Project Euler: Problem 1

`https://projecteuler.net/problem=1`
`https://github.com/oltdaniel/euler`

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"If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23. Find the sum of all the multiples of 3 or 5 below 1000."

The general idea would be to have an loop counting from 1 to below 1000. As this is still fast on most of the machines, as the range is pretty low, higher ranges will take way more computation time.

As only the sum is required as an result and not each number on its own, this routine can be heavily optimized by some simple math. But let us start with the general algorithm idea.

0.1 Basic idea

The basic idea to solve this, is counting from 1 to below 1000 and check each number, if it is evenly divisble by 3 or 5. At this point it is important to take care of the numbers evenly divisble by 15, as this numbers will be both evenly divisble by 3 and 5 too.

Complexity: $O(n-1)$

0.2 Bigger steps

The algorithm described above will increment the number by 1, which is actually very useless, as we know that numbers only numbers that are multiples of 3 and 5 will be significant to the result. So on, we can divide the code into to separate loops, counting one from 3 below the limit 1000 and increment it each time by 3, and another one from 5 below 1000 with incrementing the number each time by 5.

Complexity: $O(\lceil n \div 3 \rceil + \lceil n \div 5 \rceil)$

0.3 Reduce calculation

The general calculation done here is adding all numbers that are multiples of 3 and 5 that are below 1000. This sequence of numbers can be combined into triangular numbers, that are described by the function $f(n) = n(n+1) \div 2$.

However, as we want the sum of these numbers, we can mix up the formulat to inject a new parameter called d to introduce the divisor. This will result in replacing n with $n \div d$, the number of possible multiples, and multipling everything by k in order to sum the amount of possible multiples by their actual value (the y-th multiple of the x-th number is $y * x$).

The function f will now be written as $f(n,d) = d*(n \div d)*(n \div d+1) \div 2$. As this will not take care of the double counted multiples of 15, we need to substract these of the sum.

Complexity: $O(1)$