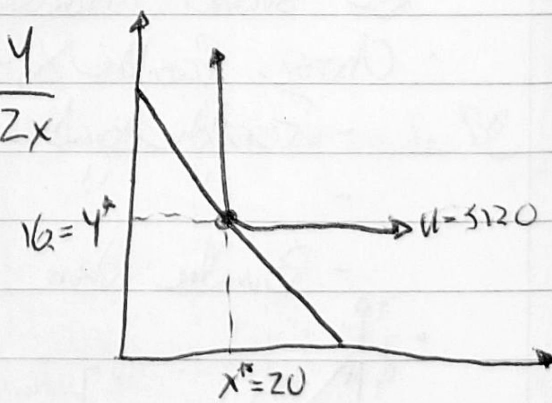
  $Y = \frac{(20)4}{5}$

  $Y = 16$

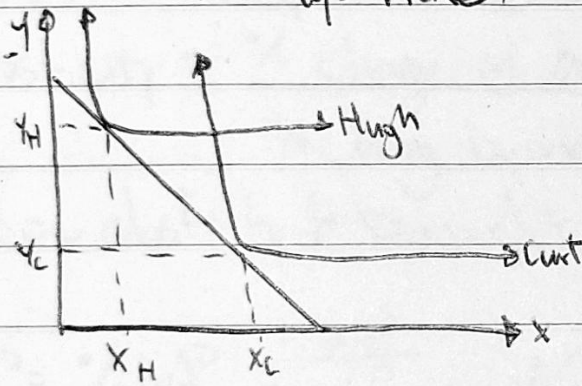
  $U = (X)(Y)^2$

  $= 20(16)^2$

  $U = 5120$

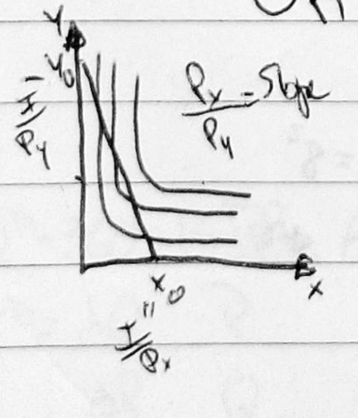


- An Implication of Utility Maximization
  - Curt, Hugh both buy X & Y
  - Curt like X more than Y 
  - Hugh like Y more than X 
  - Lets assume Curts and Hugh have the same income
  - Given the same prices for X & Y
    - Curt buys more X and hugh buys more Y.



- In consumer equilibrium they are willing to make the same trade offs
- Regardless of preferences they will both trade 1 for 1 (same MRS).

- Special Case - Corner Solutions
  - Above shows only interior solutions where some of both X & Y are chosen
  - But can get corner solution if:
    - Consumer cares more about X than Y ⇒ Steep IC
    - OR Y is expense compared to X ⇒ Flat BC.
    - Utility max, feasible bundle (Highest IC, given BC) has zero Y ⇒ spend all income on X
    - on not tangency, $MRS_{xy} > \frac{P_x}{P_y}$ at best bundle.



Individual and Market Demand

Elasticity

- This describes sensitivity or responsiveness of quantity demanded/supplied to changes in the good's own price, income, price of related goods etc.
- This is independent of units
- Elasticity = $\frac{\text{\% change in one variable}}{\text{\% chang by another variable}}$
- Price elasticity of Demand = $\frac{\text{\% change in quantity demanded}}{\text{\% change in good own price}}$
- $E^D = \frac{\%\Delta Q^D}{\%\Delta P} = \frac{\frac{\Delta Q^D}{Q^D}}{\frac{\Delta P}{P}} = \frac{\Delta Q^D}{Q^D} \cdot \frac{P}{\Delta P} = \frac{\Delta Q^D}{\Delta P} \cdot \frac{P}{Q} = \frac{dQ}{dP} \cdot \frac{P}{Q^D}$

$$E^D = \underbrace{\frac{dQ}{dP}}_{\text{Response of Q to a change in P}} \cdot \left.\frac{P}{Q^D}\right\} \text{Scale factor to keep elasticity independent of units.}$$

Example

- $Q^D = 360 - 15\sqrt{P}$. E^D if $P = 64$.

$$E^D = \frac{dQ}{dP} \cdot \frac{P}{Q}$$

$$= \frac{-15}{2\sqrt{P}} \cdot \frac{P}{Q}$$

$$= \frac{-15}{2\sqrt{64}} \cdot \frac{64}{360 - 15\sqrt{64}}$$

$$= (-0.75) \cdot \frac{64}{264}$$

$$= -0.18$$

$|E^D| = 0.18$ = Inelastic

Demand: Inelastic if $|E^D| < 1$

Elastic if $|E^D| > 1$

- Income elasticity of demand = $\frac{\text{\% change in quantity demanded}}{\text{\% change in income}}$

- $E_I^D = \frac{\%\Delta Q}{\%\Delta I} = \underbrace{\frac{dQ}{dI}}_{\text{Response = Income Effect}} \cdot \frac{I}{Q}$ } Scaling factor.

- E_I^D is negative for inferior goods ($E_I^D < 0$)
  - Quantity demanded ↓ if income ↑ (and reverse)
  - $E_I^D = \frac{dQ}{dI} \cdot \frac{I}{Q} < 0$, because $\frac{dQ}{dI} < 0$, $I > 0$, $Q > 0$

  - $E_I^D = \frac{dQ}{dI} \cdot \frac{I}{Q} > 0$, because $\frac{dQ}{dI} > 0$, $I > 0$, $Q > 0$

- E_I^D is positive for normal goods ($E_I^D > 0$)
  - Quantity demand ↑ if income ↑ (and reverse)
  - $E_I^D = \frac{dQ}{dI} \cdot \frac{I}{Q} > 0$ because $\frac{dQ}{dI} > 0$

Example

- $Q = 240 - \frac{P}{2} + 75I - 0.015I^2$ for $I < 4900$ & $P < 200$

$\frac{dQ}{dI} = 75 - 0.03I$

$0 = 75 - 0.03I$

$0.03I = 75$

$I = \frac{75}{0.03}$

$I = 2500$

$\frac{dQ}{dI} > 0$ (normal good) for $I < 2500$

$\frac{dQ}{dI} < 0$ (inferior good) for $I > 2500$

$I = 100$, $P = 50$, E_I^D?

$E_I^D = \frac{dQ}{dI} \cdot \frac{I}{Q}$

$E_I^D = (75 - 0.03(100)) \cdot \frac{100}{240 - \frac{50}{2} + 75(100) - (0.015)(100)^2}$

$E_I^D = (72)(\frac{100}{7565})$

$E_I^D = 0.952$

$E_I^D > 0$ so it is a normal good.

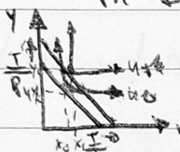
Cross-Price Elasticity of Demand

- $E_{xy}^D = \dfrac{\text{\% change in quantity demanded}}{\text{\% change in price of another good}}$
- $E_{xy}^D = \dfrac{\%\Delta Q_x}{\%\Delta P_y} = \dfrac{dQ_x}{dP_y} \cdot \dfrac{P_y}{Q_x}$
- E_{xy}^D is negative for complements ($E_{xy}^D < 0$)
  - Q^D of x ↓ if P_y ↑ (vice versa)
  - - If P_y ↑, buy less Y and X
- E_{xy}^D is positive for substitutes ($E_{xy}^D > 0$)
  - Q_x ↑ if P_y ↑ (vice versa).
    - If P_y ↑, buy less Y and more X.
- E_{xy}^D is zero for unrelated goods ($E_{xy}^D = 0$)
  - Q_x doesn't change if P_y changes.

Income Changes and Individuals Utility max choices

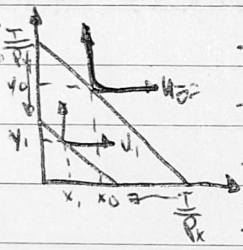
- Income Effect is: - change in the optimal utility amount of X & Y
  - from an ↑ or ↓ in income
  - hold relative prices constant ⇒ parallel shift in Budget Constraint (BC)

If Income ↑:



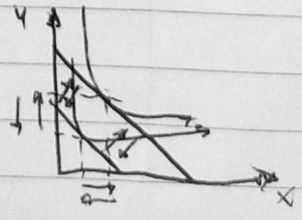
- I ↑, Y-Int ↑, X-Int ↑, U_{max} ↑,
- $U_1 > U_0$, higher utility available
- Can afford previously unaffordable bundles
- More options available

- If Income ↓:

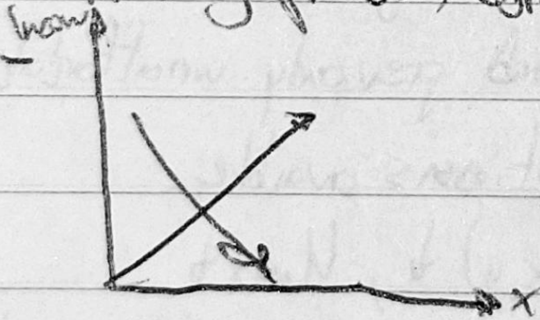


- I ↓, (x,y) ↓, U_{max} ↓
- $U_0 > U_1$, lower utility available
- Cannot afford previously affordable bundles
- Less options available

- $E_I^D = \dfrac{\%\Delta Q^D}{\%\Delta I} = \dfrac{dQ^D}{dI} \cdot \dfrac{I}{Q^D}$ = Income Elasticity of Demand (Q^D is max quantity)
- The Income Effect is $\dfrac{dQ^D}{dI}$
- Normal Good - ↑ Income ⇒ want to buy more of the good
  - ↓ Income ⇒ want to buy less
  - Most goods are normal goods
  - Income Effect $= \frac{dQ^D}{dI} > 0 \Rightarrow E_I^D > 0$
- Inferior Good - ↑ Income ⇒ Want to buy less
  - ↓ Income ⇒ " " " more
  - IE $= \frac{dQ^D}{dI} < 0 \Rightarrow E_I^D < 0$
- Both Goods cannot be inferior
  -
-
  -
- Income Expansion Path: - shows utility max bundles
  - for ~~other~~ every level of income
  - holding prices constant.
  - slopes up if both goods are normal
  - " down " " " " inferior

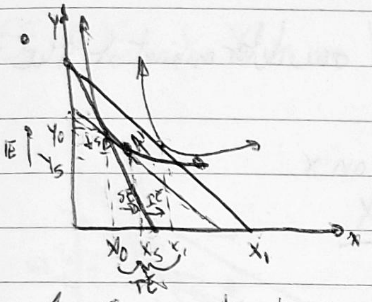


- Some Goods become inferior & further
  - IEP bends backwards if Y becomes inferior
  - IEP bends " " " " X
- The Engel Curve for a good shows consumer's
  - income on the vertical axis
  - Umax quantity of the good on horizontal axis
  - holding prices constant.

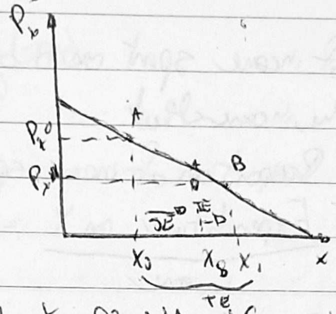


  - Engel Curve is positive for normal goods
  - Downward for inferior goods

Budget Constraint
BC



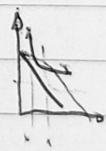
Demand Curve



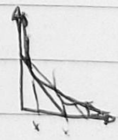
- $P_x \downarrow$
- $TE = IE + SE$

- 1- Show initial BC, highest affordable IC, and utility max bundle
  2- Draw the new BC resulting from the P↑ or ↓
  3- Shift new BC, parallel, up or down, until it is tangent to original IC curve in #1.
  4- SE = change in X & Y from tangency in #1 to tangency in #3
  5- Move pretend BC parallel up or down to its new true position resulting from the P↑↓
  6- IE = change in X & Y from the shift in #5 X, Y may ↑ or ↓ since X, Y may be normal or inferior.
  7= TE- change in the original tangency (#1) to new utility max bundle on new true BC.

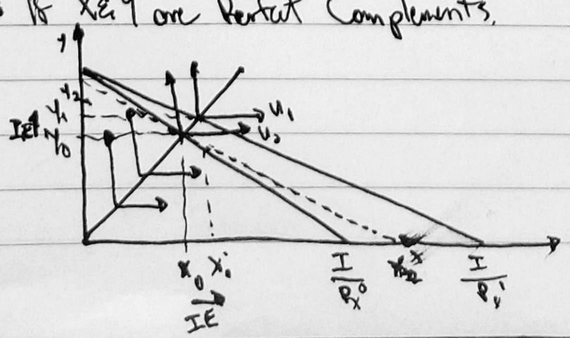
- Shape of IC is the determinant of the size of the substitution effect (SE)
- If IC are bowed, with a sharp curve
  - X & Y are complements (goods that you use together in some proportion)
  - SE is small



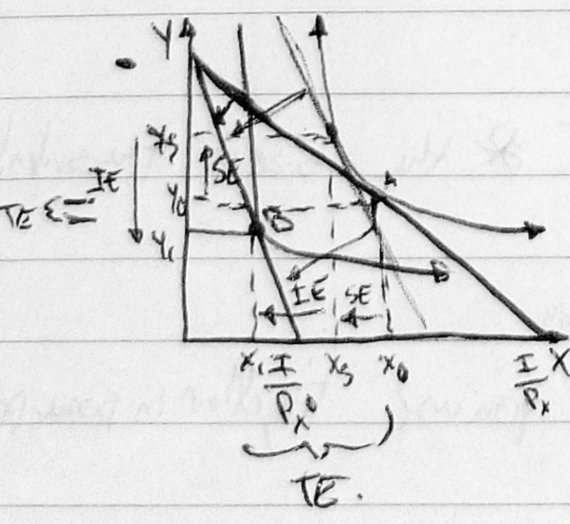
- If IC is more straight
  - X & Y are easily substituted.
  - SE is big.



- If X & Y are Perfect Complements.

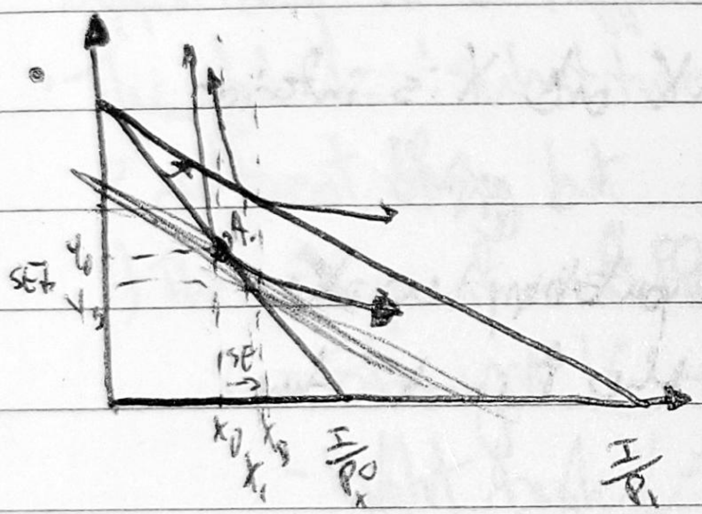


- ~~Shape~~ Share of income spent on a good is one determinant of the size of the income effect.
- $Share_x$ = ~~Proportion~~ Proportion of income spent on x
  $= \frac{\text{Expenditure on X}}{\text{Income}} = \frac{P_x \cdot X}{I}$
- If only 2 goods X, Y
  - $Share_x + Share_y = 1$.
- Eg say income = 100
  - 80% spent on rice
  - 3% " " meat.
  - 17% " " other goods.
- If P_{meat} ↓ by half
  - $1.50 increase in purchasing power to spend on more rice/meat/other goods
- If P_{rice} ↓ by half
  - $40 increase in purchasing power to spend on meat/rice/other goods



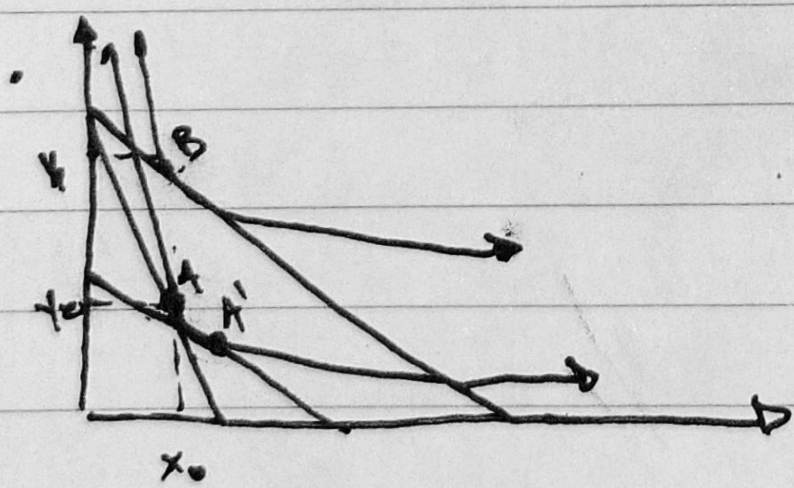
↑P_x, ↓ in PP

SE & IE if X is an Inferior Good.



Giffen Good

- If you spend most of your income on X and X is inferior then if $P_x \downarrow$ you get:
    - Very large income Effect that causes you to buy less ~~X~~
    - The big $\downarrow$ in X can outweigh the $\uparrow$ in X due to the substitution effect.
- Total Effect is $\downarrow X$ even though $P_x \downarrow$
- Demand curve for X is upward sloping if a giffen good



Section 5.4

- Specifically: What happens to demand curve for X when $P_x \uparrow$ or $\downarrow$?
- The labels 'substitutes' and "compliment's' are used to describe 2 different ~~things~~ but related things.
- 1) The shape of indifference curves
  - Straight IC → substitutes
  - Right Angle IC → complements

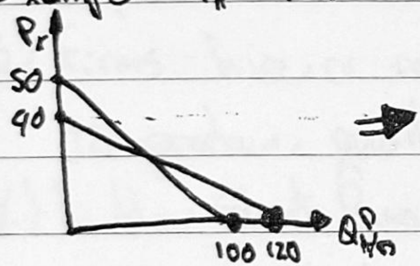
2) What happens to the demand for a good when the price of some other good ↑ or ↓

- If $P_y \uparrow$ causes an ↑ in demand for X then X, Y are substitutes
- " " " " " ↓ " " " " " " or complements.

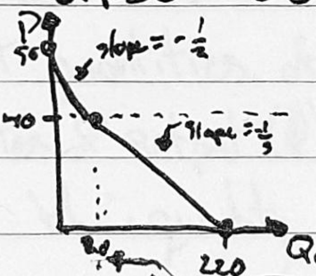
- The more straight the IC are, the more likely the goods are ~~a~~ substitutes according to the price change definition

5.5 Summing Individual Demands to Derive Market Demand Curves & Equations

- Market demand curve/equation equals (and shows) the sum or total of every individual buyer's quantity demanded at every price
- Quantity demanded is measured in the horizontal axis so market demand is the horizontal sum of every buyers demand curve
- Example: $Q_A^D = 100 - 2P$ for $P \le 50$ and $P \ge 0$, $Q_B^D = 120 - 3P$, $P \le 40$ & $P \ge 0$



Individual Demand



Market Demand

$Q_M = Q_A^D + Q_B^D$

$Q_M = 100 - 2P + 120 - 3P$

$Q_M = 220 - 5P$ $\quad P \le 40$ & $P \ge 0$

Both parties in Market

$Q_M = 220 - 5(40)$

$Q_M = 20$

6 Producer Behaviour

6.1 Simplifying Assumptions (Basics of Productions)

- 1.) Firms produce only one output (Q)

2.) There are only two inputs in the firms production process: capital (K) and labour (L).

3.) Firms hire or rent quantities of Capital (K) and labour (L) to produce quantities of output (Q)

4.) In the Short Run, the quantity of capital that firms can use in the production process is fixed.
- Only labour input is variable
- Firm chooses only optimal amount of labour.

In the Long Run, both Kapital & labour are variable
- Firm chooses the optimal amounts of both labour and capital

5.) Firm is a price taker in the input markets
- Firms can hire or rent any amount of: Labour at a fixed price of W per unit of labour and Capital at a fixed price of R per unit of ~~labour~~ capital

6.) Firms production costs are just the costs of hiring or renting the capital (K) ~~de~~ labour (L) used to produce output
- Total Cost = $C = RK + WL$

7.) Firms solve a constrained optimization problem namely:
- Choose quantities of K & L that minimizes the cost of producing a given quantity of output

8.) The production fxn shows the maximum quantity of output (Q) that can be produced by a given level of capital K and labour L $\Rightarrow Q = F(K,L)$ Eg $Q = AK^{\alpha}L^{\beta}$ where $A, \alpha, \beta < 0$

9.) Except in special cases, more K or L always produces more Q but there are diminishing marginal returns.

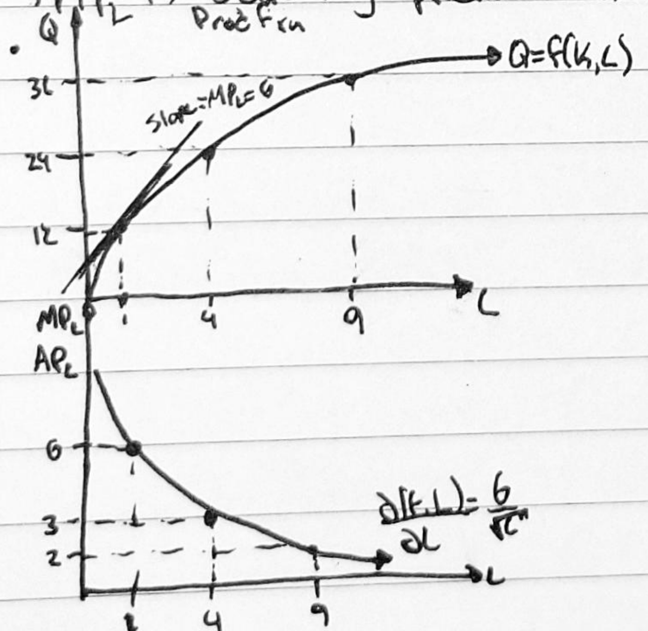
6.2 Production in the Short Run

- In the short run, K is fixed
  - the quantity K is added to the constant A in the production fxn
  - the production function is only labour
- Ex.) $K=9$, $A=4$, $\alpha=\beta=\frac{1}{2}$.

$$Q = AK^{\alpha}L^{\beta} = 4\sqrt{9}\sqrt{L} = 12\sqrt{L}$$

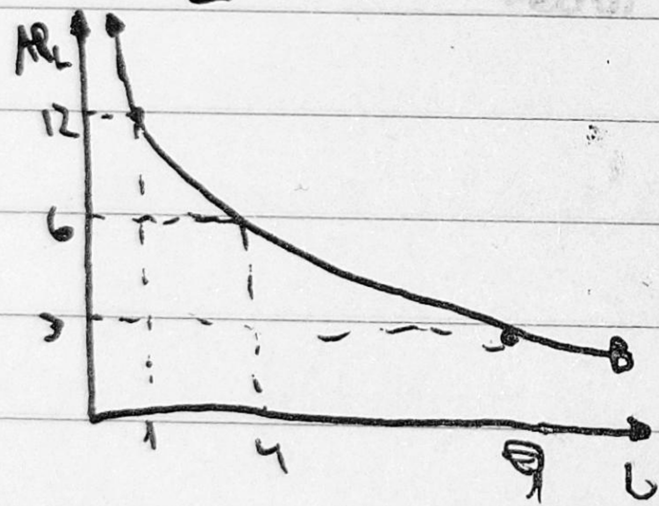
Marginal Product of Labour

- MP_L is extra output produced if one more unit of labour is hired holding everything else constant
- Ex. $\frac{\partial f(K,L)}{\partial L} = \frac{\partial AK^{\alpha}L^{\beta}}{\partial L} = AK^{\alpha}\beta \cdot L^{\beta-1} = 12\cdot\left(\frac{1}{2}\frac{1}{\sqrt{L}}\right) = \frac{6}{\sqrt{L}}$
- $MP_L > 0$ since adding more labour
- MP_L = the slope of the production function
- MP_L is decreasing production fxn is rising more slowly as L rises (diminishing returns)



Average Product of Labour

- AP_L is the average quantity of output per unit of labour
- $AP_L = \frac{f(K,L)}{L} = \frac{12}{\sqrt{L}}$



$AP_L > MP_L$

6.3 Production in the Long Run

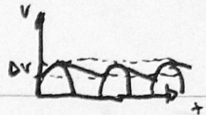
- Long Run → Both K & L are variable
  → firm chooses optimal levels of K & L
- Ex. $A = 4$, $\alpha = \beta = \frac{1}{2}$.

$$\frac{\partial(f(K,L))}{\partial L} = MP_L = \frac{2\sqrt{K}}{\sqrt{L}}$$

$$\frac{\partial(f(K,L))}{\partial K} = MP_K = \frac{\cancel{2}\sqrt{L}}{\sqrt{K}}$$

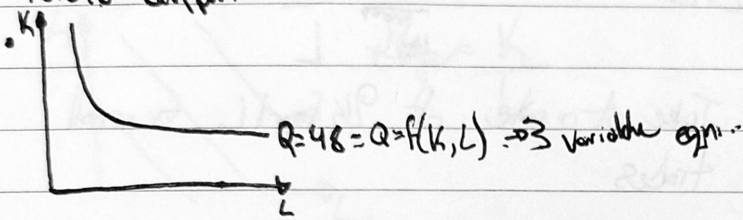
- If $MP_L = \uparrow$ in Q from hiring one more unit of L, holding K constant
- If $MP_K = \uparrow$ in Q " " " " " " K, " L constant
- $MP_L > 0$, $MP_K > 0$, adding more K & L always increases Q.
- K & L Both have diminishing returns (2nd Derivative) → $\frac{\partial^2 f(K,L)}{\partial L^2}$

Firms Cost Minimization Problem



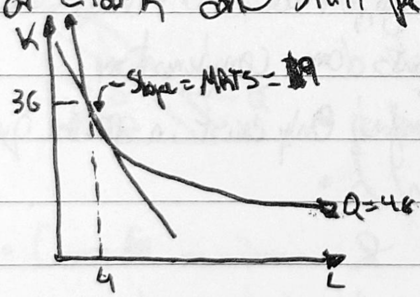
Isoquants

- Isoquants show the quantities of K and L that produce a given level of output



Marginal Rate of Technical Substitution (MRTS)

- This is minus the slope of the isoquant at a given K&L
- This is the rate at which firm can trade (substitute) K for L or L for K and still produce the same level of output



- $MRTS = -\frac{\partial K}{\partial L}$ → holding output constant
- The rate of marginal products

- For a given arbitrary function, $MRTS = \frac{MP_L}{MP_K}$
- MRTS Derivation: $Q = f(K,L)$ $\quad dQ = \frac{\partial f(K,L)}{\partial K}dK + \frac{\partial f(K,L)}{\partial L}\cdot dL$

set $dQ = 0$

$$0 = \frac{\partial f}{\partial K}dK + \frac{\partial f}{\partial L}\cdot dL$$

$$0 = MP_K dK + MP_L dL$$

$$-MP_K dK = MP_L dL$$

$$\frac{-dK}{dL} = \frac{MP_L}{MP_K}$$

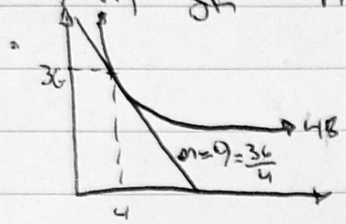
Example

- $Q = f(K,L) = 4\sqrt{K}\sqrt{L}$

$MP_L = \frac{\partial f}{\partial L} = 4\sqrt{K}\frac{1}{2}\frac{1}{\sqrt{L}}$

$MP_K = \frac{\partial f}{\partial K} = 4\sqrt{K}\frac{1}{2}\sqrt{L}$

$MRTS = \frac{2\sqrt{K}\frac{1}{\sqrt{L}}}{2\frac{1}{\sqrt{K}}\sqrt{L}} = \frac{\sqrt{K}}{\sqrt{L}} \Rightarrow \frac{K}{L}$

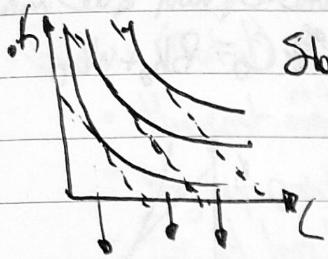


- Trade at a rate of 9K for 1L for small trades

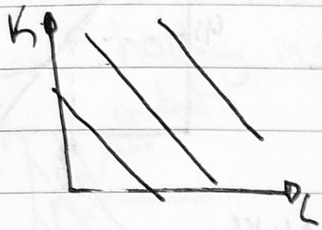
- Shape (Curviness) of the isoquant shows whether and how easily you can substitute one input for the other and still produce the same level of output
- If isoquants are straight, inputs are substitutes for each other
  - Extreme case; isoquants are straight; trade at a fixed rate
- If isoquants are more convex, inputs are complementary
  - Extreme case; isoquants are right angles; Only exist in specific quantities

Isocost Lines

- Isocost lines show the quantities of K and L that have the same total cost
- Recall ⇒ Total Cost = $C = RK + WL$
- Intercepts = x-int = $\frac{C}{W}$; y-int = $\frac{C}{R}$
- Slope = $-\frac{W}{R}$
- If R ↑, then all of the isocosts get flatter (and reverse)
- If W ↑, then all of the isocosts get steeper (and reverse)
- Isocosts are more like IC's than BC's
- They show the firms preferences where lower costs are preferred to higher
- There are an ∞ amount of isocosts.

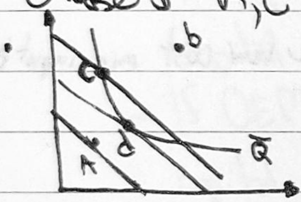


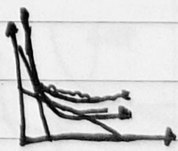
Slope $= \dfrac{MP_L}{MP_L}$



Slope $= \dfrac{K}{L}$

Hiring Inputs to Minimize Cost of Productions · Given Level of Output.

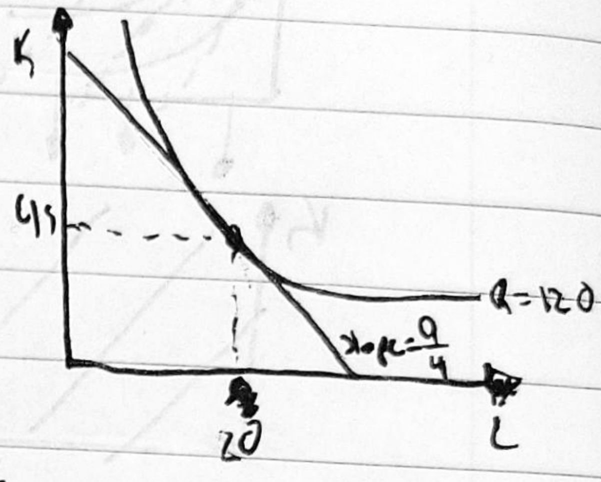
- Choose a K,L that the min cost of producing a given level of output
- 
  - a does not produce $\bar{Q}$
  - b has more K & L than needed.
  - c has $\bar{Q}$ but the K & L can be minimized
  - d has $\bar{Q}$ and at a low K/L.
- Cost minimum of a bundle is where the isocost line is tangent to the output ($\bar{Q}$) line
- At the cost minimum ~~number~~ bundle
  - Slope of isoquant = ISO cost
  - The negatives equal each other
  - $MRTS = \dfrac{MP_L}{MP_K} = \dfrac{W}{R} \Rightarrow \dfrac{MP_L}{W} = \dfrac{MP_K}{R}$.
  - If $MRTS = \dfrac{MP_L}{MP_K} > \dfrac{W}{R} \Rightarrow \dfrac{MP_L}{W} > \dfrac{MP_K}{R}$.

    - Should get more L and less K.
  - If $MRTS = \dfrac{MP_L}{MP_K} < \dfrac{W}{R} \Rightarrow \dfrac{MP_L}{W} < \dfrac{MP_K}{R}$.
    - Should get more K and and less K.

- If K_0 and L_0 are the cost min bundle then the minimum cost of producing that level of output = $C_0 = RK_0 + WL_0$
- Example: $Q = 4\sqrt{K}\sqrt{L}$, $Q = 120$, $R = 4$, $W = 9$

$$MRTS = \frac{MP_L}{MP_K} = \frac{\frac{4\sqrt{K}}{2\sqrt{L}}}{\frac{4\sqrt{L}}{2\sqrt{K}}} = \frac{K}{L}$$



Tangency = $MRTS = \frac{W}{R} = \frac{9}{4} = \frac{K}{L} \Rightarrow K = \frac{9}{4}L$

Output = $120 = 4\sqrt{K}\sqrt{L}$

$L = 20$

$K = 45.$

$C_{min} = RK + WL$

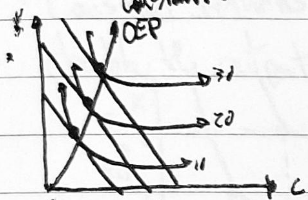
$= 9(20) + 4(45)$

$= 360$

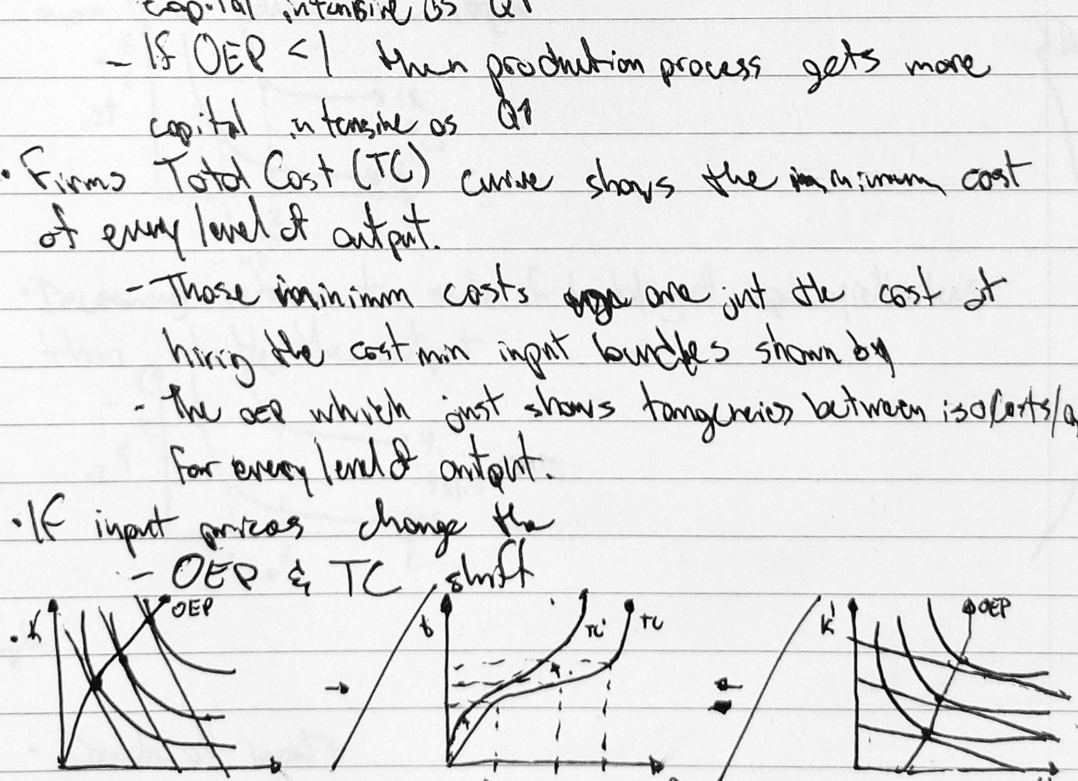
- If $W\uparrow$, then all isocosts get steeper
  - Hire less L and more K to achieve the new cost min input bundle
  - A pure substitution effect.
  - Vice Versa
- There is no income effect equivalent
  - The production equivalent of IE occurs when the firm maximizes profits.

Firms (Output) Expansion Path & Total Cost Curve

- Firms (Output) Expansion Path (OEP) shows (joins up)
  - the cost ~~maximizing~~ minimizing input combinations
  - tangency points of isocosts/quants.
  - for all levels of output
  - holding input prices and the production technology constant.



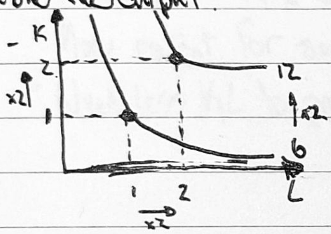
- Slope of the OEP shows how the capital-labour ratio changes as output increases.
  - If OEP > 1 then production process gets more capital intensive as Q↑
  - If OEP < 1 then production process gets more capital intensive as Q↑
- Firms Total Cost (TC) curve shows the minimum cost of every level of output.
  - Those minimum costs ~~are~~ are just the cost of hiring the cost min input bundles shown by
  - The OEP which just shows tangencies between isocosts/quants for every level of output.
- If input prices change the
  - OEP & TC shift



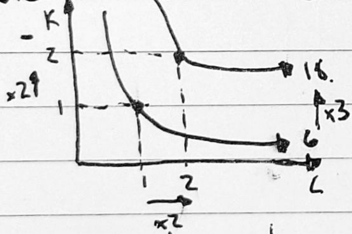
- TC shows min cost of producing each level of Q given W&R
- TC must increase if R or W↑.

Returns to Scale (RTS)

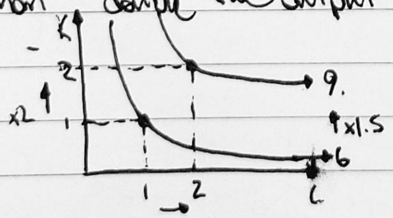
- RTS tells you how output changes if you increase all inputs by the same proportion
- Different from marginal product (MP)
  - MP tells you how output changes if you ~~than~~ increase only one input, holding all of the other inputs constant
- Constant returns to scale if doubling all inputs yeilds exactly double the output



- Increasing returns to scale if doubling alls inputs yeilds more than double the output.



- Decreasing returns to scale if doubling all inputs yields less than double the output



Doubling is just normial can be any value.

Example:

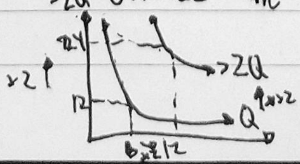
- Double all inputs

- $Q = 3K + 6L$

$2Q = 6K + 12L$

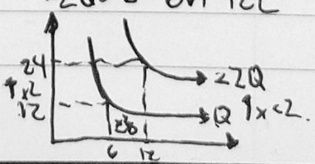
- $Q = 3K + 6L + KL$

$\to 2Q = 6K + 12L + 4KL$



- $Q = 8 + 3K + 6L$

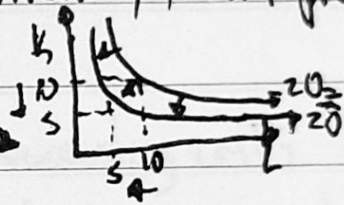
- $<2Q > 8 + 6K + 12L$



- For the Cobb Douglas function $Q = AK^{a}L^{\beta}$
  - if $a+\beta=1 \rightarrow$ Constant RTS
  - if $a+\beta>1 \rightarrow$ Increasing RTS
  - if $a+\beta<1 \rightarrow$ Decreasing RTS
- Always expect constant RTS since the firm could always double what is currently being done ~~to~~ to get double the output.

Technological Change.

- Get more output for the same inputs or equivalently, can produce the same output with fewer inputs
- Isoquants shift in



- Many different types of technological changes
  - Simplest is increases in Total Factor Productivity (TFP)
  - Given a fxn like $Q = AK^{\alpha}L^{\beta}$
    - Increases in TFP are given by A
    - More output for same input of K, L.
    - Need less K, L to produce same amount of Q.

More on Firm's Cost

Economic vs Accounting Costs

- Costcurves show economic costs, not accounting costs
- Economic costs includes opportunity costs of all inputs while accounting doesnot.
- To have zero economic profits
  - firm must be earning a positive accounting
  - firm earning enough accounting profits to cover all economic costs to pay for market exn to allignt

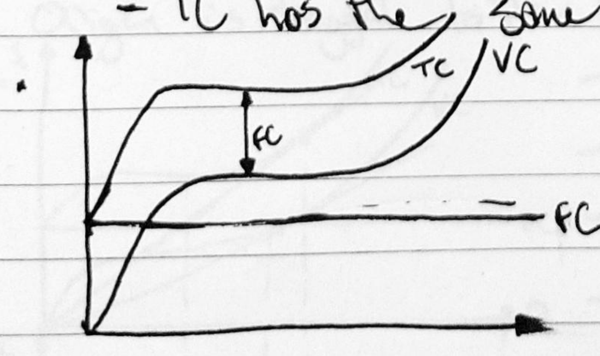
2 Ignore Sunk Costs

- Sunk costs are costs that you have already paid (and can't get back) or that you absolutely have to pay in the future.
- Sunk Costs are costs you cannot change by the choices you make
- You should ignore them when deciding what to do.

· Firm's Fixed Cost, Variable, & Total Costs

Fixed costs (FC) - are the costs of hiring/renting the firm's fixed inputs (also known as the firm's overhead)

- Fixed costs are incurred & constant for every level of output, including if firm does a shutdown (Q=0).
- Farm can avoid paying their fixed costs only if they
  - decrease their fixed input (K) to zero, and
  - exit the market (which is more than just a shutdown)
- Firms can only do this in the long run
- Variable Costs (VC) - are the costs of hiring/renting the variable inputs
- Must ↑ or ↓ the amounts of variable inputs to ↑ or ↓ output
  - VC ↑ or ↓ with output
- Usually assume that as output (Q) increases from zero VC:
  - increase at a decreasing rate, then
  - hit an inflection point (where VC stops getting flatter and gets steeper then
  - increases at in increasing rate
- Total Costs (TC) = FC + VC
  - TC has the same slope and shape as VC.

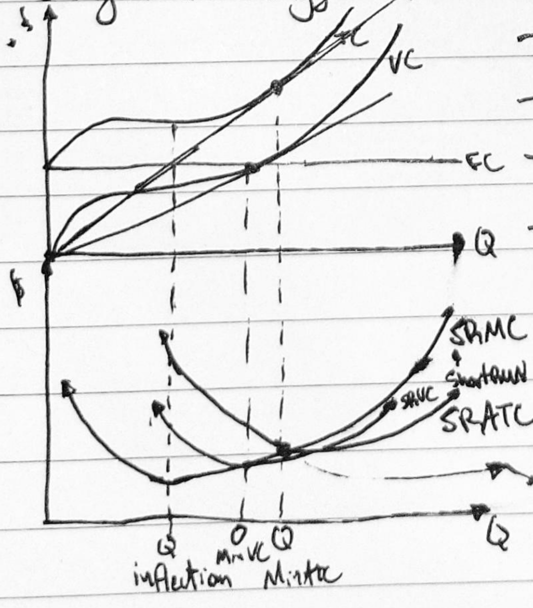


7.4 Firm's Average & Marginal Cost Curves

- Average Cost = cost per unit of output
- Average Fixed Cost (AFC) = $\frac{FC}{Q}$
- Average Variable Cost (AVC) = $\frac{VC}{Q}$
- Average Total Cost (ATC) = $\frac{TC}{Q} = \frac{FC + VC}{Q}$ = AFC + AVC

Marginal Cost

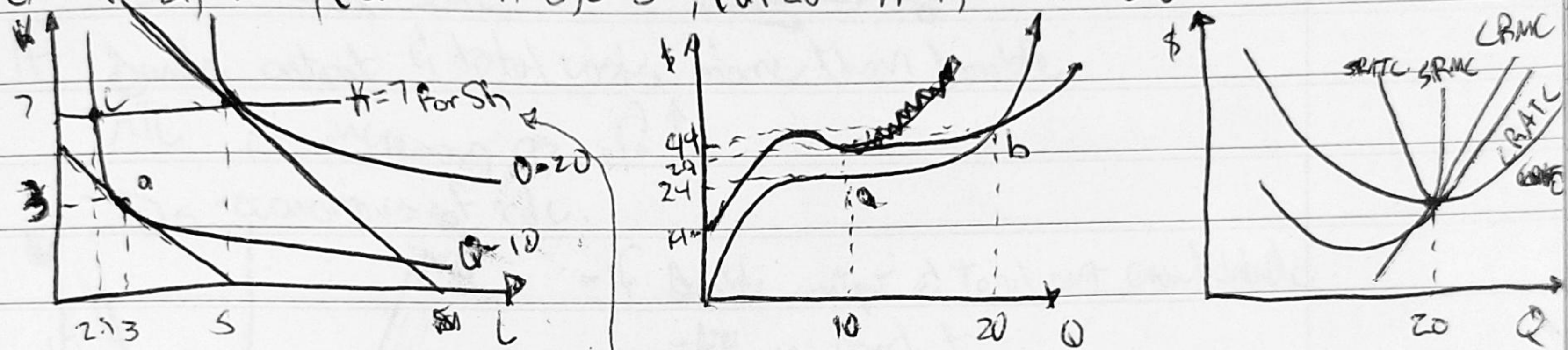
- Marginal Cost = extra cost incurred from producing one more unit of output
  - Slope of VC & TC curves
- Marginal Cost = $\frac{dTC}{dQ} = \frac{dFC}{dQ} + \frac{dVC}{dQ} = 0 + \frac{dVC}{dQ}$
- Marginal Cost = $MC = \frac{dTC}{dQ} = \frac{dVC}{dQ}$, since the change in VC is the only change in TC.
- Get minimum ATC at Q where the lowest slope ray through the origin is tangent to the TC (ATC = MC) (Tangency of TC = Slope of Ray)



- Slope of Ray = Tangency of TC
- SRATC↑
- SRATC = SRTC
- Slope of TC = MC
- SRATC = SRTC = SRMC
- If MC < AC then AC is decreasing
- If MC > AC then AC is increasing.
- Minimum Q where MC = AC.

7.5 Short Run and Long Run Cost Curves

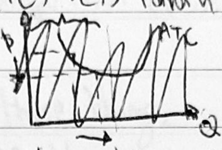
- Get different fixed, variable, average, & marginal costs for each different level of the fixed input.
- Ex. R=2, W=2, (Q=10 → K=3, L=3), (Q=20 → K=7, L=5), find LRTC.



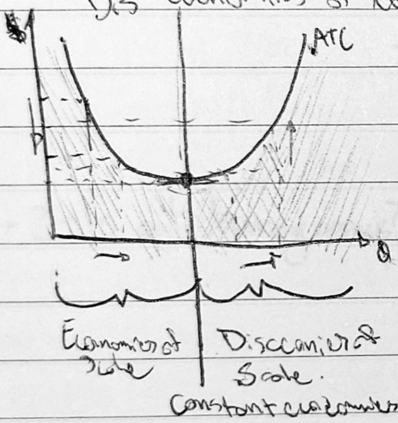
- K is fixed at 7 in the short run (SR)
  - Only two levels exists (Q=20 K=7, L=5) (Q=10, K=7, L=C)
- At Q=20, SR Min cost = LR Min cost
  At Q=10, SR min cost > LR min cost
  At Q=30, SR Min cost > LR min cost.
- SRTC is greater than LRTC at all values except for (Q=20).
- SRATC = LRATC when Q=20 SRATC > LRATC when Q≠20
- SRMC = LRMC when Q=20.
- In general there are an infinite number of SRATC, one possible for every level of K

Economies of Scale and Scope.

- If double output & total costs (TC) less than double
  - ATC is decreasing as Q↑
  - Economies of scale



- If double output & total costs more than double
  - ATC is increasing as Q↑
  - Dis-economies of scale.

- 

  - If double output & total cost exact double
    - ATC is constant
    - Constant economies of scale

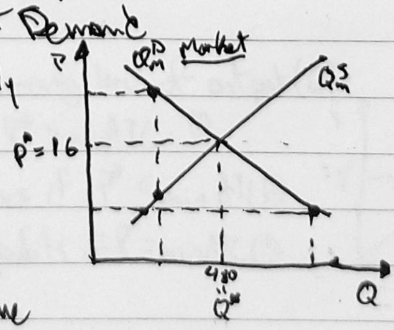
- Economies of scale (EOS) are different from returns to scale (RTS)
- RTS describes how output changes when all inputs are increased by the same population
- EOS describes how TC changes when output increases, but the inputs increased aren't increased by the same proportion.
- Economies of Scope - means that costs are lower if the firm produces more than one output at the same time as compared to costs if they produce only one output.
- Diseconomies of Scope - means that costs are higher....

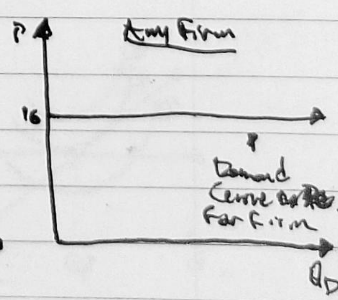
8 Supply in a Competitive Market.

market
Q_m = Quantity market producers
Q_p = Quantity Firm Produces

8.1 What is market structure & "Perfect Competition" (PC)?

- Market structure is determined by three things.
  - 1- The number of firms → In PC, there are many firms
    → Individual firms are small relative to the market
    → Any single firm can change it decisions (mainly about its output (Q)) and not affect the market price.
  - 2- Is out homogeneous? → In PC, the firms output is homogeneous
    → every firm produces identical products
    → buyers willing to buy from any firm in the markets
  - 3- Are there barriers for new firms? → In PC, there are no barriers to entry
    → Any new firm can enter the market and compete against firms
- 1 & 2 are 'price taker'
  - the market sets the price, and any firm can sell as much or as little at that price
  - firm sells at market price or not at all.
- Ex. $Q_m^D = 800 - 20P$ → Market Demand
  $Q_m^S = 30P$ → Market Supply
  $800 - 20P = 30P$
  $P = 16$ & $Q^* = 480$





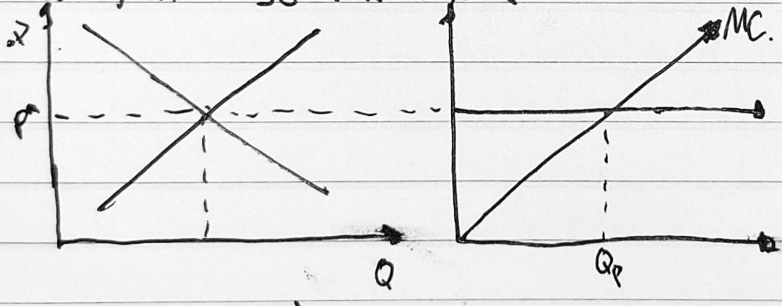
- In PC the demand curve for a single firms output is a horizontal line.

8.2 Firms Profit Maximizing Decisions in PC.

- Total Revenue for any firm = TR = $P^* \cdot Q_P$, where P^* = Price
  Q_p = Quantity produced.
- Marginal Revenue = MR = Extra revenue the firm earns from selling one more unit of output

$$MR = \frac{dTR}{dQ_p} = \frac{d(P^* \cdot Q_p)}{dQ_p} = P^*$$

- In PC, a firms MR is fixed at the market price for every output (Q_p)
- MR for a single firms output is a horizontal line at P^*
- Firm's MR curve is coincident with the demand curve for the firm's output
- Firms profit maximizing level output (Q_p^*) occurs where the firms MR = MC (marginal cost)
- But $MR = P^*$ so $MR = MC = P^*$

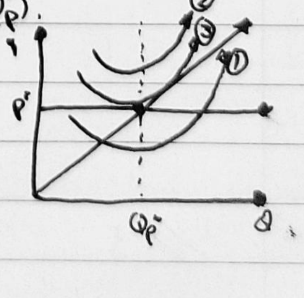


- Firms profit = $\hat{P}$ = TR − TC
  = Total Revenue − Total Cost

$$\hat{P} = P^* Q_p - ATC \cdot Q_p \leftarrow ATC = \frac{TC}{Q_p}$$

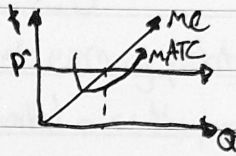
$$\hat{P} = (P^* - ATC) Q_p$$

- Given the firms profit maximizing level of output (Q_p^*),
  - firm earns positive if $P^* > \min ATC$ (1)
  - " " (economic) losses if $P^* < \min ATC$ (2)
  - " " zero (economic) profits if $P^* = \min ATC$ (3)

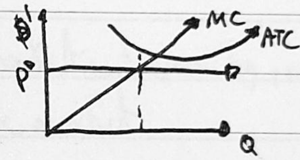


8.4 Supply in PC in the Long Run (LRS)

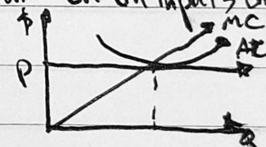
- Firms can enter and exit freely
- All inputs are costlessly variable in LR
  - → freely enter or exit
- Assume that all firms in the market and any new firms that decide to enter are all identical to one another
- If market price $P^* >$ min ATC
  - Firms are earning (economic) profits
  - New firms will enter the market.



- If market price $P^* <$ min ATC
  - Firms are earning (economic) losses
  - firms will exit the market.



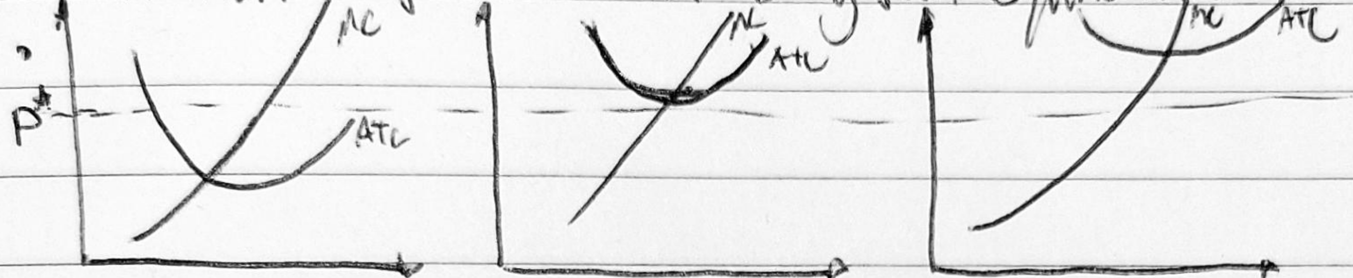
- If market price $P^* =$ min ATC
  - firms in the market earn zero (economic) profits
  - every firm is earning fair market return on all inputs and investments
  - no incentive for exit or entry
  - market is in LR equilibrium



- Recall: Each firm sets $MR = P = MC$ to max profits.
  - Each firm's supply curve is given by its marginal cost curve, above the min ATC
  - Each firm's inverse supply curve is $P = MC = 0.8Q_f$.
  - Ex: $Q_f = \left(\frac{1}{0.8}\right)P = 1.25P$
- What happens if market demand increases?
- Examples Done IN Class.

85. Market Equilbrium with Firms with different costs.

- Assume different firms have different costs.
- If at market price = p^* in long run equilibrium.



Firm A Firm B Firm C

- Firms with mATC < p^* will be in the market earning profits (firm A)
- " " mATC > p^* will not be in the market since they would have losses (Fir
- Highest cost firm will earn zero profits.
- firms with lower costs will be in the market and earning profit.
- " " high " " " not in the market.